Consider the one-dimensional wave equation
\[ u_{tt}(x, t) = c^2 u_{xx}(x, t); 0 < x < L, \ 0 < t \leq T; u(0, t) = u(L, t) = 0; \]  
\[ u(x, 0) = f(x), \ u_t(x, 0) = g(x). \]

Denote by \( \Delta t = \tau \) the step size in time, and by \( \Delta x = h \), in space; set \( m^2 = \frac{\tau^2 c^2}{h^2}, \: x_i = (i - 1)h, \: i = 1, \ldots, M + 1; \: t_j = (j - 1)\tau, \: j = 1, \ldots, N + 1. \) Note that \( L = Mh = x_{M+1}, T = N\tau = t_{N+1}. \) Write \( u_{i,j} \) for the approximation of \( u(x_i, t_j). \)

We approximate \( u_{tt}(x_i, t_j) \) by the central difference at \( x_i: \)
\[ u_{tt}(x_i, t_j) \approx \frac{1}{\tau^2} (u_{i,j+1} - 2u_{i,j} + u_{i,j-1}) \]
and \( u_{xx}(i\Delta x, j\Delta t), \) by the average of the corresponding central differences at \( t_{j+1} \) and \( t_{j-1}: \)
\[ u_{xx}(i\Delta x, j\Delta t) \approx \frac{1}{h^2} \left( \frac{1}{2} (u_{i+1,j+1} - 2u_{i,j+1} + u_{i-1,j+1}) + \frac{1}{2} (u_{i+1,j-1} - 2u_{i,j-1} + u_{i-1,j-1}) \right) \]
The result is
\[ u_{i,j+1} - 2u_{i,j} + u_{i,j-1} = \frac{1}{2} m^2 \left( (u_{i+1,j+1} - 2u_{i,j+1} + u_{i-1,j+1}) + (u_{i+1,j-1} - 2u_{i,j-1} + u_{i-1,j-1}) \right) \]
or
\[ -m^2 u_{i+1,j+1} + 2(1 + m^2)u_{i,j+1} - m^2 u_{i-1,j+1} = 4u_{i,j} + m^2 u_{i+1,j-1} - 2(1 + m^2)u_{i,j-1} + m^2 u_{i-1,j-1}. \]  \( 1 \)

With zero boundary conditions, you get \( u_{1,j} = u_{M+1,j} = 0. \) For \( i = 2, \ldots, M, \) use the initial conditions to get \( u_{i,1} = u(x_i, 0) = f((i - 1)h), \: u_{i,2} \approx u_{i,1} + \tau g((i - 1)h) + \frac{\tau^2}{2} u_{tt}(x_i, 0); \) from the equation, \( u_{tt}(x_i, 0) = c^2 u_{xx}(x_i, 0) = c^2 f''(x_i) \approx c^2 (f'(ih) - 2f((i - 1)h) + f((i - 2)h))/h^2. \)

Then, for each \( j+1 = 3, \ldots, N, \) \( 1 \) is a linear system for the unknown vector \( (u_{2,j+1}, \ldots, u_{M,j+1}), \) and all you need is to solve this system. The matrix \( A \) of this system is of size \( (M-1) \times (M-1), \) with \( 2(1 + m^2) \) on the main diagonal, \( -m^2 \) just above and below the main diagonal, and 0 elsewhere. If you write \( U(j) \) for the vector-column \( (u_{2,j}, \ldots, u_{M-1,j}), \) then the system to solve is
\[ AU(j + 1) = 4U(j) - AU(j - 1). \]