CHAPTER 7
The Cost of Production

CHAPTER OUTLINE
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7.1 Measuring Cost: Which Costs Matter?

Economic Cost versus Accounting Cost

- **accounting cost**: Actual expenses plus depreciation charges for capital equipment.

- **economic cost**: Cost to a firm of utilizing economic resources in production.

Opportunity Cost

- **opportunity cost**: Cost associated with opportunities forgone when a firm’s resources are not put to their best alternative use.

The concept of opportunity cost is particularly useful in situations where alternatives that are forgone do not reflect monetary outlays.

\[
\text{Economic cost} = \text{Opportunity cost}
\]
Sunk Costs

- **sunk cost** Expenditure that has been made and cannot be recovered.

Because a sunk cost cannot be recovered, it should not influence the firm’s decisions.

For example, consider the purchase of specialized equipment for a plant. Suppose the equipment can be used to do only what it was originally designed for and cannot be converted for alternative use. The expenditure on this equipment is a sunk cost. *Because it has no alternative use, its opportunity cost is zero.* Thus it should not be included as part of the firm’s economic costs.

A prospective sunk cost is an *investment*. Here the firm must decide whether that investment in specialized equipment is *economical*. 
Fixed Costs and Variable Costs

- **total cost (TC or C)**  
  Total economic cost of production, consisting of fixed and variable costs.

- **fixed cost (FC)**  
  Cost that does not vary with the level of output and that can be eliminated only by shutting down.

- **variable cost (VC)**  
  Cost that varies as output varies.

Fixed cost does not vary with the level of output—it must be paid even if there is no output. *The only way that a firm can eliminate its fixed costs is by shutting down.*
**SHUTTING DOWN**

Shutting down doesn’t necessarily mean going out of business.

By reducing the output of that factory to zero, the company could eliminate the costs of raw materials and much of the labor, but it would still incur the fixed costs of paying the factory’s managers, security guards, and ongoing maintenance. The only way to eliminate those fixed costs would be to close the doors, turn off the electricity, and perhaps even sell off or scrap the machinery.

**FIXED OR VARIABLE?**

How do we know which costs are fixed and which are variable?

Over a very short time horizon—say, a few months—most costs are fixed. Over such a short period, a firm is usually obligated to pay for contracted shipments of materials.

Over a very long time horizon—say, ten years—nearly all costs are variable. Workers and managers can be laid off (or employment can be reduced by attrition), and much of the machinery can be sold off or not replaced as it becomes obsolete and is scrapped.
Fixed versus Sunk Costs

Shutting down doesn’t necessarily mean going out of business. Fixed costs can be avoided if the firm shuts down a plant or goes out of business.

Sunk costs, on the other hand, are costs that have been incurred and cannot be recovered.

When a firm’s equipment is too specialized to be of use in any other industry, most if not all of this expenditure is sunk, i.e., cannot be recovered.

Why distinguish between fixed and sunk costs? Because fixed costs affect the firm’s decisions looking forward, whereas sunk costs do not. Fixed costs that are high relative to revenue and cannot be reduced might lead a firm to shut down—eliminating those fixed costs and earning zero profit might be better than incurring ongoing losses. Incurring a high sunk cost might later turn out to be a bad decision (for example, the unsuccessful development of a new product), but the expenditure is gone and cannot be recovered by shutting down. Of course a prospective sunk cost is different and, as we mentioned earlier, would certainly affect the firm’s decisions looking forward.
AMORTIZING SUNK COSTS

- **amortization**  Policy of treating a one-time expenditure as an annual cost spread out over some number of years.

Amortizing large capital expenditures and treating them as ongoing fixed costs can simplify the economic analysis of a firm’s operation. As we will see, for example, treating capital expenditures this way can make it easier to understand the tradeoff that a firm faces in its use of labor versus capital.

For simplicity, we will usually treat sunk costs in this way as we examine the firm’s production decisions. When distinguishing sunk from fixed costs does become essential to the economic analysis, we will let you know.
## Marginal and Average Cost

### TABLE 7.1 A FIRM’S COSTS

<table>
<thead>
<tr>
<th>RATE OF OUTPUT (UNITS PER YEAR)</th>
<th>FIXED COST (DOLLARS PER YEAR)</th>
<th>VARIABLE COST (DOLLARS PER YEAR)</th>
<th>TOTAL COST (DOLLARS PER YEAR)</th>
<th>MARGINAL COST (DOLLARS PER UNIT)</th>
<th>AVERAGE FIXED COST (DOLLARS PER UNIT)</th>
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MARGINAL COST (MC)

- marginal cost (MC)  Increase in cost resulting from the production of one extra unit of output.

Because fixed cost does not change as the firm’s level of output changes, marginal cost is equal to the increase in variable cost or the increase in total cost that results from an extra unit of output. We can therefore write marginal cost as

\[ MC = \frac{\Delta VC}{\Delta q} = \frac{\Delta TC}{\Delta q} \]

AVERAGE TOTAL COST (ATC)

- average total cost (ATC)  Firm’s total cost divided by its level of output.

- average fixed cost (AFC)  Fixed cost divided by the level of output.

- average variable cost (AVC)  Variable cost divided by the level of output.
### 7.2 Costs in the Short Run

#### The Determinants of Short-Run Cost

The change in variable cost is the per-unit cost of the extra labor $w$ times the amount of extra labor needed to produce the extra output $\Delta L$. Because $\Delta VC = w \Delta L$, it follows that

$$MC = \frac{\Delta VC}{\Delta q} = \frac{w \Delta L}{\Delta q}$$

The extra labor needed to obtain an extra unit of output is $\Delta L/\Delta q = 1/MP_L$. As a result,

$$MC = \frac{w}{MP_L} \quad (7.1)$$

#### DIMINISHING MARGINAL RETURNS AND MARGINAL COST

Diminishing marginal returns means that the marginal product of labor declines as the quantity of labor employed increases.

As a result, when there are diminishing marginal returns, marginal cost will increase as output increases.
The Shapes of the Cost Curves

**Figure 7.1**
**COST CURVES FOR A FIRM**

In (a) total cost TC is the vertical sum of fixed cost FC and variable cost VC.

In (b) average total cost ATC is the sum of average variable cost AVC and average fixed cost AFC.

Marginal cost MC crosses the average variable cost and average total cost curves at their minimum points.
THE AVERAGE-MARGINAL RELATIONSHIP

Marginal and average costs are another example of the average-marginal relationship described in Chapter 6 (with respect to marginal and average product).

Because average total cost is the sum of average variable cost and average fixed cost and the AFC curve declines everywhere, the vertical distance between the ATC and AVC curves decreases as output increases.

TOTAL COST AS A FLOW

Total cost is a flow: the firm produces a certain number of units per year. Thus its total cost is a flow—for example, some number of dollars per year. For simplicity, we will often drop the time reference, and refer to total cost in dollars and output in units.

Knowledge of short-run costs is particularly important for firms that operate in an environment in which demand conditions fluctuate considerably. If the firm is currently producing at a level of output at which marginal cost is sharply increasing, and if demand may increase in the future, management might want to expand production capacity to avoid higher costs.
7.3 Cost in the Long Run

The User Cost of Capital

- **user cost of capital**  Annual cost of owning and using a capital asset, equal to economic depreciation plus forgone interest.

The user cost of capital is given by the *sum of the economic depreciation and the interest (i.e., the financial return) that could have been earned had the money been invested elsewhere*. Formally,

\[
\text{User Cost of Capital} = \text{Economic Depreciation} + (\text{Interest Rate})(\text{Value of Capital})
\]

We can also express the user cost of capital as a *rate* per dollar of capital:

\[
r = \text{Depreciation rate} + \text{Interest rate}
\]
The Cost-Minimizing Input Choice

We now turn to a fundamental problem that all firms face: how to select inputs to produce a given output at minimum cost.

For simplicity, we will work with two variable inputs: labor (measured in hours of work per year) and capital (measured in hours of use of machinery per year).

THE PRICE OF CAPITAL

The price of capital is its user cost, given by $r = \text{Depreciation rate} + \text{Interest rate}$.

THE RENTAL RATE OF CAPITAL

- rental rate  Cost per year of renting one unit of capital.

If the capital market is competitive, the rental rate should be equal to the user cost, $r$. Why? Firms that own capital expect to earn a competitive return when they rent it. This competitive return is the user cost of capital.

Capital that is purchased can be treated as though it were rented at a rental rate equal to the user cost of capital.
The Isocost Line

- isocost line  Graph showing all possible combinations of labor and capital that can be purchased for a given total cost.

To see what an isocost line looks like, recall that the total cost $C$ of producing any particular output is given by the sum of the firm’s labor cost $wL$ and its capital cost $rK$:

$$ C = wL + rK $$  \hspace{1cm} (7.2)

If we rewrite the total cost equation as an equation for a straight line, we get

$$ K = C/r - (w/r)L $$

It follows that the isocost line has a slope of $\Delta K/\Delta L = -(w/r)$, which is the ratio of the wage rate to the rental cost of capital.
Choosing Inputs

Figure 7.3
Producing a Given Output at Minimum Cost

Isocost curves describe the combination of inputs to production that cost the same amount to the firm. Isocost curve $C_1$ is tangent to isoquant $q_1$ at $A$ and shows that output $q_1$ can be produced at minimum cost with labor input $L_1$ and capital input $K_1$. Other input combinations—$L_2$, $K_2$ and $L_3$, $K_3$—yield the same output but at higher cost.
Figure 7.4
Input Substitution When an Input Price Changes

Facing an isocost curve $C_1$, the firm produces output $q_1$ at point $A$ using $L_1$ units of labor and $K_1$ units of capital. When the price of labor increases, the isocost curves become steeper. Output $q_1$ is now produced at point $B$ on isocost curve $C_2$ by using $L_2$ units of labor and $K_2$ units of capital.
Recall that in our analysis of production technology, we showed that the marginal rate of technical substitution of labor for capital (MRTS) is the negative of the slope of the isoquant and is equal to the ratio of the marginal products of labor and capital:

$$\text{MRTS} = -\Delta K / \Delta L = \frac{MP_L}{MP_K}$$ \hspace{1cm} (7.3)

It follows that when a firm minimizes the cost of producing a particular output, the following condition holds:

$$\frac{MP_L}{MP_K} = \frac{w}{r}$$

We can rewrite this condition slightly as follows:

$$\frac{MP_L}{w} = \frac{MP_K}{r}$$ \hspace{1cm} (7.4)
Cost Minimization with Varying Output Levels

- **expansion path** Curve passing through points of tangency between a firm’s isocost lines and its isoquants.

The firm can hire labor $L$ at $w = $10/hour and rent a unit of capital $K$ for $r = $20/hour. Given these input costs, we have drawn three of the firm’s isocost lines. Each isocost line is given by the following equation:

$$C = (10\text{/hour})(L) + (20\text{/hour})(K)$$

The expansion path is a straight line with a slope equal to

$$\frac{\Delta K}{\Delta L} = \frac{50 - 25}{100 - 50} = \frac{1}{2}$$

The Expansion Path and Long-Run Costs

To move from the expansion path to the cost curve, we follow three steps:

1. Choose an output level represented by an isoquant. Then find the point of tangency of that isoquant with an isocost line.
2. From the chosen isocost line, determine the minimum cost of producing the output level that has been selected.
3. Graph the output-cost combination.
A FIRM’S EXPANSION PATH AND LONG-RUN TOTAL COST CURVE

In (a), the expansion path (from the origin through points A, B, and C) illustrates the lowest-cost combinations of labor and capital that can be used to produce each level of output in the long run—i.e., when both inputs to production can be varied.

In (b), the corresponding long-run total cost curve (from the origin through points D, E, and F) measures the least cost of producing each level of output.
Example 7.5 Reducing the Use of Energy

**Figure 7.7a**

Energy Efficiency Through Capital Substitution for Labor

Greater energy efficiency can be achieved if capital is substituted for energy. This is shown as a movement along isoquant $q_1$ from point $A$ to point $B$, with capital increasing from $K_1$ to $K_2$ and energy decreasing from $E_2$ to $E_1$ in response to a shift in the isocost curve from $C_0$ to $C_1$. 
EXAMPLE 7.5  REDUCING THE USE OF ENERGY

**Figure 7.7b**  
ENERGY EFFICIENCY THROUGH TECHNOLOGICAL CHANGE

Technological change implies that the same output can be produced with smaller amounts of inputs. Here the isoquant labeled $q_1$ shows combinations of energy and capital that will yield output $q_1$; the tangency with the isocost line at point $C$ occurs with energy and capital combinations $E_2$ and $K_2$. Because of technological change the isoquant shifts inward, so the same output $q_1$ can now be produced with less energy and capital, in this case at point $D$, with energy and capital combination $E_1$ and $K_1$. 

![Diagram showing energy efficiency through technological change](image)
7.4 Long-Run versus Short-Run Cost Curves

The Inflexibility of Short-Run Production

**Figure 7.8**

**THE INFLEXIBILITY OF SHORT-RUN PRODUCTION**

When a firm operates in the short run, its cost of production may not be minimized because of inflexibility in the use of capital inputs.

Output is initially at level $q_1$, (using $L_1$, $K_1$).

In the short run, output $q_2$ can be produced only by increasing labor from $L_1$ to $L_3$ because capital is fixed at $K_1$.

In the long run, the same output can be produced more cheaply by increasing labor from $L_1$ to $L_2$ and capital from $K_1$ to $K_2$. 
Long-Run Average Cost

In the long run, the ability to change the amount of capital allows the firm to reduce costs.

The most important determinant of the shape of the long-run average and marginal cost curves is the relationship between the scale of the firm’s operation and the inputs that are required to minimize its costs.

● **long-run average cost curve (LAC)**  Curve relating average cost of production to output when all inputs, including capital, are variable.

● **short-run average cost curve (SAC)**  Curve relating average cost of production to output when level of capital is fixed.

● **long-run marginal cost curve (LMC)**  Curve showing the change in long-run total cost as output is increased incrementally by 1 unit.
When a firm is producing at an output at which the long-run average cost LAC is falling, the long-run marginal cost LMC is less than LAC. Conversely, when LAC is increasing, LMC is greater than LAC. The two curves intersect at A, where the LAC curve achieves its minimum.
Economies and Diseconomies of Scale

As output increases, the firm’s average cost of producing that output is likely to decline, at least to a point.

This can happen for the following reasons:

1. If the firm operates on a larger scale, workers can specialize in the activities at which they are most productive.

2. Scale can provide flexibility. By varying the combination of inputs utilized to produce the firm’s output, managers can organize the production process more effectively.

3. The firm may be able to acquire some production inputs at lower cost because it is buying them in large quantities and can therefore negotiate better prices. The mix of inputs might change with the scale of the firm’s operation if managers take advantage of lower-cost inputs.
At some point, however, it is likely that the average cost of production will begin to increase with output.

There are three reasons for this shift:

1. At least in the short run, factory space and machinery may make it more difficult for workers to do their jobs effectively.

2. Managing a larger firm may become more complex and inefficient as the number of tasks increases.

3. The advantages of buying in bulk may have disappeared once certain quantities are reached. At some point, available supplies of key inputs may be limited, pushing their costs up.
• **economies of scale**  Situation in which output can be doubled for less than a doubling of cost.

• **diseconomies of scale**  Situation in which a doubling of output requires more than a doubling of cost.

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**Increasing Returns to Scale:** Output more than doubles when the quantities of all inputs are doubled.

**Economies of Scale:** A doubling of output requires less than a doubling of cost.

Economies of scale are often measured in terms of a cost-output elasticity, \( E_C \). \( E_C \) is the percentage change in the cost of production resulting from a 1-percent increase in output:

\[
E_C = \frac{\Delta C}{C} / \frac{\Delta q}{q}
\]

(7.5)

To see how \( E_C \) relates to our traditional measures of cost, rewrite equation as follows:

\[
E_C = \frac{\Delta C / \Delta q}{C / q} = \frac{MC}{AC}
\]

(7.6)
The long-run average cost curve LAC is the envelope of the short-run average cost curves SAC\(_1\), SAC\(_2\), and SAC\(_3\).

With economies and diseconomies of scale, the minimum points of the short-run average cost curves do not lie on the long-run average cost curve.
7.5 Production with Two Outputs—Economies of Scope

Product Transformation Curves

- **product transformation curve** Curve showing the various combinations of two different outputs (products) that can be produced with a given set of inputs.

**Figure 7.11**

**PRODUCT TRANSFORMATION CURVE**

The product transformation curve describes the different combinations of two outputs that can be produced with a fixed amount of production inputs. The product transformation curves $O_1$ and $O_2$ are bowed out (or concave) because there are economies of scope in production.
Economies and Diseconomies of Scope

- **economies of scope**  Situation in which joint output of a single firm is greater than output that could be achieved by two different firms when each produces a single product.

- **diseconomies of scope**  Situation in which joint output of a single firm is less than could be achieved by separate firms when each produces a single product.

The Degree of Economies of Scope

To measure the *degree* to which there are economies of scope, we should ask what percentage of the cost of production is saved when two (or more) products are produced jointly rather than individually.

\[
SC = \frac{C(q_1) + C(q_2) - C(q_1, q_2)}{C(q_1, q_2)} \quad (7.7)
\]

- **degree of economies of scope (SC)**  Percentage of cost savings resulting when two or more products are produced jointly rather than individually.