Durrett Chapter 4 #7.1, 7.2, 7.6, 7.7; Chapter 3 #1.6, 1.10, 1.15

HINTS:

(7.2) This is complicated by the fact that \{X_n\} is not u.i. You will need to replace \(T_1\) with \(T_1 \land n\) and let \(n \to \infty\). This requires Fatou’s Lemma and/or Dominated Convergence. Example 7.1(c) may be useful in establishing the necessary domination.

(7.7) For a normal \(N(0, \sigma^2)\) r.v. \(Z\), \(Ee^{\theta Z} = e^{\theta^2 \sigma^2/2}\). Use Exercise 7.6.

(1.10)(ii) View the sum as an integral for the measure that puts mass \(P(\alpha > m)\) at \(m\), for \(m = 0, 1, 2\ldots\)

(1.15) \(S_{T \land n}^2 - S_{T \land (n-1)}^2 = 0\) if \(T < n\), so \(S_{T \land n}^2 - S_{T \land (n-1)}^2 = \ldots \times 1_{\{T \geq n\}}\) (fill in the blank.) Also, show that for \(n > m\),

\[
E(S_{T \land n} - S_{T \land m})^2 = ES_{T \land n}^2 - ES_{T \land m}^2.
\]