Geometric Tailoring: A Design for Manufacturing Method for Rapid Prototyping and Rapid Tooling

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The goal of fabricating functional prototypes quickly is hindered by a mismatch of material properties between production materials and those used in rapid prototyping (RP) machines, such as stereolithography. Even when rapid tooling (RT) technologies are utilized for injection molded parts, differences in mold materials cause differences in molded part properties. To compensate for these material and process differences, a design for manufacturing (DFM) method is introduced, called geometric tailoring. The idea is to modify dimensions of prototype parts to match key characteristics of production parts, such as stress and deflection behaviors. For RP parts, the geometric tailoring DFM method integrates two sub-problems, one for achieving functional requirements by matching part behaviors, and one for RP process planning to incorporate manufacturing capabilities and limitations. For parts fabricated by RT, an additional sub-problem is integrated, namely injection molding process planning. Problem decomposition is critical due to the coupled nature of the sub-problems. A problem decomposition and solution procedure is presented. The geometric tailoring method is shown to enable the matching of prototype to production part behaviors, while improving manufacturability.
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1 Functional Prototypes via RP and RT

The ease, speed, and low cost of fabricating prototype parts using RP has had a very significant impact on product development, eliminating the need for sometimes complex process planning for machining prototypes or the tedious manual methods of making models. In the last 5 years, the RP industry underwent significant improvements in developing machines and materials that facilitate the fabrication of production-representative prototypes. For example, SOMOS 8120 and SOMOS 9120, materials used in StereoLithography Apparatus (SLA), have properties close to polyethylene and polypropylene, respectively. Parts fabricated through Selective Laser Sintering (SLS) process are fabricated in steel (infiltrated with copper) and have performance close to the production steel parts. Parts can be fabricated in production plastics like ABS and polystyrene through Fused Deposition Modeling process.

RP technologies are also used in fabricating tools that enable net-shape manufacturing processes, such as injection molding and investment casting, reducing tooling lead time from weeks to days. The practice of using RP technologies to fabricate tools is called Rapid Tooling (RT) [1]. Rapid Tools can be used to fabricate parts in production material. Rapid tooling techniques are categorized into direct and indirect tooling based on whether the RP technology is used in fabricating a mold or a pattern used to fabricate the mold. Injection molds fabricated in SLA (called SL tools) can be used to produce plastic parts; this is the focus in this paper.

Though parts fabricated through RP and RT technologies have properties similar to production materials, they have some limitations in being used as functional prototypes. A comparison of material properties of a typical ABS (acrylonitrile butadiene styrene) [2] and SL 7510 (a common SL resin) processed in a SLA-3500 machine [3] is presented in Table 1, while a comparison of polystyrene parts molded in steel and SL tools is presented in Table 2. 95% confidence intervals are indicated for tensile strength and Young's modulus of the molded parts. The variation in polystyrene parts is due to the differences in material properties and cooling characteristics of molds [4].

The effect of the differences in material properties is explained with an example of the tensile bar. For the same load, the tensile bar fabricated in SL 7510 would have 3.5% less elongation than that of ABS. SL 7510 bars yield at 27% more load than ABS bars. Also SL 7510 bars weigh 13% more than that of ABS. Hence, depending on the desired property(s) being tested, SL 7510 bars behave differently from ABS bars.

To explain the differences in Table 2, it should be observed that SL tools have low thermal conductivities, so that molded parts cool more slowly, leading to less crystallinity and less residual stress within parts. As a result, tensile modulus and strength of parts molded in SL tools are typically lower than for parts molded in steel, while flexural properties are enhanced. These mechanical property differences cause prototype parts to have different behaviors than production parts in their intended in-use situations, even though the parts have identical dimensions.

There are additional concerns with RP and RT parts. The material properties of the RP parts vary with the SL process variables used in their fabrication and this causes uncertainty in the values of material properties. SL molds have low strength and can fail at high injection and ejection pressure. Also, the life of these molds is low and this directly affects the fabrication cost.

These limitations of RP and RT parts limit their usefulness as functional prototypes. To overcome these limitations, a Design-For-Manufacturing (DFM) method called Material Process Geometric Tailoring is presented in this paper [5]. There are several approaches of considering manufacturing concerns and evaluating part manufacturability in the DFM phase. One approach is to evaluate the part manufacturing cost; one example for injection molding was developed by [6]. A second approach of evaluating part manufacturability is to use manufacturing guidelines to identify attributes of the geometry that are difficult to manufacture. Most leading handbooks on DFM, such as [7–9], use this approach. A third approach for evaluating manufacturability is to perform a manufacturing simulation. *Part Advisor* and *QuickFill*, developed by *Moldflow Corp.*, use this approach. All approaches

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Table 1 Comparison of material properties

	Tensile Strength (MPa)	Young's Modulus (GPa)	Elongation @ Break (%)	Density (gm/cm^3)
ABS	44.8	2.54	35	1.04
Magnum 9010, Dow Chemical				
SL 7510	57	2.63	10.1	1.18

have their advantages and limitations. Therefore many systems use a combination of the approaches. Examples include the DFM systems for metal stamping [10] and injection molding [11].

Geometric Tailoring (GT) is the modification of some non-critical geometric features of a part to lower fabrication cost and time, and to produce functional prototypes that mimic the behavior of the production parts. **Material Process Geometric Tailoring** (MPGT) is the modification of non-critical feature geometry and manufacturing process variables to lower fabrication cost and time, and to produce functional prototypes that mimic the behavior of the production parts. The difference between MPGT and GT is that the effect of process variables is considered in MPGT.

Revisiting the tensile bar example, the GT approach can be used to obtain better functional prototypes from SL 7510 by modifying the thickness of the tensile bar such that the desired properties of ABS and SL 7510 bars match. For example, if tensile modulus was the desired property, the SL 7510 tensile bar would be made thinner than the ABS molded bar.

In this paper, the authors propose a decision support system (MPGT) in which the benefits and drawbacks of design and manufacturing processes are quantified and part manufacturability is evaluated in terms of part cost and quality. With MPGT, the tradeoff between design and manufacturing requirements is used to obtain manufacturable prototypes through RP and RT processes that can be used for functional testing. Background technologies and concepts are presented in Section 2 of this paper. In Section 3, we present geometric tailoring problem formulations. Solution methods for these formulations are presented in Section 4, with an emphasis on rapid prototyping, rather than rapid tooling. In Section 5, two examples are given.

2 Background

2.1 Functional Testing Approach. Currently, the Buckingham Π theorem is the basis in prototype testing to get the correlation between physical models and products [12]. Based on the Buckingham Π theorem, a complex system can be constructed and tested by geometrically scaling models or changing materials. By doing that one can dramatically reduce the cost and time in building prototypes while getting reasonable predictive values.

Consider two systems (a test model and a product) that can be described as

$$\underline{Model}: \quad X_m = f(p_{m,1}, p_{m,2}, \ldots, p_{m,n}),$$
$$\underline{Product}: \quad X_p = f(p_{p,1}, p_{p,2}, \ldots, p_{p,n}), \tag{1}$$

where **X** is the state of interest, and p_i is a system parameter. In the equations, subscripts *m* and *p* denote the model and the product respectively [13]. In functional testing, the representation of the function *f* is usually unknown. Typically, it is desired to make $p_{m,i} = p_{p,i}$ as close as possible in functional testing to accurately predict X_p. However, it is often the case that not all system parameters are equally important in achieving the functional objectives. For example, fatigue performance of a gear is significantly affected by the maximum stress in the gear. However, small changes in the gear dimensions may have only limited effects on fatigue performance. Therefore the system parameters can be divided into two categories based on their effect on **X**. If the first *k* parameters $p_1, p_2, \ldots, p_k$ have significant effect on **X**, while parameters $p_{k+1}, p_{k+2}, \ldots, p_n$ have negligible effect on **X**, then **the principle of the approach developed in this research is to change parameters $p_{m,j}$ (for $j > k$) to better match $p_{m,i}$ and $p_{p,i}$ (for $1 \leq i \leq k$).** These parameter changes are referred to as *geometric tailoring* in this paper.

2.2 Compromise DSP. A compromise Decision Support Problem (cDSP) is a hybrid multi-objective problem formulation, incorporating concepts from both traditional mathematical programming and goal programming [14]. The objective is to explore the design space and improve a selected concept based on a set of goals, constraints, and bounds. The cDSP is often used to model decisions consisting of multiple goals that are often in conflict with one another. A satisfactory solution is one that meets both the constraints and bounds and balances the performance of the conflicting goals. In the case of process planning for SL, a part is presented in a "default" build style to serve the purpose of the existing alternative to be improved. This build style is improved by changing the build process variables. The structure of the cDSP is shown below.

Given: A feasible alternative, assumptions, parameter values, and goals.

Find: Values of design and deviation variables.

Satisfy: System constraints, system goals, and bounds on variables.

Minimize: Deviation function that measures distance between goal targets and design point.

Variables are classified into two types: system variables or deviation variables. System variables include design parameters the user can alter. System constraints must be met for the design to be feasible and are functions of the system variables. System goals model the design aspirations of the designer. The deviation variables measure how far away the actual achievement levels are from the target levels. The alternative is improved by finding a combination of system variables such that all the system constraints are satisfied while the deviation function is minimized.

The mathematical form of a goal is given in Eq. (2) for the *i*th goal. Each goal, A_i, has two associated deviation variables d_i^+ and d_i^- which indicate the extent of the deviation (overachievement and underachievement) from the target (G_i). The deviation variables, d_i^+ and d_i^-, are always non-negative, and the product constraint, $d_i^+ \cdot d_i^- = 0$, ensures that at least one of the deviation variables for a particular goal is always zero. In a cDSP, the objective that is minimized is called the deviation function, which is a weighted sum of the deviation variables (Eq. (3)).

$$A_i(X) + d_i^- - d_i^+ = G_i \tag{2}$$

$$Z = \Sigma W_i(d_i^- + d_i^+) \tag{3}$$

Table 2 Material properties of atactic polystyrene

	Tensile Strength (MPa)	95% Confidence Interval	Young's Modulus (GPa)	95% Confidence Interval	Elongation @ Break (%)	Density (gm/cm^3)
Steel mold	37.4	0.5	3.2	0.28	1.3	1.05
SLA mold	32.8	0.5	3.4	0.25	1.1	1.04

2.3 RCEM. Robust Concept Exploration Method (RCEM) [15] is a methodology obtained from the integration of robust design techniques, design of experiments, and response surface methodology within the framework of the cDSP. The RCEM facilitates an efficient and effective concept exploration process as robust top-level design specifications are identified for the design of complex systems. Thus, the focus is to minimize the effects on the conceptual design of downstream design changes.

RCEM consists of five main steps that lead to the formulation of a cDSP, the solution of which is ranged sets of specifications on system variables. (1) The first step is to identify the main factors and ranges of interest. (2) Next, a point generation step performs a design-of-experiments (DOE) to generate points in the design space for a screening experiment. (3) Analysis or simulation tools are executed to determine responses for each generated point. (4) The fourth step is to analyze the DOE results to eliminate inconsequential factors and/or to formulate a new DOE on a reduced design space. Step three may be repeated. (5) Response surfaces are created as surrogate models of the analysis/simulation codes. These surrogate models are integrated with the cDSP so that the computationally intensive analysis and simulation tools need not be used during the solution of the cDSP.

Response surface methodology is used for generating relationships between several responses and design and manufacturing variables in this research [16]. A variation of RCEM was developed to solve the MPGT problems developed in this research.

3 DFM Problem Formulation Strategy

The DFM problems for RP and RT are formulated as cDSP's. First, cDSP's are formulated for different engineering domains (designer and manufacturer) and then they are synthesized into one DFM problem by considering the coupling between the individual problems. In this section, the generic cDSP formulations for the designer (geometric tailoring), the RP engineer (RP Process Planning) and the synthesized DFM problem (MPGT) are presented, as well as the rapid tooling MPGT problem.

3.1 GT Problem Formulation. The designer's generic geometric tailoring (GT) word formulation for the RP scenario is presented in Fig. 1. The objective is to match functional properties of a prototype to properties of production parts by adjusting some of the noncritical dimensions of the part. This compensates for the difference in the material properties of prototype and production parts and helps ensure part manufacturability.

The information incorporated in the GT cDSP includes a parametric CAD model of the part, RP and production material properties, functional property models and target values of functional, geometry, process, cost and time goals. Functional property models are the quantitative models that relate functional properties to the system variables in the problem. Target values for the goals are the designer's preferred values and are used in the formulation of goals. Although the designer has a preferred value for the design variables, he/she specifies a feasible range (instead of a point) for these variables. Note that process goals correspond to the manufacturing process requirements and do not affect any of the designer's goals in the GT problem. Also, cost and time goals are affected by only the manufacturing process. However the designer specifies the targets for these goals to ensure that the part meets its project requirements. Process goals, cost, and time are integrated into the Material Process Geometric Tailoring problem to be presented in Section 3.3. The listing of these target values in the GT problem serves the purpose of transferring these requirements to the manufacturing group.

GIVEN:
- Parametric CAD model of part
- Functional property models
- Target values of functional prop's
- Target values of process goals
- RP material properties
- Production material properties
- Target values of geometry variables
- Target values of cost and time

FIND:

System Variables: Geometry variables

Deviation Variables: Deviation of goals from targets

SATISFY:

Constraints: Meet geometry and assembly constraints

Goals: Meet target functional properties; Meet targets of geometry vars

Bounds: Bounds for all system variables

MINIMIZE:

Deviation Function: Function of goal deviations

Fig. 1 cDSP word formulation of GT

GIVEN:
- $FP_k = f(G_j, M)$
- $G_{Tjn}, FP_{Tkn}, PG_{Tln}, C_{Tn}, T_{Tn}$
- M_T material properties (prototypes)
- M_P, material prop's (production)

FIND:

System Variables: G_j

Deviation Variables: $d_{i,n}^+, d_{i,n}^-$

SATISFY:

Constraints:

$$g_m(G_j, M) = 0 \quad (5)$$

$$d_{i,n}^+ * d_{i,n}^- = 0 \quad (6)$$

$$d_{i,n}^+, d_{i,n}^- \geq 0 \quad (7)$$

Goals:

$$G_{j,min}, G_{j,max}, FP_{k,min}, FP_{k,max} \quad (8)$$

Bounds:

$$G_{lbj} <= G_j <= G_{ubj} \quad (9)$$

MINIMIZE:

Archimedean Formulation: $$Z = \sum_{n=1}^{4} \sum_{i=1}^{2J+2K} w_{i,n}(d^{+}_{i,n} + d^{-}_{i,n}) \quad (10)$$

Fig. 2 cDSP math formulation of GT

The system variables in the problem are the part dimensions that the designer has identified as having a small influence on the important functional part properties. These variables can be modified within a range to obtain better functional properties. Other part dimensions significantly affect part function and are not modified as part of geometric tailoring. Deviation variables correspond to the deviation of goal achievements. Bounds for the system variables are obtained by the physical limitations on geometry variables. Designer preferences are specified through target values of goals using the Linear Physical Programming (LPP) formulation of the cDSP deviation function [17]. In this approach, the designer specifies goal values at five levels: ideal, desirable, tolerable, undesirable, and unacceptable. The unacceptable level is modeled as a constraint, while the other four levels are modeled using four goal equations.

The corresponding cDSP math formulation of the geometric tailoring problem is presented in Fig. 2. In this formulation, Young's modulus and tensile strength are the material properties provided for the target and prototype material. The quantitative models relating the functional properties to geometry variables are also formulated (Eq. (4)). In this research, response surface methodology and analytical modeling techniques are used to obtain the quantitative models. A typical example of the use of response surface models is the relationship between a part's stress and its geometry. Analytical equations are used when possible to model the exact relationships between variables and goals or constraints. A typical example is the relationship between the weight of a part and its geometry.

Equations (5)–(7) are the constraints in the problem. Equation (5) is a set of geometry constraints and Eqs. (6) and (7) are the constraints on the deviation variables. Equation (8) represents the functional property and geometry goals in the problem. Note the difference between a constraint and a goal. 'The surface finish of

GIVEN:	
• CAD model of the part	• RP material property models
• Target values of process goals	• RP process models
• Production material properties	• Targets of cost and time
FIND:	
System Variables:	**Deviation Variables**:
RP process variables	Deviation of goals from targets
SATISFY:	
Constraints:	**Goals**:
Meet RP process constraints	Meet target material properties
Bounds:	Meet targets of process goals
Bounds for all system variables	Meet target cost and time
MINIMIZE:	
Deviation Function: Function of goal deviations	

Fig. 3 cDSP word formulation of RP process planning

a surface should be $\leqslant 20\ \mu$m' is a constraint. 'I prefer the surface finish of a surface to be $\leqslant 5\ \mu$m' is a goal. A problem can have a goal and constraint for the same quantity (such as surface finish). The bounds for all the geometry variables are presented in Eq. (9). The objective function formulation is presented in Eq. (10). Note that Eqs. (5)–(8) are given in a general form in Fig. 2, but will be replaced with problem specific equations.

Goals may represent target-matching objectives (e.g., match a target dimension) or may represent cases where minimization is preferred (e.g., cost). Most goals (Eq. (8)) in this work are target-matching and are modeled as two goals: one minimization goal (e.g., $G_{j,\min}$) and one maximization goal (e.g., $G_{j,\max}$). Each of these is modeled using four goal equations, one for each preference level in the LPP formulation (except the Unacceptable level), and one constraint, for the Unacceptable level. The goal equations are presented in Eqs. (11) and (12) for minimization and maximization, respectively, and are normalized. Variables should be interpreted as follows. 'G^{+}_{Tj}' is the target value of the jth geometry goal, G_j, and FP_k is the achievement of the kth functional property goal.

$$\frac{A-\max(A-A^{+}_{T(n+1)},0)}{A^{+}_{Tn}}+d^{-}_{i,n}-d^{+}_{i,n}=1 \tag{11}$$

$$\frac{A+\min(A-A^{-}_{T(n+1)},0)}{A^{-}_{Tn}}+d^{-}_{i,n}-d^{+}_{i,n}=1 \tag{12}$$

This generic cDSP is the starting point for formulating GT problems. When formulating a specific GT problem, the first step is to identify the design freedom in the problem and hence the geometry and system variables. In the second step, the functional properties should be identified based on the primary functionality and purpose of the part. In the third step, the specific geometry constraints in the problem should be identified. In the fourth step, quantitative models should be developed to relate goals to system variables. Then, the information in the generic cDSP should be replaced with the problem specific data and models to generate the cDSP for the problem.

3.2 RP-PP Problem Formulation. The manufacturer's generic word formulation for RP process planning (RP-PP) is presented in Fig. 3. The objective of RP process planning is to achieve better quality prototypes by appropriate process planning of the RP machine. Though the RP process considered in this research is SL, this word formulation is generic to any RP process.

The information incorporated in RP process planning includes the CAD model of the part, production material properties, RP material property models, RP process models and the target values of process goals, cost and time. Production material properties and target values of process goals, cost and time serve as the targets

GIVEN:	
• CAD model of the part, N_p	• ZL, SP = f (LT)
• AC_{Ton}, SF_{Tqn}, C_{Tn}, T_{Tn}, YM_{Tn}, TS_{Tn}	• SF_q = f (PO, LT)
• AC_o = f (PO, LT, HOC, FOC, ZL, SP)	• YM_P = f (LT, HOC)
• BT = f (PO, LT, HOC, FOC, ZL, SP)	• TS_P = f (LT, HOC)
• C = f (BT)	• T = f (BT)
FIND:	
System Variables:	**Deviation Variables**:
PO (α_x, α_y, α_z), LT_r, HOC_r, FOC_r	$d_{i,n}^{+}$, $d_{i,n}^{-}$
SATISFY:	
Constraints:	**Bounds**:
$x_p \le x_b$; $y_p \le y_b$; $z_p \le z_b$	$0^0 \le \alpha_x, \alpha_y, \alpha_z \le 360^0$
$LT_r \in [2, 4, 6, 8]$	$2 \le LT_r \le 8$
$d_{i,n}^{+} * d_{i,n}^{-} = 0$, $d_{i,n}^{+}, d_{i,n}^{-} \ge 0$	$HOC_{lb} \le HOC_r \le HOC_{ub}$
Goals:	$FOC_{lb} \le FOC_r \le FOC_{ub}$
YM_{min}, YM_{max}, TS_{min}, TS_{max}, C_{min}, C_{max}, T_{min}, T_{max}, $AC_{o,min}$, $SF_{q,min}$	
MINIMIZE:	
Archimedean Formulation: $Z=\sum_{n=1}^{4}\sum_{i=1}^{O+Q+6} w_{i,n}(d^{+}{}_{i,n}+d^{-}{}_{i,n})$	

Fig. 4 cDSP math formulation of RP process planning

for the goals in the problem. The designer specifies these values in the GT problem formulation. RP material property and process models are the quantitative models that relate the goals to the system variables in the problem. Cost and time correspond to the RP process. The corresponding mathematical formulation of the RP process planning problem is shown in Fig. 4.

The system variables in the problem are the RP process variables that can be controlled by the manufacturer. These variables can be modified within the provided ranges to obtain better quality prototypes, and include part orientation (PO), slicing scheme, hatch overcure and fill overcure. Part orientation is the orientation (x, y and z orientations) in which the part is built in the SLA vat. Slicing scheme is a list of all the layer thicknesses (LT_r) used to build the part along with their corresponding z-ranges. Hatch and fill overcures are SL scanning variables. LT, HOC and FOC are an array of variables with one value for each block. Bounds for layer thickness are 2 and 8 mils (0.0508 and 0.2032 mm). Bounds on hatch and fill overcures are functions of layer thickness and the SLA machine.

Based on their capabilities and limitations, different RP processes have different process constraints. For SL, these constraints include the maximum size of the part, discrete layer thickness values, and accuracy limitations. In addition, the designer can specify problem specific constraints on, for example, the surface finish or the maximum budget for fabricating a part. These specifications are translated into constraints in the cDSP.

The goals in RP process planning arise from the requirement of matching behavior and performance of prototype and production parts. These goals fall under three categories: meeting target material properties, meeting the process goals, and meeting target cost and time. Material property goals are Young's modulus and tensile strength. Process goals include surface finish and accuracy. Cost and time goals are for the whole RP process. The intermediate responses in the problem are z-level wait, sweep period and build time, which are determined by the SL process variables.

The relationships (or quantitative models) in the RP-PP problem are process dependent and are a function of only the SL machine considered. Hence, it is possible to formulate these relationships independently of the specific problem. Empirical models are used to quantify the relationships in the RP-PP problem and are presented in [18–20].

3.3 MPGT/RP Problem Formulation. The MPGT problem is obtained by integrating GT and RP-PP problems considering the coupling between them. The objective of the MPGT/RP

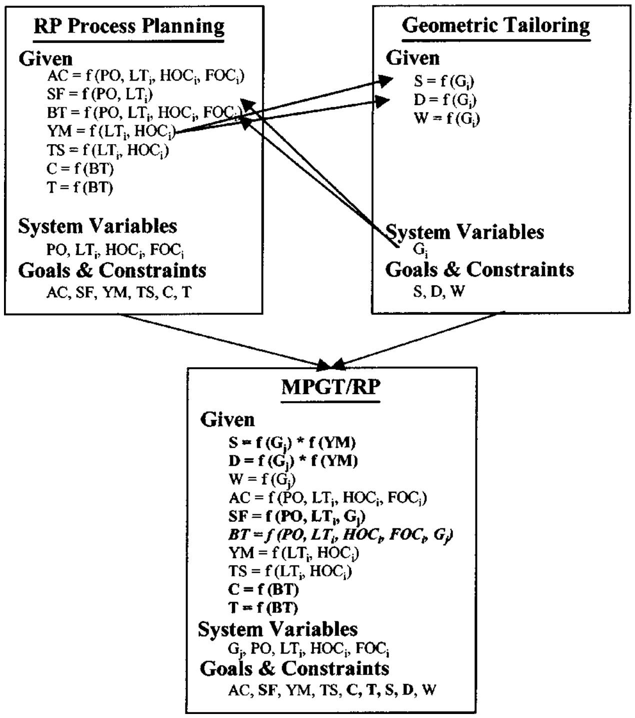

Fig. 5 Synthesis and coupling of MPGT/RP problem

problem is to achieve desired functional and material properties, part quality, cost and time by adjusting some of the non-critical dimensions of the part and by appropriate process planning of the RP machine. The MPGT/RP word and math formulations are simply the combinations of the respective GT and RP-PP problems.

The coupling between the two sub-problems (GT and RP-PP) arises due to the coupling of one or more of cost, surface finish, stress and displacement goals with system variables of the other sub-problem. The situation is illustrated in Fig. 5, where *S*, *D*, and *W* denote stress, deflection, and weight goals, respectively. Varying geometry variables modifies part size and hence affects the scan time. If part height is varied, the recoat time is also affected. Hence, modifying geometry variables of a part affects its build time. Cost and time are directly proportional to build time and hence are similarly affected by geometry variables. Surface finish can be a coupled goal under two circumstances: 1) varying geometry variables affects the orientation of surfaces in the part and affects surface finish, and 2) varying geometry variables varies the best part orientation and affects the surface orientation and surface finish.

Stress and displacement are goals in the GT problem. They are affected by the Young's modulus of the material, which is affected by RP process variables (system variables of RP-PP problem). Hence, stress and strain/displacement are coupled goals.

The synthesis of the MPGT/RP problem from its sub-problems is presented in Fig. 5. This formulation is for the worst-case scenario of coupling between the sub-problems, where Young's modulus in RP-PP is coupled with *S* and *D* in GT and the geometric variables, *G* in GT are coupled with surface finish and build time. The coupled goals and relationships are shown in bold.

3.4 MPGT/RT Problem Formulation. In the rapid tooling scenario, three engineering domains are involved: design (GT), rapid prototyping (RT-PP) and injection molding (IM-PP). These three domains comprise the three sub-problems that are integrated to form the MPGT/RT problem formulation. The GT formulation for RT scenario is very similar to that of the RP scenario. Both RP-PP and RT-PP formulations are for SL machines except that RP-PP is for process planning of parts and RT-PP is used for process planning of molds. Hence in RT-PP, material property goals are not considered (as material properties of the mold do not affect the part's properties). Also cost and time goals in RT-PP correspond to mold fabrication cost and time. IM-PP is a cDSP formulation for process planning for injection molding. Cooling time is the only system variable in this research. Part fabrication cost and time are the goals.

The coupling between GT, RT-PP and IM-PP problems is similar to the MPGT/RP problem. Geometry variables could affect surface finish if they affect surface orientation. Draft angle affects surface finish of drafted surfaces. Mold life of a mold feature is affected by its height, width, draft angle (geometric variables), layer thickness used to build the feature (RP process variables) and the cooling time (IM process variable). Additionally, mold life affects the number of molds, build time, and mold cost and time. Since the variation of mold size with geometry variables is insignificant, we assume that build time is not coupled with geometry variables.

In the MPGT/RT formulation, two additional goals, total cost (C) and total time (T) are included in the problem formulation. These goals are not part of any of the sub-problem formulations but are synthesized from mold cost and time (RT-PP) and part cost and time (IM-PP). Introducing the total cost and time goals in MPGT/RT problem increases the coupling between the sub-problems significantly since cost and time are affected by system variables from all sub-problems.

4 DFM Problem Solution Strategy

4.1 Problem Decomposition, Solution Strategy and Implementation Modules for RP. In this section, the solution strategy used to solve the MPGT/RP problem is presented. Consider a system-level cDSP that consists of two coupled cDSP's, 'cDSP for A' and 'cDSP for B.' The different steps of the solution strategy to solve the 'System cDSP' are shown in Fig. 6. Here, 'cDSP for A' and 'cDSP for B' are solved sequentially to obtain the solution to 'System cDSP'. 'Design of Experiments' is used to obtain a set of values of responses *Y* and 'cDSP for A' is solved for the obtained set of *Y* values. A vector of solutions, *Z*, results, corresponding to different values of *Y*. 'Response Surface Model' is used to generate a response surface, $Z=f_X(Y)$, through the data points (Y_i, Z_i) and this response surface forms the objective for 'cDSP for B'. Solving 'cDSP for B' results in a solution for 'System cDSP,' since solving for $f_X(Y)$ is equivalent to solving for $f(X,Y)$.

The decomposition and solution strategies used for 'System cDSP' can be applied to the MPGT/RP and MPGT/RT problems. The sub-problems of MPGT/RP obtained after decomposition are 'Modified RP-PP problem' and 'Modified MPGT/RP problem,' which correspond to 'cDSP for A' and 'cDSP for B' in Fig. 6,

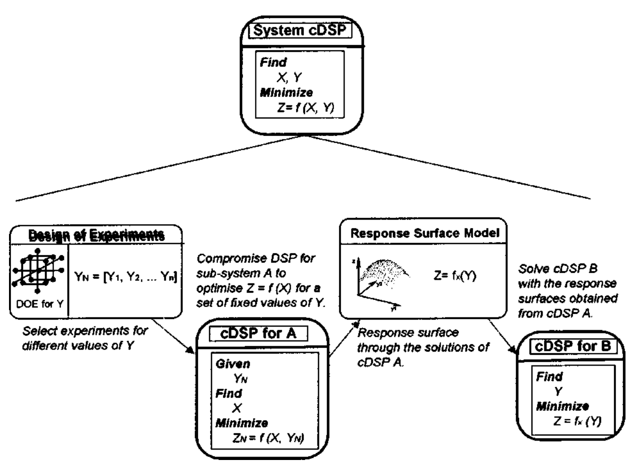

Fig. 6 Solution strategy to solve 'System cDSP'

Modified RP-PP

Given
AC = f (PO, LT_i, HOC_i, FOC_i)
SF = f (PO, LT_i)
BT = f (PO, LT_i, HOC_i, FOC_i)
YM = f (LT_i, HOC_i)
TS = f (LT_i, HOC_i)
C = f (BT) T = f (BT)
S = f (YM) D = f (YM)
G_{jN}

System Variables
PO, LT_i, HOC_i, FOC_i

Goals & Constraints
AC, SF, YM, TS, C, T, S, D

Minimize
Z_N = g(PO, LT_i, HOC_i, FOC_i, G_{jN})

Modified MPGT/RP

Given
AC = $f_{RP\text{-}PV}$ (G_i)
SF = $f_{RP\text{-}PV}$ (G_i)
W = $f_{RP\text{-}PV}$ (G_i)
YM = $f_{RP\text{-}PV}$ (G_i)
TS = $f_{RP\text{-}PV}$ (G_i)
C = $f_{RP\text{-}PV}$ (G_i)
T = $f_{RP\text{-}PV}$ (G_i)
S = $f_{RP\text{-}PV}$ (G_i)
D = $f_{RP\text{-}PV}$ (G_i)

System Variables
G_{jN}

Goals & Constraints
AC, SF, YM, TS, C, T, S, D, W

Minimize
Z_N = $g_{RP\text{-}PV}$(G_i)

Fig. 7 Modified RP-PP and MPGT/RP sub-problems

respectively. The coupling in the stress, displacement, build time, and surface finish goals is handled by separating the effects of part geometry variables from those of RP process variables. The analogy between cDSP for A and Modified RP-PP is shown in Eqs. (13)–(15).

$$X=\{PO,LT_i,HOC_i,FOC_i\} \quad \text{(RP process variables)} \tag{13}$$

$$Y_N=G_{jN} \quad \text{(part geometry variables)} \tag{14}$$

$$f(X,Y_N)=g(PO,LT_i,HOC_i,FOC_i,G_{jN}) \tag{15}$$

Note that design-of-experiments methods are applied to find sets of geometric variable values (Eq. (14)). These geometric variable values are used to modify part geometry that is utilized in the RP-PP problem. Process plans for each variant are determined. The objective function in Modified RP-PP is a weighted sum of the goal deviations (AC,SF,BT,YM,TS,C,T,S,D) and it can be expressed as a function of system variables (Eq. (15)) using the relationship between goals and system variables (Given part of the formulation).

The analogy between cDSP for B and Modified MPGT/RP is shown in Eqs. (16) and (17). The objective function in modified MPGT/RP (Eq. (17)) is a weighted sum of the deviation of the goals (AC,SF,YM,TS,C,T,S,D) and can be expressed as a function of system variables.

$$Y=G_i \tag{16}$$

$$f_x(Y)=g_{PP\text{-}PV}(G_i) \tag{17}$$

The resulting decomposition of the MPGT/RP problem into the Modified RP-PP and Modified MPGT/RP problems is shown in Fig. 7. The Modified RP-PP problem is the first sub-problem to be solved in the decomposed problem. Its problem formulation is similar to that of RP-PP problem with small differences. In Modified RP-PP problem, the variation of stress and displacement with Young's modulus is considered and these goals are included in the problem. This facilitates considering the coupling of stress and displacement goals.

The Modified MPGT/RP problem is the second sub-problem. The system variables are geometry variables (same as GT problem). These models, $f_{RP\text{-}PV}(G_i)$, are obtained by fitting response surface equations through the solutions obtained by solving the Modified RP-PP problem for different values of geometry values. In the expression $f_{RP\text{-}PV}(G_i)$, RP-PV stands for RP Process Variables. In this approach, the MPGT-RP problem can be solved by sequentially solving the Modified RP-PP problem then the Modified MPGT/RP problem. As the coupling of all goals is considered (through response surface equations), solving the modified MPGT/RP problem results in a solution for the MPGT/RP problem, though it should be noted that errors due to response surface generation exist.

The steps in the solution strategy of MPGT/RP problem are divided into three phases, based on RCEM (Sec. 2.3). Phase 1 is used to generate stress and displacement response surfaces. The first step is to formulate the required set of experiments (DOE) through consideration of factors (geometry variables) and responses (stress, displacement). Then, the designed experiments are executed. The ANSYS finite element analysis package is used to determine stress and displacement in this case. The final step is to generate the response surfaces of stress and displacement using MINITAB.

In Phase 2, a design of experiments is performed so that all response surfaces for the Modified RP-PP problem in Fig. 7 can be generated. These experiments are performed using the RP-PP software to determine the RP variables and corresponding goal achievements for all the designed experiments. RP-PP software needs the response surfaces of stress and strain obtained in Phase 1 and also requires targets for these goals. Response surfaces are generated in MINITAB. The final step is to solve the modified MPGT/RP problem. Our implementation uses OptdesX, specifically the Simulated Annealing (SAN) algorithm.

Phase 3 deals with the validation of the Modified MPGT/RP solution and the response surfaces. Estimates are made of the errors due to response surfaces and the quality of the surfaces' fit. If necessary, the design space is reduced and the solution procedure is repeated from Phase 1. Otherwise, the obtained solution is accepted as the solution for the MPGT/RP problem.

4.2 Solution Strategy for Rapid Tooling. Considering the number of coupled goals and the extent of coupling between different sub-problems, it can be seen that the MPGT/RT problem is a strongly coupled problem. Different solution strategies are used for MPGT/RP and MPGT/RT due to the differences in the nature of coupling in the two problems: 1) build time is a coupled response in MPGT/RT since build time is affected by mold life, possibly requiring multiple molds, 2) build time and surface finish are not affected by geometry variables, and 3) cost and time goals are affected by all the system variables (geometry, RP process, molding) in MPGT/RT.

The MPGT/RT problem decomposition is as follows. For default values of geometry variables, *Y*, multiple sets of feasible process plans, X_i, are generated. For each combination of *Y* and X_i, the geometric tailoring problem is solved, resulting in a vector of deviation function values $\{Z_i\}$, where $Z_i=f(X_i,Y)$. During the solution process, new values of *Y* are found that improve the mold fabrication process. The $\{Z_i\}$ are searched to find the minimum deviation function value and the combination of *Y* and X_i that corresponds to this minimum *Z* value is the solution to the MPGT/RT problem.

Not all the coupling relationships are considered in this solution strategy. Hence, the interaction of *X* and *Y* system variables is not considered, inducing an error in the obtained solution. However, in our experiments, neglecting this interaction has had minimal impact on the solutions obtained [19,21].

5 Example

In this section, the Material Process Geometric Tailoring for RP (MPGT/RP) problem is formulated and solved for a link in a parallel manipulator, shown in Fig. 8. Extensions to MPGT for rapid tooling are indicated in the final subsection.

5.1 Problem Description. Due to the light payloads used with this manipulator, production manipulator links are made of injection molded ABS plastic. For the purposes of this example, we will assume that 10 prototype links are to be fabricated in a SLA-3500 machine in SL-7510 resin. The prototype links will be tested and are to have similar behavior and performance as the production parts in terms of maximum stress and deflection.

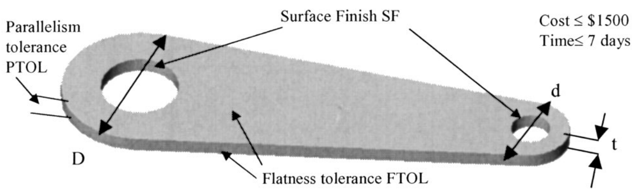

Fig. 8 Parallel manipulator link

Prototypes should be fabricated in less than 16 hours within the maximum available budget of $300. The two cylindrical holes in the robot arm mesh with other components and must have good surface finish. The loading conditions on the robot arm are as follows. The larger cylindrical hole is completely fixed. A 5N tensile load and a 20N bending load are applied uniformly on the smaller cylindrical surface in the plane of the link. Under this loading condition, the prototype parts are required to match the stress, displacement and weight characteristics of the production (ABS) parts. To represent the production part more closely, it is also required to match the mechanical properties (Young's modulus and tensile strength) of prototype and production parts. The top and bottom surfaces (flat surfaces of the robot arm) have parallelism and flatness tolerance requirements. The performance characteristics, properties, and tolerances are modeled as target-matching goals (the **FP** terms in Fig. 2).

The geometry variables, D, d and t, of the link can be modified up to $\pm 10\%$ in order to achieve desired performance. The length between the hole centerlines is 63.5 mm and cannot be modified as it affects the orientation of the links in the robotic mechanism. The diameters of the holes cannot be modified due to assembly requirements with other components. Their dimensions are 10.32 mm (large hole) and 5.16 mm.

The targets of different goals are presented in Table 3. In the table $PTOL$ is the parallelism tolerance between top and bottom flat surfaces of arm, $FTOL$ is the flatness tolerance for top and bottom flat surfaces, SF is surface finish requirement for the cylindrical surfaces of the holes, DSF is default surface finish for all the other surfaces of the part, C is cost, T is time, YM is Young's modulus, TS is tensile strength, S is stress, W is weight, YD is y-displacement, D, d and t are the geometry variables. The unacceptable values are treated as constraints. For example, $PTOL \leqslant 0.508$ mm, is a constraint. The target values of maximum stress and deflection are 5.99 MPa and 0.68 mm, respectively.

5.2 Completing the Problem Formulation. It is necessary to complete the formulation of the MPGT/RP problem. Response surface methodology is used to generate response surfaces of maximum von-Mises stress and maximum y-displacement in terms of D, d and t. A three factor—three level central composite response surface design with $\alpha = 1.6818$ (for rotatability) was used as the design for the experiments. The data points for the analysis were generated by performing FE analysis (in ANSYS) of the robot arm. MINITAB was used to perform regression analysis and ANOVA of the obtained data. The response surfaces generated are presented in Eqs. (18) and (19), along with their R^2, maximum deviation and average deviation values. The fit statistics indicate the models are very accurate. An analytical equation was developed to determine the weight of the robot arm due to its simple geometry.

$$\begin{aligned} Stress &= 68.5522 - 3.1668D - 0.8671d - 9.7250t + 0.0457D^2 \\ &\quad + 0.0150d^2 + 0.4571t^2 + 0.0072Dd + 0.1952Dt \\ &\quad + 0.0661dt \\ R^2 &= 99.0\%, \quad \text{Max. dev} = 16.2\%, \quad \text{Avg. dev} = 4.4\% \end{aligned} \tag{18}$$

$$\begin{aligned} Y_{disp} &= 11.1264 - 0.5081D - 0.3152d - 1.2971t + 0.0067D^2 \\ &\quad + 0.0039d^2 + 0.0430t^2 + 0.0061Dd + 0.0276Dt + 0.0184dt \\ R^2 &= 99.0, \quad \text{Max. dev} = 18.2\%, \quad \text{Avg. dev} = 5.0\% \end{aligned} \tag{19}$$

The MPGT/RP formulation for the manipulator link (synthesized from MPGT and RP-PP problems) is presented in Fig. 9. The problem has a total of 7 system variables, 32 goals, and 32 constraints, where the constraints correspond to unacceptable levels of the goals. The two coupled goals are build time and y-displacement. The relationships of these goals with the system variables are shown **bold** in the formulation. Equations (18) and (19) appear in Fig. 9 as the $S = f(D,d,t)$ and $YD = f(D,d,t)$ response surfaces.

5.3 Problem Decomposition and Results. The MPGT problem is decomposed into modified RP-PP and modified MPGT/RP problems. In the modified MPGT/RP formulation, the RP process variables are not included in the system variables, but their effect is considered by integrating the response surfaces ob-

Table 3 Targets for goals

Goal	Type	Ideal	Desir-	Toler-	Unde-	Unacc-	Units
$PTOL$	1S	0.0508	0.1016	0.2032	0.3302	0.508	mm
$FTOL$	1S	0.0254	0.0508	0.1016	0.1778	0.254	mm
SF	1S	0.5	1.1	2.0	3.3	5.1	μm
DSF	1S	7.6	14.0	22.9	35.6	50.8	μm
C	1S	75	100	140	200	300	$
T	1S	8	9	11	13	16	hr
YM	1S	2400	2475	2625	2800	3000	MPa
YM	2S	2400	2325	2175	2000	1800	MPa
TS	1S	44.8	45.8	48	51	56	MPa
TS	2S	44.8	43.8	41.5	38.5	34	MPa
S	1S	5.99	6.29	6.89	7.79	8.99	MPa
S	2S	5.99	5.69	5.09	4.19	3	MPa
W	1S	3.43	3.6	3.94	4.46	5.15	g
W	2S	3.43	3.26	2.92	2.4	1.72	g
YD	1S	0.68	0.71	0.78	0.88	1.02	mm
YD	2S	0.68	0.65	0.58	0.48	0.34	mm
D	1S	20.32	20.83	21.84	23.37	25.4	mm
D	2S	20.32	19.81	18.8	17.27	15.24	mm
d	1S	10.16	10.41	10.92	11.68	12.7	mm
d	2S	10.16	9.91	9.4	8.64	7.62	mm
t	1S	3.048	3.099	3.2	3.353	3.557	mm
t	2S	3.048	2.996	2.896	2.743	2.539	mm

GIVEN:

- Parametric CAD model of the robot arm
- Prototype parts built on SLA 3500 with SL 7510 resin
  - Young's modulus, YM = f (LT, HOC) → RP-PP problem
  - Tensile strength, TS = f (LT, HOC) → RP-PP problem
  - Density = $1.22 * 10^{-6}$ kg/mm^3
- Production parts injection molded with ABS – Molded material
  - Young's modulus = 2540 MPa
  - Tensile strength = 44.8 MPa
  - Density = $1.04 * 10^{-6}$ kg/mm^3
- Required number of parts (N_P) = 10
- Allowable layer thicknesses (LT): 2, 4 and 8 mils
- Response surfaces:
  - ZL, SP = f (LT) → RP-PP problem
  - PTOL, FTOL = f (PO, LT, HOC, FOC, ZL, SP)→ RP-PP problem
  - SF, DSF = f (PO, LT) → RP-PP problem
  - **BT = f (D, d, t, PO, LT, HOC, FOC, ZL, SP) → Coupled goal**
    - BT = f (PO, LT, HOC, FOC, ZL, SP) → RP-PP problem
  - C = f (BT) → RP-PP problem
  - T = f (BT) → RP-PP problem
  - **YD = f (D, d, t, YM) → Coupled goal**
    - YD = f(D, d, t) → GT problem
  - S = f (D, d, t) → GT problem
  - W = f (D, d, t) → GT problem

FIND:

System variables

- Geometry variables: D, d, and t
- RP process variables: PO, LT, HOC, and FOC

Deviation variables

d_{in}^{+}, d_{in}^{-} i = 1,…,32 n = 1,…,4

SATISFY:

Goals

- C, T, BT, PTOL, FTOL1, FTOL2, SF1, SF2, DSF are minimization goals
- TS, YM, S, W, YD, D, d and t are target-matching goals and each is split into min. and max. goals.

Bounds

- $17.86 \le D \le 21.82$
- $8.52 \le d \le 10.42$
- $2.61 \le t \le 3.19$
- PO – Discrete variable
- $2 \le LT \le 8$ (mils) – Discrete variable
- $0.002 \le HOC_2 \le 0.006$ (mils)
- $0.003 \le HOC_4 \le 0.007$ (mils)
- $0.001 \le HOC_8 \le 0.005$ (mils)
- $0.012 \le FOC_2 \le 0.016$ (mils)
- $0.004 \le FOC_4 \le 0.008$ (mils)
- $0.002 \le FOC_8 \le 0.006$ (mils)

MINIMIZE:

$$Z = \sum_{n=1}^{4} \sum_{i=1}^{32} w_{i,n+1}\left(d_{i,n}^{+} + d_{i,n}^{-}\right)$$

Fig. 9 cDSP of MPGT/RP for robot arm example

tained from solving the modified RP-PP problem. The same experiments used to generate von Mises stress and *y*-displacement response surfaces in the MPGT problem are also used to generate the response surfaces for the modified MPGT problem, including: accuracy, surface finish, Young's modulus, tensile strength, and cost. The RP process planning software is used to perform experiments. The solutions obtained for each experiment are similar, yielding almost flat horizontal planes as response surfaces.

The modified MPGT/RP problem was solved in OptdesX using the Simulated Annealing (SAN) algorithm. Different starting points are investigated to test theconvergence of the solution. SAN was run for 5000 cycles for each starting point, with each run requiring less than 30 seconds of execution time on a SGI Impact 10,000 workstation. To test the accuracy of the SAN algorithm, a grid search algorithm was used, resulting in the solution: $D=19.34$ mm, $d=10.02$ mm, and $t=2.899$ mm. This is approximately the same solution reached using the SAN algorithm.

Since response surfaces were used to solve the modified MPGT-RP problem, it is necessary to validate this solution without utilizing response surfaces. The actual values of the stress and *y*-displacement goals are obtained by running ANSYS with the values of geometry variables corresponding to the solution. Actual values of RP process goals are obtained by running the RP process planning software. For part weight, the actual value is determined from the volume of the original CAD model. These values along with the errors (from estimated values) are presented in Table 4. The errors for all the goals are less than 1.2%. The error values for the response surfaces and the result are very small and hence this solution is accepted as the solution for the MPGT problem.

The solution of MPGT problem was also validated through physical experiments. The results of all the physical experimentation are summarized in Table 5 along with their expected values. Ten links were built in our SLA-3500 machine; their dimensions, surface finishes, and weight were measured. Tensile bars were also built and tested to determine Young's modulus and tensile strength of the material. From the values in the table, we can see that the variables *D*, *d* and *t* have the least error $<0.5\%$. The Young's modulus, tensile strength, weight, surface finish and build time variables have an error of $<12\%$. These errors are small and can be attributed to response surface generation error and experimental error. The build time error is substantial, but can be attributed to the omission of *z* compensation effect on the build time

Table 4 Estimated and actual values for the solution

	D	*d*	*t*	*LT*	*HOC*	*FOC*	*SF*	Cost	*YM*	*TS*	*YD*	Stress	Weight
Actual							1.6	182	2460	47.7	0.77	6.9	3.62
Estimated	19.34	10.02	2.899	2	4	12	1.6	181	2475	47.9	0.76	6.9	3.62
% Error							0.0	0.5	0.6	0.4	1.2	0.2	0.0

Table 5 Physical validation results

Goal	Actual	Estimated	% Error
D [mm]	19.24	19.34	0.5
d [mm]	10.04	10.02	−0.2
t [mm]	2.89	2.9	0.3
Young's Modulus [MPa]	2688	2460	−8.5
Tensile Strength [MPa]	46.5	47.7	2.6
Flatness Tol1 [mm]	0.025	0.043	70.0
Flatness Tol2 [mm]	0.020	0.041	112.5
Parallelism Tol [mm]	0.020	0.040	100.0
Large hole SF [mm]	1.8	1.6	−11.1
Small hole SF [mm]	1.6	1.6	1.6
Build Time [min.]	126	140	11.1
Weight [g]	3.52	3.6	2.3

models used in the RP-PP method. Inclusion of z-compensation would have reduced the build time error to less than 1%. The errors of tolerance variables are very high, but indicate that SL accuracy was better than expected from the response surface models.

5.4 Results and Discussion for RT Scenario. In addition to RP, a rapid tooling scenario was also investigated. For this problem, it was assumed that fifty functional prototypes of the robot arms fabricated in the production material are required for functional testing in the later phases of product realization. The production robot arms are injection molded in atactic polystyrene material and the functional prototypes should have very similar behavior and performance as the production parts. Prototypes should be fabricated in less than a week within the maximum available budget of $1500.

The RT solution procedure involves formulating GT, RT-PP, IM-PP problems including response surfaces, then synthesizing the MPGT problem considering coupling. The solution of the decomposed MPGT problem is presented in Table 6. Using the results, one set of molds was fabricated in our SLA-3500 machine and 50 polystyrene parts were molded. These parts were measured.

The values of the different goals and variables obtained from physical experimentation are summarized in Table 7 along with their estimated values. From the values in the table, we can see that the variables D, d, Young's modulus, tensile strength, build time and density have error $<1.5\%$. This error is very small and

Table 6 Solutions of MPGT/RT problem

Goal	D	d	t	TA	Draft1	Draft2	CT
Sol'n	20.68	9.92	2.99	0	0.61	0.01	300
Stress	Y-Disp	Weight	SF1	SF2	PTol	FTol1	FTol2
5.99	0.49	3.36	1.7	1.7	0.051	0.051	0.051
Cost			# Molds				Obj Fn.
$570			1				0.03542

Table 7 Physical validation results for MPGT/RT

Goal	Actual	Estimated	% Error
D	20.71	20.64	−0.3
d	10.04	9.9	−1.4
t	3.65	2.99	−18.1
Young's Modulus	3486	3400	−2.5
Tensile Strength	32.3	32.8	1.5
Flatness Tol1	0.018	0.043	142.9
Flatness Tol2	0.048	0.043	−10.5
Parallelism Tol	0.056	0.041	−27.3
Large hole SF	1.7	1.6	−8.7
Small hole SF	1.8	1.6	−12.0
Build Time	290	293	1.0
Weight	4.09	3.34	−18.3
Density	1.025	1.04	1.5

can be considered as experimental or part building error. The surface finish values have an error of ≈10%. Though not very small, this error is also reasonable. It should be noted that the surface finish models are developed for the SLA molds and are applied for the injection-molded parts. Part fabrication and ejection could cause small variations in surface finish values. Weight and part thickness have an error of 18%. This can be attributed to the flash generated due to mold warpage. The error in the tolerance values could be due to the small surface area used for the measurement. Also, the mold life was validated by injection-molding 50 parts from the fabricated mold.

6 Conclusions and Future Work

The Material Process Geometric Tailoring framework was developed to assist designers in fabricating functional prototypes, where mismatches in materials and mechanical properties cause prototypes to have different behaviors than production parts. With this framework, design requirements, design freedom, and manufacturing capabilities are integrated for Rapid Prototyping and Rapid Tooling scenarios. The problem of producing functional prototypes is cast as a DFM problem. Design requirements and design freedom are modeled in one compromise Decision Support Problem (cDSP), while manufacturing and material capabilities are modeled as a second cDSP. The MPGT problem is formulated by integrated these cDSP's into one decision problem. A problem decomposition and solution method was presented, where decomposition was achieved by separating the effects on responses of interest of part dimensions from those of manufacturing process variables. Response surface methodology is used to model coupling effects among these responses. The MPGT problem is solved by sequentially solving the process planning problem, followed by the MPGT problem with only part dimensions as variables. One rapid prototyping example was presented in detail and a rapid tooling extension was also summarized.

As a result of this work, we can conclude that the compromise DSP formulation was effective at integrating design requirements, design freedom, and manufacturing capabilities. The geometric tailoring, process planning, and MPGT cDSP formulations are generic to model functional prototype DFM problems, and can be instantiated for particular problems, such as the robot arm example presented in Section 5. Decoupling of part dimensions from process variables using response surfaces was effective, but introduced errors into the modeling of stress, deflection, etc. goals. By reducing the design space and re-solving the problem, errors were shown to become greatly reduced [18]. Through this decomposition strategy, a sequential solution method was developed that was efficient. A series of four example problems have been solved using the methods presented in this paper, including both RP and RT scenarios.

Since design requirements and freedom are integrated into MPGT problems, the framework presented here facilitates the transfer of DFM responsibility to the manufacturer. Conversely, by integrated manufacturing capabilities into a MPGT problem, the designer is enabled to perform better DFM. With this method, we have demonstrated that design and manufacturing activities can be separated, facilitating the use of this method in geographically distributed product development organizations.

Although the MPGT method was developed for functional prototypes, it is applicable to the design of production parts and their manufacture using conventional process (as opposed to RP machines). Consider DFM activities associated with changing a part's material. The MPGT method could be applied to automatically determine part dimensions, given the differences in material properties. Furthermore, differences in injection molding processes can also be included in the problem formulation so that part dimensions and molding processes can be fine-tuned.

Future work will involve developing improved models of material properties and process capabilities, particularly for modeling tolerances as functions of process variables. The second-order response surface models utilized in problem decomposition did not always accurately model relationships among variables and responses. Improved surrogate models for MPGT goals will be investigated. Also, methods exist in the Multi-Disciplinary Optimization literature for solving coupled problems simultaneously, rather than in the sequential manner utilized in our work. We will investigate this literature for suitable solution methods.

Acknowledgments

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Nomenclature

A = value of goal [various]
AC = accuracy [mm]
BT = build time [hr]
C = cost [$]
d_i^-, d_i^+ = deviation variables [−]
FOC = fill overcure [mils]
FP = functional properties [various]
G = geometric dimensions, Goal target value [mm]
HOC = hatch overcure [mils]
LT = layer thickness [mil]
M = material properties [various]
p = parameter [mm]
PO = part orientation [radians]
S = stress [MPa]
SP = sweep period [sec]
T = time [hr]
t = thickness of feature [mm]
TS = tensile strength [MPa]
W = weight of part [g]
YD = Y Deflection [mm]
YM = Young's Modulus [MPa]
ZL = Z-level wait [sec]

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