Centralization versus Decentralization: An Application to Price Setting by a Multi-Market Firm.*

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Abstract

This paper compares centralized and decentralized price setting by a firm that sells a single product in two markets, but is constrained to set one price (e.g. due to arbitrage). Each market is characterized by a different linear demand function, and demand conditions are privately observed by a local manager. This manager only cares about profits in his own market and, as a result, communicates his information strategically. Our main results link organizational design to market demand. First, if pricing is decentralized, it is always delegated to the manager who faces the flattest inverse demand function, regardless of the size of market demand. Second, even when pricing can be allocated to an unbiased headquarters, decentralization is optimal when markets differ sufficiently in how flat the inverse demand functions are. Finally, decentralization is more likely when, in expectations, local managers disagree more about prices.

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1 Introduction

When discussing issues such as pricing by a monopolist or oligopolistic firm, economic textbooks typically assume that firms act as profit maximizers. In contrast, much recent economic research has focussed on agency problems that may push the managers of a firm away from profit maximization, and how organizations are optimally designed to mitigate these. Perhaps surprisingly, at least given the historic importance of Price Theory, price elasticities or product market competition do not play a role in most of these theories. The aim of this paper is to link one of the most basic questions in organizational economics – *should decision-making be centralized or delegated?* — with one of the most basic questions in price theory — *how is pricing affected by demand conditions?*

Ever since Hayek (1945), economists have invoked "local knowledge" as a reason to decentralize decision-making. In a world in which technological advances have all but eliminated communication costs, one may wonder whether local knowledge still matters. In response to this, Dessein (2002) and Alonso, Dessein and Matouschek (forthcoming) have proposed a informational rationale for decentralization that does not rely on physical communication constraints. As they show, if a principal cannot commit to a formal mechanism to elicit information, decentralization to biased but better informed agents is often optimal as this avoids that agents distort their information in order to influence decision-making. The present paper builds on these insights to analyze whether pricing decisions are optimally made by an unbiased headquarters, which then bases its decision on information communicated by local division managers, or whether pricing should be delegated. In particular, we are interested how the shape of the demand function faced by each division may affect whether or not pricing decisions are decentralized.

In our model, a firm sells a product in two markets, but can only charge one price due to arbitrage by retailers or consumers, legal constraints on price discrimination or a "most favored customer" clause. The two markets differ in their (linear) demand, and demand conditions are privately observed by a local manager who only cares about profits in his market (for example, due to career concerns). Finally, the firm can only commit to an ex ante allocation of the pricing decision. Our results the link organization design with the shape of the demand function: (i) Conditional on decentralization, the pricing decision is always delegated to the manager who faces the market with the flattest demand curve, even if that market is much smaller (that is, the intercept is smaller).[2] (ii) If both (inverse) demand functions have the same slope, pricing is always centralized. (iii) If the difference between the slopes of the demand function is

[2] The only restriction is that it must be optimal to sell in both markets.

beyond a threshold, it is always optimal to decentralize pricing to the division with the flattest demand function. (iv) When division managers differ more in their preferred price, decentralization is more likely.

2 The Model

A firm produces one good at zero costs and sells it in two markets. In each market the firm faces uncertainty about the demand conditions. In particular, the inverse demand function in Market $j = 1, 2$ is given by $p_j = a_j - b_j q_j$, where a_j is independently drawn from a uniform distribution with support $\left[\mu_j - s, \mu_j + s\right]$, where $\mu_j - s > 0$ and $(\mu_1 - \mu_2)^2 < s^2$.[3] The slopes of the inverse demand functions satisfy $b_2 \geq b_1 > 0$. Due to arbitrage, legal restrictions on price discrimination or contractual commitments to customers (such as most favored customer clauses), the firm needs to set the same price in both markets, i.e. $p_1 = p_2 = p$. The profits in Market $j = 1, 2$ are then given by $\pi_j = pq_j$.

The firm consists of two operating divisions and, potentially, one headquarter division. Each division is run by one manager. Manager 1 is in charge of the first market and privately observes the realization of a_1 while Manager 2 is in charge of the second market and privately observes the realization of a_2. Manager HQ is in charge of headquarters and observes neither a_1 nor a_2. Manager $j = 1, 2$ only cares about the profits of division j, π_j, for example due to (unmodelled) career concerns. Manager HQ, in contrast, cares equally about both divisions and thus maximizes $\pi_1 + \pi_2$.

We follow the property rights literature (Grossman and Hart 1986) in assuming that contracts are highly incomplete. In particular, the organization only commits to an ex ante allocation of the right to make the pricing decision.[4] There are three possible allocations of the decision right: under Centralization Manager HQ has the right to make the decision while under Decentralization-$j = 1, 2$ Manager j has the right to make the decision. Once the decision right has been allocated, it cannot be transferred before the decision is made.

The lack of commitment implies that the decision-makers are not able to commit to paying transfers that depend on the information they receive or to

[3] The latter assumption ensures that the variance of the intercepts $\sigma^2 = s^3/3$ is sufficiently large relative to the average differences in the intercepts and facilitates the analysis of the communication game that we describe below.

[4] Internal contracting on pricing may be difficult as courts are often unwilling to interfere with internal firm matters. Negotiating a mechanism with division managers may also result in costly bargaining or influence activities. As an empirical matter, Marin and Verdier (2007), Table A1,2, report that product pricing and price increases are among the most decentralized decisions in their sample of firms: only 4 out of 16 decisions (German firms) and 2 out of 13 decisions (Austrian firms) are more decentralized.

make their decisions depend on such information in different ways. Communication therefore takes the form of an informal mechanism: cheap talk. For simplicity we assume that this informal communication occurs in one round of communication. In particular, under Centralization, Managers 1 and 2 simultaneously send messages $m_1 \in M_1$ and $m_2 \in M_2$ to headquarters. Since division managers do not care about each other's profits – and hence each other's private information — communication plays no role under Decentralization-$j = 1, 2$.

The timing of the game is as follows. First, the right to make the pricing decision is allocated to maximize the total expected profits $\mathrm{E}[\pi_1 + \pi_2]$. Second, the division managers become informed about demand conditions in their markets, that is, they learn a_1 and a_2 respectively. Third, if pricing is centralized, the division managers communicate with the decision-maker. Finally, the pricing decision is made.

3 Decision-Making

We start by analyzing the decisions that headquarters would make if it were perfectly informed about the demand conditions in both markets. If Manager HQ could observe a_1 and a_2 herself, she would set the price that maximizes total firm profits

$$\pi_1 + \pi_2 = \sum_{j=1,2} \left[\frac{a_j^2}{4b_j} - \frac{1}{b_j} \left(p - \frac{a_j}{2} \right)^2 \right]. \tag{1}$$

To understand this expression, note that $a_j^2/4b_j$ are the maximum profits that can be earned in Market $j = 1, 2$. To realize these profits, the price needs to be set equal to $a_j/2$, which is the optimal price for Market j. If the firm sets a different price, then profits in Market j are reduced by an amount equal to the second term on the RHS. In this sense the second term represents the cost of setting $p \neq a_j/2$. The price that maximizes (1) is given by

$$p^{FB} = \frac{1}{1 + b_1/b_2} \frac{a_1}{2} + \frac{b_1/b_2}{1 + b_1/b_2} \frac{a_2}{2},$$

where $b_1/b_2 \leq 1$. The first best price is therefore a convex combination of the prices that maximize profits in Market 1 and Market 2 respectively. Note that the weights that are put on the two prices depend on the slopes of the inverse demand functions b_1 and b_2 but not on the intercepts a_1 and a_2. This feature, which will play an important role later on, is due to the fact that the cost of setting a price $p \neq a_j/2$ that is sub-optimal for Market j is decreasing in b_j but independent of a_j. In particular, (1) shows that the smaller b_j, the bigger the cost of setting $p \neq a_j/2$.

Suppose now that Manager HQ does not observe the demand conditions but retains the right to set prices. In this case Manager HQ first receives messages m_1 and m_2 from the division managers and then sets the price that maximizes $\mathrm{E}[\pi_1 + \pi_2 \mid m]$, where $m = (m_1, m_2)$. This price is given by

$$p^C = \mathrm{E}\left[p^{FB} \mid m\right] = \frac{1}{1 + b_1/b_2} \frac{\mathrm{E}[a_1 \mid m]}{2} + \frac{b_1/b_2}{1 + b_1/b_2} \frac{\mathrm{E}[a_2 \mid m]}{2}. \tag{2}$$

Suppose next that the pricing decision is delegated to Manager 1, then the chosen price maximizes π_1, and will thus be given by $p^{D_1} = a_1/2$. Note that the discrepancy in the preferences between Manager HQ and Manager 1 is decreasing in $b_1/b_2 \leq 1$. Thus, the smaller b_1/b_2, the less biased is Manager 1's decision-making. The analysis of decision-making under Decentralization 2 is analogous. In particular, $p^{D_2} = a_2/2$.

4 Strategic Communication

To understand strategic communication under Centralization, we first analyze the division managers' incentives to misrepresent information under this organizational structure.

4.1 Incentives to Misrepresent

Consider the incentives of Manager 1 to misrepresent his information under Centralization. When she sets the price, Manager HQ puts less weight on setting it equal to $\mathrm{E}[a_1 \mid m]/2$ and more weight on setting it equal to $\mathrm{E}[a_2 \mid m]/2$ than Manager 2 would like her to. Since $\mathrm{E}[a_2] = \mu_2$ this implies that if Manager 1 truthfully communicated his state, he would expect headquarters to set a price that is too close to $\mu_2/2$. To induce headquarters to set a price that is further away from $\mu_2/2$, he therefore exaggerates the difference between his state and μ_2, that is, he reports $m_1 > a_1$ if $a_1 > \mu_2$ and $m_1 < a_1$ if $a_1 < \mu_2$. Only if $a_1 = \mu_2$ does Manager 1 report his state truthfully.

To see this more formally, let $\nu_1 = \mathrm{E}[a_1 \mid m_1]$ be Manager HQ's posterior belief about a_1 and suppose that Manager 1 can simply choose any ν_1. Ideally, Manager 1 would like Manager HQ to have the posterior that maximizes his expected payoff which is given by

$$\nu_1^* = \arg\max_{\nu_1} \mathrm{E}\left[p^H q_1\left(p^H\right) \mid a_1\right], \tag{3}$$

where p^H depends on v_1 as defined in (2). In equilibrium the expected value of the posterior of a_2 is equal to μ_2. Assuming that this relationship holds we can use (3) to obtain

$$\nu_1^* = a_1 + \frac{b_1}{b_2}(a_1 - \mu_2). \tag{4}$$

Since $b_1/b_2 \geq 0$ this confirms the above intuition that Manager 2 exaggerates the difference between his state a_1 and μ_2. Only when $a_1 = \mu_2$ does he have an incentive to communicate truthfully. Moreover, it can be seen that his incentives to exaggerate are increasing in $|a_1 - \mu_2|$ and in b_1/b_2. Essentially, the smaller b_1/b_2, the more responsive price setting by headquarters is to the information it receives about the demand conditions in the first market. Thus, the smaller b_1/b_2, the less Manager 1 has to exaggerate to influence decision-making in his favor. The analysis of communication for Manager 2 is analogous.

4.2 Communication Equilibria

We can now build on our understanding of the managers' incentives to misrepresent their information to characterize the communication equilibria. We provide an informal description of these equilibria in this section and relegate the formal characterization to the Appendix.

FIGURE

As in Crawford and Sobel (1982), all communication equilibria are interval equilibria in which the state spaces are partitioned into intervals and the division managers only reveal which interval their local conditions a_1 and a_2 belong to. In this sense the managers' communication is noisy and information is lost. Consider Manager 1's communication strategies as illustrated in Figure 1. The states space $[\mu_1 - s, \mu_1 + s]$ is then partitioned into $N \geq 1$ intervals that are small close to $a_1 = \mu_2$ and grow larger as $|a_1 - \mu_2|$ increases. Moreover, the stronger the incentives to misrepresent information, as characterized by (4), the faster the intervals grow as $|a_1 - \mu_2|$ increases and, hence, the more noisy is communication. To see this, let k_i and k_{i+1} denote the endpoints of $i + 1's$ interval and suppose that this interval lies to the right of μ_2. It can be shown that the size of this interval $(k_{i+1} - k_i)$ is equal to the length of the preceding interval $(k_i - k_{i-1})$ plus $4(b_1/b_2)(k_i - \mu_2)$.

There are many equilibria of the communication game. Indeed, any number of partitions can be sustained in equilibrium. For the remainder of the paper we focus on the equilibrium for which the number of partitions goes to infinity.

LEMMA 1. *The most efficient communication equilibrium under Centralization is the one in which the number of partitions goes to infinity. For this equilibrium the residual variances are given by*

$$V_j^C \equiv \mathrm{E}\left[(a_j - \mathrm{E}[a_j|m_j])^2\right] = \left(\sigma^2 + (\mu_1 - \mu_2)^2\right)\frac{b_j/b_i}{3 + 4b_j/b_i},$$

where $i, j = 1, 2$ and $i \neq j$.

Proof: The proof of this proposition is analogous to those of Lemma 1 and Proposition 2 in Alonso, Dessein and Matouschek (forthcoming). Details are available from the authors upon request. ■

The residual variance is therefore directly related to the division managers' incentives to misrepresent information as defined in (4). In particular, as b_1/b_2 increases, less information is communicated in equilibrium by Manager 1 to headquarters and the residual variance regarding a_1 under Centralization increases. At the same time, as b_1/b_2 increases, more information is communicated by Manager 2. Since no informative communication takes place under decentralization, the residual variances are respectively given by $V_1^{D_1} = 0$ and $V_2^{D_1} = \sigma^2$ under Decentralization-1 and $V_1^{D_2} = \sigma^2$ and $V_2^{D_2} = 0$ under Decentralization-2.

5 Organizational Performance

Now that we have analyzed decision-making and communication, we can work out the expected profits for each organizational structure.

LEMMA 2 *Let Π^* denote first best expected profits. Then, expected profits are given by*

$$\Pi^l = \Pi^* - A^l V_1^l - B^l V_2^l - C^l \left((\mu_1 - \mu_2)^2 + 2\sigma^2 \right) \tag{5}$$

for $l = C$, D_1, D_2, where A^l, B^l and C^l are functions of b_1, b_2 and λ only and are defined in the proof.

We can use this proposition to compare the relative performance of the different organizational structures. For this purpose, it is useful to decompose the benefit $\Pi^{D_1} - \Pi^C$ of delegating the pricing decision to Manager 1.[5] The first component is the *gain in local information* which captures the increase in expected profits that is due to the pricing decision being based on better information about the demand conditions in Market 1. Formally, the gain in local information is given by $A^C V_1^C \geq 0$. The second component is the *loss of non-local information.* This component captures the reduction in expected profits that is due to the pricing decision being based on worse information about the demand conditions in Market 2. Formally, the loss in non-local information is given by $B^C \left(\sigma^2 - V_2^C \right) \geq 0$. The third and final component is the *loss of control* which is due to biased decision-making by Manager 1 and is given by $2C^{D_1}\sigma^2 - \left(B^C - B^{D_1} \right) \sigma^2 \geq 0$. With this decomposition in hand, we now turn to a pairwise comparison of the different organizational structures.

[5] The analysis for Manager 2 is analogous.

Decentralization-1 versus Decentralization-2: Suppose first that the slopes of the inverse demand functions are the same in both markets, that is, $b_1/b_2 = 1$. In this case headquarters is indifferent between delegating to Manager 1 and delegating to Manager 2. Note that this is the case even if one market is on average much larger than the other, that is, even if there are large differences between μ_1 and μ_2. To understand this, recall from our discussion of the profit function (1) that the reduction in profits in Market $j = 1, 2$ from setting the price different from the price that maximizes profits in Market j is independent of a_j and only depends on b_j. When $b_1 = b_2$ it is therefore as important to adapt the price to the conditions in a small market as to those in a larger market.

While the two decentralized structures perform equally well when $b_1/b_2 = 1$, headquarters is better off delegating to Manager 1 than delegating to Manager 2 whenever $b_1/b_2 < 1$, that is, whenever the inverse demand function in Market 1 is flatter than that in Market 2. This too can be understood by recalling our discussion of the profit function (1) which showed that the cost of not adapting prices to the demand conditions in Market $j = 1, 2$ is decreasing in b_j. Thus, when $b_1/b_2 < 1$ it is more important to adapt the pricing decisions to the conditions in Market 1 than to those in Market 2.

PROPOSITION 1. *Delegating pricing to Manager 1 is strictly preferred over delegating pricing to Manager 2 if and only if $b_1 < b_2$, regardless of μ_1 or μ_2.*[6]

Centralization versus Decentralization-1: To compare the centralized with the decentralized structure, it is useful to define

$$R \equiv \left((\mu_1 - \mu_2)^2 + \sigma^2\right) / \left((\mu_1 - \mu_2)^2 + 2\sigma^2\right)$$

Inspection of (5) shows that the relative performance of the different organizational structures depend on μ_1, μ_2 and σ^2 only through R.

Suppose first that $\mu_1 = \mu_2$ in which case $R = 1/2$. Note that in this case the relative performance of the different organizational structures depend only on the relative slopes b_1/b_2 and not on the variance σ^2. In particular, Decentralization-1 is then optimal whenever b_1/b_2 is sufficiently small. In contrast, Centralization is optimal if the flatness in demand is relatively similar in both markets. Intuitively, delegating control to Manager 1 results in a gain of local information (regarding Market 1), but a loss in non-local information (regarding Market 2) and a loss of control (as Manager 1 is biased). For $b_1/b_2 = 1$, the gain in local information equals the loss in non-local information. The additional loss of control under delegation then implies that Centralization is op-

[6] Recall that we assume that $\mu_i - s \geq 0$ and $(\mu_i - \mu_j)^2 \leq s^2$, which puts restrictions how small each market can be and on differences in demand. Proposition 1 further assumes that it is always optimal to sell in both markets.

timal. When b_1/b_2 becomes smaller, however, this loss of control decreases as there is less discrepancy between the price preferred by headquarters and that by Manager 1. Similarly, as b_1/b_2 becomes smaller, the gain in local information increases relative to the loss in non-local information. Whenever $\dot{b}_1/b_2$ is sufficiently small, decentralization is then optimal.

Suppose next that $\mu_1 \neq \mu_2$. The term $(\mu_1 - \mu_2)^2$ can then be seen as a *measure of conflict* between division managers regarding the preferred price p. From Lemma 1, increased conflict between division managers reduces the quality of communication under Centralization. As such, it both increases the gain of local information and it reduces the loss in non-local information associated with delegating to Manager 1, two factors favoring delegation. On the downside, increased conflict also leads to more biased decision-making by Manager, which is reflected in an increase in the loss of control. As the next proposition shows, the increase in the gain of local information and reduction in the loss of non-local information dominates, so that more conflict always makes it weakly more attractive to delegate to Manager 1 :

PROPOSITION 2. *Delegating pricing to Manger* 1 *is strictly preferred over centralized price setting if and only if* $b_1/b_2 < \beta$, *where* $\beta \in (0, 1)$ *is increasing in* R *and thus also in* $(\mu_1 - \mu_2)^2$.

6 Conclusion

This paper links organizational design to market demand. When markets are characterized by a flatter demand curve, getting the price "right" is more important. When local managers are better informed about local demand conditions, delegating pricing to the manager who faces the flattest demand may then be optimal, even if his market has less expected sales. Increased conflict between managers also makes decentralized pricing more likely. Intuitively, more conflict makes communication with headquarters more noisy, and hence central decision-making less effective. In our set-up, the need for coordination is absolute: only one price can be charged. In future work, we aim to relax this assumption by analyzing price coordination between substitute or complement products.

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7 Appendix

A communication equilibrium under Centralization is characterized by communication rules for the division managers and by a decision rule and belief functions for Manager HQ. The communication rule for Manager $j = 1, 2$ specifies the probability of sending message $m_j \in M_j$ conditional on observing state a_j and we denote it by $\mu_j (m_j \mid a_j)$. The decision rules map messages $m_1 \in M_1$ and $m_2 \in M_2$ into prices and we denote it by by $p(m)$. Finally, the belief functions are denoted by $g_j (a_j \mid m_j)$ for $j = 1, 2$ and state the probability of state a_j conditional on observing message m_j. We focus on Perfect Bayesian Equilibria of the communication subgame which require that (i.) communication rules are optimal for the division managers given the decision rule, (ii.) the decision rule is optimal for Manager HQ given the belief functions and (iii.) the belief functions are derived from the communication rules using Bayes' rule whenever possible.

PROPOSITION A1 (Communication Equilibria). *For every positive integers* $\underline{N}_j$ and $\overline{N}_j$, $j = 1, 2$, *there exists at least one equilibrium* $(\mu_1(\cdot),\ \mu_2(\cdot),\ p(\cdot),\ g_1(\cdot),\ g_2(\cdot))$, *where*

i. $\mu_j(m_j \mid a_j)$ *is uniform, supported on* $[k_{j,i-1}, k_{j,i}]$ *if* $a_j \in (k_{j,i-1}, k_{j,i})$,

ii. $g_j(a_j \mid m_j)$ *is uniform supported on* $[k_{j,i-1}, k_{j,i}]$ *if* $m_j \in (k_{j,i-1}, k_{j,i})$,

iii.a. $k_{1,i+1} - k_{1,i} = k_{1,i} - k_{1,i-1} + 4b_1/b_2 \left(k_{1,i} - \mu_2\right)$ *for* $i = 1, ..., \ \overline{N}_1 - 1$

$k_{1,-i} - k_{1,-(i+1)} = k_{1,-(i-1)} - k_{1,-i} + 4b_1/b_2 \left(\mu_2 - k_{1,i}\right)$ *for* $i = 1, ..., \underline{N}_1 - 1$,

iii.b. $k_{2,i+1} - k_{2,i} = k_{2,i} - k_{2,i-1} + 4b_2/b_1 \left(k_{2,i} - \mu_1\right)$ *for* $i = 1, ..., \ \overline{N}_2 - 1$

$k_{2,-i} - k_{2,-(i+1)} = k_{2,-(i-1)} - k_{2,-i} + 4b_2/b_1 \left(\mu_1 - k_{2,i}\right)$ *for* $i = 1, ..., \underline{N}_2 - 1$

iii.c. $k_{1,-\underline{N}_1} = k_{2,-\underline{N}_2} = \mu - s$, $k_{1,\overline{N}_1} = k_{2,\overline{N}_2} = \mu + s$, $k_{1,0} = \mu_2$ and $k_{2,0} = \mu_1$.

iv. $p(m) = p^H$, *where* p^H *is given by (2).*

Moreover, all other finite equilibria have relationships between a_1 *and* a_2 *and Manager HQ's choice of* p *that are the same as those in this class for some value of* $\underline{N}_1$, $\underline{N}_2$, $\overline{N}_1$ *and* $\overline{N}_2$; *they are therefore economically equivalent.*

Proof: The proof of this proposition is analogous to that of Proposition 1 in Alonso, Dessein and Matouschek (forthcoming). Details are available from the authors upon request. ■

Proof of Lemma 2: Substituting $p = p^{Dj}$, $j = 1, 2$, into (1) and taking expectations gives

$$\Pi^{D_j} = \Pi^* - \frac{b_j}{4b_i \left(b_1 + b_2\right)} \left(\left(\mu_1 - \mu_2\right)^2 + 2\sigma^2 \right) \text{ for } i = 1, 2,\ i \neq j.$$

Thus, $A^{D_j} = B^{D_j} = 0$ and $C^{D_j} = b_j / \left[4b_i \left(b_1 + b_2\right)\right]$.

Similarly, substituting $p = p^H$ into (1) and taking expectations gives, after some cumbersome manipulations,

$$\Pi^H = \Pi^* - \frac{b_2}{4b_1 \left(b_1 + b_2\right)} V_1^C - \frac{b_1}{4b_2 \left(b_1 + b_2\right)} V_2^C.$$

Thus $A^C = b_2 / \left[4b_1 \left(b_1 + b_2\right)\right]$, $B^C = b_1 / \left[4b_2 \left(b_1 + b_2\right)\right]$ and $C^C = 0$. ■

Proof of Proposition 1: From Lemma 2 it follows that

$$\Pi^{D_1} - \Pi^{D_2} = \frac{b_2 - b_1}{4b_1 b_2} \left(\left(\mu_1 - \mu_2\right)^2 + 2\sigma^2 \right). \blacksquare$$

Proof of Proposition 2: From Lemma 2 it follows that

$$\Pi^H - \Pi^{D_1} = \frac{\left(\mu_1 - \mu_2\right)^2 + 2\sigma^2}{4\left(b_1 + b_2\right)} \left[\left(b_1/b_2\right) - 2R \frac{3\left(b_1/b_2\right) + 2\left(b_1/b_2\right)^2 + 2}{\left(3\left(b_1/b_2\right) + 4\right)\left(4\left(b_1/b_2\right) + 3\right)} \right].$$

The term in squared brackets is negative at $b_1/b_2 = 0$, positive at $b_1/b_2 = 1$ and strictly increasing in b_1/b_2. Thus, there exists a unique $\beta \in (0,1)$ such that $\Pi^H - \Pi^{D_1} < 0$ if $b_1/b_2 < \beta$, $\Pi^H - \Pi^{D_1} = 0$ if $b_1/b_2 = \beta$ and $\Pi^H - \Pi^{D_1} > 0$ if $b_1/b_2 > \beta$. Moreover, it is easily shown that β is increasing in R. ■