

# Stability of Information-Sharing Alliances in a Three-Level Supply Chain

Greys Sošić

*Marshall School of Business, University of Southern California, Los Angeles, California*  
*sosic@marshall.usc.edu*

January 2010

## **Abstract**

In their recent paper, Leng and Parlar (L&P) (2009) analyze information-sharing alliances in a three-level supply chain (consisting of a manufacturer, a distributor, and a retailer) that faces a nonstationary end demand. Supply chain members can share demand information, which reduces information distortion and thus decreases their inventory holding and shortage costs. We expand the results from L&P by considering dynamic (farsighted) stability concepts. We use two different allocation rules and show that under some reasonable assumptions there should always be some information sharing in this supply chain. We also identify conditions under which the retailer in a stable outcome shares his demand information with the distributor, with the manufacturer, or with both remaining supply chain members.

# 1. Introduction

It is well established in the operations literature that lack of information sharing among supply chain members leads to supply chain inefficiencies. Lee et al. [19] describe a demand distortion that occurs in a supply chain when demand variability amplifies along a supply chain, from the retailers to the upstream supply chain members. This phenomenon is usually referred to as the bullwhip effect, and it can lead to significant problems for the upstream supply chain members, such as inaccurate forecasting, poor capacity utilization, poor customer service, etc. Illustrations of these problems, and attempts to resolve them through information sharing, can be found in several well-known Harvard Business School cases (see, e.g., Clark [9], Hammond [14]). A number of research papers have studied the effects that information sharing has on supply chain members, and the impact different parameters have on the benefits achieved in a supply chain. We briefly review some of this work in the next section.

In their recent paper, Leng and Parlar (L&P) [21] consider a multi-period model of a three-level supply chain, that consists of a manufacturer, a distributor, and a retailer. The retailer in period  $t$  faces customer demand that follows the one-period autoregressive model, and can be described by the equation  $D_t = d + \rho D_{t-1} + \varepsilon_t$ , where  $d > 0$  denotes the average demand,  $0 < \rho \leq 1$  is the autocorrelation coefficient, and  $\varepsilon_t$  are i.i.d. error terms normally distributed with mean zero and variance  $\sigma^2$ . L&P consider the savings that various supply chain members can achieve as a result of cooperation, reflected through information sharing. Demand information shared by a coalition is the demand data faced by the downstream member in the coalition. The base case considers holding and shortage costs incurred by each party in the supply chain in which no information is shared, and the model is then compared with four different information-sharing scenarios. In three cases, the retailer is the one sharing the demand information, while the partner(s) may be the manufacturer, the distributor, or both. In the fourth case, the distributor shares information about her<sup>1</sup> demand pattern with the manufacturer, while the retailer does not cooperate with either of them.

L&P assume that the manufacturer is the most upstream member, so his production quantity can be determined by his own production schedule, and his expected cost can be computed in closed form. However, the distributor's and the retailer's orders may not be completely fulfilled in some periods. While L&P find the expressions for their expected profits under the 100%-fill-rate assumption, they also conduct extensive numerical experiments which show that this assumption is often violated in practice. Thus, they use simulation to find the expected costs incurred by the retailer and the distributor in different information-sharing coalitions. Note that the downstream supply chain member, who shares his or her information with the upstream member(s), observes only indirect savings (through fewer stockouts at his or her immediate supplier). Consequently,

---

<sup>1</sup>Note that hereinafter manufacturers and retailers will be considered male, while distributors will be considered female.

some of the savings from the parties who benefit from information sharing have to be passed to the parties who reveal their information, in order to incentivize their cooperation. L&P consider several solution concepts and identify situations under which different information-sharing outcomes may be stable.

In this paper, we extend their result by investigating dynamically (farsighted) stable outcomes under the Shapley value allocation and the constrained nucleolus. While L&P show that the Shapley value often falls outside the core (and thus leads to myopic instability of the grand coalition), we provide conditions (which hold for examples from L&P) under which the grand coalition is always stable if the players are farsighted (that is, when players commit to a certain course of action, they take into account how the others may respond to their moves). In addition, while L&P show that information sharing between the retailer and the distributor is myopically stable only for very low values of  $\rho$ , we show that this outcome is stable for farsighted players under mild conditions and for arbitrary values of the autocorrelation coefficient. We also provide conditions under which the retailer shares his information with the manufacturer in a stable outcome.

The plan of the paper is as follows: the next section provides a literature review, while Section 3 introduces the L&P model. In Section 4, we define two farsighted stability concepts, the largest consistent set (LCS) and the equilibrium process of coalition formation (EPCF), and we identify stable outcomes for our model in Section 5. Some concluding remarks are given in Section 6, and the proofs are in the appendix.

## 2. Literature Review

Several research papers have studied the value of information sharing in a supply chain. Among them, Gavirneni et al. [10] analyze a two-level supply chain with a single manufacturer and a single retailer. They evaluate the benefit of obtaining additional information about the retailer's inventory level and compare the effects of different demand distributions. Cachon and Fisher [4] show the benefits that the manufacturer can obtain when he uses information about the retailer's inventory levels in a model with a single supplier and  $n$  identical retailers. Both of these papers assume demand processes that are independent and identically distributed over time. Chen et al. [6] analyze the effects of the bullwhip effect with and without information sharing in a two-level supply chain in which the retailer faces customer demand that is autocorrelated. Thus, the underlying demand process can be represented with a one-period autoregressive model, AR(1). Chen et al. [7] extend the analysis to multiple-level supply chains and show that the bullwhip effect can be reduced (but not completely eliminated) in the presence of information sharing. A similar model of customer demand is used by Lee et al. [20], who analyze the impact of the autocorrelation coefficient and the lead time on the benefits from information sharing in a two-stage supply chain. Simchi-Levi and Zhao [30] analyze the value of information sharing in a finite-horizon model of a single manufacturer–single retailer supply chain, in which the retailer faces i.i.d. demands and the

manufacturer has a finite production capacity. They show that the sharing of demand information leads to a reduction of inventory cost for the manufacturer, while maintaining the same service level to the retailer. Simchi-Levi and Zhao [31] extend the analysis to the infinite-horizon model, in which they consider both the discounted and the average cost criteria.

All of the models discussed above use a noncooperative framework, and hence the supply chain members make their decisions unilaterally and incur their individual costs. Thus, it appears that the operations management (OM) literature so far has paid limited attention to the cooperative setting, in which cooperating parties allocate savings generated through information sharing among themselves. For a good review of the use of game theory in supply chain analysis, see Cachon and Netessine [5], and for the use of a cooperative game framework in supply chains, see Nagarajan and Sošić [26]. Application of cooperative games in OM papers is usually limited to the use of allocations that satisfy the core property (see, for instance, Anupindi et al. [1], Hartman et al. [15]). Although allocations that belong to the core discourage players' deviations from the grand coalition, because such a defection does not increase their allocations, the core can, in general, be empty. In addition, a nonempty core may contain a large number of allocations, so one may need a procedure to select a unique core member. On the other hand, the Shapley value is an allocation rule that always exists and is uniquely determined. Moreover, it satisfies a number of monotonicity criteria, which makes it appealing in a cost and/or profit allocation framework. Recently, some authors have used the Shapley value as a solution concept in supply chain management models (Granot and Sošić [12], Kemahlioglu Ziya and Bartholdi [17], Plambeck and Taylor [27], Sošić [33]).

In the area of vertical information sharing, papers that use a cooperative game framework include Raghunathan [28] and L&P [21]. Raghunathan [28] considers a two-level supply chain, in which he extends the model developed in Lee et al. [20] so that one supplier sells to an arbitrary number of retailers. He assumes that the savings are allocated among the supply chain members according to the Shapley value, and shows that the demand correlation affects the allocations of the manufacturer and the retailers in different directions.

L&P [21] build on the model developed in [20] and analyze the allocation of savings from information sharing among the members of a three-level supply chain. They provide analytical expressions for the expected holding and shortage costs incurred by the manufacturer, and they use simulation to find expected costs of the distributor and the retailer. They propose the analysis of the problem by using a cooperative game in coalitional form, and use the constrained core as the stability concept. L&P calculate the expected savings for each supply chain member in all possible information-sharing alliances, and derive the necessary conditions for stability of different coalitions. They provide unique allocation schemes for stability of two-player coalitions, and provide a condition under which the constrained core of the game is nonempty. They show that allocating the savings among supply chain members according to the Shapley value does not result in a core allocation, while the use of the constrained nucleolus yields a stable outcome. In a numerical study,

the authors provide sensitivity analyses that study the impact of the autocorrelation coefficient, the unit shortage and holding costs, and different allocation schemes.

As mentioned above, L&P [21] use the constrained core to study stability of different information-sharing alliances. One possible criticism to this concept is its myopia; that is, it assumes that the players do not consider possible reactions of their supply chain partners when they commit to a certain course of action. However, it is likely in real-life settings that there will be changes in coalition structures (through defections and regrouping) before some sort of stability is attained. We believe that incorporating a notion of stability that captures some of these dynamics is of theoretical importance and practical value. Toward this end, we propose in this paper two different concepts: the farsighted stability concept, resulting in the largest consistent set (LCS), as proposed by Chwe [8]; and the equilibrium process of coalition formation (EPCF), proposed by Konishi and Ray (K&R) [18]. Using the above framework, we are interested in analyzing what the stable information-sharing alliances would be if supply chain members are allowed to freely form information-sharing coalitions and are farsighted.

### 3. The Leng and Parlar (2009) Model

We now briefly describe the model used in [21].

#### 3.1 Ordering process

Before the end of period  $t$ , after demand  $D_t$  has been realized, the retailer observes his inventory level and places an order with the distributor. He receives his order at the beginning of period  $t+1$  and can use it to fulfill demand in that period. All excess demand is backlogged, and the retailer incurs a penalty when he is not able to meet demand. At the end of time period  $t$ , the distributor receives an order from the retailer, and ships to him either the ordered amount or the available inventory on hand. The distributor uses the size of the received order to forecast the next period's demand and, before the end of period  $t$ , places an order with the manufacturer. The distributor receives her order at the beginning of period  $t+1$ , and can use it to fulfill the order received from the retailer at the end of that period. Finally, at the end of period  $t$ , the manufacturer receives an order from the distributor and ships either the ordered amount or the available inventory on hand. The manufacturer uses the distributor's order to forecast the next period's demand. Because he produces the final product, it is assumed that his production quantity is fully realized.

#### 3.2 Costs in different information-sharing alliances

In the analysis of supply chain costs, L&P consider inventory holding and shortage costs incurred at every stage of the supply chain, when there is no fixed order-placing cost and unit inventory holding and shortage costs are stationary over time.  $h^i, p^i, i = M, D, R$ , denote the unit holding

and shortage costs for the manufacturer (M), the distributor (D), and the retailer (R), respectively. When excess demand can be met from alternative sources, shortage cost may represent the cost of expediting the shortage, using alternative sources, etc.; when excess demand cannot be met, shortage cost may represent the cost of lost goodwill, the opportunity cost of lost profit, etc. The goal of each supply chain member is to minimize his or her own cost. L&P consider the impact that the sharing of demand information with upstream supply chain members has on cost reduction.

In order to identify different information-sharing outcomes, let us briefly introduce some terminology from game theory. We denote by  $N = \{1, 2, \dots, n\}$  the set of all players in a game. A subset  $Z \subseteq N$  is called a *coalition*, and  $N$  is called the *grand coalition*. A *coalition structure*,  $\mathcal{Z}$ , is a partition on  $N$ , and  $\mathbf{Z}$  is the set of all coalition structures. We now apply this terminology to our game.

The retailer can share his demand information with either one of the remaining two supply chain members, or with both of them, while the distributor can share her information only with the manufacturer. We identify these information-sharing agreements with corresponding alliances: if there is no information sharing and each party acts independently, we denote the resulting coalition structure by  $I$ ; if  $i$  shares information with  $j$ , where  $i \in \{D, R\}, j \in \{D, M\}, i \neq j$ , the resulting coalition structure is denoted by  $ji$ ; if the retailer shares his demand information with both remaining players, the resulting coalition structure is the grand coalition,  $G$ .

For the manufacturer, the expected cost in the different coalition structures is given by

$$C_M^{\mathcal{Z}} = \sigma [h^M k^M + (h^M + p^M)L(k^M)] \begin{cases} \sqrt{(1 + \rho)^2 [(1 + \rho)^2 + 4\rho^2] + \rho^4}, & \text{for } \mathcal{Z} = I, \\ \sqrt{(1 + \rho + \rho^2)^2 + \rho^2(1 + \rho^2)}, & \text{for } \mathcal{Z} = DR, \\ (1 + \rho)\sqrt{(1 + \rho)^2 + \rho^2}, & \text{for } \mathcal{Z} = MD, \\ (1 + \rho)^2, & \text{for } \mathcal{Z} = MR, \\ (1 + \rho + \rho^2), & \text{for } \mathcal{Z} = G, \end{cases}$$

where  $k^i = \Phi^{-1}(p^i/(p^i + h^i)), i = M, D, R$ ,  $\Phi$  is the standard normal distribution function with zero mean and unit variance, and  $L(z)$  denotes the normal loss function,  $L(z) = \int_z^\infty (x - z)d\Phi(z)$ . Proposition 1 in L&P shows that

$$C_M^I > C_M^{MD} > C_M^{MR} > C_M^{DR} > C_M^G. \quad (1)$$

If we assume a 100% fill rate (that is, each order placed by the retailer or by the distributor is completely fulfilled), the supply chain members who share information with their upstream partners do not observe any savings as a result of that cooperation. Thus, the retailer's cost does not change in any of the coalition structures, while for the distributor we have

$$\tilde{C}_D^{\mathcal{Z}} = \sigma [h^D k^D + (h^D + s^D)L(k^D)] \begin{cases} \sqrt{(1 + \rho)^2 + \rho^2}, & \text{for } \mathcal{Z} \in \{I, MR, MD\}; \\ (1 + \rho), & \text{for } \mathcal{Z} \in \{DR, G\}, \end{cases}$$

where we use  $\tilde{C}$  to denote the cost under the 100%-fill-rate assumption. It is easy to verify that the cost observed by the distributor when she does not receive any information from the retailer strictly exceeds the cost she incurs after information sharing. Thus, we can denote the distributor's expected savings from information sharing by

$$\Delta\tilde{C}^D = \sigma [h^D k^D + (h^D + s^D)L(k^D)] \left[ \sqrt{(1 + \rho)^2 + \rho^2} - (1 + \rho) \right].$$

Note that the distributor's cost does not change when she shares information with the manufacturer.

However, the distributor's and the retailer's orders may not be completely fulfilled by the manufacturer and the distributor, respectively. Under the more realistic assumption of less than 100% fill rates, the model becomes rather complex, and the resulting expressions become too intractable to analyze. As a result, L&P use simulation to estimate expected costs of the retailer and the distributor.

### 3.3 Cooperative game framework

As noted above, supply chain members who share information with their upstream partners do not observe any direct savings as a result of that cooperation (only indirect savings through fewer stockouts at their suppliers when fill rates are less than 100%), while the costs of their partners decrease. Consequently, in order to induce information sharing within the supply chain, members with reduced costs have to allocate some of the savings to their partners, and hence the players' payoffs will, in general, differ from the savings that they observe. These allocations will have an impact on the coalitions that will be formed, and on possible defections from coalitions. We model this in the framework of game theory.

A pair  $(N, \nu)$ , where  $N$  is the set of players and  $\nu : 2^N \rightarrow \mathbb{R}$ ,  $\nu(\emptyset) = 0$ , is called a *cooperative game*, and  $\nu$  is called the characteristic function. A mapping  $\Phi$  that assigns to every game a vector  $\varphi = (\varphi_1, \dots, \varphi_n) \in \mathbb{R}^N$  is called an *allocation rule*, and  $\varphi$  is called an *allocation*. An allocation  $\varphi$  is a member of the *core* (see [11]) of  $(N, \nu)$  if it satisfies  $\sum_{i \in Z} \varphi_i \geq \nu(Z) \forall Z \subseteq N$ , and  $\sum_{i=1}^n \varphi_i(\nu) = \nu(N)$ . The core possesses a myopic stability property—under core allocations no subset of players has an incentive to secede from the grand coalition and form its own coalition.

We define a marginal contribution of player  $i$  with respect to a particular ordering of players as its marginal worth to the coalition formed by the players before him in the order. Thus, if  $1, 2, \dots, i-1$  are the players preceding  $i$  in the given ordering, then  $i$ 's marginal contribution is  $\nu(\{1, 2, \dots, i-1, i\}) - \nu(\{1, 2, \dots, i-1\})$ . The *Shapley value* (see [29]) is an allocation rule obtained by averaging the marginal contributions for all possible orderings,

$$\xi_i(\nu) = \sum_{\{Z: i \in Z\}} \frac{(|Z| - 1)!(n - |Z|)!}{n!} (\nu(Z) - \nu(Z \setminus \{i\})).$$

We now define two allocations in the context of our game. If we denote by  $\zeta^Z$  total savings

realized by members of coalition  $Z$ , then

$$\zeta^Z = \sum_{i \in Z} (C_i^I - C_i^Z).$$

Let us denote by  $\xi_i^Z, i \in \{S, M, R\}$ ,  $i$ 's payoff based on the Shapley value in coalition structure  $Z$ . When there is no information sharing, there are no savings, so each supply chain member in  $I$  receives zero payoff,  $\xi_i^I = 0, i \in \{S, M, R\}$ . Next, suppose that two supply chain members share information. Then, the player who does not participate in information sharing keeps his savings for himself. More formally,

$$\xi_i^{ij} = \zeta_i^{ij}/2, \quad \xi_k^{ij} = C_k^I - C_k^{ij}, \quad i \neq j \neq k.$$

Finally, with three players,

$$\xi_i^G = \frac{\zeta^G}{3} + \frac{\zeta^{ij} - \zeta^{jk}}{6} + \frac{\zeta^{ik} - \zeta^{jk}}{6}, \quad i, j, k \in \{M, D, R\}, \quad i \neq j \neq k. \quad (2)$$

Next, denote by  $\psi$  the allocation rule that assigns to each grand coalition member an amount described by the expression for the constrained nucleolus when the constrained core is empty, introduced in Proposition 5 of L&P (hereinafter referred to only as the *constrained nucleolus*). Then,

$$\psi_i^G = \frac{\zeta^G + \zeta^{ij} + \zeta^{ik} - 2\zeta^{jk}}{3}, \quad i, j, k \in \{M, D, R\}, \quad i \neq j \neq k, \quad (3)$$

while  $\psi_i^Z = \xi_i^Z, Z \neq G$ . We note here that  $\psi$  is used by L&P only when the constrained core is empty,<sup>2</sup> while we impose no such condition.

L&P define a stable coalition as one from which players have no incentive to leave (which defines a myopic concept of stability), and they provide conditions (Proposition 2) under which each of the five possible coalition structures may be stable. In their numerical analysis, L&P show that the Shapley value usually does not belong to the core (see, for instance, Table 4). To induce stability of the grand coalition, it is often necessary to use LP to calculate allocations. The resulting allocation still does not assure that the grand coalition remains stable if the players consider how the others may react to their moves. In addition, L&P show that a two-player coalition between the retailer and the distributor exhibits stability only for very low values of the autocorrelation coefficient ( $\rho < 0.02$ ). In what follows, we discuss farsighted stability concepts, and show that the Shapley value and the constrained nucleolus induce stability of  $G$  and/or  $DR$  under some reasonable conditions for arbitrary values of  $\rho$ .

### 3.4 Model Limitations

Before proceeding with our analysis, we note here some assumptions made in both the L&P model and our model and the resulting limitations.

---

<sup>2</sup>When the constrained core is non-empty, L&P use an LP to find the constrained nucleolus.

- First, both models assume that firms adopt myopic ordering policies. In general, a myopic ordering policy may not always be feasible; that is, the starting inventory in a given period may be above the myopic order-up-to level. There have been several papers analyzing dependent demand models that provided conditions under which myopic policies are optimal for the underlying models. Johnson and Thompson [16] provide sufficient conditions for an inventory model with lost sales and an autoregressive moving average (ARMA) demand process to have a myopic optimal policy. Their assumptions are rather simple—that demand is always positive, and that demand can be no more than four standard deviations from the mean (which is usually true for a normal distribution)—and rather reasonable. Sobel [32] analyzes conditions for a Markov decision process to have a myopic optimum and for a stochastic game to possess a myopic equilibrium point. The principal conditions are that (1) each single period reward is the sum of terms due to the current state and action, (2) each transition probability depends on the action taken but not on the state from which the transition occurs, and (3) an appropriate static optimum (or equilibrium point) is ad infinitum repeatable. Lovejoy [22] provides an additional set of assumptions required for a myopic optimal policy in a rather general inventory model. Similarly to many other authors (see, for instance, Lee et al. [20]), we assume that standard deviation is significantly smaller than mean demand and that a myopic policy can be used. Note that the problem of having the starting inventory higher than the myopic order-up-to level can also be dealt with by assuming that excess inventory can be returned without cost.
- Second, both models assume that the lead time between the manufacturer and the distributor and the lead time between the distributor and the retailer are one period. We acknowledge that previous work with two-level supply chains often considers longer lead times, and shows that longer lead times increase the benefits from information sharing. However, the main goal of our analysis is not to quantify the savings from information savings, but to identify stable outcomes. Because arbitrary long lead times significantly complicate our expressions and related analysis, we do not extend our model to arbitrary long lead times. We note, however, that one of our results shows that it is unlikely that no information is exchanged among supply chain members; we can, therefore, conjecture that longer lead times, which make information sharing more valuable, make formation of information-sharing coalitions even more likely.
- Third, when we assume a 100% fill rate, we are able to express conditions for stability of various outcomes in terms of primitives of the model (shortage and holding costs). However, when this assumption is relaxed and a fill rate of less than 100% is allowed, both papers express conditions for stability of different information-sharing alliances in terms of intermediate results (relationships among costs incurred in different alliances). As stated by L&P, while it may be possible to formulate our models with the assumption of less than 100% fill rates, the resulting expressions become too intractable to analyze. Thus, we use simulations instead of

analytical expressions to estimate expected costs of the retailer and the distributor, and we use relationships between these costs to identify conditions for stability.

- Fourth, we assume that all supply chain members have equal bargaining power. Clearly, this assumption may not be realistic in many scenarios, and it may have an impact on resulting stable outcomes. In particular, when the stable outcome is not unique and theoretical results do not identify a single outcome as “favorite,” it is likely that the outcome preferred by the strongest member will actually be realized.
- Finally, we assume that the players can join and leave coalitions at will, and that all coalitions are legally possible. While it may happen in real life that alliances of some firms are considered illegal collusion and are prohibited by government, we make no such assumption—players in our model only consider their monetary benefits when determining whether to join or to leave a coalition. Recall that alliance membership in our case implies information sharing among alliance members; clearly, there are many situations in which this type of alliances is possible and legal, as witnessed by information sharing along different stages of numerous supply chains.

## 4. Farsighted stability concepts

We assume that all supply chain members can communicate and can join or leave alliances freely. Thus, they may consider both unilateral and joint deviations from any coalition structure. Deviations of this type are allowed in the *strong Nash equilibrium* (see [2]), the *core*, and the *coalition structure core* (see [3]). All of these solutions are myopic in the sense that they only look at one-step effects of players’ moves. Consequently, they may not be appropriate for many real-life situations. For instance, consider an arbitrary coalition structure,  $\mathcal{Z}$ , and suppose that a set of players,  $Z$ , can increase their payoffs by deviating and forming a different coalition structure. For a myopic view of stability, this would make  $\mathcal{Z}$  unstable. However, we should consider possible further defections from the initial deviation. Another coalition may decide to deviate from the current status quo, which may benefit a possibly different set of players. In fact, any defection may trigger a sequence of further moves and eventually culminate with an outcome in which some players that initially deviated, members of  $Z$ , receive a lower payoff than the one they obtained in  $\mathcal{Z}$ . Under such a scenario, farsighted players may prefer not to move in the first place, and thus  $\mathcal{Z}$ , which initially appeared unstable, may actually prove to be stable. A solution concept that allows players to consider further deviations by arbitrary coalitions is *the largest consistent set*, which was introduced by Chwe (see [8]). We define it below and use it as a stability criterion in our analysis.

## 4.1 The largest consistent set (LCS)

Let us denote by  $\prec_i$  the players' strong preference relations, described as follows: for  $\mathcal{Z}_1, \mathcal{Z}_2 \in \mathbf{Z}$ ,

$$\mathcal{Z}_1 \prec_i \mathcal{Z}_2 \iff \varphi_i^{\mathcal{Z}_1} < \varphi_i^{\mathcal{Z}_2},$$

where  $\varphi_i^{\mathcal{Z}}$  is a player  $i$ 's share of savings in the coalition structure  $\mathcal{Z}$ , and

$$\mathcal{Z}_1 \prec_{S_0} \mathcal{Z}_2 \iff \mathcal{Z}_1 \prec_i \mathcal{Z}_2 \quad \forall i \in S_0.$$

For a given coalition  $S_0$ , let  $F_{S_0}(\mathcal{Z})$  denote the set of coalition structures achievable by a one-step coalitional move by  $S_0$  from  $\mathcal{Z}$ . Denote by  $\rightarrow_{S_0}$  the following relation:

$$\mathcal{Z}_1 \rightarrow_{S_0} \mathcal{Z}_2 \implies \mathcal{Z}_2 \in F_{S_0}(\mathcal{Z}_1).$$

We say that  $\mathcal{Z}_1$  is *directly dominated* by  $\mathcal{Z}_2$ ,  $\mathcal{Z}_1 < \mathcal{Z}_2$ , if

$$\exists S_0 \text{ such that } \mathcal{Z}_2 \in F_{S_0}(\mathcal{Z}_1) \text{ and } \mathcal{Z}_1 \prec_{S_0} \mathcal{Z}_2.$$

We say that  $\mathcal{Z}_1$  is *indirectly dominated* by  $\mathcal{Z}_m$ ,  $\mathcal{Z}_1 \ll \mathcal{Z}_m$ , if

$$\exists \mathcal{Z}_1, \mathcal{Z}_2, \dots, \mathcal{Z}_m \text{ and } S_1, S_2, \dots, S_{m-1} \text{ such that } \mathcal{Z}_{i+1} \in F_{S_i}(\mathcal{Z}_i) \text{ and } \mathcal{Z}_i \prec_{S_i} \mathcal{Z}_m \text{ for } i = 1, 2, \dots, m-1.$$

A set  $Y$  is called *consistent* if the following holds:

$$\mathcal{Z} \in Y \iff \forall S_0, \forall \mathcal{V} \in F_{S_0}(\mathcal{Z}), \exists \mathcal{B} \in Y, \text{ such that } \mathcal{V} = \mathcal{B} \text{ or } \mathcal{V} \ll \mathcal{B} \text{ and } \mathcal{Z} \not\prec_{S_0} \mathcal{B}.$$

Chwe proves the existence, uniqueness, and non-emptiness of the largest consistent set (LCS). Because every coalition considers the possibility that, once it reacts, another coalition may react, and then yet another, and so on, the LCS incorporates *farsighted* coalitional stability. The LCS describes all possible stable outcomes and has the merit of "ruling out with confidence": if  $\mathcal{Z}$  is not contained in the LCS,  $\mathcal{Z}$  cannot be stable. For a more detailed analysis of farsighted stability, see [8]. Some applications of stability analysis using the LCS criterion include [13], [23], [24], [25] and [33].

## 4.2 Equilibrium process of coalition formation (EPCF)

While Chwe establishes the existence of an LCS, a criticism of this solution concept is that it may be too inclusive. Konishi and Ray [18] propose an alternate dynamic approach to stability of coalition structures, which they call the *equilibrium process of coalition formation*, and they establish a possible link between the LCS and the limit states of absorbing deterministic EPCFs.

For each player, let  $\delta_i$  denote his discount factor. Then,  $i$ 's profit from a sequence of coalition structures  $\{\mathcal{Z}_t\}$  may be written as  $\sum_{t=0}^{\infty} \delta_i^t \varphi_i^{\mathcal{Z}_t}$ . Clearly,  $\delta \rightarrow 0$  will correspond to myopic solution concepts, and being concerned with farsightedness, we are more interested in values of  $\delta$  close to one.

A process of coalition formation (PCF) is a transition probability  $p : \mathbf{Z} \times \mathbf{Z} \rightarrow [0, 1]$ , so that  $\sum_{\mathcal{V} \in \mathbf{Z}} p(\mathcal{Z}, \mathcal{V}) = 1 \quad \forall \mathcal{Z} \in \mathbf{Z}$ . A PCF  $p$  induces a value function  $v_i$  for each player  $i$ , which represents the infinite-horizon payoff to a player starting from any coalition structure  $\mathcal{Z}$  under the Markov process  $p$ , and is the unique solution to the equation

$$v_i(\mathcal{Z}, p) = \varphi_i^{\mathcal{Z}} + \delta_i \sum_{\mathcal{V} \in \mathbf{Z}} p(\mathcal{Z}, \mathcal{V}) v_i(\mathcal{V}, p).$$

We say that  $S_0$  has a *strictly profitable* move from  $\mathcal{Z}$  under  $p$  if there is  $\mathcal{V} \in F_{S_0}(\mathcal{Z}), \mathcal{V} \neq \mathcal{Z}$ , such that  $v_i(\mathcal{V}, p) > v_i(\mathcal{Z}, p) \quad \forall i \in S_0$ . A move  $\mathcal{V}$  is called *efficient* for  $S_0$  if there is no other move  $\mathcal{W}$  for  $S_0$  such that  $v_i(\mathcal{W}, p) > v_i(\mathcal{V}, p) \quad \forall i \in S_0$ . A PCF is an equilibrium PCF (EPCF) if:

1. Whenever  $p(\mathcal{Z}, \mathcal{V}) > 0, \mathcal{V} \neq \mathcal{Z}$ , then  $\exists S_0$  such that  $\mathcal{V}$  is a profitable and efficient move for  $S_0$  from  $\mathcal{Z}$ , and
2. If there is a strictly profitable move from  $\mathcal{Z}$ , then  $p(\mathcal{Z}, \mathcal{Z}) = 0$  and there is a strictly profitable and efficient move  $\mathcal{V}$  with  $p(\mathcal{Z}, \mathcal{V}) > 0$ .

Thus, a deviation from one coalition structure to another occurs only if all members of the deviating coalition agree to move and they cannot find a strictly better alternative. In addition, the deviation from a coalition structure must occur if there is a strictly profitable move. Notice that this definition does not require that every strictly profitable move has a positive probability. Konishi and Ray [18] show that there is an equilibrium process of coalition formation.

If  $p(\mathcal{Z}, \mathcal{V}) \in \{0, 1\} \quad \forall \mathcal{Z}, \mathcal{V}$ , a PCF is called deterministic.  $\mathcal{Z} \in \mathbf{Z}$  is said to be absorbing if  $p(\mathcal{Z}, \mathcal{Z}) = 1$ , while a PCF  $p$  is absorbing if,  $\forall \mathcal{V} \in \mathbf{Z}$ , there is some absorbing  $\mathcal{Z} \in \mathbf{Z}$  such that  $p^{(k)}(\mathcal{Z}, \mathcal{V}) > 0$ , where  $p^{(k)}$  denotes the  $k$ -step transition probability. Konishi and Ray demonstrate that, when  $\delta$ 's are large enough, the set of all absorbing states under all deterministic EPCFs is a subset of the LCS. In addition, under the LCS different coalitions may possess different conjectures about what will happen at a given state. If all players have common beliefs (as they must in an EPCF), this possibility cannot arise. This is one reason why the set of all absorbing states (under all deterministic absorbing EPCFs) is typically a strict subset of the LCS. Some applications of stability analysis by using the EPCF include [23], [24], [25], and [33].

### 4.3 The LCS and the EPCF

When players form coalitions, they reach some kind of an agreement. One can think of a contract that is signed by players binding them to their affiliation. Even if there are no legal obstacles, defections and re-formations are costly. The LCS assumes that the players receive their payoffs once a stable outcome is reached—players take into account all possible defections by fellow players before agreeing on an outcome. Thus, if the  $n$  players are involved in some kind of a negotiation process that will eventually determine a mutually agreeable outcome (a coalition structure), the

LCS takes into account the possible scenarios of threats and counter-threats involving defections and counter-defections that the players will consider before signing a binding agreement.

The EPCF takes a completely different approach—it allows players to defect from any state and receive a state-dependent payoff. Thus, at any stage, the coalition outcomes are non-binding. Individual players in a status quo coalition structure can calculate the expected discounted rewards if the coalition structures evolved along any sample path. The absorbing states of the EPCF—the stable outcomes—can be interpreted as coalition structures from which no further movements are likely. It is interesting to note that, though the two approaches are completely different, the absorbing states of the EPCF may provide a refinement of the LCS.

Our main motivation for using these two very different approaches is to check the robustness of our results and provide some sort of validation for the predictions of stable outcomes. In addition, EPCF helps in removing the “inefficient” outcomes from the set of stable coalition structures. For example, consider a game with four players in which the players’ allocations in different coalition structures are given as follows: each player in the grand coalition,  $G$ , receives a payoff of 5; if a coalition of three players is formed,  $\{i, (jkl)\}$ , then each coalition member receives a payoff of 3, while the independent player,  $i$ , receives a payoff of 7 units; if two two-player coalitions are formed,  $\{(ij), (kl)\}$ , then each payer receives a payoff of 4 units; and each player in any of the remaining coalition structures receives a payoff of 1 unit. Note that all players strictly prefer  $G$  to coalition structures of the form  $\{(ij), (kl)\}$ , but each outcome  $\{(ij), (kl)\}$  nevertheless belongs to the LCS. Note that a possible joint move of all players to the grand coalition, which would make all of them better off, is deterred by a possible subsequent move by a player, say  $i$ :

$$\{(ij), (kl)\} \rightarrow_{\{i,j,k,l\}} G \rightarrow_{\{i\}} \{i, (jkl)\},$$

because  $\{i, (jkl)\}$  also belongs to the LCS and makes players  $j, k$ , and  $l$  worse off (they receive 4 each in  $\{(ij), (kl)\}$ , while only 3 each in  $\{i, (jkl)\}$ ). Thus,  $j, k$ , and  $l$  would not make the initial move. However,  $\{(ij), (kl)\}$  cannot be an absorbing state of the EPCF when  $\delta$  is large enough, because we cannot find a profitable and efficient move from the grand coalition that eventually leads to  $\{(ij), (kl)\}$ .

## 5. Stable Outcomes

As mentioned earlier, allocation of savings among the coalition partners has an impact on their willingness to join a particular information-sharing coalition. Players receive highest payoffs in different information-sharing agreements. Thus, while one player prefers the grand coalition, another may prefer a two-player alliance. We next investigate which outcomes may be considered stable—that is, when information-sharing members would have no incentive to defect. We assume that all defections take place before information sharing occurs. Players make their decisions with a cognizance that their actions are likely to start a series of possible reactions by others, and they

therefore approach the stability problem from a farsighted perspective. The questions we address are the following:

1. If we look at the alliance stability from a farsighted perspective, is the information sharing among all supply chain members a stable outcome under the allocation rules defined above?
2. What other information-sharing alliances may be stable in a farsighted sense?

In our analysis, we consider the case with the assumption of a 100% fill rate, as well as the case in which this assumption is relaxed, and we compare our results.

## 5.1 100% fill rate

Let us first consider the costs that supply chain members incur under a 100%-fill-rate assumption. As discussed earlier, L&P show (see Appendix A) that the retailer observes the same cost in all coalition structures, while for the distributor's cost  $\tilde{C}_D^G = \tilde{C}_D^{DR} < \tilde{C}_D^{MD} = \tilde{C}_D^{MR} = \tilde{C}_D^I$ . The savings in different coalition structures are given by

$$\begin{aligned}\tilde{\zeta}^{MR} &= C_M^I - C_M^{MR}, & \tilde{\zeta}^{MD} &= C_M^I - C_M^{MD}, \\ \tilde{\zeta}^{DR} &= \Delta\tilde{C}_D, & \tilde{\zeta}^G &= \Delta\tilde{C}_D + C_M^I - C_M^G.\end{aligned}$$

### 5.1.1 Shapley value

By using expression (2), we can calculate players' allocations for all different outcomes; we present them in Table 1.

Coalition structure $\mathcal{Z}$	$\tilde{\xi}_M^{\mathcal{Z}}$	$\tilde{\xi}_D^{\mathcal{Z}}$	$\tilde{\xi}_R^{\mathcal{Z}}$
$I$	0	0	0
$MR$	$\frac{C_M^I - C_M^{MR}}{2}$	0	$\frac{C_M^I - C_M^{MR}}{2}$
$DR$	$C_M^I - C_M^{DR}$	$\frac{\Delta\tilde{C}_D}{2}$	$\frac{\Delta\tilde{C}_D}{2}$
$MD$	$\frac{C_M^I - C_M^{MD}}{2}$	$\frac{C_M^I - C_M^{MD}}{2}$	0
$G$	$\frac{2C_M^I}{3} - \frac{C_M^{MR} + C_M^{MD} + 2C_M^G}{6}$	$\frac{\Delta\tilde{C}_D}{2} + \frac{C_M^I - C_M^{MD}}{6} + \frac{C_M^{MR} - C_M^G}{3}$	$\frac{\Delta\tilde{C}_D}{2} + \frac{C_M^I - C_M^{MR}}{6} + \frac{C_M^{MD} - C_M^G}{3}$

Table 1: Shapley Allocations in Different Coalition Structures

It is easy to verify that for the manufacturer's Shapley allocations in different coalition structures,  $\tilde{\xi}_M^{\mathcal{Z}}$ , the following inequalities hold:

$$\tilde{\xi}_M^I < \tilde{\xi}_M^{MD} < \tilde{\xi}_M^{MR} < \tilde{\xi}_M^G < \tilde{\xi}_M^{DR}. \quad (4)$$

Note that (1) implies that the manufacturer's individual savings are the highest in the grand coalition. However, when only the distributor and the retailer form a coalition, the manufacturer keeps all the savings for himself, while in the grand coalition he has to share his savings with the retailer and the distributor to induce their participation.

While the manufacturer's ordering of payoffs does not depend upon the value of  $\rho$ , this is not always the case for the remaining two players. Let  $i, j \in \{D, R\}, i \neq j$ . Then, regardless of the value of  $\rho$ , it holds that

$$\tilde{\xi}_i^I = \tilde{\xi}_i^{Mj} = 0 < \min \left\{ \tilde{\xi}_i^{Mi}, \tilde{\xi}_i^{DR} \right\}, \quad \text{and} \quad \tilde{\xi}_i^{DR} < \tilde{\xi}_i^G, \quad (5)$$

while we can have

$$\tilde{\xi}_i^{DR} \leq \tilde{\xi}_i^{Mi} \text{ or } \tilde{\xi}_i^{DR} \geq \tilde{\xi}_i^{Mi}, \quad \text{and} \quad \tilde{\xi}_i^G \leq \tilde{\xi}_i^{Mi} \text{ or } \tilde{\xi}_i^G \geq \tilde{\xi}_i^{Mi}.$$

Let us denote

$$\Gamma = \frac{h^D k^D + (h^D + p^D)L(k^D)}{h^M k^M + (h^M + p^M)L(k^M)}. \quad (6)$$

In most supply chains, it is reasonable to assume that the distributor faces higher stockout and holding costs than the supplier; that is,  $\Gamma \geq 1$ . When this is true, we can evaluate that  $\tilde{\xi}_i^{Mi} < \tilde{\xi}_i^G$ . Thus, the autocorrelation coefficient only has an impact on  $i$ 's preferences for  $DR$  over  $Mi$ —in determining when the information sharing with only the manufacturer is better than information sharing with the third player.

Taking all of these different preferences into account, we investigate stable outcomes under the Shapley allocations. Our analysis is summarized in the following result.

**Theorem 1** *Suppose that the members in a three-level supply chain with 100% fill rate agree to distribute their savings from information sharing according to the Shapley value. Then, whenever  $\Gamma \geq 1$ , the only stable outcomes are achieved if the retailer shares his demand information with both the distributor and the manufacturer ( $G$ ), or only with the distributor ( $DR$ ).*

Thus, under the assumption that the savings are allocated according to the Shapley value, we have shown that with 100% fill rate the grand coalition always satisfies stability criteria in a farsighted sense. While it is known that information sharing always benefits supply chain members, it follows from the above that there will, indeed, always be some information sharing in a stable outcome (that is,  $I$  is never stable). In addition, the manufacturer never receives demand information from only one other supply chain member. We also show that, regardless of the value of the autocorrelation coefficient or supply chain members' unit costs, it is always stable for the retailer to share his demand information with the distributor, or with both the distributor and the manufacturer. It is interesting to note, as discussed in the proof of Theorem 1, that  $MR$  may be in the LCS even when all players individually prefer the grand coalition to  $MR$ , but the EPCF removes such an inefficient outcome from the set of stable coalition structures.

### 5.1.2 Constrained nucleolus

Next, we consider the constrained nucleolus. By using expression (3), we can calculate players' allocations for all different outcomes; we present them in Table 2.

Coalition structure $\mathcal{Z}$	$\tilde{\psi}_M^{\mathcal{Z}}$	$\tilde{\psi}_D^{\mathcal{Z}}$	$\tilde{\psi}_R^{\mathcal{Z}}$
$I$	0	0	0
$MR$	$\frac{C_M^I - C_M^{MR}}{2}$	0	$\frac{C_M^I - C_M^{MR}}{2}$
$DR$	$C_M^I - C_M^{DR}$	$\frac{\Delta\tilde{C}_D}{2}$	$\frac{\Delta\tilde{C}_D}{2}$
$MD$	$\frac{C_M^I - C_M^{MD}}{2}$	$\frac{C_M^I - C_M^{MD}}{2}$	0
$G$	$\frac{3C_M^I - C_M^G - C_M^{MR} - C_M^{MD} - \Delta\tilde{C}_D}{3}$	$\frac{2\Delta\tilde{C}_D + 2C_M^{MR} - C_M^G - C_M^{MD}}{3}$	$\frac{2\Delta\tilde{C}_D + 2C_M^{MD} - C_M^G - C_M^{MR}}{3}$

Table 2: Constrained Nucleolus in Different Coalition Structures

The ordering of the manufacturer's preferences in this case is not as straightforward as under the Shapley value. As the manufacturer's payoffs in two-player alliances are the same as under the Shapley value, we have  $\tilde{\psi}_M^I < \tilde{\psi}_M^{MD} < \tilde{\psi}_M^{MR} < \tilde{\psi}_M^{DR}$ , but the relationships between his preference for the grand coalition and two-player coalitions have to be investigated further because they involve the relationships between the savings observed by the manufacturer and the savings observed by the distributor, and hence the relationships between the costs at the two levels have to be taken into account. Recall that  $\Gamma$  is defined by equation (6). We can evaluate that for  $\Gamma \leq 12$  (which seems like a reasonable assumption), the ordering is given by

$$\tilde{\psi}_M^I < \tilde{\psi}_M^{MD} < \tilde{\psi}_M^{MR} < \tilde{\psi}_M^G < \tilde{\psi}_M^{DR}; \quad (7)$$

hence, the manufacturer's preferences are the same as under the Shapley allocations. In addition, for  $i, j \in \{D, R\}, i \neq j$ , we have

$$\tilde{\psi}_i^I = \tilde{\psi}_i^{Mj} = 0 < \min \left\{ \tilde{\psi}_i^{Mi}, \tilde{\psi}_i^{DR} \right\}, \quad \text{and} \quad \tilde{\psi}_i^{DR} < \tilde{\psi}_i^G, \quad (8)$$

while we can have

$$\tilde{\psi}_i^{DR} \leq \tilde{\psi}_i^{Mi} \text{ or } \tilde{\psi}_i^{DR} \geq \tilde{\psi}_i^{Mi}, \quad \text{and} \quad \tilde{\psi}_i^{Mi} \leq \tilde{\psi}_i^G \text{ or } \tilde{\psi}_i^{Mi} \geq \tilde{\psi}_i^G.$$

Thus, the preferences of supply chain members for different outcomes when  $\Gamma \leq 12$  are the same as under the Shapley allocation, except for the possibility of having  $\tilde{\psi}_i^{Mi} \geq \tilde{\psi}_i^G$ . Because  $MD$  is preferred by  $MR$  for both the retailer and the manufacturer, we can show that  $MD$  cannot be stable. However, when  $\tilde{\psi}_R^{MR} \geq \tilde{\psi}_R^G$ , which can happen for large values of  $\rho$  if  $\Gamma \leq 1.5$ ,<sup>3</sup> it can be shown that  $MR$  becomes stable. As a result, we state the following result without a proof.

<sup>3</sup>If  $\Gamma = 1$ , it happens for  $\rho \geq 0.84$ .

**Theorem 2** *Suppose that the members in a three-level supply chain with a 100% fill rate agree to distribute their savings from information sharing according to the constrained nucleolus. Then, whenever  $1.5 < \Gamma \leq 12$ , the only stable outcomes are achieved if the retailer shares his demand information with both the distributor and the manufacturer ( $G$ ), or only with the distributor ( $DR$ ). When  $\Gamma \leq 1.5$ ,  $G$  and  $DR$  are stable, and  $MR$  becomes stable for large values of  $\rho$ .*

As shown in the above analysis, in a model with a 100% fill rate in which  $1 \leq \Gamma \leq 12$ , the outcomes in which the retailer shares the information with the distributor or with both remaining supply chain members are stable regardless of the allocation rule used to distribute the savings (the Shapley value or the constrained nucleolus). In addition, under the constrained nucleolus, information sharing between the retailer and the manufacturer may become stable for large values of  $\rho$  if the costs of the manufacturer and the distributor do not differ too much ( $\Gamma \leq 1.5$ ), while it is never stable under the Shapley allocations..

## 5.2 Fill rate less than 100%

Now, if we relax the 100%-fill-rate assumption, the cost for the retailer and the distributor may change in different coalition structures (depending upon the stockouts). L&P provide two examples in which they look at the different values of the autocorrelation coefficient. Despite a significant change in  $\rho$ , in both examples

$$\xi_M^I < \min\{\xi_M^{MD}, \xi_M^{MR}\} < \max\{\xi_M^{MD}, \xi_M^{MR}\} < \xi_M^G < \xi_M^{DR},$$

and

$$\xi_i^I < \xi_i^{Mj} < \xi_i^{Mi} < \xi_i^{DR} < \xi_i^G, i \neq j \neq M.$$

Therefore, many players' preferences carried over when the assumption of the 100% fill rate was relaxed. In particular, the preferences for the manufacturer are almost identical, while the distributor and the retailer both prefer  $G$  to  $DR$ , and do not like to be left out of information-sharing coalitions. We now further investigate the conditions for different players' preferences when the 100%-fill-rate assumption does not hold.

### 5.2.1 Shapley value

Clearly, as  $I$  is the base case,  $\xi_i^I = 0, i = R, D, M$ . It is likely that each supply chain member's cost is the highest if there is no information sharing, as improved information leads to a better match of supply and demand, and thus lower holding and shortage costs. This is equivalent to

$$C_i^Z < C_i^I, \quad i = R, D, M, \quad Z \neq I. \quad (9)$$

Note that (9) implies  $\zeta^Z > 0, Z \neq I$ . We acknowledge that information sharing implies some additional cost for the supply chain. However, if these costs exceed the benefits, it is likely that

information sharing will not occur at all. Thus, we think it is reasonable to concentrate on cases in which (9) holds.

Let us first consider the manufacturer. Assuming that (9) holds, for  $i \in \{D, R\}$  we have

$$\xi_M^{Mi} = \zeta^{Mi}/2 > 0 = \xi_M^I.$$

Next, for  $i \neq j \neq M$ ,

$$\xi_M^G - \xi_M^{Mi} = \frac{2\zeta^G - 2\zeta^{Mi} + \zeta^{Mj} - 2\zeta^{DR}}{6},$$

and hence  $\xi_M^G > \xi_M^{Mi}$  holds when

$$\zeta^G + \frac{\zeta^{Mj}}{2} > \zeta^{Mi} + \zeta^{DR}. \quad (10)$$

In addition,

$$\xi_M^{DR} - \xi_M^G = \frac{6(C_M^I - C_M^{DR}) - 2\zeta^G - \zeta^{MR} - \zeta^{MD} + 2\zeta^{DR}}{6},$$

and hence  $\xi_M^{DR} > \xi_M^G$  when

$$6(C_M^I - C_M^{DR}) + 2\zeta^{DR} > 2\zeta^G + \zeta^{MR} + \zeta^{MD}. \quad (11)$$

Next, we look at the distributor's and the retailer's preferences. Assuming that (9) holds, we have

$$\xi_i^{DR} = \zeta^{DR}/2 > 0 = \xi_i^I, i \neq j \neq M.$$

In addition,

$$\xi_j^G - \xi_j^{DR} = \frac{2\zeta^G - 2\zeta^{Mi} + \zeta^{Mj} - 2\zeta^{DR}}{6}, \quad \xi_j^G - \xi_j^{Mj} = \frac{2\zeta^G - 2\zeta^{Mj} + \zeta^{DR} - 2\zeta^{Mi}}{6},$$

and hence for  $i \neq j \neq M$ ,  $\xi_j^G > \xi_j^{DR}$  when (10) holds, and  $\xi_j^G > \xi_j^{Mj}$  holds when

$$\zeta^G + \frac{\zeta^{DR}}{2} > \zeta^{Mj} + \zeta^{Mi}. \quad (12)$$

Note that (10) and (12) hold for all players (that is,  $\xi_i^G > \max\{\xi_i^{ij}, \xi_i^{ik}\}$ ,  $i \neq j \neq k$ ) if

$$\zeta^G + \frac{\zeta^{ij}}{2} > \zeta^{ik} + \zeta^{jk}, \quad (13)$$

and that (11) holds if

$$C_M^I - C_M^{DR} > \frac{\zeta^G}{3}, \quad \text{and} \quad (14a)$$

$$\zeta^{DR} > \frac{\zeta^{MR} + \zeta^{MD}}{2}. \quad (14b)$$

We now discuss conditions (13) and (14).

Inequality (13) implies that the total savings generated by an all-inclusive information sharing,  $\zeta^G$ , together with the average coalition-member saving in an arbitrary two-member information-sharing coalition,  $\zeta^{ij}/2$ , exceed the sum of coalition savings for the remaining two two-member information-sharing coalitions,  $\zeta^{ik} + \zeta^{jk}$ . Clearly, an all-inclusive information sharing should lead to the best match of supply and demand and maximize supply chain savings. In both examples from L&P, these savings alone exceed the sum of savings generated by any pair of two-member information-sharing coalitions ( $\zeta^G > \zeta^{ij} + \zeta^{ik}, i \neq j \neq k$ ). In condition (13),  $\zeta^G$  is augmented by the average member saving in a two-member coalition,  $\zeta^{ij}/2$ . Therefore, on both sides of the inequality we have four average coalition-member savings—on the left-hand side, three of those savings belong to the all-inclusive information sharing ( $3 \cdot \zeta^G/3 + \zeta^{ij}/2$ ), while all of the savings on the right-hand side are incurred in two-member coalitions ( $2 \cdot \zeta^{ij}/2 + 2 \cdot \zeta^{jk}/2$ ). Thus, we believe that it is reasonable to assume that conditions (13) hold in many real-life cases. Observe that whenever the grand coalition generates the highest average coalition-member savings ( $\zeta^G/3 > \zeta^{ij}/2$ ), at least one of the conditions described by (13) holds (that is, at least one of the supply chain members prefers to be in the grand coalition rather than in a two-player information-sharing alliance with another player). Finally, we note that (13) holds under the 100%-fill-rate assumption whenever  $\Gamma \geq 1$ .

Next, we consider inequalities (14). Both of these conditions seem quite reasonable: the manufacturer can observe a smoother demand (reduced bullwhip effect) when the retailer shares information with the distributor, and reduction in his cost,  $C_M^I - C_M^{DR}$ , can easily exceed the average cost reduction stemming from an all-inclusive information sharing,  $\zeta^G/3$ . In addition, as the retailer possesses the most accurate demand information, it is likely that the savings of a coalition in which he shares his demand information with the distributor,  $\zeta^{DR}$ , exceed the average coalition savings when the manufacturer receives information from one of the remaining players,  $(\zeta^{MR} + \zeta^{MD})/2$ . While it can happen that  $\zeta^{MR} > \zeta^{DR}$ , it is also likely that lower savings in the coalition without the retailer's information,  $\zeta^{MD}$ , can offset any possible advantages of  $MR$  over  $DR$ . It is also worthwhile to notice that (14) holds for both examples in L&P, and it is our belief that it will hold in many realistic situations. If we now consider the 100% fill rate, we note that (14a) holds whenever  $\Gamma \geq 1$ , while (14b) holds only for large values of  $\Gamma$ . However, recall that the ordering of the manufacturer's preferences requires a weaker condition, (11), and it can be shown that this condition is satisfied under the 100%-fill-rate assumption whenever  $\Gamma \geq 1$ .<sup>4</sup>

We can conclude that whenever inequalities (9), (13), and (14) hold, the manufacturer's preferences are similar to the ones obtained in the case with a 100% fill rate, given by (4):

$$\xi_M^I < \min\{\xi_M^{MD}, \xi_M^{MR}\} < \max\{\xi_M^{MD}, \xi_M^{MR}\} < \xi_M^G < \xi_M^{DR}. \quad (15)$$

---

<sup>4</sup>Under the less-than-100%-fill-rate assumption, the party that shares information observes cost reduction through fewer stockouts, while it sees no savings under the 100%-fill-rate assumption. Thus, the benefits of coalition  $DR$  when we do not assume perfect fill rate are higher, and the condition (14b) is more likely to hold in this case.

In addition, whenever (9) and (13) hold, the distributor's and the retailer's preferences are similar to the ones obtained in the case with a 100% fill rate:

$$\xi_i^I = 0 < \xi_i^{DR} < \xi_i^G, \quad \xi_i^I = 0 < \xi_i^{Mi} < \xi_i^G, \quad i \in \{D, R\}. \quad (16)$$

As mentioned earlier, when the manufacturer receives demand information from both supply chain partners, he has to share with them part of the savings, while by staying independent the manufacturer can keep all of them for himself. Thus, whenever (14) holds, the manufacturer may always increase his payoff by defecting from the grand coalition and leaving the retailer to share information with the distributor ( $\xi_M^G < \xi_M^{DR}$ ). This implies myopic instability of information sharing among all supply chain members under the Shapley allocations. It is not clear, however, whether the two remaining supply chain members will decide to share information or not, because one defection may trigger a sequence of further deviations. Our analysis leads to the following theorem, which is one of the main results of our paper.

**Theorem 3** *Suppose that conditions (9), (13), and (14) hold when the fill rate is less than 100%, and the savings are distributed according to the Shapley value, (2). Then, the grand coalition and DR are the only outcomes stable in the farsighted sense.*

Thus, if conditions (9), (13), and (14) hold, the stability results are equivalent to the ones obtained under the 100%-fill-rate assumption. We note that the stability of both the grand coalition and DR is obtained by using two very different concepts, the LCS and the EPCF, which contributes to the robustness of our results. Observe also that conditions in Proposition 2 of L&P imply that myopic stability of the grand coalition implies instability of two-member alliances (and vice versa), which, as was shown above, is not the case when the players are farsighted and the conditions from Theorem 3 hold.

While we think that conditions (13) and (14) (which imply (10), (11), and (12)) are rather reasonable and will hold in most realistic situations, we also analyze what happens if they are not satisfied.

**Proposition 1** *Suppose that (9) holds, and the savings are distributed according to the Shapley value, (2).*

- *If (11) does not hold and (10) holds for  $i = R, D$ , then the grand coalition is stable, while DR is not. If, in addition, (12) does not hold and  $\zeta^{Mj} > \max\{\zeta^{DR}, \zeta^{Mi}\}$ , then  $Mj$  is stable, while  $Mi$  is not.*
- *If (10) does not hold for  $i = R, D$  and (11) holds, then DR is stable, while the grand coalition is not. If  $\zeta^G/3 > \zeta^{ij}/2$ , then MR and MD cannot be stable.*

Thus, when the assumption of the 100% fill rate is relaxed, there may be instances in which either  $G$  or  $DR$  is stable, but not both of them. For  $DR$  to be the only stable outcome, we would need the grand coalition to provide little additional savings as compared with the two-player alliances, while  $DR$  will not be stable if it generates low savings compared with both remaining two-player alliances and/or low savings to the manufacturer due to a smoother demand pattern.

Similarly to the case with 100% fill rate, it is unlikely that the manufacturer is better off if he receives information from the distributor instead of the retailer, and hence we can expect  $\zeta^{MD} < \zeta^{MR}$ , which makes the stability of  $MD$  unlikely. Recall that  $MR$  is never stable under the 100%-fill-rate assumption, while it can be stable in this case if it yields the highest savings among all of the two-player alliances.

Although in Example 1 from L&P the Shapley value does not satisfy conditions for myopic stability (although the distributor and the retailer prefer  $G$  to  $DR$ , the manufacturer does not, and hence the grand coalition is not stable when the players do not consider the reactions of their partners), we show that farsighted players using  $\xi$  do not want to defect from an all-inclusive information-sharing alliance. In addition, although  $DR$  is not myopically stable in Example 1, we show that a distributor and the retailer who enter an information-sharing agreement do not want to secede if they take into account future consequences of their moves.

To get better insights into applicability of our results, we use the data from Example 1 in L&P ( $\rho = 0.5, (p_r, p_d, p_m) = (5, 3, 2), (h_r, h_d, h_m) = (2, 1.5, 1)$ ) as a base case and conduct an extensive numerical analysis by changing the values of different parameters (similarly to the analysis conducted in §5 of L&P). Namely, we use the following ranges: because we assumed that  $\rho$  is nonnegative, we let  $\rho \in [0.01, 0.9]$ ; because shortage penalty cost usually exceeds holding cost, we let  $p_r \in [2, 6], p_d \in [1.5, 5.5], p_m \in [1, 5.5]$  and  $h_r \in [0.5, 5], h_d \in [0.5, 2.75], h_m \in [0.2, 2]$ . Our results confirmed many of our assumptions:

- Most supply chain members see the highest cost in models without any information sharing; that is, (9) holds in almost all of the cases that we analyzed. The sole exceptions in which (9) was not satisfied were:
  - the cases with  $\rho \leq 0.2$ , in which the retailer's cost in  $MR$  exceeds his cost in  $I$ . However, the total coalition cost decreased,  $\zeta^{MR} > 0$ . In general, the coalition total costs decrease in all cases,  $\zeta^Z > 0 \forall Z$ , except when  $\rho = 0.01$  (when only  $MR$  leads to overall coalition savings,  $\zeta^{MR} > 0$ ); and
  - the cases with  $h_m \leq 0.4$ , in which the distributor's cost in  $DR$  exceeds her cost in  $I$ . When  $h_m = 0.2$ , the entire coalition cost increases,  $\zeta^{DR} < 0$ .
- Inequality (13) holds in all examples, except when  $\rho = 0.01$ .
- Inequalities (14) are usually not satisfied for the extreme parameter values (very high or very low). However, as mentioned earlier, our results require only a weaker condition, (11), which holds in all cases except when  $p_m = 1$ .

Thus, our analysis indicates that Theorem 3, which implies stability of the grand coalition and alliance  $DR$ , holds in almost all of the cases analyzed. We list the notable exceptions:

- When the values of  $\rho$  are small, the value of information sharing decreases; while the retailer's individual costs may increase in  $DR$ , we still have  $\zeta^{DR} > 0$  for reasonable values of  $\rho$ , and both  $G$  and  $DR$  remain stable. However, when  $\rho$  approaches zero, we have  $MR$  as the only stable outcome.
- When the value of  $h_m$  is small, coalition  $DR$  may cease to be stable, with the result that the grand coalition becomes the only stable outcome. A similar result holds for small values of  $p_m$ . This is consistent with our previous observation, that  $DR$  is not stable if it brings low savings to the manufacturer.

While L&P show that  $DR$  is myopically stable only for very low values of the correlation coefficient, we show that with farsighted players this outcome is more likely to be stable for arbitrary values of  $\rho$  (except when  $\rho$  approaches zero). We also note that in both examples from L&P inequality  $\zeta^{DR} > \max\{\zeta^{Mj}, \zeta^{Mi}\}$  holds, and hence  $MR$  is not stable. However, we found that it may be stable when  $\rho$  is very low.

### 5.2.2 Constrained nucleolus

Next, we assume that the savings are distributed according to the allocation rule  $\psi$ . Similarly as above, we can show that  $\psi_M^G < \psi_M^{DR}$  when

$$3(C_M^I - C_M^{DR}) + 2\zeta^{DR} > \zeta^G + \zeta^{MR} + \zeta^{MD}, \quad (17)$$

while  $\psi_M^G > \psi_M^{Mi}$  when

$$\zeta^G + \zeta^{Mj} > \frac{\zeta^{Mi}}{2} + 2\zeta^{DR}, \quad i \neq j \neq M. \quad (18)$$

For the distributor and the retailer,  $\psi_i^G > \psi_i^{DR}$  holds when

$$\zeta^G + \zeta^{Mi} > \frac{\zeta^{DR}}{2} + 2\zeta^{Mj}, \quad i \neq j \neq M. \quad (19)$$

Once again, we can verify that conditions (17)-(19) hold for both examples in L&P. We now discuss these requirements further.

It is easy to verify that (17) holds whenever (14) is satisfied, so we can use arguments similar to the ones used in the previous section to justify why (17) is likely to hold. In addition, while condition (14b) in general does not hold under the 100%-fill-rate assumption (except when  $\Gamma > 6$  and  $\rho$  is high), recall that we need a weaker condition, (17), which is satisfied for arbitrary values of  $\Gamma$ .

Note that both sides of (18) and (19) consist of the average savings of five players. However, the left-hand side contains three players who belong to the coalition with the highest total savings

$(3 \cdot \zeta^G/3 + 2 \cdot \zeta^{Mi}/2)$ , while all members on the right-hand side belong to two-member information-sharing coalitions  $(4 \cdot \zeta^{DR}/2 + \zeta^{Mj}/2, \frac{\zeta^{DR}}{2} + 4 \cdot \zeta^{Mj}/2)$ , so it is reasonable to expect that these conditions are often satisfied (they hold for both examples in L&P). In addition, under the 100%-fill-rate assumption, conditions (18) and (19) hold whenever  $\Gamma \leq 12$ .

Theorem 1 in L&P states that the constrained core is non-empty when  $2\zeta^G > \zeta^{DR} + \zeta^{MD} + \zeta^{MR}$ . It is easy to verify that this condition is satisfied whenever (19) holds. We also note that  $\zeta^G/3 > \zeta^{ij}/2$  implies that (19) has to hold for at least one player; that is, the retailer and the distributor cannot both prefer  $DR$  to  $G$ .

Our first stability results for the constrained nucleolus are given in the following theorem (we omit the proof, as it follows steps similar to the ones described in the proof of Theorem 3).

**Theorem 4** *Suppose that conditions (9), (14), (18), and (19) hold, and that the savings are distributed according to the constrained nucleolus, (3). Then, the grand coalition and  $DR$  are always stable in the farsighted sense, while  $I$  is never stable.*

By comparing Theorems 3 and 4, we can conclude that conditions for the stability of the grand coalition and alliance  $DR$  resemble those used when savings were allocated based on the Shapley value. Thus, if conditions (9) and (14) hold and the savings in an all-inclusive information-sharing alliance are high enough, it is likely that the grand coalition and  $DR$  are stable regardless of the allocation rule used to allocate the savings. We note that allocations based on the constrained nucleolus imply that a two-member information-sharing alliance including the manufacturer may be stable along with  $DR$  and  $G$  (if the autocorrelation coefficient is sufficiently high), while this does not happen under Shapley allocations.

Although we assume that conditions from Theorem 4 are reasonable and likely to hold under most scenarios, we next consider the cases in which these conditions do not hold. We introduce the following inequality:

$$C_M^I - C_M^{DR} > \frac{\zeta^{Mi}}{2}, i \in \{D, R\}. \quad (20)$$

(20) implies that the manufacturer prefers  $DR$  to both remaining two-member coalitions. As mentioned before, when the retailer shares his information with the distributor, the manufacturer can keep all savings for himself, which is likely to exceed his share of savings in either two-member coalition.<sup>5</sup> Thus, it seems reasonable to expect that (20) will hold in most cases. We also want to consider the cases in which the retailer or the distributor prefers the grand coalition to a two-retailer alliance with the manufacturer,  $\psi_i^G > \psi_i^{Mi}, i \in \{D, R\}$ :

$$\zeta^G + \zeta^{DR} > \frac{\zeta^{Mj}}{2} + 2\zeta^{Mi}, \quad i \neq j \neq M. \quad (21)$$

---

<sup>5</sup>In both examples in L&P, we have an even stronger relationship,  $C_M^I - C_M^{DR} > \zeta^{Mi}$ .

**Proposition 2** *Suppose that conditions (9) and (20) hold and the savings are distributed according to the constrained nucleolus, (3).*

*If (14) holds, then*

- *DR is a stable outcome;*
- *if (19) is satisfied for both D and R, then the grand coalition is also stable;*
- *if (18) holds, then Mj cannot be stable; if (18) does not hold for R and D and  $\zeta^{Mj} > \max\{\zeta^{DR}, \zeta^{Mi}\}$ , then Mj is also stable, while Mi is not.*

*If (17) does not hold, then*

- *if  $\zeta^G/3 > \zeta^{ij}/2$ , G is a stable outcome;*
- *if (19) is satisfied for both D and R, then DR cannot be stable; if (19) is satisfied for D or R, then DR is stable;*
- *if (21) holds, then Mj cannot be stable; if (21) does not hold for R and D and  $\zeta^{Mj} > \max\{\zeta^{DR}, \zeta^{Mi}\}$ , then Mj is also stable, while Mi is not.*

As in the case with the Shapley allocations, when the assumption of the 100% fill rate is relaxed, there may be instances in which either  $G$  or  $DR$  is stable, but not both of them (the conditions are similar).

Once again,  $MR$  and  $MD$  cannot be stable at the same time, and by using the reasoning similar to that in the previous subsection, we can conclude that  $MR$  is more likely to be stable than  $MD$ . In addition,  $MR$  cannot be the only stable outcome (similarly to the case with the Shapley allocations), but when the savings are allocated according to the constrained nucleolus, we can have instances in which the grand coalition,  $DR$ , and  $MR$  can all be stable. Note that  $MR$  can be stable only if the savings of the coalition  $MR$  exceed the savings of both remaining two-member coalitions. If we analyze the model with a 100% fill rate, we can see that  $\zeta^{MR} > \max\{\zeta^{DR}, \zeta^{MD}\}$  happens under many realistic scenarios ( $\Gamma \leq 3$ ); exceptions are the cases in which the distributor faces costs much higher than the manufacturer. High distributor's cost seems to be a reasonable condition for instability of  $MR$  in the case in which the fill rate is lower than 100% as well—it implies that the savings that can be generated when the distributor receives demand information are high,  $\zeta^{DR} > \zeta^{MR}$ .

We again turn to numerical simulation to evaluate our results. Note that Theorems 3 and 4 use similar conditions and that while Theorem 3 requires (13) to hold, Theorem 4 uses conditions (18) and (19). We found that (18) and (19) hold in all of our examples, except when  $\rho = 0.01$ , which is the case when condition (13) is also not satisfied. Thus, our conclusions are similar to those obtained for the Shapley allocations.

## 6. Concluding Remarks

As shown above, when the players are farsighted, the grand coalition and/or  $DR$  are stable outcomes under rather mild conditions, regardless of the allocation rule used to allocate savings from information sharing, with or without the assumption of 100% fill rate. The choice of the allocation rule and the possibility of a less-than-100% fill rate can have an impact on the stability of the alliance  $MR$ —while it is never stable under a 100% fill rate and the Shapley allocations, it becomes stable when we allow for the possibility of less-than-perfect fill rates, or when the constrained nucleolus is used. Note that conditions for the stability of two-member alliances are weaker than the corresponding conditions in the myopic case, which require that total supply-chain savings in outcomes in which only two members share information exceed the savings in the all-inclusive information-sharing alliance and implied instability of the grand coalition (see Proposition 2 in L&P). In addition, when players are farsighted, two- and three-member alliances may be stable simultaneously.

We have shown that under most scenarios some information sharing will occur in a supply chain ( $I$  is never stable when the cost of information sharing is not too high), regardless of which of the two allocation rules,  $\xi$  or  $\psi$ , is used. In addition, although the grand coalition is likely to be myopically unstable under either of the two allocation rules, it is often stable when the players are farsighted. It is easy to evaluate that  $\xi_M^G - \psi_M^G = \frac{2\xi^{DR} - \xi^{MD} - \xi^{MR}}{6}$ , and hence whenever (14b) holds, the manufacturer prefers  $\xi$  to  $\psi$ . Thus, if the manufacturer is the strongest supply chain member (e.g., Intel), he may encourage his partners to adopt the Shapley value in their savings allocation.

As shown above, under some mild conditions, the retailer shares his demand information with one or both supply chain partners in a stable outcome. The stable coalition structure that will actually be realized may depend upon the power of different supply chain members, or upon the cost of information sharing. For instance,  $DR$  may be a more desirable outcome than the grand coalition when information sharing is costly. In addition, if the retailer is the strongest among the three supply chain members (e.g., Wal-Mart), he may influence the formation of the outcome that he likes best (say, the grand coalition<sup>6</sup>), while a strong manufacturer may encourage the formation of an information-sharing agreement that includes only the retailer and the distributor.<sup>7</sup> Recall that we assumed equal bargaining power among all supply chain members and did not take this type of relationships into account; relaxing this assumption would imply that players' preferences for different coalition structures may change as well, which could ultimately lead to different stable outcomes. We feel that this may be an interesting area of future research. We have also shown that the stability of alliance  $MD$  is highly unlikely. Thus, if the supply chain invests in information sharing, the best results are obtained if one of the information-sharing parties possesses the true end-demand information. We conjecture that similar stability behavior will be observed in longer

---

<sup>6</sup>Indeed, Wal-Mart integrates the functions of the distributor and the retailer, and shares the end-demand information with its suppliers (manufacturers).

<sup>7</sup>The preferred outcomes depend on the conditions satisfied in a particular case.

supply chains as well—information-sharing alliances should include members that are closer to the end-demand points, which results in a smoother demand pattern at the higher supply chain echelons, and significant benefits may be realized even without passing demand information along the entire supply chain (which may also be cheaper to implement). This corresponds to some real-life observations. Dell, for example, performs the functions of the manufacturer and the retailer, and shares the demand information with its suppliers. In VMI settings, the manufacturer receives the end-demand information from the retailer, and may pass it further up the supply chain to its suppliers.

## Appendix

**Proof of Theorem 1:** Because we have two possible orderings of both the retailer’s and the distributor’s savings allocations, we have to consider four different combinations of players’ preferences. Suppose, for instance, that their preferences are described with the following relations:

$$I \prec_M MD \prec_M MR \prec_M G \prec_M DR, \quad I \sim_D MR \prec_D DR \prec_D MD \prec_D G, \quad I \sim_R MD \prec_R DR \prec_R MR \prec_R G,$$

where  $Z \sim_i \mathcal{V}$  indicates that player  $i$  receives equal allocations in coalition structures  $Z$  and  $\mathcal{V}$ . We consider each coalition structure and analyze whether it belongs to the LCS. Recall that, in order to show stability of a particular coalition structure, we have to consider all possible defections from that coalition structure. Thus, we need to show that a defection from an outcome,  $Z_0$ , by a coalition,  $Z$ , is deterred for all possible  $Z$ . For each  $Z$ , it is enough to find one possible sequence of defections by coalitions  $Z_1, \dots, Z_{m-1}$  that ends in a stable outcome,  $Z_m$ , which is not strictly preferred over  $Z_0$  by some members in  $Z$ . In addition, the sequence of defections must be constructed so that all members of every deviating coalition strictly prefer  $Z_m$  over the current status quo; that is, they benefit from making the move. Thus, if a set of players contemplates a defection, they will not deviate if there is a possibility that their move may be followed by a sequence of moves that will end in an outcome that may make some of the defecting players worse off, or in which they receive exactly the same payoff as they did before the move. Note that this does not imply that a particular sequence of moves *will* occur; it implies that it is *possible* for it to happen, and farsighted players want to avoid such a possibility.

- Suppose that the current coalition structure is  $I$ . Then, there is no way to deter a possible joint deviation by the manufacturer and the retailer,  $I \rightarrow_{\{M,R\}} MR$ . The distributor cannot defect from the current coalition, in which it is independent—alone. Thus, any defection has to include the manufacturer or the retailer. Recall that the definitions of the LCS and indirect dominance assume that every deviating player must prefer the final outcome over the current status quo. Thus, if the manufacturer defects from his current alliance with the retailer, the final outcome must be either the grand coalition or the alliance of the distributor and the retailer,  $DR$ . But, in either of these two coalition structures, allocations to the

manufacturer and the retailer, who were the players who initiated the defection from  $I$ , exceed their allocations in  $I$ ,  $I \prec_{M,R} G, I \prec_{M,R} DR$ . Thus, if either of the two sequences,

$$I \rightarrow_{\{M,R\}} MR \rightarrow \dots \rightarrow G \quad \text{or} \quad I \rightarrow_{\{M,R\}} MR \rightarrow \dots \rightarrow DR,$$

occurs, the final outcome indirectly strictly dominates the initial status quo for both players who initiated the sequence of defections. We can show that a defection by the retailer is deterred by following a similar approach.

A similar analysis shows that the coalition structure  $MD$  is not in the LCS.

- Next, consider the coalition structure  $DR$ . Since it is the coalition structure most preferred by the manufacturer, we do not need to consider the manufacturer's defections from this outcome. However, suppose that the distributor defects alone:  $DR \rightarrow_{\{D\}} I$ . It is easy to see that this defection is deterred by a further joint deviation of the distributor and the retailer,

$$DR \rightarrow_{\{D\}} I \rightarrow_{\{D,R\}} DR.$$

The allocation that the distributor receives in  $DR$  does not exceed her allocation in  $DR$ , while both the distributor and the retailer receive a larger share than in  $I$ . Thus, if we use the notation from the definition of the LCS, we have  $m = 2, \mathcal{Z} = \mathcal{B} = \mathcal{Z}_2 = DR, S_0 = \{D\}, S_1 = \{D, R\}$ , and  $\mathcal{Z}_1 = \mathcal{V} = I$ . Then,

$$\mathcal{Z} = DR \not\prec_{S_0=\{D\}} \mathcal{B} = DR, \quad \text{and} \quad \mathcal{Z}_1 = \mathcal{V} = I \prec_{S_1=\{D,R\}} \mathcal{B} = DR.$$

A similar analysis shows that a defection by the retailer is deterred as well. Thus,  $DR \in LCS$ .

- We next look at  $MR$ . Because the distributor does not belong to any alliance, it cannot deviate alone. A defection by the manufacturer or the retailer alone can be deterred in a way similar to that shown in the analysis of the coalition structure  $DR$ . Now, consider a joint defection by the manufacturer and the distributor,  $MR \rightarrow_{\{M,D\}} MD$ . Because the manufacturer prefers any information-sharing alliance over this outcome, we can show that this defection can be deterred by the following sequence of deviations:

$$MR \rightarrow_{\{M,D\}} MD \rightarrow_{\{M\}} I \rightarrow_{\{M,R\}} MR.$$

A similar approach can be used to show how the joint deviation of the retailer and distributor may be deterred.

Finally, consider a deviation by all three players,  $MR \rightarrow_{\{S,M,R\}} G$ . If we recall that the retailer prefers to share his information with the manufacturer rather than with the distributor, we can show that this deviation may be deterred by a further defection of the manufacturer,

$$MR \rightarrow_{\{M,D,R\}} G \rightarrow_{\{M\}} DR.$$

Here,  $m = 2$ ,  $\mathcal{Z} = MR$ ,  $\mathcal{B} = \mathcal{Z}_2 = DR$ ,  $S_0 = \{M, D, R\}$ ,  $S_1 = \{M\}$ , and  $\mathcal{Z}_1 = \mathcal{V} = G$ . Then,

$$\mathcal{Z} = MR \not\prec_{S_0=\{M,D,R\}} \mathcal{B} = DR, \quad \text{and} \quad \mathcal{Z}_1 = \mathcal{V} = G \prec_{S_1=\{M\}} \mathcal{B} = DR.$$

Thus,  $MR$  is in the LCS, too.

It is interesting to note that  $MR$  is in the LCS even though each of the players prefers the grand coalition to that outcome. However, if all players decide to move from  $MR$  and form the grand coalition, this deviation may lead to a further defection by the manufacturer, which would make the retailer worse off than in the  $MR$ , because  $DR \prec_R MR$ .

- Finally, consider the grand coalition. This is the outcome most preferred by the retailer and the distributor, so that they do not have an incentive to deviate. Suppose that the manufacturer decides to defect,  $G \rightarrow_{\{M\}} DR$ . Because the retailer prefers the grand coalition, he may trigger the following sequence:

$$G \rightarrow_{\{M\}} DR \rightarrow_{\{R\}} I \rightarrow_{\{M,D,R\}} G.$$

Thus, individual deviation by the manufacturer is deterred, and  $G \in LCS$ .

Note that the manufacturer's allocation increases if he deviates, which means that the grand coalition is not stable in the myopic sense. However, we show above that such defection by a farsighted supplier is deterred by a further deviation of the retailer.

We now investigate whether the above outcomes are stable under the EPCF. We first check the stability of the  $DR$ , and define PCF  $p$  as follows:

$$I \rightarrow_{\{D,R\}} DR; \quad MR \rightarrow_{\{M\}} I; \quad MD \rightarrow_{\{M\}} I; \quad DR \rightarrow DR; \quad G \rightarrow_{\{M\}} DR.$$

It is easy to verify that each defection benefits deviating players. Note that the manufacturer initially decreases his allocation when he defects from  $MR$  and  $MD$ , but this move is followed by a move to  $DR$ , in which he receives a strictly higher allocation over infinitely many periods. Thus, when the discount factor is large enough, the manufacturer benefits from such moves, and  $DR$  is an absorbing state for an EPCF. To see that  $G$  is stable, we define  $p$  as

$$I \rightarrow_{\{M,D,R\}} G; \quad MR \rightarrow_{\{M\}} I; \quad MD \rightarrow_{\{M\}} I; \quad DR \rightarrow_{\{D\}} I; \quad G \rightarrow G.$$

Finally, we check the stability of  $MR$ . As mentioned above, all players prefer the grand coalition to the outcome  $MR$ , so that if  $G$  is the current status quo, we cannot find a profitable and efficient move from  $G$  that eventually leads to  $MR$ . Thus,  $MR$  cannot be an absorbing state of the EPCF.

A similar analysis has to be performed for the remaining three preference orderings. ■

**Proof of Theorem 3:** We will first show that  $G$  and  $DR$  belong to the LCS, and then we will check to see whether they can be obtained as the absorbing states of an EPCF. When conditions (9), (13), and (14) hold, the preferences are described with the following relations:

$$\begin{aligned} I &\prec_M \min\{MD, MR\} \prec_M \max\{MD, MR\} \prec_M G \prec_M DR, \\ I &\prec_D \min\{MD, DR\} \prec_D \max\{MD, DR\} \prec_D G, \\ I &\prec_R \min\{MR, DR\} \prec_R \max\{MR, DR\} \prec_R G. \end{aligned}$$

It is easy to show that  $I \notin LCS$  following steps similar to those used in the proof of Theorem 1.

Next, consider the coalition structure  $DR$ . Since it is the coalition structure most preferred by the manufacturer, we do not need to consider his defections. Because both the retailer and the distributor prefer  $DR$  to  $I$ , an individual defection from  $DR$  by either of them is deterred by their joint return to  $DR$ ,

$$DR \rightarrow_{\{i\}} I \rightarrow_{\{D,R\}} DR, \quad i \in \{D, R\}.$$

Thus,  $DR$  belongs to the LCS.

Finally, consider the grand coalition and suppose that the manufacturer decides to defect. Because the retailer prefers the grand coalition, he may trigger the sequence

$$G \rightarrow_{\{M\}} DR \rightarrow_{\{R\}} I \rightarrow_{\{M,D,R\}} G,$$

which deters the original move. The same argument applies to a joint defection of  $D$  and  $R$ . Next, suppose that the retailer or the distributor defects:

$$G \rightarrow_{\{i\}} Mj, \quad i \neq j \neq M.$$

Because the manufacturer prefers the grand coalition, he may trigger the sequence

$$G \rightarrow_{\{i\}} Mj \rightarrow_{\{M\}} I \rightarrow_{\{M,D,R\}} G,$$

which deters the original move. The same argument applies to a joint defection of  $M$  and  $D$  or  $R$ . Because all possible defections are deterred,  $G \in LCS$ , too.

To show that  $G$  and  $DR$  are stable under the EPCF, we use steps similar to those described in the proof of Theorem 1. In addition, because all players prefer  $G$  to  $Mj$ , we can use EPCF to show that  $Mj$  cannot be stable. ■

### Proof of Proposition 1:

- Let us first suppose that (11) does not hold. Then, information sharing between the distributor and the retailer leads to low savings  $(6(C_M^I - C_M^{DR}) + 2\zeta^{DR} < 2\zeta^G + \zeta^{MR} + \zeta^{MD})$ , and as a result  $\xi_M^{DR} < \xi_M^G$ . If, in addition, (10) holds, then the manufacturer prefers  $DR$  to the remaining two-member coalitions (hence, the grand coalition is his preferred outcome), and

the distributor and the retailer prefer the grand coalition to  $DR$ . Consequently, a defection from  $DR$  to  $G$  cannot be deterred, and  $DR$  is not stable.

If (12) does not hold and  $\zeta^{Mj} > \max\{\zeta^{DR}, \zeta^{Mi}\}$ , then  $\xi_i^G < \xi_i^{Mi}$  for  $i = D, R$ ,  $\xi_M^{Mi} < \xi_M^{Mj}$ , and  $\xi_j^{DR} < \xi_j^{Mj}$ . Now, a joint defection from  $Mj$  by  $i$  and another player is easily deterred because both  $M$  and  $j$  prefer  $Mj$  to the alliance with the remaining player. A joint defection from  $Mj$  by all players to the grand coalition is deterred by the following sequence:

$$Mj \rightarrow_{\{M,D,R\}} G \rightarrow_{\{j\}} Mi \rightarrow_{\{M\}} I \rightarrow_{\{M,j\}} Mj.$$

Thus,  $Mj$  is stable.

- Next, suppose that (10) does not hold. Then, the savings generated in the grand coalition are small compared with the savings in two-member alliances  $\left(\zeta^G + \frac{\zeta^{Mj}}{2} < \zeta^{Mi} + \zeta^{DR}\right)$ , and the distributor and the retailer prefer  $DR$  to the grand coalition, while the manufacturer prefers receiving information from only one partner to an all-inclusive information-sharing arrangement. If, in addition, (11) holds, then the manufacturer prefers  $DR$  to  $G$ , and a defection by the manufacturer from  $G$  to  $DR$  cannot be deterred.

Recall that at least one of the inequalities (13) has to hold when  $\zeta^G/3 > \zeta^{ij}/2$ . Thus, if (10) does not hold for  $R$  and  $D$ , inequality (12) has to be satisfied, and hence  $\xi_i^G > \xi_i^{Mi}$  for  $i = D, R$  and the sequence

$$Mj \rightarrow_{\{M,D,R\}} G \rightarrow_{\{M\}} DR$$

cannot be deterred. Thus,  $MR$  and  $MD$  cannot be stable. ■

### Proof of Proposition 2:

- Suppose that (14) holds. Then, (21) also holds, and  $\psi_M^{DR} > \psi_M^G$  and  $\psi_i^{Mi} < \psi_i^G, i \in \{D, R\}$ . In addition, (20) implies that  $\psi_M^{DR} > \psi_M^{Mi}, i \in \{D, R\}$ . Then, the manufacturer prefers  $DR$  to all other coalition structures, and it is easy to verify that it is a stable outcome.

If, in addition, (19) is satisfied, then  $\psi_i^{DR} < \psi_i^G, i \in \{D, R\}$ , and it can be verified that  $G$  becomes stable— $D$  and  $R$  do not want to defect from it, because it is their most-preferred outcome, while a defection by  $M$  can be deterred by the following sequence:

$$G \rightarrow_M DR \rightarrow_{\{j\}} I \rightarrow_{\{M,D,R\}} G.$$

Condition (18) implies that the manufacturer prefers the grand coalition to two-player alliances with either of the remaining two players. Because the retailer and the distributor also prefer  $G$  to an alliance with the manufacturer, a two-retailer alliance containing the manufacturer cannot be stable if (18) holds.

If (18) does not hold and  $\zeta^{Mj} > \max\{\zeta^{DR}, \zeta^{Mi}\}$ , then  $\xi_i^G < \xi_i^{Mi}$  for  $i = D, R$ ,  $\xi_M^{Mi} < \xi_M^{Mj}$ , and  $\xi_j^{DR} < \xi_j^{Mj}$ . We can show that  $Mj$  is stable by using steps similar to the ones used in the proof of Proposition 1.

- Suppose now that (17) does not hold. Therefore, the manufacturer's preferences are given by  $\psi_M^G > \psi_M^{DR}$ , and recall that (20) implies  $\psi_M^{DR} > \psi_M^{Mi}, i \in \{D, R\}$ . Whenever  $\zeta^G/3 > \zeta^{ij}/2$ , at least one of the remaining two supply chain members prefers the grand coalition over  $DR$ , and it is easy to show that the grand coalition is stable.

If (19) holds, then all players prefer  $G$  to  $DR$ , and it is easy to show that  $DR$  cannot be stable. However, when exactly one player, say  $j$ , prefers  $DR$  to  $G$ , then a defection from  $DR$  to  $G$  can be deterred by the sequence

$$DR \rightarrow_{\{M,D,R\}} G \rightarrow_{\{j\}} Mi \rightarrow_M I \rightarrow_{\{D,R\}} DR,$$

and hence it is easy to show that  $DR$  is stable.

When (21) holds, then all players prefer the grand coalition over two-player alliances, including the manufacturer, so  $Mj$  cannot be stable.

If (21) does not hold and  $\zeta^{Mj} > \max\{\zeta^{DR}, \zeta^{Mi}\}$ , then we can show that  $Mj$  is stable by using steps similar to the ones used in the proof of Proposition 1. ■

## References

- [1] ANUPINDI, R., Y. BASSOK, E. ZEMEL. 2001. A General Framework for the Study of Decentralized Distribution Systems, *M&SOM* **3**, 349-368.
- [2] AUMANN, R.J. 1959. Acceptable Points in General Cooperative  $n$ -person Games, in *Contributions to the Theory of Games IV*, A.W. Tucker and R.D. Luce eds., Princeton University Press, N.J., 287-324.
- [3] AUMANN, R.J., J.H. DREZE. 1974. Cooperative Games with Coalition Structures, *International J. Game Theory*, **3** 217-237.
- [4] CACHON, G., M. FISHER. 2000. Supply Chain Inventory Management and the Value of Shared Information, *Management Sci.* **46**, 1032-1048.
- [5] CACHON, G., S. NETESSINE. 2004. Game Theory in Supply Chain Analysis, in *Supply Chain Analysis in the eBusiness Era*, D. Simchi-Levi, S.D. Wu, and M. Shen eds., Kluwer.
- [6] CHEN, F., Z. DREZNER, J. K. RYAN, D. SIMCHI-LEVI. 2000a. Quantifying the Bullwhip Effect in a Simple Supply Chain: The Impact of Forecasting, Lead Times, and Information, *Management Sci.* **46**, 436-443.
- [7] CHEN, F., J.K. RYAN, D. SIMCHI-LEVI. 2000b. The Impact of Exponential Smoothing Forecasts on the Bullwhip Effect, *Naval Res. Logistics* **47**, 269-286.
- [8] CHWE, M. S.-Y. 1994. Farsighted Coalitional Stability, *J. Econ. Theory* **63**, 299-325.

- [9] CLARK, T. 1994. Campbell Soup: A Leader in Continuous Replenishment Innovations. HBS Case, Boston, MA.
- [10] GAVIRNENI, S., R. KAPUSCINSKI, S. TAYUR. 1999. Value of Information in Capacitated Supply Chains, *Management Sci.* **45**, 16-24.
- [11] GILLIES, D.B. 1959. Solutions to General Non-zero Sum Games, in *Contributions to the Theory of Games IV*, A.W. Tucker and R.D. Luce eds., Princeton U. Press, 47-83.
- [12] GRANOT, D., G. SOŠIĆ. 2003. A Three Stage Model for a Decentralized Distribution System of Retailers, *Oper. Res.* **51**, 771-784.
- [13] GRANOT, D., G. SOŠIĆ. 2005. Formation of Alliances in Internet-Based Supply Exchanges, *Management Sci.* **51**, 92-105.
- [14] HAMMOND, J. 1995. Barilla SpA (A), (B), (C), and (D). HBS Case, Boston, MA.
- [15] HARTMAN, B.C., M. DROR, M. SHAKED. 2000. Cores of Inventory Centralization Games, *Games and Econ. Behavior* **31**, 26-49.
- [16] JOHNSON, G., H. THOMPSON. 1975. Optimality of Myopic Inventory Policies for Certain Dependent Demand Processes, *Management Sci.* **21**, 1303-1307.
- [17] KEMAHLIOGLU ZIYA, E., J.J. BARTHOLDI III. 2003. Inventory Management for a Supply Chain with Coalition-Forming Players, Working Paper, Georgia Inst. of Tech., Atlanta, GA.
- [18] KONISHI, H., D. RAY. 2003. Coalition Formation as a Dynamic Process, *J. Econ. Theory* **110**(1), 1-41.
- [19] LEE, H.L., P. PADMANABHAN, S. WHANG. 1997. Information Distortion in a Supply Chain: The Bullwhip Effect, *Management Sci.* **43** 546558.
- [20] LEE, H.L., K.C. SO, C.S. TANG. 2000. The Value of Information Sharing in a Two-Level Supply Chain, *Management Sci.* **46**, 626-643.
- [21] LENG, M., M. PARLAR. 2009. Allocation of Cost Savings in a Three-Level Supply Chain with Demand Information Sharing: A Cooperative-Game Approach, *Oper. Res.* **57**(1), 200-213.
- [22] LOVEJOY, W.S. 1990. Myopic policies for some inventory models with uncertain demand distributions, *Management Sci.* **36**(6), 724-738.
- [23] NAGARAJAN, M., Y. BASSOK. 2008. A Bargaining Framework in Supply Chains (The Assembly Problem), *Management Sci.* **54**(8), 1453-1466.
- [24] NAGARAJAN, M., G. SOŠIĆ. 2007. Stable Farsighted Coalitions in Competitive Markets, *Management Sci.* **53**(1), 29-45.
- [25] NAGARAJAN, M., G. SOŠIĆ. 2009. Coalition Stability in Assembly Models, *Oper. Res.* **57**(1), 131-145.
- [26] NAGARAJAN, M., G. SOŠIĆ. 2008. Game-Theoretical Analysis of Cooperation Among Supply

Chain Agents: Review and Extensions, *European J. Oper. Res.* **187**(3) 719-745.

- [27] PLAMBECK, E.L., T.A. TAYLOR. 2007. Implications of Breach Remedy and Renegotiation Design for Innovation and Capacity, *Management Sci.* **53**, 1859-1871.
- [28] RAGHUNATHAN, S. 2003. Impact of Demand Correlation on the Value and Incentives for Information Sharing in a Supply chain, *European J. Oper. Res.* **146**, 634-649.
- [29] SHAPLEY, L.S. 1953. A Value for N-Person Games, *Contribution to the Theory of Games vol.2*, Princeton U. Press, Princeton, N.J. 307-317.
- [30] SIMCHI-LEVI, D., Y. ZHAO. 2003. The Value of Information Sharing in a Two-Stage Supply Chain with Production Capacity Constraints, *Naval Res. Logistics* **50**, 888-916.
- [31] SIMCHI-LEVI, D., Y. ZHAO. 2004. The Value of Information Sharing in a Two-Stage Supply Chain with Production Capacity Constraints: The Infinite Horizon Case, *Probability in the Engineering and Informational Sci.* **18**, 247-274.
- [32] SOBEL, M. 1981. Myopic Solutions of Markov Decision Processes and Stochastic Games, *Oper. Res.* **29**, 995-1009.
- [33] SOŠIĆ, G. 2006. Transshipment of Inventories Among the Retailers: Myopic vs. Farsighted Stability, *Management Sci.* **52**(10), 1493-1508.