Name-Your-Own-Price as a Competitive Distribution Channel in the Presence of Posted Prices

Xiao Huang
John Molson School of Business, Concordia University, Montréal, Québec, Canada, H3G 1M8, xiaoh@jmsb.concordia.ca

Greys Sošić
Marshall School of Business, University of Southern California, Los Angeles, California 90089, sosic@marshall.usc.edu

Priceline.com patents the innovative marketing strategy, Name-Your-Own-Price (NYOP), that sells opaque products through customer-driven pricing. In this paper, we study how competitive suppliers with substitutable, non-replenishable goods may sell their products (1) as regular goods through a direct channel at posted prices, and/or (2) as opaque goods through a third-party channel, which allows for the NYOP approach. We model the third-party channel as an intermediary firm that collects the difference between the customers’ bids and reservation prices set by the suppliers, and discuss different channel strategies and customers’ bidding strategies. We show that high-end customers may demonstrate low-end behavior (that is, name their prices prior to attending the direct channel, making an even lower bid than the low-end customers), and that the intermediary firm benefits more from horizontally differentiated goods than from vertically differentiated ones. We also use dynamic programming approach to analyze how should suppliers competitively determine channel prices for given initial inventory levels with the goal of maximizing the average expected profit, and show that time and inventory levels have very different impact in dual-channel and single-channel settings. Our results suggest that the suppliers may not benefit from the existence of a profit-maximizing NYOP channel. In particular, a monopolist would opt out of the NYOP channel and sell at posted prices only, which implies that NYOP is not appropriate for customer discrimination in the regular-goods market. Numerical experiments show that suppliers are able to generate higher expected profits in the absence of the NYOP channel.

Key words: Competition; Distribution channels; Dynamic pricing; e-Commerce; Name-Your-Own-Price; Nash equilibrium; Opaque goods; Priceline;

History: First Version: October 15, 2009; This Version: September 2, 2010;

1. Introduction

In the last decade, an increased variety of selling mechanisms arose in the consumer domain as e-business and online shopping have become vibrant parts of our lives. On one hand, sellers, from airlines and hotels to apparel and groceries stores, are more then ever engaged in adjusting posted
prices throughout the time. The buyers, on the other hand, enjoy some new purchasing opportunities. Beside the old-fashioned posted-price purchase, buyers can now, say, use auctions to buy football tickets from eBay.com, and thus not know the actual prices at the beginning of the purchasing process, or book hotels through “Name-Your-Own-Price” at Priceline.com, where even the actual products can be unknown. The latter behavior has been amplified by the recent economic downturn. As Forbes (Feb 19, 2009) reported, “With the economy failing, … (consumers)’re looking for deep discounts. … Expedia … is losing business while its competitor Priceline.com thrives on its ‘Name Your Own Price’ model.” During the summer of 2009, “Priceline earnings increased 35%. … Revenue climbed 18% to $603.7 million. Gross margin rose to 50.6% from 49.4%” (Wall Street Journal, Aug 11, 2009). At the end of 2009, Priceline overtook Expedia in market capitalization (tnooz.com Nov 17, 2009).

The evolution in pricing allows consumers to have more impact on how much they want to pay. Current pricing schemes can be roughly divided into two categories: seller-driven pricing and buyer-driven pricing. Under seller-driven pricing, the seller sets the price of the goods, and the buyer simply makes a take-it-or-leave-it decision. This type of pricing is the one most commonly observed, and it has been referred to in literature as “posted-price” or “list-price.” Under buyer-driven pricing, the roles of the two parties are reversed: a buyer announces the price that she (hereinafter, buyers will be referred to with female pronouns, sellers with male pronouns) is willing to pay, while the seller decides whether and at what price to let the goods go. Auctions and name-your-own-price (NYOP) models belong to this pricing category; the former has been used by, for instance, eBay, Google, and compUSA, while the latter is being adopted by companies like Priceline.com.

At the same time, information availability about the products on sale has been changing as well. Sellers used to provide as much information about an item as possible, whether through displaying samples in brick-and-mortar stores, or by listing detailed specifications/figures in online stores. However, in recent times some sellers choose to strategically withhold information from the customers. For instance, if a customer decides to purchase an air ticket at Priceline.com through “Name-Your-Own-Price,”¹ she will not be able to learn the details of her trip (e.g., the airline name, departure time,² etc.) until the deal is finalized. Thus, the exact features of the product at the time of purchase are rather vague to the customers. Such products, whose characteristics are not fully revealed at the point of payment, have been referred to as opaque goods in industry (one may refer to Fay 2008, for a detailed description of opaque goods and Anderson 2008, for information on how Priceline.com works with its suppliers). As opposed to opaque goods, we will refer to the goods for which buyers know all features at the time of purchase as regular goods.

¹ Starting in 2005, Priceline also has products with posted price (USA Today, Apr 11, 2005).
² The O/D dates and airports have to be confirmed, though.
<table>
<thead>
<tr>
<th>Seller-Driven</th>
<th>Regular Goods</th>
<th>Opaque Goods</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expedia, Orbitz</td>
<td>Hotwire, BookIt</td>
<td></td>
</tr>
<tr>
<td>Buyer-Driven</td>
<td>eBay, Google</td>
<td>(Hann and Terwiesch 2003, Terwiesch et al. 2005)</td>
</tr>
<tr>
<td>Auction:</td>
<td>Unrevealed German NYOP firm</td>
<td>Priceline</td>
</tr>
<tr>
<td>NYOP:</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1 Pricing Schemes for Regular and Opaque Goods

Table 1 summarizes how regular and opaque goods have been sold under different pricing schemes.

While auctions have been studied extensively in the literature as one of the buyer-driven pricing schemes, NYOP has to date received limited attention, despite the fact that it possesses some nice characteristics:

- First, processing is fast—after a bid is submitted (that is, after a customer named her price), the customer is normally informed about the outcome within minutes (or, in the worst case, within 24 hours). Hence, unlike auctions, which may take days or weeks to finalize, NYOP does not discourage impatient customers from participating.

- Second, NYOP does not intensify the competition among customers. Unlike in an auction, whether a particular bid is accepted or not in NYOP depends only on the amount of the bid and the reservation prices of the suppliers, but not on the behavior of other customers. A customer is in this case more likely to use the NYOP channel as an additional opportunity to get a good deal, since she can always buy from the direct channel if her bid was rejected. This also allows us to focus more on the competition between suppliers than that among customers.

- Third, NYOP is simple for suppliers to implement. The inventory is depleted by at most one unit per decision epoch, and can be immediately followed by suppliers’ updates of posted and reservation prices. When compared with the uncertain number of winners in an auction, NYOP gives firms more power in their inventory control, and allows for dynamic pricing in both channels.

- Finally, NYOP provides a platform of competition in terms of opaque goods. Products sold through the NYOP channel do not possess some relevant information (e.g., airline, hotel brand) that can be used by customers for product discrimination. The suppliers are under the “brand shield” and “price shield” that the direct channels cannot offer (Dolan 2000)—the NYOP channel essentially provides a space in which suppliers compete head-to-head to win customers, who in turn give up the power to exercise their preferences.

Taking this into account, we are interested in analyzing how the the NYOP channel could be integrated with the traditional selling channel in a competitive market, and what would be its impact on the business environment. In particular, we investigate the following research questions:

1. How would customers react to the NYOP-opaque-product channel coupled with the posted-price-regular-product channel?
2. How should suppliers determine their optimal pricing/channel strategies in the presence of competition and the NYOP channel?

3. What would be the impact of the NYOP channel on the entire industry?

In our analysis of the problem, we set up a framework with three stakeholders: a sequence of customers, two competing suppliers, and one intermediary NYOP firm. We assume that the suppliers sell substitutable, perishable, and non-replenishable products. The products could be sold either as regular goods through their direct channels (stores or websites) at posted prices, or as opaque goods through a third-party intermediary firm that conducts NYOP service among the customers (e.g., Priceline.com). Thus, each supplier chooses his own set of pricing mechanisms (seller-driven pricing, buyer-driven pricing, or both at the same time) and the format of his products (regular, opaque, or both). At the beginning of each time epoch, the suppliers set their posted prices in direct channels. Then, the suppliers inform the intermediary firm about their reservation prices, defining the lowest price at which they are willing to let a unit go. A customer arriving thereafter may decide to (1) buy at the posted price from her preferred supplier, or (2) go to the intermediary firm and name-her-own-price. If a bid is submitted to the NYOP channel, the intermediary firm benefits from the difference between the customer’s offer and the lowest reservation price from the suppliers (Dolan 2000). In case the bid fails to meet the lowest reservation price, the customer is rejected by the intermediary firm, but she still has a chance to buy the product at posted price. Consistently with prior literature on this topic (e.g., Amaldoss and Jain 2008) and with Priceline’s policy (Dolan 2000), we assume that repeated bidding is prohibited, hence any strategic behavior on the customers’ side is not within the scope of this paper (see §8 for more discussion).

Our analysis shows that high-end customers may demonstrate low-end behavior (that is, name their prices prior to attending the direct channel, making an even lower bid than the low-end customers), and that the intermediary firm benefits more from horizontally differentiated goods than from vertically differentiated ones.

We also use dynamic programming approach to analyze how should suppliers competitively determine channel prices for given initial inventory levels with the goal of maximizing the average expected profit, and show that remaining time and inventory levels have very different impact in dual-channel and single-channel settings. For example, while the revenue can be decreasing with one’s inventory level in the presence of the NYOP channel, this would never happen if suppliers use only direct channels. We further show that an NYOP channel is less attractive for regular products—i.e., a monopolist would not use NYOP if he can sell in a direct channel at posted prices. However, in a duopoly market in which opaque products are available, both suppliers will try to utilize the NYOP channel. Interestingly, our numerical results indicate that the suppliers may see higher revenues in the absence of the NYOP channel. Thus, while the existence of the NYOP channel may negatively impact the revenue of regular products, it may positively impact the revenue of opaque products.

An earlier policy at Priceline.com forbid customers to re-bid within the next seven days after the initial offer.
NYOP channel may be a result of the competition in the market, its presence further intensifies the competition (that is, posted prices and expected revenues are lower in the presence of NYOP than without it). This result is consistent with some ongoing empirical studies and should be taken into account by the NYOP firm when deciding how to handle its suppliers.

The paper is organized as follows. In §2, we review the literature. In §3 we introduce our model. Customers’ purchasing/bidding behavior is analyzed in §4, and results for the purchasing channel are shown in §5. In §6, we propose a dynamic programming approach for solving the equilibrium pricing decision. Numerical results are provided in §7. We discuss the results and extensions in §8.

2. Literature Review

Our work is related to two different streams of literature, which we analyze separately: the first stream captures the papers in the area of NYOP channels and opaque products, while the second deals with dynamic pricing strategies.

2.1. Name-Your-Own-Price and Opaque Products

The business model of NYOP and opaque goods has drawn increased attention in the recent operations management and marketing literature. Table 2 presents a selection of papers relevant to either NYOP or opaque products.

<table>
<thead>
<tr>
<th>Posted Prices</th>
<th>Regular Goods</th>
<th>Opaque Goods</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Fay (2009)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Our Paper</td>
<td></td>
</tr>
</tbody>
</table>

Table 2 Literature of Regular vs. Opaque Products

4 Some of the models may be applied to both regular and opaque products—we put them under the column “Regular Goods,” while the second column contains papers targeting only opaque products.
a firm can learn customers’ willingness to pay and the transaction costs of repeated bidding. Hann and Terwiesch (2003) estimate the transaction cost per bid of the on-line customers in naming their own prices. Hinz and Spann (2008) model how social network may facilitate individuals in learning the reservation price of the seller. Some articles also shed light on the NYOP retailer’s policy. Terwiesch et al. (2005) analyze the repeated bidding behavior and suggest the threshold price for monopoly firms. Joint bidding for multiple items has been studied by Amaldoss and Jain (2008), while Wang et al. (2009) analyze a model for the airline industry in which only posted price is used in the first period, while both posted price and NYOP might be adopted in the second period.

Problems involving opaque goods usually demand a very different model structure. As it usually requires two or more suppliers/products to make an opaque product, competition issues may arise (for monopolistic case see Jiang 2007; Fay and Xie 2008). Fay (2008) models selling opaque products at posted prices only. The suppliers have to precommit with the NYOP firm on the number of units to be sold as opaque products. The intermediary firm handles all of the pricing and product allocations with the customers. No bid/auction is involved in the model, and the intermediary firm adopts seller-driven pricing to set a firm price for the opaque products. Conditions under which the opaque good may bring down the price in the traditional channel and harm the profits are identified. Jerath et al. (2009) consider the competition between two firms selling limited inventories in a two-period model. They allow the firms to simultaneously sell through an opaque channel instead of the direct channels for the second period, and analyze expected revenue in both cases.

If NYOP is introduced in a model with opaque products, the notion of “NYOP firm” needs further clarification. In the presence of opaque goods, the NYOP system usually consists of an NYOP intermediary firm and a number of suppliers, in which the former acts as an agent that delegates the NYOP service for the latter. This is different from many models in the third quadrant of Table 2, which deal with only one “NYOP retailer”—a centralized supplier that also provides NYOP service. With opaque products, the suppliers and NYOP-service provider are usually separated. The suppliers determine their reservation prices, and the NYOP-service provider selects the supplier that accepts the lowest payment. Fay (2009) studies the competition between Priceline.com and Hotwire.com. Both companies deal with opaque products, but the former uses a buyer-driven NYOP scheme while the latter uses a seller-driven, posted-price scheme.

2.2. Dynamic Pricing: Channels and Competitions
Most of the NYOP literature discussed so far did not put any restriction on the supply. In other words, the papers mentioned above assumed that the products are replenishable, so that the entire demand may be satisfied. In our model, the notion of limited supply is incorporated by considering

5 “…maximizes the spread between the customer’s ‘named-price’ and the necessary payment to the airline partner…” (Dolan 2000).
non-replenishable products. In maximizing the expected revenue, suppliers have to dynamically adjust their posted/reservation prices throughout the sales horizon. There has been a rich literature that applies dynamic pricing methods to improve the seller’s profit. One may refer to McGill and van Ryzin (1999) and Elmaghraby and Keskinocak (2003) for a general review of applications in perishable-yet-non-replenishable goods (such as hotels and airlines) and replenishable ones (in which retailers can make inventory ordering decisions), respectively. However, most of the existing literature has been focusing on seller-driven pricing, while we are also interested in buyer-driven pricing.

As electronic markets became more convenient and easier to join, several review papers discussed the possibility of extending the models towards a broader aspect of pricing (e.g., Bitran and Caldentey 2003; Elmaghraby and Keskinocak 2003b; Pinker et al. 2003). The use of auctions as the unique sales channel has been discussed in Vulcano et al. (2002) and van Ryzin and Vulcano (2004). While the former looks at sales of a finite quantity of non-replenishable items, the latter considers replenishable products with an infinite sales horizon.

Recognizing that many firms have multiple sales channels (in particular, a combination of a brick-and-mortar store and an online store), a number of articles study how a firm should set up its dual-channel strategy—namely, having seller-driven prices and buyer-driven prices at the same time. Etzion et al. (2006) model how to use auctions and posted prices at the same time, with infinite supply in a limited time. Caldentey and Vulcano (2007) study a similar problem but with limited supply. Jiang (2007) considers the market strategy a monopolistic airline should adopt if it can either sell some itineraries as regular products and/or pack them together and sell as opaque products. Huh and Janakiraman (2008) show that when products are replenishable, the optimality of \((s,S)\) inventory policies still holds under various pricing schemes, including buyer-driven ones (like auctions and NYOP). Wang et al. (2009) consider a two-period model for the airline industry in which dual channel is allowed in the second period. They find out that decisions about adoption of an NYOP channel depend on the uncertainty of high-fare demand, rather than on the amount of excess capacity.

The papers mentioned so far consider a single firm only. However, firms offering the same airline itineraries, nearby hotel rooms, same-size car rentals, or any other substitutable products may face even greater challenges in finding the right pricing policies and inventory control policies when competing with their rivals. The competition could even be the key driver for the selection of a certain pricing policy (Bitran and Caldentey 2003). In recent years, researchers have begun to study competition in revenue management. Netessine and Shumsky (2005) analyze the airline seat-allocation problem for both horizontal competition (firms offering the same single-leg itinerary) and vertical competition (firms offering different legs in a multi-leg itinerary) between two firms. Gallego and Hu (2007) model dynamic pricing under competition as a stochastic control problem.
in a continuous-time differential game. Lin and Sibdari (2008) study competitive dynamic pricing for multiple firms in which customers’ choice follows the multinomial logit model depending on the current prices of the market. Levin et al. (2009) present a model of oligopolistic dynamic pricing in which customers may strategically choose their purchase time.

In traditional revenue management (RM), the optimal pricing policy largely depends upon inventory availability. This increases the complexity of modeling competition in RM models, as rivals’ inventory levels may not be observable and customers’ preferences may be unknown. For analytical ease, many works (see, e.g., Gallego and Hu 2007; Lin and Sibdari 2008; Levin et al. 2009) assume perfect information; that is, they assume that inventory levels and distributions of random factors are public knowledge. Still, there exists a number of papers that take different approaches to avoid such assumptions. In solving competitive pricing and the booking-limits problem, Perakis and Sood (2006) apply robust optimization to address the unknown distribution in demand. Zhang and Kallesen (2008) propose an MDP model for companies to estimate their rival’s prices through some “belief matrix.” Levina et al. (2009) allow a monopolist to learn the demand characteristics of his customers from the dynamic pricing process.

Due to the potential complexity, there is limited RM literature in the dual-channel multiple-firm quadrant. Jerath et al. (2009) allow competing firms to participate in both direct and opaque selling channels, yet in a sequential manner. In particular, their focus is on how last-minute opaque selling (at seller-driven prices) and strategic customer behavior might affect the revenue under competition. However, we are interested in how firms would balance between the two channels—one seller-driven priced and the other buyer-driven priced—throughout the time.

Our model adds a new block to the current literature in the following aspects. (1) We look at dual channel–dynamic pricing strategy in a duopoly. Specifically, we allow the firms to adopt two channels at the same time. To our knowledge, this has not been extensively studied in the

<table>
<thead>
<tr>
<th>Single-Channel</th>
<th>Dual-Channel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monopolists</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Wang et al. (2009)</td>
</tr>
<tr>
<td>Duopolists/Oligopolists</td>
<td></td>
</tr>
<tr>
<td>Gallego and Hu (2007)</td>
<td></td>
</tr>
<tr>
<td>Lin and Sibdari (2008)</td>
<td></td>
</tr>
<tr>
<td>Zhang and Kallesen (2008)</td>
<td></td>
</tr>
<tr>
<td>Levin et al. (2009)</td>
<td></td>
</tr>
</tbody>
</table>

Table 3 Brief RM Literature Review with Single or Dual Channel
literature thus far. (2) We apply NYOP to opaque products by decentralizing the suppliers and the intermediary NYOP firm. The suppliers manage their own seller-driven pricing channels, while the intermediary firm adopts buyer-driven pricing. (3) Our focus is more supply-oriented, through the analysis of the suppliers’ equilibrium decisions under inventory constraints. Few NYOP papers have considered dual-channel strategy or limited supply, which happens to be how NYOP is being applied in practice. We believe that these distinctions make this problem/model a relevant and interesting research topic.

3. The Model
In this section, we first describe the flow of our game, and then discuss some of our assumptions in more detail.

3.1. Game Flow
Consider a model with two suppliers selling substitutable products, an intermediary firm, and a sequence of customers. A supplier and its product have the same index $i$, where $i \in \{1, 2\}$. Assume that supplier $i$ holds $\bar{x}_i$ units of initial inventory, which will expire after $T$ periods and is not replenishable. We use a backward time index $t = T, T - 1, \ldots, 1$ to denote the current period, so a smaller number indicates that we are closer to the ending time. At each period, exactly one customer arrives, and she possesses random valuation $v_i$ for product $i$. We assume $v_i$’s are independently distributed on $[0, 1]$ with density $f_i(\cdot)$ and cumulative distribution function (cdf) $F_i(\cdot)$. Each customer demands at most one unit. This sequence of customers with constant arrival rate 1 can easily be extended to a sequence of homogeneous customers with a fractional arrival rate by adjusting the distributions of valuations accordingly.

At the beginning of each period, the suppliers publicly announce their posted prices, $p = (p_1, p_2)$, at which customers can buy from their direct channels. After the posted prices are announced, the intermediary firm conducts an English auction with the suppliers to determine which one will fulfill the demand for the opaque product in this period. The suppliers take turns in submitting their desired reservation prices until someone is unwilling to decrease any further. The supplier with the lower reservation price, $r$, is awarded the opportunity to supply the opaque product in this period. In the case of a tie, one supplier is randomly chosen with 50% probability. Neither these processes nor the inventory levels are observable to the customers, who have to adopt their own priors in estimating which supplier will be assigned to them, as well as the reservation price $r$. Different types of priors will be discussed further in §3.2.

After the prices are set, a customer arrives with valuation $v = (v_1, v_2)$ as her private information. Neither the suppliers nor the intermediary firm has any knowledge of $v$ except its distribution function, $F_{1,2}(\cdot)$. Based on the posted prices ($p$), her valuation ($v$), and her prior in estimating
the NYOP channel (the opaque-product provider and reservation price, \( r \)), the customer decides if she wants to buy directly from her preferred supplier or name her price (which we will refer to as “make a bid” for the reminder of the paper) \( b \) with the intermediary firm. In the second case, she may be matched up with either of the two suppliers if the bid is accepted. In case the bid is rejected, the customer can always return to her preferred supplier and buy the product at its posted price. However, making a bid is considered a commitment to buy, and thus a customer cannot decline a product assigned by the intermediary firm if she discovers \textit{ex-post} that being awarded the alternative product or buying at the posted price would make her better off. We also assume that the customer is not allowed to make a second bid with the NYOP channel after the first bid is rejected. Repeated bidding would certainly generate more research questions that are worth analyzing. However, as our goal is to characterize the generic policy and impact of the dual distribution channel (NYOP and posted prices), the effects of repeated bidding (or any strategic customer behavior) are not within the scope of this paper (see discussion in §8), and we simplify the scenario by applying a one-time bidding restriction\(^6\). This assumption aligns with the policy used by Priceline, that “only one offer was permitted in a seven-day period...” (Dolan 2000).\(^7\) Readers may refer to Fay (2004), Spann (2004), Terwiesch et al. (2005), and Hinz and Spann (2008) for problems involving repeated bidding/bid learning.

The NYOP intermediary firm is modeled as a profit maximizer. After receiving a bid, \( b \), for the opaque product, the intermediary firm simply compares \( b \) with the lower reservation price, \( r \): if \( b \geq r \), the bid is accepted. The intermediary firm collects \( b \) from the customer, assigns her a unit of product from the supplier that requests payment \( r \), and pays that supplier his requested amount. Otherwise, if \( b < r \), the bid is rejected. This process aligns with the description in Dolan (2000).

Figure 1 illustrates the sequence of events for each period of the game.

![Figure 1](image.jpg)

Figure 1  Game sequence in each period.

Figure 2 depicts the problem confronting each customer in Stage 3. We use squares to depict decision nodes (in which customers need to pick from one of the branches) and circles to depict

---

\(^6\) Frictional cost related to this one-time bidding is considered small and will not be taken into account in this model.

\(^7\) The restrictions may have been updated, but the spirit is that such sites may not encourage repeated bidding within a short period.
probability nodes, with potential outcomes that are not under customers’ control. The last column shows the cash flow of each stakeholder under each scenario.

![Diagram of Customer's Problem in Stage 3]

Figure 2  Customer’s Problem in Stage 3

3.2. Assumptions
We now provide a more detailed discussion of our main assumptions.

Reservation Price. We assume that the intermediary firm imposes the restriction which limits the reservation price levels to be not higher than the respective posted prices. A plausible reason for this assumption lies in the fact that customers usually perceive the NYOP channel as a way to save money (as a tradeoff for the more restricted information), and this is exactly how the intermediary firms market themselves. For example, Priceline.com consistently advertises on the website “save 40% on flights, 50% on hotels, 30% on cars.” This assumption directly leads to

\[ r \leq p = \min\{p_1, p_2\}, \]  

(1)

which will be used throughout the paper.

Information Structure. The general information structure is characterized in Table 4. As mentioned in the model, posted prices \( p \) are public information, while customer valuation \( v \) is known to the customers only.

We use \( A \) to indicate the supply availability at the suppliers; i.e., if supplier \( i \) is out of stock, then \( A_i = 0 \), otherwise \( A_i = 1 \). It is more realistic to assume that customers can observe supply availability of the suppliers \( (A) \) than their real-time inventory levels \( (x) \), as the suppliers of non-replenishable products are usually more willing to signal a stock-out than to let customers be
aware of how many units \(x\) they have yet to sell. In addition, it is not unrealistic to assume that customers might know the initial inventory levels of the suppliers \(\bar{x}\): for instance, in hotel and airline industries, the total number of rooms or seats is known to everybody.

The model also assumes that both the opaque-product provider \(I\) and the reservation price \(r\) are decision variables which are known to the intermediary firm and the suppliers at the beginning of each time epoch. However, this information is by no means accessible to the customers (since \(x\) is unobservable), who have to adopt certain prior in estimating these factors. We assume that this prior, which can be reached through consumer research or survey, is public information to all stakeholders (different priors will be discussed in the next paragraph). We use “customer type” to refer to a class of customers with the same prior, and denote it by \(K\).

Throughout the table, IR is used when the availability of information has little or no importance to the intermediary firm. This irrelevance stems from the fact that the intermediary firm does not hold any physical inventory—it only needs to verify whether an NYOP bid should be accepted (if it is above the reservation price, \(r\)) and which supplier/product should be assigned to the customer in case of an acceptance (the opaque-product provider, \(I\), where \(I \in \{1, 2\}\)). As the latter has been already answered by the English auction conducted between the suppliers, inventory levels have little value to the intermediary firm.

**CUSTOMER TYPES.** As discussed earlier, each customer upon arrival adopts certain prior in estimating the opaque-product provider and the reservation price\(^8\), which have already been determined through the English auction\(^9\). In other words, the reservation price and opaque-product provider do not depend upon one’s bid, but may be impacted by the remaining time, inventory

---

\(^8\) A special instance occurs in the last period \(t = 1\), at which time neither the reservation price or the opaque-product provider depend upon inventory levels, and the customers can make their best guess.; this will be discussed in §6.2. In periods other than \(t = 1\), the prior discussed above applies.

\(^9\) The model is not affected if the timing of the game is different, i.e., intermediary firm conducts the English auction only upon each NYOP request, which may be more suitable for real-life applications as it is less costly than auction at each time epoch.
levels, posted prices, and the prior that customers adopt.

We assume that customers estimate the reservation price as a random variable $\xi(p, A, t)$ with the support $[0, p]$, $p = \min\{p_1, p_2\}$, with density $g(\cdot|p, A, t)$ and cdf $G(\cdot|p, A, t)$. Thus, the estimation can be updated as the remaining time or information available to the customers (e.g., whether a supplier runs out of stock) change. Further, we assume that $\xi$ has decreasing reversed hazard rate (DRHR); i.e., $g/G$ is decreasing. This is equivalent to assuming that the cdf $G$ is log-concave. Many (truncated) distributions satisfy this condition (e.g., uniform, normal, exponential, chi-squared, logit, etc.), hence this assumption is not too restrictive (Bagnoli and Bergstrom 2005).

The estimator for the opaque-product provider, $I(p, A, t) \in \{1, 2\}$, is modeled in a similar way. More specifically, if $A_i = 0$ and $A_j > 0$, the product at the intermediary firm can no longer be opaque and $I = j$. When $A_1A_2 > 0$, the distribution of $I$ may be related to $(p, A, t)$ in different ways; we will consider two alternatives in our analysis. If we denote the probability $P(I(p, A, t) = 1) = \alpha(p, A, t)$, then we consider two types of customers, which we will call Natural and Price-Oriented:

- **Natural** ($K = N(\alpha)$): customers assume that $\alpha(p, A, t) = \alpha$, which is static over time. In particular, $\alpha$ can be related to initial inventory levels, so that

$$P(I = j) = \frac{x_j}{x_1 + x_2}.$$

This fits well with the assumption that the supplier with higher initial inventory levels is more likely to have lower threshold price throughout the time, and is, thus, more likely opaque-product provider.

- **Price-Oriented** ($K = P$): customers estimate the opaque-product provider through suppliers’ posted prices, i.e., $\alpha(p, A, t) = p_2/(p_1 + p_2)$. Unlike the previous estimator, this one is likely to vary with time.

Most of our analysis in §7 is conducted under the assumption that the customers are of Normal type; we do, however, conduct some analysis for the case with Price-Oriented type as well.

**Supplier Participation.** To the best of our knowledge, Priceline.com does not publicly announce the suppliers that they partner with, or potential suppliers for a particular opaque product\(^{10}\). It is also rare for suppliers themselves to publicize that they are currently in business with an intermediary firm (say, Priceline.com). While the benefit for Priceline.com of not revealing this information is obvious (otherwise, the customers would be in a better position to get a low price\(^{11}\)), the reasons for the suppliers to keep this information hidden can go far beyond short-term monetary benefits/losses. One potential concern might be that claims of this type are costly to uphold—committing not to partner with a particular business unit is not a common business practice and

\(^{10}\) Other websites that sell opaque product at posted prices, such as Hotwire.com, may have a different policy and provide a general list of the suppliers they partner with.

\(^{11}\) On the other hand, at Hotwire.com customers do not have a say about the prices they would pay.
may hinder future opportunities. In some instances, this can be more detrimental than committing to the partnership, which, in our problem, comes with the flexibility of either being or not being the opaque-product provider. Further, if the “you-will-not-get-our-product-through-NYOP” guarantee is only intended for a limited period, the suppliers may later incur large switching cost in convincing their customers that the opposite is now true. Most retail businesses prefer to avoid such complexities and be silent on this issue. We, therefore, make the assumption that, unless a supplier runs out of stock, customers are not aware of the actual participation of that supplier.

4. The Customers

We analyze the problem in a backward manner, starting in this section with solving the customers’ problem. As can be seen from Figure 2, customers with valuation $v$ have three options to choose from: (1) buy directly from supplier 1 at $p_1$, (2) buy directly from supplier 2 at $p_2$, or (3) bid $b(v)$ with the intermediary firm. Even if a customer loses her bid with the intermediary firm, she can always go back and buy directly from one of the suppliers. According to our assumptions, customers observe the posted prices, $p$, and supplier availability, $A$, and estimate the reservation price as $\xi$ with density $g(\cdot|p,A,t)$ and cdf $G(\cdot|p,A,t)$. In this section, we omit the signal notations $(p,A,t)$ and use $g(\cdot)$ and $G(\cdot)$ for brevity.

4.1. One Supplier Runs Out of Stock

When one of the suppliers runs out of stock, customers observe $A = (0,1)$ or $(1,0)$. Thus, the product at the intermediary firm ceases to be an opaque product and becomes a regular one. It is straightforward that a customer will always use the NYOP channel in this scenario. Given that there is now only one product, with posted price $p = p_j$ and reservation price $r \sim \xi(p,t)$, the customer has to determine her bid, $b$, which maximizes her ex-ante surplus, $V$, where

$$V(v, b) = G(b)(v-b) + G(b)\max\{v-p, 0\} = \begin{cases} G(b)(p-b) + (v-p), & \text{if } v \geq p \\ G(b)(v-b), & \text{if } v < p. \end{cases}$$

The above holds regardless of the customer type. This leads to our first result.

**Proposition 1.** At time $t$, if only one supplier carries some stock in the market with posted price $p$, a customer would always attend the NYOP channel first. If the customer’s valuation for the in-stock supplier is $v$, the optimal bidding, $b^*$, should satisfy $b^* = \min\{v, p\}$. Specifically, 

(i). there exists a unique $b^* \leq \min\{v, p\}$ at which the customer will bid;

(ii). $b^*$ is non-decreasing in $v$ and $p$;

(iii). if $\xi \sim U[p, p]$ for some $p \in [0, p]$, the optimal bid $b^* = \frac{\min\{p, v\}}{2} + \frac{p}{2} \cdot (v > p)$.

Note that leaving without purchase is implicitly considered as bidding $b(v) = 0$ with the intermediary firm.
Thus, when granted the chance to get a “bargain” (at the NYOP channel) for a regular product, the customers would always attend the NYOP channel first and bid at a level lower than the posted price. A higher tag price raises the upper bound of customers’ estimation of the reservation price, and hence it has a positive impact on the bidding amount. Likewise, higher valuation suggests fewer financial constraints, which also leads to more generous bids.

Consider $p$ to be the supplier’s marginal cost for one unit of the product. When customers make a naive estimation on the reservation price ($\xi \sim U[p, p]$), (iii) suggests that the bid reflects sharing at least 50% of the supplier’s marginal profit. When the marginal cost is rather low, $p \to 0$ (e.g., theater tickets, airline seats, or hotel rooms), customers ask for at least a 50% discount from the posted price ($b^* \to \min\{p, v\}/2$). Such bold bidding behavior stems from the fact that (1) there is no risk involved with the kind of product that one would receive (opaqueness is gone), (2) one can always buy from the direct channel in case of a rejection. In section §6.1 we show that a monopolistic supplier offset such advantage-taking behavior by offering products through the direct channel only.

4.2. Both Suppliers Present

When both suppliers have available inventory, the intermediary firm can sell opaque products. Customers in this scenario have to be aware that there is a chance that they may be assigned a less-favored product when bidding in the NYOP channel.

Denote $\Omega_N = \{v : v_i - p_i < 0, i = 1, 2\}$ and $\Omega_i = \{v : v_i - p_i \geq 0, v_i - p_i \geq v_j - p_j\}$. In the absence of the intermediary NYOP firm, customers in $\Omega_N$ have no way to obtain either product, while customers in $\Omega_i$ prefer buying from supplier $i$ than from supplier $j$. When an NYOP channel is available, all customers in region $\Omega_N$ bid with the NYOP channel, as this is the only chance that they might get any product. Customers in the other two regions have to decide whether to name their own price first (and buy at the posted price if the bid is not accepted) or to buy directly from their preferred supplier.

Consider a type-$K$ customer with valuation $v$ who chooses to bid $b$. Let $\alpha^K_i = 1 - \alpha^K_j$ denote her belief that supplier $i$ is the opaque-product provider, which may depend upon $p$, $\bar{x}$, and $t$, but not on the bid $b$. We, therefore, in this section abbreviate $\alpha^K_i(p, \bar{x}, t)$ as $\alpha^K_i$, $i = 1, 2$. The customer’s expected payoff is then

$$V(v, b) = G(b)(\alpha^K_1 v_1 + \alpha^K_2 v_2 - b) + \bar{G}(b) \max\{v_1 - p_1, v_2 - p_2, 0\}.$$ 

The following proposition characterizes the optimal bid for each customer if she is forced to attend the NYOP channel.

**Proposition 2.** At time $t$, suppose that both suppliers carry some stock with posted price $(p_1, p_2)$, and that customer has valuation $v$. Then,
(i). there exists a unique $b^*(\mathbf{v}) \in [0, p]$ at which the customer will bid;

(ii). if $\mathbf{v} \in \Omega_N$, $b^*(\mathbf{v})$ increases with $\alpha^K_i v_1 + \alpha^K_2 v_2$; if $\mathbf{v} \in \Omega_i$ for $i \in \{1, 2\}$, $b^*(\mathbf{v})$ decreases with $v_i - v_j$;

(iii). $b^*(\mathbf{v}) < \alpha^K_1 v_1 + \alpha^K_2 v_2$.

(iv). $b^*(\mathbf{v}) = 0$ if and only if $\mathbf{v} \in \Omega_i$, and $v_i - v_j > p_i/\alpha^K_j$ for $i \in \{1, 2\}$;

Note that Proposition 2 implies that not every customer would attend NYOP channel. (i) identifies how customers value the opaque product: the optimal bidding, $b^*(\mathbf{v})$, satisfies\(^{13}\)

$$b^* + \frac{G(b^*)}{g(b^*)} = \begin{cases} \alpha^K_i v_1 + \alpha^K_2 v_2 & \text{if } \mathbf{v} \in \Omega_N \\ \alpha^K_j (v_j - v_i) + p_i & \text{if } \mathbf{v} \in \Omega_i \text{ where } i = 1, 2. \end{cases}$$

From (ii), it is interesting to note that if a customer cannot afford either product at the posted price (i.e., $\mathbf{v} \in \Omega_N$), her bid is related to her expected valuation of the opaque product, $\alpha^K_i v_1 + \alpha^K_2 v_2$; on the other hand, if a customer has the option to use the direct channel (i.e., $\mathbf{v} \in \Omega_i$ for $i \in \{1, 2\}$), her bid is only related to the difference between her valuations of the two products, $v_i - v_j$. Indeed, when NYOP is the only channel through which one can obtain some product, the customer’s bid has to reflect the expected value that she may receive. However, when a customer participates in the NYOP channel only to “gamble” and see if she can get her preferred product at a lower price, she needs to take into account how much more she values one product over the other, as NYOP comes with the risk of receiving the less-preferred product.

(iii) implies that a customer will always bid below her valuation of the opaque product. In other words, there is a non-trivial information rent that each customer would deduct from the expected value of the opaque product. This rent is $G(b^*)/g(b^*)$ for customers in $\Omega_N$, and $G(b^*)/g(b^*) + v_i - p_i$ for those who belong to $\Omega_i$, $i = 1, 2$. In the former case, the rent is increasing with the expected value, $\alpha^K_i v_1 + \alpha^K_2 v_2$, while in the latter case it may also increase with $v_i$, customer’s valuation of the product that she prefers in the direct channel.

Recall that (ii) implies that customers place a relatively low bid in the NYOP system when $v_i - v_j$ is high, which makes it unlikely that they will meet the reservation price. This leads to (iv), which implies that customers participate in the NYOP channel only when $v_i - v_j$ is below some threshold; otherwise, they buy directly from their preferred supplier to avoid the risk of receiving the less-preferred product.

Let

$$\Omega_i(\delta) = \begin{cases} \Omega_i \cap \{\mathbf{v} : v_i - v_j \leq \delta\}, & \text{if } i = 1, 2 \\ \Omega_i \cap \{\mathbf{v} : \alpha^K_i v_1 + \alpha^K_2 v_2 \geq \delta\}, & \text{if } i = N, \end{cases}$$

then $\Omega_i(\delta)$, $i = 1, 2$, denotes the set of customers who prefer supplier $i$ in the absence of the NYOP channel, but have the valuation difference less than $\delta$, while $\Omega_N(\delta)$ denotes those who cannot afford

\(^{13}\) If the solution is out of the the range of $[0, p]$ then corner solution applies.
either product at the posted price, but their valuations of the opaque product exceed \( \delta \). We then have the following result.

**Proposition 3.**
(i). Customers with \( \nu \in \Omega_N \) will always attend the NYOP channel;
(ii). For \( i \in \{1, 2\} \), there exists \( \delta^*_i \) such that a customer with valuation \( \nu \in \Omega_i(\delta^*_i) \) will use the NYOP channel. Specifically, \( \delta^*_i = p_i/\alpha^K_j \).

Proposition 3 and Proposition 2 (iv) jointly suggest that whenever \( b(\nu) \) is nonzero, a customer will choose to name her own price first, before buying directly from a supplier. We next provide an illustrative example for our results.

**Example 1.** Assumes that customers are of type \( K = N(0.5) \) (i.e., \( \alpha^K_1 = \alpha^K_2 = 0.5 \)), and \( G(b) = \frac{2bp - b^2}{p^2} \). It can be verified that \( \frac{\partial G(b)}{\partial b} < 0 \), so \( G(b) \) is DRHR. Figures 3 and 4 depict how the customers’ choice of the channel and bidding amount changes with posted prices and their personal valuations, respectively.

![Figure 3](image-url)  
**Figure 3** Channel selection for customers with different valuations given posted prices.

When the posted prices are equally small, say \( p_1 = p_2 = 0.2 \), Proposition 3 suggests that \( \delta^*_1 = \delta^*_2 = p_i/\alpha^K_j = 0.4 \). In this case, only customers who significantly discriminate between the two products buy from direct channels; customers who cannot afford either product at the posted price or those who have similar valuations for the two products use the NYOP system. The maximum bid is less than 50% of the lowest posted price. As one of the posted prices increases, say \( p_1 = 0.2 \) and \( p_2 = 0.4 \), \( \delta^*_1 = 0.4 \) and \( \delta^*_2 = 0.8 \). We can see that more customers choose to bid at the NYOP channel before direct purchase, and the maximum bid increases as well. When \( p_1 = 0.2 \) and \( p_2 = 0.8 \), \( \delta^*_1 \) stays the same while \( \delta^*_2 = 1 \). Customers who prefer supplier 1 still prefer to buy directly from him; however, those buying directly from supplier 2 abandon the posted price channel and use NYOP instead. The maximum bid can be as much as 75% of the lowest posted price.

It can also be noticed from Figure 4 that when \( p_i \) is fixed, each customer’s bid is non-decreasing with \( p_j \). For example, throughout the three cases, customers with \( v_1 - v_2 \in \{0, 0.4\} \) consistently choose to buy from supplier 1 if NYOP does not work out (partially due to the fact that the change...
in \( p_2 \) does not affect their acceptance probability for a given bid). Their bids are, therefore, not affected by the posted price of the other supplier, \( p_2 \). The bids of customers with \( v_1 - v_2 \in \{-0.8, 0\} \) increase when \( p_2 \) is 0.4 instead of 0.2. However, the underlying reasons for the increase differ across the customers. For example, for a customer with \( v_1 - v_2 \in \{-0.2, 0\} \) the exit option (in case the bid for NYOP is not accepted) shifts from supplier 2 to supplier 1 as \( p_2 \) increases. Hence, the NYOP channel becomes more attractive to this type of customers, especially those who do not value supplier 1 too much. On the other hand, for a customer with \( v_1 - v_2 \in \{-0.8, -0.2\} \) the exit option is consistently supplier 2, despite of the increase in \( p_2 \). However, her optimal payoff from the direct channel decreases, which motivates her to place a higher bid and thus increase the chance of getting the product through the NYOP channel and mitigate the loss in her bottom-line payoff.

5. Final Sales Channel

In the previous section, customers made their initial channel selections based on their private valuations, observed posted prices, and their estimates of reservation price and opaque-product provider. The disparity between the estimated and the real reservation price/opaque-product provider may result in some unsuccessful bids. As the direct channel is always open to all customers, the channel participation will change after some customers learn that their bids were not accepted by the NYOP channel. In this section, we discuss how the final sales channel is affected by the suppliers’ posted price, \( p \), and reservation price, \( r \). The following notation will be used throughout the rest of the paper:

- \( \mathcal{B}_0(\mathbf{r}, \mathbf{p}, \mathbf{x}) = \{ \mathbf{v} : b(\mathbf{v}, \mathbf{p}, \mathbf{x}) \geq r \} \) — the set of customers whose bids are accepted at NYOP.
- \( \mathcal{B}_i(\mathbf{r}, \mathbf{p}, \mathbf{x}) = \{ \mathbf{v} : b(\mathbf{v}, \mathbf{p}, \mathbf{x}) < r, v_i - v_j \geq p_i - p_j, v_i \geq p_i \} \) — the set of customers who will purchase from the direct channel of supplier \( i \). This includes both the customers who buy at the posted price in the first place (\( b = 0 \)) and those who lose their bid with the NYOP channel and go back to the direct channel afterwards (\( 0 < b < r \)).
Huang and Sošić: Name-Your-Own-Price as a Competitive Distribution Channel in the Presence of Posted Prices

- \( B_N(r, p, x) = \{v : b(v, p, x) < r, v_i < p_i, v_j < p_j\} \) — the set of customers who will leave empty-handed (or, equivalently, those whose bids were rejected at the NYOP channel and who cannot afford any of the posted prices).

Recall that customers’ bidding behavior changes as the number of in-stock suppliers varies; thus, we again analyze two different scenarios corresponding to possible values of \( A \).

5.1. One Supplier Runs Out of Stock

When one of the suppliers runs out of stock, customers observe \( A = (0, 1) \) or \( (1, 0) \), and there is no competition. Customers observe a unique product (without loss of generality, assume it is product \( j \)) with posted price \( p \), and adopt certain prior, \( \xi \), for the reservation price \( r \). According to Proposition 1, all customers bid with the NYOP firm first, and consider buying at the posted price only if their bid is rejected.

**Proposition 4.** When one supplier is out of stock, the final sales channel is characterized by
1. if \( r > r_0 \), then
   \[ B_0(r, p, x) = \emptyset, \quad B_j(r, p, x) = \{v : v_j > p\}, \quad B_N(r, p, x) = \{v : v_j < p\}; \]
2. if \( r \leq r_0 \), then
   \[ B_0(r, p, x) = \{v : v_j > r + G(r) g(r)\}, \quad B_j(r, p, x) = \emptyset, \quad B_N(r, p, x) = \{v : v_j < r + G(r) g(r)\}. \]

where \( r_0 \) uniquely satisfies \( r_0 + \frac{G(r)}{g(r)} = p \).

Since only one supplier is in stock, the NYOP channel is practically selling regular goods, and customers bid according to Proposition 1. In this case, if the reservation price is high \( (r > r_0) \), all bids could not go through the NYOP channel as they will not meet the reservation price. Then, the customers with valuation above the posted price \( p \) will buy from the direct channel, while the remaining customers leave empty-handed. On the other hand, if the reservation price is low \( (r \leq r_0) \), a fraction of the bids will be accepted and no one will buy through the direct channel (since anyone who is able to afford the product at posted price will make a bid above the reservation price). From the supplier’s point of view, one channel will be idle under either kind of pricing schemes.

**Example 2.** Suppose the prior is defined as in Proposition 1 (iii). Then, \( r_0 = \frac{p + p}{2} \) and \( r + \frac{G(r)}{g(r)} = 2r - p \). If the reservation price is set to be above the prior, no one will win her bid from NYOP; otherwise, if the reservation price is below the prior, those customers with valuation above \( 2r - p \) will win their bids, and the others will leave without any purchase.

5.2. Both Suppliers Present

When both suppliers have some inventory (i.e., \( A = (1, 1) \)), the customers bidding below the reservation price \( r \) will be rejected and will join the direct channel. The final channel realization is characterized as follows.
Proposition 5. When both suppliers are in stock, the final sales channel is characterized by

\[
\mathcal{B}_0(r, p, x) = \bigcup_{i=1,2,N} \Omega_i(\delta_i(r))
\]

(4)

\[
\mathcal{B}_i(r, p, x) = \Omega_i \setminus \Omega_i(\delta_i(r)), \quad i = 1, 2, N
\]

(5)

where

\[
\delta_i(r) = \begin{cases} 
\frac{p_i - r - G(r)/g(r)}{\alpha_j} & \text{if } i = 1, 2 \\
 r + G(r)/g(r) & \text{if } i = N.
\end{cases}
\]

\(\Omega_i(\delta_i(r))\) is the set of customers that bid higher than \(r\) yet will buy from supplier \(i\) in the absence of the NYOP channel. \(\Omega_N(\delta_N(r))\) are the customers naming a price higher than \(r\), but who cannot afford either of the two posted prices.

The following example illustrates the general idea of Proposition 5 and allows a closer look into how the market segmentation varies with reservation price \(r\).

Example 3. Assume customers adopt the same prior as in Example 1. Figure 5 illustrates the channels at which the customers will obtain their products in the end. Throughout the examples, we let \(p = (0.2, 0.4)\). By varying the minimum reservation price from 0.04 through 0.08 to 0.1, we depict the sets of customers that will win the product from the NYOP channel, buy it from supplier 1/supplier 2, or leave empty handed. The dotted lines correspond to the second graph in Figure 3, showing the initial channel selection of the customers. Hence, when \(r = 0.04\), customers with \(v\) in the polygon bounded by \((0, 0.63), (0, 0.77), (0.23, 1), (0.37, 1)\), which we will denote as \(v \in \mathcal{P}\{ (0, 0.63), (0, 0.77), (0.23, 1), (0.37, 1) \}\), choose to name their own prices \(b(v)\). By Proposition 2, their bids are all less than 0.04, and are therefore rejected. These customers eventually buy from supplier 2 after learning about the rejection. White areas represent the set of customers who will be awarded a unit of the products from the NYOP channel by bidding above \(r\), while black areas represent customers who can neither afford the posted prices nor bid above \(r\).

![Figure 5](image.png)
6. Optimal Pricing Decisions for the Suppliers and NYOP Firm

We now analyze the first stage in Figure 1 and study how suppliers determine their posted prices as well as the reservation price, taking into account the customers’ strategies in the second stage.

6.1. One Supplier Runs Out of Stock

When only one supplier, say \( j \), offers his products through the intermediary firm, then at the beginning of each time epoch he makes a decision on \( p \) and \( r \) without taking competitive factors into consideration. Let us denote by \( \Pi_j(p,r;x,t) \) the expected profit supplier \( j \) will receive when the posted and reservation prices for period \( t \leq T \) are \( p \) and \( r \), respectively. The customers’ bidding behavior and final channel realization were characterized in Proposition 1 and Proposition 4, respectively. Then, supplier \( j \)’s spot revenue is given by

\[
\pi_j(p,r) = \begin{cases} 
\bar{F}_j(p)p, & \text{if } r > r_0 \\
\bar{F}_j\left(r + \frac{G(r)}{g(r)}\right)r & \text{if } r \leq r_0,
\end{cases}
\]

where \( r_0 \) is defined in Proposition 4.

At \( t = 1 \), supplier \( j \) will seek a pair \((p,r)\) that maximizes \( \pi_j(p,r;x) \). At \( t > 1 \), we use dynamic programming to determine the best \((p,r)\):

\[
\Pi_j(p,r;x,t) = \max_{p \geq r \geq 0} \left\{ \bar{F}_j(p) [p + \Pi_j(x - e_j, t - 1)] + F_j(p) \Pi_j(x, t - 1), \quad \text{if } r > r_0 \right. \\
\left. \bar{F}_j\left(r + \frac{G(r)}{g(r)}\right) [r + \Pi_j(x - e_j, t - 1)] + F_j\left(r + \frac{G(r)}{g(r)}\right) \Pi_j(x, t - 1) \quad \text{if } r \leq r_0, \right\}
\]

where \( e_j \) is a \( 1 \times 2 \) vector with the \( j \)th component equal to 1, and zeros everywhere else.

**Theorem 1.** A monopolistic supplier maximizes his expected revenue by using only posted prices in the direct channel.

In other words, if a monopolist supplier can sell his products directly at posted prices, or as opaque goods through the NYOP channel, or both, he will always choose to go with posted prices only. As Proposition 4 implies, a combination of the two channels is not necessary because there will always be one channel that is idle. Hence, the problem is reduced to selecting the channel that brings in more revenue (direct channel with posted prices or NYOP). A closer comparison of (2) and (3) suggests that the supplier has access to the same set of customers under either channel when \( r = r_0 \). However, if the NYOP channel is chosen, the supplier would collect less than he could with a posted price \((r = r_0 \leq p)\) because the third-party (intermediary firm) will share a portion of the revenue.

Uniformly distributed valuation \( v_j \) allows us to provide an exact solution for the pricing strategy, as well as the expected revenue, as a function of the remaining time and available inventory. This is shown in the following example:
Example 4. Assume \((v_i, v_j) \sim U[0,1] \times [0,1]\), \(x_i = 0\) and \(x_j > 0\). Supplier \(j\)'s spot revenue can be expressed as

\[
\pi_j(p, r; x) = \begin{cases} 
p - p^2, & \text{if } 2r > p \\
r - 2r^2, & \text{if } 2r \leq p.
\end{cases}
\]

Then, for \(t = 1\), \(x_i = 0\), and \(x_j = 1\), it can be verified that the optimal pair \((p, r)\) equals \((0.5, 0.5)\) and \(\Pi_j(x, 1) = 0.25\). At \(t > 1\), optimal pair \((p, r)\) is given by

\[
p^* = r^* = \frac{1 + \Pi_j(x, t-1) - \Pi_j(x - e_j, t-1)}{2}
\]

and

\[
\Pi_j(x, t) = \Pi_j(x - e_j, t-1) + \left(\frac{1 + \Pi_j(x, t-1) - \Pi_j(x - e_j, t-1)}{2}\right)^2.
\]

It is not hard to verify that \(\Pi_j(x, t)\) is nondecreasing with \(t\). It can also be shown that \(\lim_{t \to \infty} \Pi_j(x, t) = 1\) when \((x_i, x_j) = (0,1)\): as time goes by, the optimal \(p^* = r^*\) drops from 1 to 0.5. The NYOP channel does not bring in any additional profit in an environment characterized by lack of competition.

6.2. Both Suppliers Present

Note that our discussion so far (customer behavior and final channel realization) does not require any particular assumption about the customer valuation distribution. Although we are more interested in two-dimensional general distributions of \(v\), some simplified analysis provides helpful insights as well. We begin this subsection by looking at two special sets of customers and derive some relevant managerial insights; the general problem is then described and characterized in \S 6.2.2.
6.2.1. Vertically vs. Horizontally Differentiated Customers

For customers with belief \((\alpha^K_1, \alpha^K_2)\), we will say that the customers are:

- *vertically differentiated* if \(v_i - v_j = \bar{v}\) for some constant \(\bar{v} > 0\) and \(i \in \{1, 2\}, j = 3 - i\);
- *horizontally differentiated* if \(\alpha^K_i v_i + \alpha^K_j v_j = \bar{v}\) for some constant \(\bar{v} > 0\) and \(i, j \in \{1, 2\}\).

The horizontal differentiation allows customers to have a common expectation regarding the opaque product, but potentially very different opinions about the regular ones. In other words, should customer A value product 1 more than customer B does, A’s valuation of product 2 would be lower than that of B. In some degree, this is similar to the Hotelling’s model of horizontal differentiation. Vertical differentiation, on the other hand, implies that one’s valuation of a product grows along with her valuation of the other one. Thus, a high-end customer is of high-end type for both products, and vice versa. These special instances lead to following results.

**Proposition 6.**

1. If the products are vertically differentiated (i.e., \(v_i - v_j = \bar{v}\) for some constant \(\bar{v} > 0\)), then supplier \(i\) sets a posted price such that all customers buy from its direct channel. The intermediary firm collects zero rents.

2. If the products are horizontally differentiated (i.e., \(\alpha^K_i v_i + \alpha^K_j v_j = \bar{v}\) for some constant \(\bar{v} > 0\)), then there exists \(p_0 > 0\) such that (a) when the lower posted price, \(p\), satisfies \(p > p_0\), all customers are covered, and the intermediary firm earns positive profit; (b) when \(p \leq p_0\), some customers will not receive any of the products, and no customers will use NYOP.

The implication of the preceding result is that the intermediary firm should be careful when selecting regular products to construct opaque products in the NYOP channel. If it has been recognized that product candidates have a significant quality difference (a flight departing at 1 am vs. a flight on the same route that leaves at noon) or one product has higher brand recognition (such that all customers may prefer one particular product, although the quality of the other is the same), then the intermediary firm may not benefit much from these differences. The main reason is that the supplier with “better quality/image” can obtain the deterministic difference in customer valuation by offering a posted price that customers “cannot refuse”. Under this posted price, the customers are convinced that the opaque product is not worth bidding on. On the other hand, if the products are horizontally differentiated (e.g., hotels in the same region and with the same star ranking), the valuation for the ex-post product will be much more diversified. The suppliers would then need the intermediary firm to shoulder part of this uncertainty such that their direct channels can target their own high-end customers. These results are consistent with the observations in Perkins (2006).
6.2.2. **General Customers** At the beginning of each time epoch \( t \), the suppliers observe their inventory positions \( \mathbf{x} = (x_1, x_2) \). For analytical convenience, we assume that the inventory position \( x_i \) is visible to both the supplier \( i \) itself and his competitor \( j \); similar assumptions have been made by many papers analyzing competition in revenue management (e.g., Gallego and Hu 2007; Levin et al. 2009; Lin and Sibdari 2009). For imperfect information on parameters such as demand distributions or inventory levels, one may refer to Perakis and Sood (2006), Levin et al. (2008), Zhang and Kallesen (2009), etc. As our primary goal is to study how the presence of an NYOP channel may affect the strategic decisions of suppliers, this assumption falls within the scope of the paper.

After observing inventory level \( \mathbf{x} \), the suppliers competitively determine their posted prices, \( \mathbf{p} = (p_1, p_2) \). Denote by \( \Pi_i(\mathbf{p}; \mathbf{x}, t) \) the expected revenue for supplier \( i \) at the beginning of period \( t \) given posted prices \( \mathbf{p} \); by \( \mathbf{p}^*(\mathbf{x}, t) \) the equilibrium posted price for period \( t \), if one exists; and by \( \Pi_i(\mathbf{x}, t) = \Pi_i(\mathbf{p}^*(\mathbf{x}, t); \mathbf{x}, t) \) the expected revenue for supplier \( i \) at the beginning of period \( t \), given that posted prices correspond to the equilibrium ones (i.e., \( \mathbf{p} = \mathbf{p}^*(\mathbf{x}, t) \)). Obviously, \( \Pi_i(\mathbf{x}, 0) = 0 \) and \( \Pi_i(\mathbf{x}, t)|_{x_i=0} = 0 \). When submitting the reservation price to the intermediary firm and assuming his competitor does not change his decision, a supplier has to decide whether he may accept a reservation price lower than the current one, \( r - \epsilon \) instead of \( r \), where \( \epsilon \) is the minimum decrement designed by the intermediary firm.

Suppose that supplier 2 has reservation price \( r \); then, supplier 1’s problem is given by

\[
\Pi_1(\mathbf{x}, t) = \max_{r \leq p} \left\{ \left[ p_1 + \Pi_1(x_1 - 1, x_2, t - 1) \right] Pr\{v \in \mathcal{B}_1(r, \mathbf{p}, \mathbf{x}) \} + \Pi_1(x_1, x_2 - 1, t - 1) Pr\{v \in \mathcal{B}_0(r, \mathbf{p}, \mathbf{x}) \} + \Pi_1(x_1, x_2 - 1, t - 1) Pr\{v \in \mathcal{B}_n(r, \mathbf{p}, \mathbf{x}) \} \right\}.
\]

(6)

At time \( t \), given posted prices \( \mathbf{p} \), inventory level \( \mathbf{x} \), and properly designed \( \epsilon \), there exists \( r^*(\mathbf{p}, \mathbf{x}, \epsilon, t) \in [0, p] \) and a supplier \( i \in \{1, 2\} \) who is willing to accept reservation price \( r^* \) when the other supplier, \( -i \), does not. Define

\[
\tilde{\Pi}_1(\mathbf{x}, t) = \Pi_1(x_1, x_2 - 1, t) - \Pi_1(x_1 - 1, x_2, t), \quad (7a)
\]

\[
\tilde{\Pi}_2(\mathbf{x}, t) = \Pi_2(x_1 - 1, x_2, t) - \Pi_2(x_1, x_2 - 1, t); \quad (7b)
\]

(7) provides the threshold prices below which a particular supplier would drop from the auction. Obviously, the one with a higher threshold, say 1, is less tolerant in this auction and will give up
his candidacy as the opaque provider at an earlier time. His rival, supplier 2, will stay to serve the NYOP channel. Similarly to the second-price auction, the NYOP firm rewards supplier 2 (assuming he stays in the game) as high as \( \tilde{\Pi}_1 \), even though his actual threshold price may be lower. We can now state the following result.

**Proposition 7. (Reservation Prices)**

\[
r^*(x, t) = \lim_{\epsilon \to 0} r^*(p, x, \epsilon, t) = \min\{p, \tilde{\Pi}_i(x, t - 1) \},
\]

where \( i = \arg\max_{i=1,2} \tilde{\Pi}_i(x, t - 1) \). Specifically, at \( t = 1 \) and in the presence of competition \( (x_i > 0, x_j > 0) \), the reservation price is given by \( r^* = 0 \).

Note that if \( v_1 \) and \( v_2 \) are identically distributed and \( x_1 = x_2 \), we have \( \tilde{\Pi}_1(x, t) = \tilde{\Pi}_2(x, t) \) and both suppliers stop at \( r^* \), unwilling to accept lower reservation prices. The intermediary firm can then allocate the customers with a half-half probability to the suppliers with the same reservation price, \( r^* \). One can verify that Proposition 7 still holds (after appropriate modification of (6)).

Although the suppliers determine their reservation prices after the posted prices are announced, Proposition 7 suggests that, besides being bounded above by it, \( r \) depends very little on the minimum posted price, \( p \). In addition, Proposition 7 implies that it is possible for the suppliers to post non-trivial tag prices even at the last minute, albeit the fact that they are offering last-minute sales leads to arbitrary bids through NYOP.

It is also worth mentioning that although customers in general cannot observe real inventory level, hence they have to adopt certain prior in estimating the reservation price and opaque-product provider, this is not true when \( t = 1 \). Proposition 7 suggests that \( r^* = 0 \) in the last selling period regardless of the inventory level; customers may well take this fact into account when deciding their bid\(^ {14} \). In this case, it can be verified that the customers in \( \Omega_i(\epsilon_i) \) will buy directly from supplier \( i \); if a customer is not located in these areas, she will bid 0 at the NYOP channel and be matched with either product. Thus, at \( t = 1 \) the suppliers should choose the posted price that maximize their revenue in the direct channel (as no revenue will come from the opaque channel). For example, if \( (v_1, v_2) \sim U[0, 1] \times [0, 1] \), the last-period posted price for either supplier should be \( p_1 = p_2 = \arg\max_{0 \leq p \leq 0.5} p(1 - 2p)^2 = 1/6 \).

To verify the existence of pure-strategy Nash equilibrium (NE) \((p_1^*(x, t), p_2^*(x, t))\), we need to check if the payoff function in (6) is quasi-concave in \( p \). While this result seems analytically intractable for general distributions of \( v \) or for Price-Oriented customers \((K = P)\), it can be proved that it holds for uniformly distributed valuations \((v_1, v_2) \sim [0, 1] \times [0, 1] \) with Natural type of customers \((K = N(\alpha))\). Numerical experiments suggests that similar results hold in more general instances, for other distributions and customer types.

\(^ {14} \) In our model, this is the only moment in which customers do not need the prior.
Theorem 2. Consider Natural type of customers ($K = N(\alpha)$ for some $0 < \alpha < 1$) and assume that their valuations $v$ are uniformly distributed on $[0,1] \times [0,1]$. Then,

1. pure-strategy NE in posted prices exists;
2. reservation price $r^*$ is zero when both suppliers oversupply (i.e., $x_i \geq t$ for $i = 1, 2$).

Theorem 2 (i) allows us to perform numerical analysis in the next section, and Theorem 2 (ii) extends the result from Proposition 7, which states that oversupply at both suppliers at any time leads to zero reservation price. We discuss our computational results in more detail in the next section.

7. Numerical Analysis

We next look at numerical solutions for reservation prices and equilibrium decisions in posted prices, given the current state $(x_1, x_2, t)$. In this section, customer valuation $(v_1, v_2)$ is assumed to be uniformly distributed on $[0,1] \times [0,1]$. Unless otherwise indicated, the customer type is Natural with $K = N(0.5)$; we provide comparison between Natural and Price-Oriented types of customers at the end of the section. Both dual-channel (posted prices and NYOP) and single-channel (posted price only) strategies are examined. We will denote the dual-channel case by “DC” and the single-channel case by “SC” in this section.

7.1. Expected Revenue

We discuss the expected revenue for a particular supplier from two perspectives: we first assume that the competitor’s inventory level is given and examine how one’s revenue would change with the remaining number of selling periods and his own inventory level; we then fixed the time point and look at how the competitor’s inventory level would affect one’s expected revenue. Throughout this section we also compare revenues under dual-channel (DC) versus single-channel (SC).

7.1.1. Varying Time

Figure 6 depicts how the supplier’s expected revenue varies with the remaining sales time $t$ and his own inventory level $x_i$, while his rival’s inventory level $x_j$ is fixed.

- Effect of Time. Not surprisingly, the expected revenue grows with the length of the remaining time in both cases. For SC, the marginal revenue of one additional sales period, $\Pi_1(x_1, 5, t) - \Pi_1(x_1, 5, t - 1)$, decreases with $t$. For DC, the marginal revenue of time is decreasing only when some supplier does not oversupply (i.e., $\min\{x_1, x_2\} < t$). As Theorem 2 (ii) indicates, if both suppliers oversupply (i.e., $x_{1,2} \geq t$), each supplier strives to sell his inventories, and the NYOP channel intensifies the competition, which leads to a zero reservation price. In these instances, one more sales period (or equivalently, one more unit of random-valued demand) is contributing

15 We have also conducted the same analysis based on normal distribution. The results are similar to those obtained for uniform distribution and are omitted for brevity.
positively to the expected revenue, as this unit has a non-trivial chance to be sold at some posted price. This time-effect depends only on the fact that there is currently an over-supply, not on the specific inventory levels.

- **Effect of Inventory Levels.** While in SC the expected revenue grows with one’s own inventory, $x_1$, the same does not hold in DC, especially when $x_1 + x_2 \geq t$. Hence, while in SC “more is better,” in DC “more may be worse, especially when total supply exceeds total demand.” The diminishing margin of inventory holds under similar conditions for the two cases.

- **Comparison between DC and SC.** When comparing the two graphs in Figure 6, one may notice that the expected revenue is higher in SC than in DC. The difference in expected revenue is more significant as competition becomes more intense (i.e., $x_1$ becomes higher). The low revenue in DC generally comes from two sources. First, the NYOP channel provides a platform on which suppliers compete to sell at a low price without damaging the integrity of their direct channel. As competition becomes more intense, the reservation price at the NYOP channel could become rather low (eventually becoming $r = 0$ in case of over-supply). Thus, there is a reasonable chance that some product quantity will be sold at lower prices in DC than SC. Second, the posted price competition becomes more intense as well. In the absence of the NYOP channel, suppliers compete *publicly* through their posted prices only, and there is less uncertainty about whose product the customer will eventually buy. On the other hand, with the NYOP channel each supplier is aware that the other seller may “steal” his customer through NYOP. Hence, each supplier is competing with the part of the opaque product that comes from his rival, as well as with other regular products in the market. Figure 8 also shows that posted prices in the presence of the NYOP channel usually have lower values than when this channel is absent. This provides experimental evidence that the NYOP channel intensifies the competition between the suppliers.
7.1.2. Fixed Time  In Figure 7, we examine how the expected revenue varies with inventory levels, $x_1$ and $x_2$, at fixed time $t = 8$. Once again, we note that SC generates higher expected revenue than DC.

![Figure 7](image)

**Figure 7**  Expected revenue for supplier 1 as a function of available inventories, $(x_1, x_2)$, at $t = 8$.

- **Effects of Inventory Level.** The marginal revenue of one’s own inventory is decreasing, but it remains non-negative in SC. On the other hand, the margin in DC is also decreasing but can become negative when the inventory level is within a certain range. Specifically, when $x_2 < \left[ \frac{t}{2} \right] + 1 = 5$, the margin for $x_1$ is non-negative, but when $x_2 \geq \left[ \frac{t}{2} \right] + 1 = 5$, the margin can be negative for $\left[ \frac{t}{2} \right] + 1 = 5 \leq x_1 \leq 8 = t$. For example, when $x_2 = 6 > 5$, $\Pi_1$ increases with $x_1$ as long as $x_1 \leq 4$, it decreases as $x_1$ changes from 5 to 8, and becomes zero for $x_1 > 8$; when $x_2 = 2 \leq 5$, $\Pi_1$ is non-decreasing with $x_1$ all the time. Because we assume $t = 8$, there will be a total of 8 arriving customers, each with unknown valuation. Since we assumed that $v$ is uniformly distributed, there is practically no difference between the two suppliers, and one can think of inventory level 4 as the “quota” level that splits the potential demand. When the rival stocks no more than this quota ($x_2 < 5$), the game is less competitive, and the suppliers’ expected revenue functions have similar shapes in DC and SC. When the rival stocks more than the quota ($x_2 \geq 5$) but the supplier himself is within the quota ($x_1 < 5$), the rival estimates that the supplier will behave less aggressively because it seems reasonable to expect that the supplier with less inventory will “sell less at a high price” instead of “price low and sell more.” When both suppliers stock over the quota ($x_{1,2} \geq 5$), each supplier believes his rival will take some competitive actions, and each has the motivation to do the same, as he has the resource (high inventory levels) for achieving a greater market share. Both posted prices and reservation prices become low as a result of the severe competition, which leads to the counterintuitive result in which more inventory may lead to lower revenue.
7.2. Posted Price in Equilibrium

Similarly to the expected revenue, we discuss the equilibrium posted price from two different perspectives as well.

7.2.1. VARYING TIME

Figure 8 is a counterpart of Figure 6, as it depicts equilibrium prices in the direct channel under the same conditions.

- **Effect of Time and Inventory Level.** The posted price in SC decreases with the rival’s inventory level and elapsed time; the marginal value of time can be either decreasing (when the rival’s inventory level is low) or increasing (when the rival’s inventory level is high). In DC, we observe the same general trend with respect to posted prices. Nevertheless, we may observe significant differences between the two graphs. According to Theorem 2 (ii), reservation price will be zero (also shown in Figure 11) if both suppliers oversupply (i.e., \( x_2 = 5 \geq t \) and \( x_1 \geq t \)). When this happens, the posted price also takes a very low value (around 0.167). This also explains why all of the lines with \( x_1 \in \{5, 6, 7, 8, 9, 10\} \) coincide when \( t \in [1, 5] \). Moreover, when a supplier has a greater inventory \( (x_1 > x_2 = 5) \) and his rival does not \( (t > x_2 = 5) \), his posted price stays at a moderate level. On one hand, he does not price too low because the rival is not oversupplying; on the other hand, it is not necessary to price too high since he has enough inventory. As one may be aware, many of the inconsistencies stem from oversupply. If we remove the oversupply effects by concentrating on smaller inventory levels, the graphs for SC and CD may look more alike (Figure 9).

- **Comparison between DC and SC.** Although the expected revenues are overall higher in SC than in DC, the same is not true for posted prices. For example, Figure 8 shows that when \( x_2 = 5 \), the posted prices in SC are generally higher when \( t \leq 5 \). However, as \( t \) becomes greater than 5 (supplier 2 is no longer oversupplying), posted prices in SC are lower than those in DC for some large values of \( x_1 \). Thus, with enough periods to go, the overstocking supplier will target his direct
7.2.2. Fixed Time At a given time period, say $t = 8$ (Figure 10), a supplier’s posted prices are rather sensitive to inventory level in DC when his rival stocks over the “quota” ($\lfloor \frac{t}{2} \rfloor$). When it is, indeed, the case that $x_2 > \lfloor \frac{t}{2} \rfloor = 4$, a supplier’s posted price first decreases with his inventory, $x_1$, until it hits $x_2$; then it begins to increase with $x_1$ after $x_1$ exceeds $x_2$. One of the possible reasons for this can be found in our discussion in §7.4, which shows that when $x_2 \geq \lfloor \frac{t}{2} \rfloor$, the opaque-product provider will be “supplier 2 → supplier 1 → supplier 2” as $x_1$ increases. This result suggests that a supplier’s posted price should be higher when he is not the opaque provider than when he is, because if his competitor (the opaque-product provider) is getting the low-end customers or the customers who do not differentiate much between the products, then it is better for the supplier to focus on his own loyal market (those customers who have strong preferences for him over his competitor). One effective way to identify such customers, and to improve the revenue, is through charging a higher price.

7.3. Reservation Price
Recall that Proposition 7 shows that the reservation price is the lower marginal value of a unit of either product. As can be seen below, reservation price shows much more consistency than posted prices. In general, it decreases with remaining time or inventory levels. Figure 11 is a counterpart of Figures 6 and 8, and it shows the corresponding reservation prices when oversupply is likely to happen ($x_2 = 5$); Figure 12 is a counterpart of 9 when oversupply is less likely to happen ($x_2 = 1$). In both cases, reservation prices are zero when $t \leq \min\{x_1, x_2\}$, and they then increase with the remaining time.

In Figure 11, when $x_1 > 5$, the total supply exceeds potential demand, and the reservation prices...
are very small. The shape of ratio $r/p$ is close to that of the reservation price, $r$, when $x_1 \leq 5$, and the discount is deeper as the remaining time decreases. However, for $x_1 \geq 5$, $r/p$ increases with time; this is again due to oversupply.

In Figure 12, when the inventory level is relatively small compared with potential demand ($x_2 = 1$), the reservation prices show more consistency. Both the reservation price, $r$, and the degree of discount, $r/p$, decrease with the remaining time and available inventory. With the exception of the case in which the opaque product can be obtained for free (that is, when $t = 1$), the discount that one may get from NYOP is no less than 50% of the lower posted price, which matches some real-life observations.
Figure 12  Reservation price at the intermediary firm as a function of the remaining time, $t$, and supplier 1’s inventory level, $x_1$, when $x_2 = 1$.

7.4. Opaque-Product Provider

Besides analyzing the reservation price itself, which immediately determines the income of the intermediary firm, it is also interesting to consider which supplier would win the auction and provide the opaque product. We refer to this supplier as the **opaque-product provider**.

As demonstrated in Proposition 7, besides using auctions, the opaque-product provider can be determined by looking at the inventory level at a given time point; we illustrate this in Figure 13.

When both firms oversupply (i.e., $x_1 \geq t$ and $x_2 \geq t$), both parties have zero reservation price, and the opaque-product provider is randomly drawn. Additionally, whenever $x_1 = x_2$, the two firms have the same decisions about their reservation prices. While the final reservation price may be positive, each of the two firms has the same chance of being the opaque-product provider. $t = n$
implies that the total number of upcoming customers is \( n \), and the “quota” for each supplier could be set at \( n/2 \). Figure 13 suggests that if the inventory level of one supplier is less than the quota (i.e., \( x_i \leq n/2 \) for some \( i \in \{1, 2\} \)), then the opaque-product provider will be the one with more inventory; otherwise, if both inventories exceed the quota (i.e., \( x_{1,2} > n/2 \)), the supplier with less inventory will supply the intermediary firm. The first case is rather straightforward—when \( x_i \leq n/2 \) and \( x_i < x_j \), supplier \( i \) is not motivated to sell more or to sell cheaply because of his low inventory levels. The second case is more interesting in that it reverses the previous claim. In this case, competition becomes more intense compared with the first case because neither supplier is expected to sell all of his inventory. Both suppliers may prefer a less competitive environment, and the fastest way to get there is to grant the supplier with less inventory more opportunity to sell his units. The supplier with more inventory will not supply the intermediary firm until his rival’s inventory level hits \( n/2 \).

7.5. Customer Types

Our numerical analysis so far has been conducted for customer with type \( N(0.5) \). We also conducted numerical analysis for other types of Natural customers (\( \alpha \neq 0.5 \)). Our results show that when customers believe that one supplier is less likely to be the opaque-product provider, then that supplier will receive a higher profit. For example, supplier 1 gets a higher revenue in \( K = N(0.2) \) then \( K = N(0.5) \); since 0.2 is significantly apart from 0.5, the opaque product is less “opaque” (that is, the customer believes it is much more likely that she will receive product 2 in the NYOP channel). This outcome is consistent with our finding that suppliers profit is lower in the existence of NYOP channel.

Although we did not formally prove the existence of NE in posted prices for Price-Oriented customers (\( K = P \)), through a number of simulations that we have conducted, it seems that results similar to those obtained in Theorem 2 hold for \( K = P \) as well. Moreover, most of the solution structures characterized in previous subsections carry over to \( K = P \) case. For brevity, we only compare the expected revenue for the three problems in Figure 14: the model without the NYOP channel, the NYOP channel with customer type \( K = P \), and the NYOP channel with customer type \( K = N(0.5) \).

In many instances, suppliers’ profit is slightly lower under \( K = P \) (which becomes more universal for \( N(\alpha) \)-type customers with \( \alpha < 0.5 \)), potentially because the price-oriented customers (\( K = P \)) are “smarter.” From our discussions in previous subsections, one can conclude that suppliers increase their posted prices when they are not the opaque-product provider. Price-oriented customers can incorporate this reasoning into their analysis and thus achieve better outcome.

Note that we do not assume that our customers are “strategic,” a modeling assumption that is gaining a lot of attention in recent operations management literature (see, for instance, Su (2007), Shen and Su (2007), Aviv and Pazgal (2008), Su and Zhang (2008), Jerath et al. (2009), to name
8. Discussion and Extensions

The Internet opened an entirely new world for both the sellers and the consumers. As a result, businesses are under an increased pressure to redesign their distribution channels as well as their pricing and inventory-control strategies. Our model blends the elements of dual-channel and competitive dynamic pricing models in trying to understand how a new sales format—Name-Your-Own-Price (NYOP)—may influence the industry. We model two suppliers who compete with each other in selling regular goods at their direct channels and opaque goods by contracting with an intermediary NYOP firm. The intermediary firm acts like an agent that hides brand and some other information about the products to ensure direct channel integrity. It benefits from the difference between customers’ willingness-to-pay for the opaque product and the lowest price at which a supplier would let his product go.

With respect to the research questions posed at the beginning of the paper, we believe that our results provide relevant managerial insights for each stakeholder in our model. In our analysis of customers’ channel and bidding behavior, we find out that low-end customers tend to look at their average valuation of the products—their bids depend on how much they appreciate the “expected” product. However, high-end customers focus on differences in their valuation of the two products, and their bids decrease with the degree of differentiation. Thus, an intermediary NYOP firm should avoid bundling vertically differentiated products, but concentrate on horizontally differentiated just a few. Thus, we do not assume that our customers are capable of making intertemporal tradeoffs (such as postponing their purchase for the sake of a better deal) or any sophisticated calculations to get their prior closer to the true $r$ or $I$. While these assumptions are beyond the scope of this paper, they are certainly interesting problems to explore in the area of customer behavior (see §8.1). Our analysis so far indicates that allowing for such strategic behavior on the customers’ side will only further deteriorate the benefit that the suppliers can obtain from the NYOP channel.
ones in choosing a set of potential opaque-goods suppliers.

From the suppliers’ perspective, we provide a dynamic programming approach in determining the final reservation prices, equilibrium posted prices, and expected revenues. Our numerical results illustrate how dual-channel and single-channel can behave in the opposite way with respect to their sensitivity to the number of remaining selling periods and inventory levels. Most importantly, our analysis implies that NYOP does not make either the suppliers or the industry better off—the existence of an intermediary firm results from the competition between the suppliers. However, with an NYOP channel the suppliers are worse off in the intensified competition.

To the best of our knowledge, studies related to opaque selling have so far neglected to properly address the impact of NYOP on competing suppliers. The opaque channel seems like a two-sided sword that, on one hand, may help in increasing market penetration, while on the other hand, it could just cannibalize the existing channels. Granados et al. (2010) provide empirical evidence that the opaque channel may not expand the market. If we further take the pricing factor into account, it is not surprising that the existence of NYOP may hurt the suppliers.

Having said that, one question remains unanswered: if NYOP is not benefiting the suppliers, why are they partnering with the intermediary firm? We propose some possible explanations:

• Information asymmetry. As discussed in §3.2 (Supplier Participation), membership in an NYOP channel is generally not public information. If an in-stock supplier (say, supplier 2) chooses not to join an NYOP channel, he is giving up the customers that might buy from him through the opaque channel (to his competitor, supplier 1). It is, then, in the best interest of supplier 1 to join the NYOP channel in this scenario, as the opaqueness still holds due to information asymmetry. This can be shown by a slight modification of (6):

$$
\Pi_1(x, t) = \max_{0 \leq r \leq p_1} \left\{ \left[ p_1 + \Pi_1(x_1 - 1, x_2, t - 1) \right] \Pr\{v \in B_1(r, p, x)\} \\
+ \left[ r + \Pi_1(x_1 - 1, x_2, t - 1) \right] \Pr\{v \in B_0(r, p, x)\} \\
+ \Pi_1(x_1, x_2 - 1, t - 1) \Pr\{v \in B_2(r, p, x)\} + \Pi_1(x_1, x_2, t - 1) \Pr\{v \in B_N(r, p, x)\} \right\}.
$$

Supplier 1 will chose the optimal reservation price, $r^*$, given that he is the sole opaque-product provider to the NYOP channel. There are non-trivial instances in which $r^*$ is below $p_1$, implying that supplier 1 will participate in the NYOP channel even though his competitor decides not to. It is also not hard to show that supplier 1 can do at least as good as when there is no NYOP channel (by setting $r = p_1$ always). In such scenario, he is taking advantage of supplier 2’s suboptimal channel decision. Note that we assume only two suppliers in the market; in a more realistic setting with multiple suppliers, this scenario becomes even more plausible. Thus, the suppliers suffer from the fact that their participation is invisible to the customers.

• Competition vs. collusion. Throughout our analysis, we assume that suppliers are competitors and rule out any possibility of collusion. One may assume that, if the suppliers were allowed
to communicate and reach agreements that would remove the intermediaries from the market, they might generate higher profits. These are, however, strong assumptions, which have not been observed in industry as of yet, and which may also have legal implications.

- **NYOP firm as a profit maximizer.** If the sole purpose of NYOP intermediary is to maximize its own welfare, it would choose the mechanism that ignites severe competition among suppliers. Indeed, during the early years of Priceline.com, it went through a lot of resistance from the airlines due to potential low bids in the opaque channel (Pederson 2004). This is not surprising: even from the theoretical perspective, a profit-maximizing strategy may lack long term sustainability, as the suppliers may decide to collude by not participating and thus driving the intermediary firm out of business. In particular, policies in Anderson (2009) suggest that Priceline.com may be more interested in expanding its supplier base than in maximizing its profits with a selected, smaller number of suppliers. This also implies a rich set of possible model extensions, and will be discussed in more details below.

### 8.1. Future Extensions

While our paper deals with a stylized model in analyzing the impact of the NYOP channel, there are many directions in which this work can be extended (e.g., Granados et al. 2009); we list a few of them below.

**Opaque Product Line.** The intermediary firm chooses the underlying products for the opaque goods. For a two-supplier-two-product problem, our results indicates that it is better to have horizontally differentiated products (e.g., hotels with different brand names but at the same star-level and nearby locations) than vertically differentiated ones (e.g., flights of the same route but one departs at 1 am while the other departs at noon). As more suppliers participate in the model, there is more room for the intermediary firm to increase the bundling assortment; the firm may also inform the customers about the underlying products that comprise the opaque product they are bidding for. There are many interesting approaches with regard to how such decisions could be made.

**Contracting.** As has been discussed earlier, the relationships between the intermediary firm and suppliers can have many alternatives as well. The short-term goal for the intermediary firm needs not be restricted to maximizing its profit; even if it was, the process of determining the opaque-product provider can take various alternatives. Our current setting assumes that the intermediary firm uses a descending auction to determine the reservation price. One realistic alternative may be to ask the sellers to privately submit their own reservation prices, but it may add a new level of competition and require some additional prior assumptions. Conversely, the problem could be simplified by letting the intermediary firm set the reservation price and then randomly select a
supplier that is willing to participate. In addition, revenue-sharing contacts in which the suppliers receive a portion of the spread between the customer bid and the reservation price, \( b - r \), may be adopted.

**INFORMATION AVAILABILITY.** Information structure assumed in the model drives some of our results, and is applicable to some examples observed in practice. It is in the interest of the intermediary firm to explore the consequences of offering various degrees of information to its customers. One potential extension is to consider Hotwire.com (regular goods) and Priceline.com (opaque goods), and compare the optimal information-offering for both models.

We have also assumed that each supplier knows the capacity at both suppliers; the problem can be modified by looking at instances with imperfect information. On one hand, players may not know demand distribution, and we may allow suppliers/NYOP firms to learn from the bidding data. On the other hand, suppliers may rely on their own mechanisms in estimating the remaining inventories of their competitors through their posted prices or their reservation prices at the intermediary firm.

**CUSTOMER BEHAVIOR.** In the presence of multiple sales channels and a finite number of sales periods, customers can strategically plan their purchasing/bidding timing across a long horizon. Shen and Su (2007) review the recent literatures in strategic customer behavior in RM and auctions. While our model assumes a static customer set, it would be of practical value to examine the problem in which strategic customers can make intertemporal decisions on channel selection and bidding, or in which returning customers can repeat their bids after the “frozen” period has elapsed.

**References**


Appendix

Proof of Proposition 1  (i). The FOC for \( V(v, b) \) is given by

\[
g(b) \left[ \min\{v, p\} - b \right] - G(b) = g(b) \left[ \min\{v, p\} - b - \frac{G(b)}{g(b)} \right].
\]

As \( \min\{v, p\} - b \) decreases with \( b \), \( G(b)/g(b) \) increases with \( b \), and \( G(0)/g(0) = 0 \), there exists a unique \( b^* \in [0, \min\{v, p\}] \) that satisfies

\[
\min\{v, p\} - b^* - \frac{G(b^*)}{g(b^*)} = 0.
\]

It is easy to verify that the extreme solutions (\( b = 0 \) and \( b = \min\{v, p\} \)) are not optimal; therefore, \( V(v, b) \) is maximized at \( \text{FOC}=0 \), i.e., when \( b = b^* \).

(ii) Since \( \min\{v, p\} \) is non-decreasing in both \( v \) and \( p \), and \( b + G(b)/g(b) \) increases with \( b \), \( b^* \) is non-decreasing in \( v \) and \( p \).

(iii) When \( \xi \sim U[\underline{p}, \overline{p}] \),

\[
G(b)/g(b) = \begin{cases} 
0, & \text{if } b \leq \underline{p} \\
b - \underline{p}, & \text{if } \underline{p} < b \leq \overline{p} \\
p - \underline{p}, & \text{if } b > \overline{p},
\end{cases}
\]

hence the solution to \( \text{FOC}=0 \) is

\[
b^* = \begin{cases} 
\min\{v, p\}/2, & \text{if } \underline{p} \leq v \leq \overline{p} \\
(p + \min\{v, p\})/2, & \text{if } \underline{p} < b \leq p.
\end{cases}
\]

□

Proof of Proposition 2:  The FOC can be derived as

\[
\frac{\partial V(v, b)}{\partial b} = \begin{cases} 
g(b)(\alpha_1^K v_1 + \alpha_2^K v_2 - b) - G(b), & \text{if } v \in \Omega_N \\
g(b) \left[ \alpha_j^K (v_j - v_i) - b + p_i \right] - G(b), & \text{if } v \in \Omega_i, \ i = 1, 2.
\end{cases}
\]

For \( v \in \Omega_N \), there exists a unique \( b^* \in [0, p] \) that satisfies \( \alpha_1^K v_1 + \alpha_2^K v_2 - b^* = G(b^*)/g(b^*) \). Moreover, as \( b + G(b)/g(b) = 0 \) at \( b = 0 \), there must be \( b^*(v) > 0 \) in this case. For \( v \in \Omega_i, i \in \{1, 2\} \), the FOC suggests \( \alpha_j^K (v_j - v_i) + p_i - b^* = G(b^*)/g(b^*) \). A valid solution exists if and only if \( \alpha_j^K (v_j - v_i) + p_i \leq 0 \), which is equivalent to \( v_i - v_j \leq p_i/\alpha_j^K \); otherwise, \( V(v, b) \) is maximized at \( b = 0 \). This proves (i).

Recall that \( b^* \) is the unique solution to \( \alpha_1^K v_1 + \alpha_2^K v_2 = b^* + G(b^*)/g(b^*) \) when \( v \in \Omega_N \), and \( \alpha_j^K (v_j - v_i) + p_i = b^* + G(b^*)/g(b^*) \) when \( v \in \Omega_i \). Because \( b + G(b)/g(b) \) increases with \( b \), it is straightforward that \( b^* \) increases with \( \alpha_1^K v_1 + \alpha_2^K v_2 \) (resp., \( v_j - v_i \)) when \( v \in \Omega_N \) (resp., \( \Omega_i \)). This proves (ii).

When \( v \in \Omega_N \), it is obvious that \( b^* < \alpha_1^K v_1 + \alpha_2^K v_2 \). Also, note that \( \alpha_j^K (v_j - v_i) + p_i = \alpha_1^K v_1 + \alpha_2^K v_2 - (v_i - p_i), \) and \( v_i - p_i > 0 \) for \( v \in \Omega_i \), so we should have \( \alpha_j^K (v_j - v_i) + p_i < \alpha_1^K v_1 + \alpha_2^K v_2 \). Thus, \( b^* < \alpha_j^K (v_j - v_i) + p_i < \alpha_1^K v_1 + \alpha_2^K v_2 \), which proves (iii). □
Proof of Proposition 3: For customers in $\Omega_N$, NYOP is the only channel in which they might get some product, so they will always attend NYOP. For those with an external choice (i.e., $v \in \Omega_i$ for $i = 1, 2$), denote $V^N = V(v, b^*(v))$. A customer with $v \in \Omega_i$ will choose the NYOP channel if and only if $V^N > v - p_i$, where

$$V^N(v, b^*) = G(b^*)(\alpha_1^K v_1 + \alpha_2^K v_2 - b^*) + G(b^*)(v_i - p_i).$$

We then have

$$v_i - p_i - V^N = G(b^*)(v_i - p_i) - G(b^*)(\alpha_1^K v_1 + \alpha_2^K v_2 - b^*) = G(b^*)[b^* - p_i - \alpha_j^K (v_j - v_i)].$$

By the proof of Proposition 2, $b^*$ is solved through $p_i + \alpha_j^K (v_j - v_i) = b^* + \frac{G(b^*)}{g(b^*)}$ when $v \in \Omega_i$. Thus, when $p_i + \alpha_j^K (v_j - v_i) \geq 0$, we have $b^* \leq p_i + \alpha_j^K (v_j - v_i)$ and equality holds if and only if $p_i + \alpha_j^K (v_j - v_i) = 0$. On the other hand, if $p_i + \alpha_j^K (v_j - v_i) < 0$, corner solution is optimal and $b^* = 0 > p_i + \alpha_j^K (v_j - v_i)$. Therefore, $V^N > v_i - p_i$ if and only if $p_i + \alpha_j^K (v_j - v_i) \leq 0$, and hence $\delta_i^* = p_i/\alpha_j^K$. □

Proof of Proposition 4: By Proposition 1 the optimal bid satisfies $b^* + \frac{G(b^*)}{g(b^*)} = \min\{v_j, p\} \leq p$. Hence, for all $v$ there is $b^*(v_j) < r_0$. Thus, if $r > r_0$, all bids will be rejected ($\mathcal{B}_0 = \emptyset$) and customers who can afford the posted price ($v \geq p$) will attain the product through direct channel ($\mathcal{B}_j = \{v : v_j > p\}$).

If $r \leq r_0$, then bid $b^*(v_j)$ for the customers with $v_j \geq r + \frac{G(r)}{g(r)}$ will exceed $r$ and be accepted by the NYOP firm (i.e., $\mathcal{B}_0 = \left\{v : v_j \geq r + \frac{G(r)}{g(r)}\right\}$). The bids of customers with $v_j < r + \frac{G(r)}{g(r)} < p$ are rejected and they cannot afford the posted price $p$ either; thus, $\mathcal{B}_N = \left\{v : v_j < r + \frac{G(r)}{g(r)}\right\}$ and no one buys from the direct channel (i.e., $\mathcal{B}_j = \emptyset$). □

Proof of Proposition 5: By Proposition 2, if $v \in \Omega_i$ for $i \in \{1, 2\}$, the optimal bid $b^*$ is determined through $v_j - v_i$; it exceeds $r$ if and only if $\alpha_j^K (v_j - v_i) + p_i > r + G(r)/g(r)$, which can be written as $[p_i - r - G(r)/g(r)]/\alpha_j^K > v_i - v_j$. This defines $\delta_i(r)$ for $i = 1, 2$. Similarly, if $v \in \Omega_N$, $b^* > r$ if and only if $\alpha_1^K v_1 + \alpha_2^K v_2 > r + G(r)/g(r)$, which gives $\delta_N(r)$. □

Proof of Proposition 6: (i) Proposition 5 implies that, when products are vertically differentiated, then customers who eventually receive a unit will all buy either from supplier $i$, or from supplier $j$, or from the NYOP channel. We first argue that they would not buy directly from supplier $j$, as there is always a strategy in which supplier $i$ can set his posted prices as $p_i = p_j + v_i - v_j - \varepsilon = p_j + \bar{v} - \varepsilon$ for some $\varepsilon > 0$ such that all customers prefer to buy from supplier $i$ at $p_i$. With this in mind, supplier $j$ would accept any reservation price $r > 0$. Moreover, by setting $p_i$ such that $\delta_i(r) \leq v$, supplier $i$ has a strategy that may induce all customers to buy at his posted
prices for all possible \( r \). This is feasible since \( \delta_i(r) \) decreases with \( r \) and increases with \( p_i \) for both \( p_i = p \) and \( p_j = p \). Given that it can win all customers at posted prices, supplier \( i \) does not have an incentive to bid a positive reservation price with the intermediary firm because supplier \( j \) would always react with a lower \( r \).

(ii) By Proposition 5, if \( \delta_N(r) < \bar{v} \), either (a) none of the customers will buy anything, or (b) some customers receive a unit of product from supplier 1, some from supplier 2, and the rest from the NYOP channel. We next prove that \( G/g \) is decreasing in \( p \), so that solving \( \delta_N(r) = r + G(r|p)/g(r|p) = \bar{v} \) gives the proper \( p^0 \).

**Lemma 1.** \( G(r|p)/g(r|p) \) is decreasing and convex in \( p \).

**Proof.** Let \( G \) and \( \hat{g} \) denote the respective distribution function and density of \( \xi \) when the support is on \([0,1] \), then \( g(r|p)/G(r|p) = q \frac{\hat{g}(r)q}{G(r)q} \), where \( q = 1/p \). For all the DRHR distributions (truncated to \([0,1] \) if necessary) listed in Table 1 of Bagnoli and Bergstrom (2005), it can be verified that \( x\hat{g}(x)/\hat{G}(x) \) decreases in \( x \). This implies that we should have \( rg(r)/G(r) = rq \frac{\hat{g}(r)q}{G(r)q} \) decreasing with \( rq = r/p \). For any given \( r \), this further implies that \( G/g \) decreases with \( p \).

The convexity result will be used in later proofs. \( \square \)

**Proof of Theorem 1:** For \( t = 1 \), we need to optimize \( \pi_j(p,r;x) \). Denote \( p^* = \arg\max \bar{F}_j(p)p \). In other words, \( F_j(p^*)p^* \) is the best that the supplier can achieve with the direct channel only. If the supplier uses the NYOP channel as well \((r \leq r_0)\), the objective becomes

\[
\max_{r \leq r_0} \bar{F}_j \left( r + \frac{G(r)}{g(r)} \right) r = \max_{r \leq r_0} \frac{r}{r + G(r)/g(r)} \bar{F}_j \left( r + \frac{G(r)}{g(r)} \right) \left( r + \frac{G(r)}{g(r)} \right) \\
\leq \max_{r \leq r_0} r + G(r)/g(r) F_j(p^*)p^* \\
< F_j(p^*)p^*.
\]

Therefore, in the last time epoch it is always optimal not to let customers purchase from the NYOP channel when the supplier is alone.

If \( t > 1 \), let \( r^* \) be the infimum of the range of values in \([0,r_0] \) that maximize

\[
\bar{F}_j \left( r + \frac{G(r)}{g(r)} \right) \left[ r + \Pi_j(x - e_j, t - 1) \right] + F_j \left( r + \frac{G(r)}{g(r)} \right) \Pi_j(x, t - 1).
\]

Let \( p = r^* + \frac{G(r^*)}{g(r^*)} - \epsilon < r^* + \frac{G(r^*)}{g(r^*)} \); it is not hard to verify that

\[
\bar{F}_j(p + \Pi_j(x - e_j, t - 1)) + F_j(p)\Pi_j(x, t - 1) \\
= \bar{F}_j \left( r^* + \frac{G(r^*)}{g(r^*)} \right) \left[ r^* + \Pi_j(x - e_j, t - 1) \right] + F_j \left( r^* + \frac{G(r^*)}{g(r^*)} \right) \Pi_j(x, t - 1) + \bar{F}_j \left( r^* + \frac{G(r^*)}{g(r^*)} \right) G(r^*)/g(r^*) \\
+ \left[ \bar{F}_j \left( r^* + \frac{G(r^*)}{g(r^*)} - \epsilon \right) - \bar{F}_j \left( r^* + \frac{G(r^*)}{g(r^*)} \right) \right] \left[ r^* + \frac{G(r^*)}{g(r^*)} + \Pi_j(x - e_j, T - 1) - \Pi_j(x, t - 1) \right]
\]
When $\epsilon$ is small enough, we have
\[
\left[ \tilde{F}_j \left( r^* + \frac{G(r^*)}{g(r^*)} - \epsilon \right) - \tilde{F}_j \left( r^* + \frac{G(r^*)}{g(r^*)} \right) \right] \left( r^* + \frac{G(r^*)}{g(r^*)} + \Pi_j(x-e_j,t-1) - \Pi_j(x,t-1) \right) \\
-\epsilon \tilde{F}_j \left( r^* + \frac{G(r^*)}{g(r^*)} - \epsilon \right) \rightarrow 0.
\]

Hence,
\[
\tilde{F}_j(p) \left[ p + \Pi_j(x-e_j,t-1) \right] + F_j(p) \Pi_j(x,t-1) \\
\geq \tilde{F}_j \left( r^* + \frac{G(r^*)}{g(r^*)} \right) \left[ r + \Pi_j(x-e_j,t-1) \right] + F_j \left( r^* + \frac{G(r^*)}{g(r^*)} \right) \Pi_j(x,t-1).
\]

It is, therefore, better not to use NYOP at any time $t$. \hfill \Box

**Proof of Proposition 7:** (6) implies that supplier $i$ would not accept a reservation price that is lower than $\tilde{\Pi}_i(x,t-1)$. On the other hand, a reservation price that is higher than the minimum posted price, $p$, will lead to $Pr\{v \in \mathcal{B}_0(r,p,x)\} = 0$. To rule out the trivial cases, we let $r \leq p$.

Assume that the current reservation price satisfies $\max_{i=1,2} \tilde{\Pi}_i(x,t-1) \leq r \leq p = \min\{p_1, p_2\}$. We want to show that there is always an $\epsilon_0$ such that when $\epsilon \leq \epsilon_0$ at least one supplier is willing to accept a lower reservation price, $(r - \epsilon)$, if the other is not. Comparing the two terms in (6), it can be verified that for supplier 1 the gain from accepting $r - \epsilon$ is

\[
\left[ r - \epsilon - \tilde{\Pi}_1(x,t-1) \right] Pr\{v \in \mathcal{B}_0(r,p,x)\} - \left[ r - \epsilon + \Pi_1(x_1-1,x_2,t-1) \right] \Delta Pr\{v \in \mathcal{B}_0(r,p,x)\} \\
- \left[ p_1 + \Pi_1(x_1-1,x_2,t-1) \right] \Delta Pr\{v \in \mathcal{B}_1(r,p,x)\} \\
- \Pi_1(x_1,x_2-1,t-1) \Delta Pr\{v \in \mathcal{B}_2(r,p,x)\} - \Pi_1(x_1,x_2,t-1) \Delta Pr\{v \in \mathcal{B}_N(r,p,x)\}, \tag{A1}
\]

where $\Delta Pr\{v \in \mathcal{B}_i(r,p,x)\} = Pr\{v \in \mathcal{B}_i(r,p,x)\} - Pr\{v \in \mathcal{B}_i(r-\epsilon,p,x)\}$ is positive for $i = 1, 2, N$, and negative for $i = 0$. As $\epsilon \to 0$,

\[
\lim_{\epsilon \to 0} (A1) = \lim_{\epsilon \to 0} \left\{ \left[ r - \epsilon - \tilde{\Pi}_1(x,t-1) \right] Pr\{v \in \mathcal{B}_0(r,p,x)\} \right. \\
- \left[ r - \epsilon + \Pi_1(x_1-1,x_2,t-1) \right] \frac{\partial Pr\{v \in \mathcal{B}_0(r,p,x)\}}{\partial r} \epsilon \\
- \left[ p_1 + \Pi_1(x_1-1,x_2,t-1) \right] \frac{\partial Pr\{v \in \mathcal{B}_1(r,p,x)\}}{\partial r} \epsilon \\
- \Pi_1(x_1,x_2-1,t-1) \frac{\partial Pr\{v \in \mathcal{B}_2(r,p,x)\}}{\partial r} \epsilon - \Pi_1(x_1,x_2,t-1) \frac{\partial Pr\{v \in \mathcal{B}_N(r,p,x)\}}{\partial r} \epsilon \left\} \\
= \left[ r - \tilde{\Pi}_1(x,t-1) \right] Pr\{v \in \mathcal{B}_0(r,p,x)\} > 0.
\]

In fact, to guarantee the positivity of (A1), it is sufficient to have $\epsilon \leq \left( r - \tilde{\Pi}_1(x,t-1) \right) Pr\{v \in \mathcal{B}_0(r,p,x)\} / \max\{M,0\}$, where

\[
M = Pr\{v \in \mathcal{B}_0(r,p,x) + [r + \Pi_1(x_1-1,x_2,t-1)] \frac{\partial Pr\{v \in \mathcal{B}_0(r,p,x)\}}{\partial r}
\]

\[
\begin{align*}
\text{posted price,} \\
i, \therefore \text{post NYOP at any time} \\
\text{want to show that there is always an} \\
\text{can be verified that for supplier 1 the gain from accepting} \\
\text{Proof of Proposition 7:} \text{ (6) implies that supplier i would not accept a reservation price that is lower than} \\
\text{minimum posted price, } p, \text{ will lead to } Pr\{v \in \mathcal{B}_0(r,p,x)\} = 0. \text{ To rule out the trivial cases, we let } r \leq p. \\
\text{Assume that the current reservation price satisfies } \max_{i=1,2} \tilde{\Pi}_i(x,t-1) \leq r \leq p = \min\{p_1, p_2\}. \text{ We want to show that there is always an } \epsilon_0 \text{ such that when } \epsilon \leq \epsilon_0 \text{ at least one supplier is willing to accept a lower reservation price, } (r - \epsilon), \text{ if the other is not. Comparing the two terms in (6), it can be verified that for supplier 1 the gain from accepting } r - \epsilon \text{ is}
\]
which implies that supplier 1 will accept the reservation price \( r - \epsilon \) if supplier 2 will not. Thus, by properly designing \( \epsilon \), the intermediary firm can at its best attain reservation price \( \max_{i=1,2} \Pi_i(x, t - 1) \) (if it is not exceeding \( p \)). If \( \max_{i=1,2} \Pi_i(x, t - 1) > p \), neither of the suppliers will sell through the intermediary firm, and we can simply let \( r = p \).

At \( t = 1 \), \( \Pi_1(x, t - 1) = \Pi_1(x, 0) = \Pi_1(x_1, x_2 - 1, 0) - \Pi_1(x_1 - 1, x_2, 0) = 0 \). Similarly, \( \Pi_2(x, t - 1) = 0 \). Hence, we have \( r^* = \min\{p, \max\{\Pi_1(x, t - 1), \Pi_2(x, t - 1)\}\} = 0 \). \( \square \)

**Proof of Theorem 2:** (i). We need to prove that \( \Pi_i \) is quasi-concave in \( p_i \) for \( i = 1, 2 \). For uniform distribution, it is sufficient to prove concavity for any \( v \in [0, 1] \), if \( v \) follows the distribution \( \alpha_1^K v_1 + \alpha_2^K v_2 = v \) and \( \alpha_1^K v_1 - \alpha_2^K v_2 \sim U[-\min\{v/\alpha_1^K, (1 - v)/\alpha_2^K\}, \min\{v/\alpha_1^K, (1 - v)/\alpha_2^K\}] \) (e.g., the two products are pseudo-horizontally-differentiated).

Let us first assume \( \iota = 1 \); hence, supplier 2 is the opaque-product provider. Then,

\[
\Pi_1(x, t) = [p_1 + \Pi_1(x_1 - 1, x_2, t - 1)] Pr\{v \in \mathcal{B}_1(r, p, x)\} + \Pi_1(x_1, x_2 - 1, t - 1) Pr\{v \in \mathcal{B}_0(r, p, x)\} + \Pi_1(x_1, x_2 - 1, t - 1) Pr\{v \in \mathcal{B}_2(r, p, x)\} + \Pi_1(x_1, x_2 - 1, t - 1) Pr\{v \in \mathcal{B}_N(r, p, x)\}.
\]

- If \( p_1 \geq r_2 = p \), Proposition 5 implies that both \( \mathcal{B}_2(r, p, x) \) and \( \mathcal{B}_N(r, p, x) \) are unaffected by \( p_1 \), and \( \frac{\partial \pi_0(r, p, x)}{\partial p_1} = -\frac{\partial \pi_1(r, p, x)}{\partial p_1} > 0 \). So,

\[
\frac{\partial \Pi_1(x, t)}{\partial p_1} = Pr\{v \in \mathcal{B}_1(r, p, x)\} + \left[\Pi_1(x_1, x_2, t - 1) - p_1\right] \frac{\partial Pr\{v \in \mathcal{B}_0(r, p, x)\}}{\partial p_1} \tag{A2a}
\]

\[
-2 \frac{\partial^2 Pr\{v \in \mathcal{B}_0(r, p, x)\}}{\partial p_1^2} + \left[\Pi_1(x_1, x_2, t - 1) - p_1\right] \frac{\partial^2 Pr\{v \in \mathcal{B}_0(r, p, x)\}}{\partial p_1^2}. \tag{A2b}
\]

- If \( p_1 < r_2 \),

\[
\frac{\partial Pr\{v \in \mathcal{B}_0(r, p, x)\}}{\partial p_1} = \frac{\partial Pr\{v \in \mathcal{B}_1(r, p, x)\}}{\partial p_1} + \frac{\partial Pr\{v \in \mathcal{B}_2(r, p, x)\}}{\partial p_1} + \frac{\partial Pr\{v \in \mathcal{B}_n(r, p, x)\}}{\partial p_1},
\]

thus

\[
\frac{\partial \Pi_1(x, t)}{\partial p_1} = Pr\{v \in \mathcal{B}_1(r, p, x)\} - \left[\Pi_1(x_1, x_2, t - 1) - p_1\right] \frac{\partial Pr\{v \in \mathcal{B}_1(r, p, x)\}}{\partial p_1} - \left[\Pi_1(x_1, x_2, t - 1) - \Pi_1(x_1, x_2, t - 1)\right] \frac{\partial Pr\{v \in \mathcal{B}_n(r, p, x)\}}{\partial p_1} \tag{A3a}
\]

\[
\frac{\partial^2 \Pi_1(x, t)}{\partial p_1^2} = 2 \frac{\partial Pr\{v \in \mathcal{B}_1(r, p, x)\}}{\partial p_1} - 2 \left[\Pi_1(x_1, x_2, t - 1) - p_1\right] \frac{\partial^2 Pr\{v \in \mathcal{B}_1(r, p, x)\}}{\partial p_1^2} - \left[\Pi_1(x_1, x_2, t - 1) - \Pi_1(x_1, x_2, t - 1)\right] \frac{\partial^2 Pr\{v \in \mathcal{B}_n(r, p, x)\}}{\partial p_1^2}. \tag{A3b}
\]

Proposition 5 implies that, given \( \alpha_1^K v_1 + \alpha_2^K v_2 = v \) for some particular \( v \), exactly one of \( \mathcal{B}_0 \) or \( \mathcal{B}_N \) is 0.
\[ \begin{align*}
\bullet & \; Pr\{v \in \mathcal{B}_N(r, p, x)\} = 0.
\text{For } p_1 \geq p_2 = p, \text{ it can be verified that } Pr\{v \in \mathcal{B}_0(r, p, x)\} \text{ is an affine function of } p_1 \text{ (i.e., } A p_1 + B \text{ with } A \geq 0). \text{ Thus, } \frac{\partial^2 Pr\{v \in \mathcal{B}_0(r, p, x)\}}{\partial p_1^2} = 0 \text{ and } \frac{\partial^2 \Pi_1(x,t)}{\partial p_1^2} \leq 0. \text{ By (A2), the payoff function is concave.}

\text{For } p_1 < p_2, \text{ it can be verified that } Pr\{v \in \mathcal{B}_1(r, p, x)\} \text{ takes the form } -A p_1 + B + C \delta_N(r \mid p_1) \text{ with } A, C \geq 0. \text{ Then,}
\begin{align*}
\frac{\partial^2 \Pi_1(x,t)}{\partial p_1^2} &= 2 \frac{\partial^2 Pr\{v \in \mathcal{B}_1(r, p, x)\}}{\partial p_1^2} + (p_1 - r) \frac{\partial^2 Pr\{v \in \mathcal{B}_1(r, p, x)\}}{\partial p_1^2} \\
&= \left[-2A + 2C \frac{\partial (G/g)}{\partial p_1} + (p_1 - r)C \frac{\partial^2 (G/g)}{\partial p_1^2}\right]/v.
\end{align*}
\text{Recall that in the proof in Proposition 6 (ii),} \quad \frac{G(r \mid p_1)}{rg(r \mid p_1)} = \frac{\hat{G}(r/p_1)}{p_1} := \hat{\delta}(r/p_1). \text{ Thus}
\begin{align*}
2 \frac{\partial (G/g)}{\partial p_1} + (p_1 - r) \frac{\partial^2 (G/g)}{\partial p_1^2} &= 2 \frac{\partial r}{\partial p_1} \hat{\delta}'(r/p_1) + (p_1 - r) \left[ \hat{\delta}''(r/p_1) \frac{\partial r}{\partial p_1} + \hat{\delta}'(r/p_1) \frac{\partial^2 r}{\partial p_1^2} \right] \\
&= -2 \frac{p_1^2}{p_1^2} \hat{\delta}'(r/p_1) + \frac{(p_1 - r)^2}{p_1^2} \hat{\delta}''(r/p_1) \\
&\sim -2 \hat{\delta}'(r/p_1) + (1 - r/p_1) \hat{\delta}''(r/p_1).
\end{align*}
\text{For all the DRHR distributions (truncated to } [0, 1] \text{ if necessary, with distribution function } \hat{G} \text{ and density } \hat{g} \text{ listed in Table 1 of Bagnoli and Bergstrom (2005), it can be verified that } \hat{\delta}(x) = \frac{\hat{G}(x)}{x \hat{g}(x)} \text{ satisfies } \hat{\delta}' \geq 0 \text{ and } \hat{\delta}'' \leq 0. \text{ Thus, } (A4) \leq 0, \text{ which implies the concavity of } \frac{\partial^2 \Pi_1(x,t)}{\partial p_1^2}.
\bullet & \; Pr\{v \in \mathcal{B}_p(r, p, x)\} = 0.
\text{For } p_1 \geq p_2 = p, \text{ (A2b) } \frac{\partial^2 \Pi_1(x,t)}{\partial p_1^2} = 0, \text{ so concavity holds.

\text{For } p_1 < p_2, \text{ } Pr\{v \in \mathcal{B}_1(r, p, x)\} \text{ is an affine function of } p_1 \text{ with the form } -A p_1 + B, \text{ where } A \geq 0, \text{ and } \frac{\partial^2 \Pi_1(x,t)}{\partial p_1^2} \leq 0 \text{ and } \frac{\partial^2 Pr\{v \in \mathcal{B}_N(r, p, x)\}}{\partial p_1^2} = 0. \text{ By (A3b), } \frac{\partial^2 \Pi_1(x,t)}{\partial p_1^2} \leq 0, \text{ and the payoff function is concave.}

\text{The case } t = 2 \text{ can be proved similarly.}

\text{(ii). We can show that, for any } t, \text{ there exits a } \pi_t \text{ such that for any } x_1 \geq t \text{ and } x_2 \geq t,
\begin{align*}
\Pi_1(x_1, x_2, t) = \Pi_2(x_1, x_2, t) = \pi_t,
\end{align*}
\text{which immediately leads to } r^* = 0 \text{ for any } x_1 \geq t \text{ and } x_2 \geq t. \text{ We show (A5) by induction. First, it is easy to see that (A5) holds for } t = 0. \text{ Now, suppose (A5) holds for all } t < T. \text{ We then have}
\begin{align*}
\Pi_1(x_1 - 1, x_2, T - 1) &= \Pi_1(x_1 - 1, x_2 - 1, T - 1) = \Pi_2(x_1, x_2 - 1, T - 1) = \Pi_2(x_1 - 1, x_2 - 1, T - 1) \text{ for any } x_1 \geq T \text{ and } x_2 \geq T. \text{ Hence, } \Pi_i(x_1, x_2, T - 1) = 0 \text{ for } i = 1, 2 \text{ and}
\begin{align*}
r^*(x_1, x_2, T) = 0, \quad \forall x_1 \geq T, x_2 \geq T.
\end{align*}
(A6) implies that, when both suppliers oversupply, the price of the opaque goods will remain zero until one supplier’s inventory becomes lower than the potential demand. The expected profit function for supplier 1 is then

\[
\Pi_1(x, T) = [p_1 + \Pi_1(x_1 - 1, x_2, T - 1)] Pr\{v \in \mathcal{B}_1(0, p, x)\} + \Pi_1(x_1 - 1, x_2, T - 1) Pr\{v \in \mathcal{B}_0(0, p, x)\} + \Pi_1(x_1, x_2 - 1, T - 1) Pr\{v \in \mathcal{B}_2(0, p, x)\} + \Pi_1(x_1, x_2, T - 1) Pr\{v \in \mathcal{B}_N(0, p, x)\} + \pi_{T-1},
\]

which does not depend on \( x \) for \( x_1 \geq T \) and \( x_2 \geq T \). Similarly, \( \Pi_2(x, T) = p_2 Pr\{v \in \mathcal{B}_2(0, p, x)\} + \pi_{T-1} \). When \( r = 0 \), the expressions for \( \mathcal{B}_1(0, p, x) \) and \( \mathcal{B}_2(0, p, x) \) are the same. It is then straightforward that in an equilibrium there should be \( p_1 = p_2 \) and \( \Pi_1(x, T) = \Pi_2(x, T) \). Thus, (A5) also holds for \( t = T \). This completes the proof. \( \square \)