Selling Through Priceline? Managing Name-Your-Own-Price and Direct Channels Simultaneously in a Competitive Market

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Abstract

Priceline.com patents the innovative marketing strategy, Name-Your-Own-Price (NYOP), that sells opaque products through customer-driven pricing. In this paper, we study how competitive retailers with substitutable, non-replenishable goods may sell their products (1) as regular goods through a direct channel at posted prices, and/or (2) as opaque goods through a third-party channel, which allows for the NYOP approach. We model the NYOP channel as an intermediary firm that collects the difference between the customers’ bids and reservation prices set by the retailers, and discuss different channel strategies and customers’ bidding strategies. We find that high-valued customers may demonstrate low-valued behavior, and that the intermediary firm benefits more from horizontally differentiated goods than from vertically differentiated ones. We also analyze how competing retailers should dynamically determine channel prices in maximizing the average expected revenue. Our results suggest that time and inventory levels can have very different impact in dual-channel versus single-channel settings. In particular, more inventory may reduce one’s expected revenue in presence of the NYOP channel, which never happens when there is direct channel only. It can be shown that retailers do not benefit from the existence of a revenue-maximizing NYOP channel.

Keywords: Competition; Distribution channels; e-Commerce; Game Theory; Opaque Product; Pricing; Name-Your-Own-Price;
In recent years, an increased variety of selling mechanisms arose in the consumer domain as e-business and online shopping have become vibrant parts of our lives. On one hand, sellers, from airlines and hotels to apparel and groceries stores, are more than ever engaged in opening new market channels and adjusting posted prices throughout the time. The buyers, on the other hand, enjoy some new purchasing opportunities. Beside the old-fashioned posted-price purchase, buyers can now, say, use auctions to buy football tickets from eBay.com, and thus not know the actual prices at the beginning of the purchasing process, or book hotels through “Name-Your-Own-Price” (NYOP) at Priceline.com, where even the actual products can be unknown. The latter behavior has been amplified by the earlier economic downturn. As Forbes (Feb 19, 2009) reported, “With the economy failing, ... (consumers)’re looking for deep discounts. ... Expedia ... is losing business while its competitor Priceline.com thrives on its ‘Name Your Own Price’ model.” During the summer of 2009, “Priceline earnings increased 35%. ... revenue climbed 18% to $603.7 million. Gross margin rose to 50.6% from 49.4% ” (Wall Street Journal (Aug 11, 2009)). At the end of 2009, Priceline overtook Expedia in market capitalization (tnooz.com (Nov 17, 2009)). The recent Priceline.com Annual Report (2011) also confirms that booming NYOP sales of hotel nights and rental car days contribute to the company’s annual increase in domestic bookings as well as merchant revenues.

The traditional posted price (or listed price, tag price) can be deemed as seller-driven pricing — the seller sets the price of the goods, and the buyer simply makes a take-it-or-leave-it decision. Under auction or NYOP, however, the roles of the two parties are reversed: a buyer announces the price that she (hereinafter, buyers will be referred to with feminine pronouns, sellers with masculine pronouns) is willing to pay, while the seller decides whether and at what price to let the goods go. We then refer to these mechanisms as buyer-driven pricing.

On a further note, information availability about the products on sale has been chang-
Sellers used to provide as much information about an item as possible, whether through displaying samples in brick-and-mortar stores, or by listing detailed specifications/figures in online stores. However, these days some sellers may choose to strategically withhold information from the customers; this occurs at Hotwire.com and Priceline.com. For instance, if a customer decides to purchase an air ticket at Priceline.com through “Name-Your-Own-Price,”\(^1\) she will not be able to learn the details of her trip (e.g., the airline name, departure time,\(^2\) etc.) until the deal is finalized. Thus, the exact features of the product at the time of purchase are rather vague to the customers. Such products, whose characteristics are not fully revealed at the point of payment, have been referred to as *opaque* goods in industry (Anderson (2008), Fay (2008)). As opposed to *opaque* goods, we will refer to the goods for which buyers know all features at the time of purchase as *regular* goods. Table 1 summarizes how *regular* and *opaque* goods have been sold under aforementioned pricing schemes. [Insert Table 1 here]

While auctions have been studied extensively in the literature as one of the *buyer-driven* pricing schemes, NYOP has to date received less attention, despite the fact that it possesses some nice characteristics. For one thing, processing is faster—after a customer named her price, she is normally informed about the outcome within minutes, while auction outcomes may be known only after several days, or even weeks. Thus, NYOP does not forego impatient customers. Second, NYOP does not intensify the competition among customers—while in an auction successful bid greatly depends upon the behavior of other customers, NYOP only cares whether the bid is over an invisible threshold or not. Therefore, it allows us to focus more on the competition between retailers than that among customers. Third, NYOP is simple for retailers to implement—the inventory is depleted by at most one unit per decision epoch, and can be immediately followed by retailers’ updates of posted and reservation prices, while the number of winners in an auction can be uncertain. Consequently, NYOP gives firms more leverage in the control of inventory and pricing across channels. Finally, NYOP

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\(^1\)Starting in 2005, Priceline also has products with posted price (USA Today, Apr 11, 2005).

\(^2\)The O/D dates and airports have to be confirmed, though.
provides a platform of competition in terms of *opaque* goods. Products sold through the NYOP channel do not possess some relevant information (e.g., airline, hotel brand) that can be used by customers for product discrimination. The retailers are under the “brand shield” and “price shield” that the direct channels cannot offer (Dolan (2000))—the NYOP channel essentially provides a space in which retailers compete head-to-head to win customers, who in turn give up the power to exercise their preferences.

In general, the NYOP channel seems like a two-sided sword that, on one hand, may help in increasing market penetration, while on the other hand, it could just cannibalize the existing channels. These contrasting features motivated our interest in analyzing how the NYOP channel could be integrated with the traditional selling channel in a competitive market, and what would be its impact on the business environment. In particular, we investigate the following research questions:

1. How would customers react to the NYOP-opaque channel coupled with the posted-price-regular channel?
2. How should retailers dynamically determine their optimal pricing/channel strategies in the presence of competition and the NYOP channel?
3. What would be the impact of the NYOP channel on the entire industry?

In our analysis of the problem, we set up a framework with three stakeholders: a sequence of customers, two competing retailers, and one intermediary NYOP firm. We assume that the retailers sell substitutable, perishable, and non-replenishable products. The products could be sold either as *regular* goods through their direct channels (stores or websites) at posted prices, or as *opaque* goods through a third-party intermediary firm that conducts NYOP service among the customers (e.g., Priceline.com). Thus, each retailer chooses his own set of pricing mechanisms (seller-driven pricing, buyer-driven pricing, or both at the same time) and the format of his products (regular, opaque, or both). Specifically, they can dynamically adjust these decisions over the entire selling season, in accordance to the current inventory level and leftover sales time. On the market side, a customer may (1) buy
at the posted price from her preferred retailer, or (2) go to the intermediary firm and name-her-own-price. In the latter case, the intermediary firm benefits from the difference between the customer’s offer and the lowest reservation price from the retailers (Dolan (2000)). If the bid fails to meet the lowest reservation price, the customer is rejected by the intermediary firm, but she still has a chance to buy the product at posted price.

Our analysis shows that high-valued customers may demonstrate low-valued behavior (that is, name their prices prior to attending the direct channel, making an even lower bid than the low-end customers), and that the intermediary firm benefits more from horizontally differentiated goods than from vertically differentiated ones. For the retailers, dual-channel equilibria can differ significantly compared to single-channel outcomes. In particular, more inventory may reduce one’s expected revenue in presence of the NYOP channel, which never happens with direct channel only. Interestingly, it can also be shown that the existence of NYOP channel does not benefit the retailers.

Our model adds a new block to the current knowledge of NYOP in the following aspects. (1) We study dual channel – dynamic pricing strategy in a competitive market. Specifically, we allow the competing firms to adopt two channels throughout the time. The value of the NYOP channel is then analyzed in a more thorough study; to the best of our knowledge, this has not been extensively studied in the literature thus far. (2) Given that many existing NYOP works are more customer-behavior oriented, our focus is more retailer-oriented. We analyze the retailers’ equilibrium decisions under inventory/time constraints. We believe that these distinctions make this problem/model a relevant and interesting research topic.

The paper is organized as follows. In §1, we review the literature. In §2 we introduce our model. Customers’ purchasing/bidding behavior is analyzed in §3, and results for the purchasing channel are shown in §4. In §5, we propose a dynamic programming approach for solving the equilibrium pricing decision. In characterizing the exact solution, we analyze some special games in §6. The impact of the NYOP channel is studied in §7, which also confirms the robustness of the results in §6. We discuss the results and extensions in §8.
1. Literature Review

The business model of NYOP and opaque goods has drawn increased attention in the recent literature.

Many papers regarding NYOP consider regular products. They tend to be consumer-oriented and emphasize consumer bidding behavior and/or restrictions that retailers may put on bidding. More specifically, most of the work assumes NYOP as the only sales channel (i.e., there is no parallel seller-driven, posted-price channel that customers may turn to). Fay (2004) analyzes whether an NYOP retailer should encourage or discourage repeated bidding. Spann et al. (2004) analyze how a firm can learn customers’ willingness to pay and the transaction costs of repeated bidding. Hann and Terwiesch (2003) estimate the transaction cost per bid of the on-line customers in naming their own prices. Hinz and Spann (2008) model how social network may facilitate individuals in learning the reservation price of the seller. Some articles also shed light on the NYOP retailer’s policy. Terwiesch et al. (2005) analyze the repeated bidding behavior and suggest the threshold price for monopoly firms. Joint bidding for multiple items is then studied by Amaldoss and Jain (2008). Cai et al. (2009) study the value of double-bidding as well as direct channel with an NYOP retailer. Fay and Laran (2009) examine how the frequency of changing reservation price on the supply side would affect bidding behavior on the market side. Wang et al. (2009) allow a firm to adopt NYOP in last minute sales, and find out that the value for an NYOP channel depends on the uncertainty of high-fare demand, rather than on the amount of excess capacity.

Problems involving opaque goods usually demand a very different model structure. As it usually requires two or more retailers/products to make an opaque product, competition issues may arise (for monopolistic case see Jiang (2007), Fay and Xie (2008)). In addition, other questions also arise as to whether the pricing of the opaque product should be seller-driven (posted price) or buyer-driven (auction, NYOP, etc.). Specifically for the latter, the

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3 Some of the models do not particularly differentiate regular versus opaque products—we put them under the column “Regular Goods,” while the second column contains papers targeting only opaque products.
notion of “NYOP firm” needs further clarification. In the presence of opaque goods, the NYOP system usually consists of an NYOP intermediary firm and a number of retailers, in which the former acts as an agent that delegates the NYOP service for the latter. This is different from many aforementioned papers, which deal with only one “NYOP retailer”—a centralized retailer that also provides NYOP service. With opaque products, the retailers and NYOP-service provider are usually separated. The retailers determine their reservation prices, and the NYOP-service provider selects the retailer that accepts the lowest payment.4 Fay (2008) models selling opaque products at posted prices only. Conditions under which the opaque good may bring down the price in the traditional channel and harm the revenues are identified. Fay (2009) studies the competition between Priceline.com and Hotwire.com. Both companies deal with opaque products, but the former uses a buyer-driven NYOP scheme while the latter uses a seller-driven, posted-price scheme.

Most of the NYOP/opaque literature discussed so far did not put much restriction on the supply; the papers that did usually assumed that there can be only one distribution channel. However, it should be noted that in industries like airlines, hotels, and rental cars (where NYOP is usually applied), quantity/time limits have strong implications on the revenue (McGill and van Ryzin (1999)), and that these products/services are constantly available in multiple sales channels (Bitran and Caldentey (2003), Elmaghraby and Keskinocak (2003), Caldentey and Vulcano (2007)).

In order to achieve a more realistic setting, we consider a general model in which competing retailers can dynamically determine their channel strategy, pricing strategy, and inventory control strategy during a finite time horizon. To our best knowledge, very few papers have incorporated all of these factors thus far.

4“...maximizes the spread between the customer’s ‘named-price’ and the necessary payment to the airline partner...” (Dolan (2000)).
2. The Model

In this section, we first describe the flow of our game, and then discuss some of our assumptions in more detail.

2.1 Game Flow

Consider a model with two retailers (each of whom will be referred to as “he”) selling substitutable products, an intermediary NYOP firm (referred to as “it”), and a sequence of customers (each of whom referred to as “she”). A retailer and its product have the same index \( i \), where \( i \in \{1, 2\} \). Assume that retailer \( i \) holds \( \bar{x}_i \) units of initial inventory, which will expire after \( T \) periods and is not replenishable. We use a backward time index \( t = T, T - 1, \ldots, 1 \) to denote the current period, so a smaller number indicates that we are closer to the ending time. At each period, exactly one customer arrives, and she possesses random valuation \( v_i \) for product \( i \). We assume \( v_i \)’s are independently distributed on \([0, 1]\) with density \( f_i(\cdot) \) and cumulative distribution function (cdf) \( F_i(\cdot) \). Each customer demands at most one unit. This sequence of customers with constant arrival rate 1 can easily be extended to a sequence of homogeneous customers with a fractional arrival rate by adjusting the distributions of valuations accordingly.

Figure 1 illustrates the sequence of events for each period of the game. [Insert Figure 1 here] At the beginning of each period, the retailers *publicly* announce their posted prices, \( \mathbf{p} = (p_1, p_2) \), at which the coming customer can buy from their direct channels. After the posted prices are announced, the intermediary firm will work with the retailers to determine which one may fulfill the demand if the customer desires an opaque product in this period. The NYOP intermediary firm is modeled as a revenue maximizer that benefits from the spread of the customer’s bid \( b \) and its payment to the opaque product provider \( r \), also called the “reservation price”). Thus, it will find a lowest reservation price \( r \) at which at least one retailer is willing to release his next unit through the NYOP channel \(^5\). Then, if a

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\(^5\)This process can be implemented by conducting an English auction with the retailers. The retailers take turns in submitting their desired reservation prices until someone is unwilling to decrease any further. The
customer bids at $b$, the intermediary firm simply compares $b$ with the lower reservation price, $r$: if $b \geq r$, the bid is accepted. The intermediary firm collects $b$ from the customer, assigns her a unit of product from the retailer that requests payment $r$, and pays that retailer his requested amount. Otherwise, if $b < r$, the bid is rejected. This process aligns with the description in Dolan (2000).

After all the prices are set, a customer arrives with valuation $\mathbf{v} = (v_1, v_2)$ as her private information. Neither the retailers nor the intermediary firm has any knowledge of $\mathbf{v}$ except its distribution function, $F_{1,2}(\cdot)$. Based on the posted prices ($\mathbf{p}$), her valuation ($\mathbf{v}$), and her prior in estimating the NYOP channel (the opaque-product provider and reservation price, $r$)\(^6\), the customer decides if she wants to buy directly from her preferred retailer or name her price (which we will refer to as “make a bid” for the reminder of the paper) $b$ with the intermediary firm. In the second case, she may be matched up with either of the two retailers if the bid is accepted. In case the bid is rejected, the customer can always return to her preferred retailer and buy the product at its posted price. However, making a bid is considered a commitment to buy, and thus a customer cannot decline a product assigned by the intermediary firm if she discovers ex-post that being awarded the alternative product or buying at the posted price would make her better off. We also assume that the customer is not allowed to make a second bid with the NYOP channel after the first bid is rejected. Repeated bidding would certainly generate more research questions that are worth analyzing. However, as our goal is to characterize the generic policy and impact of the dual distribution channel (NYOP and posted prices), the effects of repeated bidding (or any strategic customer behavior) are not within the scope of this paper (see discussion in §8), and we simplify the scenario by applying a one-time bidding restriction\(^7\). This assumption

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\(^6\)The prior can be reached through consumer research or survey. Note that the NYOP channel decisions are pre-determined in each period. Thus a customer’s belief can never be accurate in the short run. However, we will show in §6 that the accuracy/rationality for one’s belief can be achieved in the long run, and it would not affect the outcome of the model.

\(^7\)Frictional cost related to this one-time bidding is considered small and will not be taken into account in this model.
aligns with the policy used by Priceline, that “only one offer was permitted in a seven-day period...” (Dolan (2000)).\textsuperscript{8} Readers may refer to Fay (2004), Spann (2004), Terwiesch et al. (2005), and Hinz and Spann (2008) for problems involving repeated bidding/bid learning.

Figure 2 depicts the problem confronting each customer in Stage 3. We use squares to depict decision nodes (in which customers need to pick from one of the branches) and circles to depict probability nodes, with potential outcomes that are not under customers’ control. The last column shows the cash flow of each stakeholder under each scenario. [Insert Figure 2 here]

2.2 Assumptions

We now provide a more detailed discussion of our main assumptions. Readers less interested in model/technical details may skip this subsection and resume from §4 directly.

**Reservation Price.** We assume that the intermediary firm imposes the restriction which limits the reservation price levels to be not higher than the respective posted prices, i.e., \( r \leq p = \min\{p_1, p_2\} \). A plausible reason for this assumption lies in the fact that customers usually perceive the NYOP channel as a way to save money (as a tradeoff for the more restricted information).

**Information Structure.** The general information structure is characterized in Table 2. As mentioned in the model, posted prices (\( p \)) are public information, while customer valuation (\( v \)) is known to the customers only. For analytical convenience, we assume that the inventory position \( x \) is visible to both retailers; similar assumptions have been made by many papers analyzing competition in revenue management (e.g., Gallego and Hu (2007); Levin et al. (2009); Lin and Sibdari (2009))\textsuperscript{9}. As our primary goal is to study how the presence of an NYOP channel may affect the strategic decisions of retailers, this assumption falls within the scope of the paper. We use \( A \) to indicate the supply availability at the

\textsuperscript{8}The restrictions may have been updated, but the spirit is that such sites may not encourage repeated bidding within a short period.

\textsuperscript{9}For imperfect information on parameters such as demand distributions or inventory levels, one may refer to Perakis and Sood (2006), Levin et al. (2008), Zhang and Kallesen (2009), etc.
retailers; i.e., if retailer $i$ is out of stock, then $A_i = 0$, otherwise $A_i = 1$. It is more realistic to assume that customers can observe supply availability of the retailers ($A$) but not their real-time inventory levels ($x$). In addition, it is not unrealistic to assume that the initial inventory levels of the retailers ($\bar{x}$) are public information: for instance, in hotel and airline industries, the total number of rooms or seats is known to everybody. The opaque-product provider ($I$) and the reservation price ($r$) are decision variables of the NYOP firm and the retailers at the beginning of each time epoch. However, this information is by no means accessible to the customers (since $x$ is unobservable), who will adopt subjective priors in estimating these factors.\footnote{Similar assumptions upon consumers adopting subject priors in estimating the reservation prices with the NYOP channel were made across relevant literature including Hann and Terwiesch (2003), Spann et al. (2004), Terwiesch et al. (2005) and Cai et al. (2009).} We assume that these subjective priors, which can be reached through consumer research or survey, are public information to all stakeholders (different priors will be discussed in §8). [Insert Table 2 here]

**CUSTOMER BELIEF.** As discussed earlier, each customer upon arrival adopts certain prior in estimating the opaque-product provider and the reservation price\footnote{A special instance occurs in the last period $t = 1$, at which time neither the reservation price or the opaque-product provider depend upon inventory levels, and the customers can make their best guess; this will be discussed in §6.2. In periods other than $t = 1$, the prior discussed above applies.}, which have already been determined on the supply side.

The priors might depend upon observed information. We assume that customers estimate the reservation price as a random variable $\xi(p, A, t)$ on the support $[0, p]$, where $p$ is the minimum posted price $\min\{p_1, p_2\}$. Its density is denoted as $g(\cdot|p, A, t)$ and c.d.f. as $G(\cdot|p, A, t)$. For convenience, we use $\bar{G}$ to denote $1 - G$. Thus, the estimation can be updated as the remaining time or information available to the customers (e.g., whether a retailer runs out of stock) change. Further, we assume that $\xi$ has decreasing reversed hazard rate (DRHR); i.e., $g/G$ is decreasing. This is equivalent to assuming that $G$ is log-concave. Many (truncated) distributions satisfy this condition (e.g., uniform, normal, exponential, chi-squared, logit, etc.), hence this assumption is not too restrictive (Bagnoli and Bergstrom (2005)). The estimator for the opaque-product provider, $I(p, A, t) \in \{1, 2\}$, is modeled in a similar way.
More specifically, if only one retailer is in-stock, i.e., $A_i = 0$ and $A_j > 0$, the product at the intermediary firm can no longer be opaque and $I = j$. When both retailers are in-stock ($A_1 A_2 > 0$), the distribution of $I$ may be related to $(p, A, t)$ in different ways. We will denote the probability for retailer $i$ to be the opaque provider as $\alpha_i(p, A, t) = Pr(I(p, A, t) = i)$.

Retailer Participation. To the best of our knowledge, Priceline.com does not publicly announce the retailers that they partner with, or potential retailers for a particular opaque product\(^{12}\). It is also rare for retailers themselves to publicize that they are currently in business with an intermediary firm (say, Priceline.com). While the benefit for Priceline.com of not revealing this information is obvious (otherwise, the customers would be in a better position to get a low price\(^ {13}\)), the reasons for the retailers to keep this information hidden can go far beyond short-term monetary benefits/losses. One potential concern might be that claims of this type are costly to uphold—committing not to partner with a particular business unit is not a common business practice and may hinder future opportunities. In some instances, this can be more detrimental than committing to the partnership, which, in our problem, comes with the flexibility of either being or not being the opaque-product provider. Further, if the “you-will-not-get-our-product-through-NYOP” guarantee is only intended for a limited period, the retailers may later incur large switching cost in convincing their customers that the opposite is now true. Most retail businesses prefer to avoid such complexities and be silent on this issue. We, therefore, make the assumption that, unless a retailer runs out of stock, customers are not aware of the actual participation of that retailer.

3. The Customers

We analyze the problem in a backward manner, starting in this section with solving the customers’ problem. As can be seen from Figure 2, customers with valuation $v$ have three options to choose from: (1) buy directly from retailer 1 at $p_1$, (2) buy directly from retailer

\(^{12}\)Other websites that sell opaque product at posted prices, such as Hotwire.com, may have a different policy and provide a general list of the retailers they partner with.

\(^{13}\)On the other hand, at Hotwire.com customers do not have a say about the prices they would pay.
2 at $p_2$, or (3) bid $b(v)$ with the intermediary firm$^{14}$. In this section, we omit the signal notations $(p, A, t)$ for the valuation distribution.

### 3.1 One Retailer Runs Out of Stock

When one of the retailers runs out of stock, customers observe $A = (0, 1)$ or $(1, 0)$. Thus, the product at the intermediary firm ceases to be an opaque product and becomes a regular one. It is straightforward that a customer will always use the NYOP channel in this scenario. Given that there is now only one product, with posted price $p = p_j$ and reservation price $r \sim \xi$, the customer has to determine her bid, $b$, which maximizes her ex-ante surplus, $V$, where

$$V(v, b) = G(b)(v - b) + G(b)\max\{v - p, 0\} = \begin{cases} G(b)(p - b) + (v - p), & \text{if } v \geq p \\ G(b)(v - b), & \text{if } v < p. \end{cases}$$

This leads to our first result:

**Proposition 1. (Customer channel participation and bid; one in-stock retailer)**

Suppose that only one retailer carries some stock in the market with posted price $p$,

(i). a customer would always attend the NYOP channel first;

(ii). there exists a unique $b^*$ at which the customer will bid, which is non-decreasing in one’s valuation $v$ and the posted price $p$;

(iii). particularly, if the customers estimate the reservation price $\xi \sim U[p, p]$ for some $p \in [0, p]$, the optimal bid $b^* = \frac{\min\{p, v\}}{2} + \frac{p}{2} \cdot 1(v > p)$.

Thus, when granted the chance to get a “bargain” (at the NYOP channel) for a regular product, the customers would always attend the NYOP channel first and bid at a level lower than the posted price. A higher tag price raises the upper bound of customers’ estimation of the reservation price, and hence it has a positive impact on the bidding amount. Likewise, higher valuation suggests fewer financial constraints, which also leads to more generous bids.

$^{14}$Note that leaving without purchase is implicitly considered as bidding $b(v) = 0$ with the intermediary firm.
When customers make a naive estimation on the reservation price \( \xi \sim U[p, p] \) and the marginal cost \( p \) is rather low (e.g., theater tickets, airline seats, or hotel rooms), (iii) suggests that customers ask for at least a 50% discount from the posted price. Such bold bidding behavior stems from the fact that (1) there is no risk involved with the kind of product that one would receive (opaqueness is gone), (2) one can always buy from the direct channel in case of a rejection. In section §6.1, we will show that a monopolistic retailer offset such advantage-taking behavior by offering products through the direct channel only.

3.2 Both Retailers Present

When both retailers have available inventory, the intermediary NYOP firm can sell opaque products. Customers in this scenario have to be aware that there is a chance that they may be assigned a less-favored product when bidding in the NYOP channel. Consider a customer with valuation \( v \). Her expected payoff from the NYOP channel is then

\[
V(v) = \max_b V(v, b) = \max_b \left[ G(b)(\alpha_1 v_1 + \alpha_2 v_2 - b) + \bar{G}(b) \max\{v_1 - p_1, v_2 - p_2, 0\}\right],
\]

and her payoff from the direct channels is simply \( \max\{v_1 - p_1, v_2 - p_2, 0\} \).

Denote the set \( \Omega_N = \{v : v_i - p_i < 0, i = 1, 2\} \); we will refer to members of \( \Omega_N \) as budget customers. Further, let \( \Omega_i = \{v : v_i - p_i \geq 0, v_i - p_i \geq v_j - p_j, j \neq i\}, i \in \{1, 2\} \); members of \( \Omega_i \) will be referred to as \( i \)-lovers. In the absence of the NYOP firm, budget customers have no way to obtain either product, while \( i \)-lovers would buy from the direct channel of retailer \( i \). When an NYOP channel is available, all budget customers bid with the NYOP channel, as this is the only chance that they might get any product. Other customers have to decide whether to name their own price first (and buy at the posted price if the bid is not accepted) or to buy directly from their preferred retailer. In the following proposition, we find that the initial channel selection for \( i \)-lovers is related to \( v_i - v_j \), which we shall refer to as the degree of preference.

Proposition 2. (Customer channel participation; two in-stock retailers) Suppose that both retailers carry some stock with posted price \((p_1, p_2)\). Then,
(i). budget customers will always attend the NYOP channel;

(ii). an i-lover will use the NYOP channel if and only if her degree of preference, $v_i - v_j$, is below some threshold level $\delta_1^* = p_i/\alpha_j$.

While NYOP participation is universal for budget customers, as neither direct channel is affordable to them, (ii) suggests that the same does not hold for i-lovers. An i-lover’s willingness to use NYOP depends upon her “neutrality” level for the underlying outcomes. In general, the NYOP channel captures the set of customers with low degrees of preference (i.e., those who are relatively indifferent between the two retailers).

The individual bids of the customers that choose to use NYOP might vary, depending on their valuations. The following proposition characterizes the optimal bid for each NYOP participant.

**Proposition 3. (Customer’s bid at the NYOP channel; two in-stock retailers)** At time $t$, suppose that both retailers carry some stock with posted price $(p_1, p_2)$, and that a customer with valuation $v$ chooses to use NYOP. Then, there exists a unique $b^*(v) \in [0, p]$ at which the customer will bid. Specifically,

(i). for budget customers, the optimal bid $b^*(v)$ increases with the estimated value of the opaque product, $\alpha_1 v_1 + \alpha_2 v_2$;

(ii). for i-lovers, the optimal bid $b^*(v)$ decreases with the degree of preference, $v_i - v_j$;

(iii). all customers bid below the estimated value of the opaque product; that is, $b^*(v) < \alpha_1 v_1 + \alpha_2 v_2$.

From (i), it is interesting to note that if a customer cannot afford either product at the posted price (e.g., budget customers), her bid is related to her expected valuation of the opaque product, $\alpha_1 v_1 + \alpha_2 v_2$. On the other hand, if a customer has the option to use the direct channel (e.g., i-lovers), (ii) implies that her bid is only related to the difference between her valuations of the two products, $v_i - v_j$. Indeed, when NYOP is the only channel through which one can obtain some product, the customer’s bid has to reflect the expected
value that she may receive. However, when a customer participates in the NYOP channel only to “gamble” and see if she can get her preferred product at a lower price, she needs to take into account how much more she values one product over the other, as NYOP comes with the risk of receiving the less-preferred product. (iii) implies that there is a non-trivial information rent that each customer would deduct from the estimated value of the opaque product.

4. Final Sales Channel

In the previous section, customers made their initial channel selections based on their private valuations, observed posted prices, and their estimates of reservation price and opaque-product provider. The disparity between the estimated and the real reservation price/opaque-product provider may result in some unsuccessful bids. As the direct channel is always open to all customers, the channel participation will change after some customers learn that their bids were not accepted by the NYOP channel. In this section, we discuss how the final sales channel is affected by the retailers’ posted price, \( p \), and reservation price, \( r \). The following notation will be used throughout the rest of the paper:

- **NYOP buyers** \( B_0(r, p, A) \) — the set of customers whose bids are accepted at NYOP. These are the \( i \)-lovers and budget customers that outbid the reservation price.
- **\( i \)-Direct buyers** \( B_i(r, p, A) \) — the set of customers who will end up purchasing from the direct channel of retailer \( i \). It consists of \( i \)-lovers that bear relatively high degree of preference between the retailers, such that they would buy directly from retailer \( i \) in the first place, or place an insufficient bid at the NYOP channel and turn to the direct channel after being rejected.
- **Empty hands** \( B_N(r, p, A) \) — the set of customers who will leave without purchase. These are budget customers with low estimated value of the opaque product.
For simplicity, we will again omit the signal notation. Note that $B_0 \cup B_1 \cup B_2 \cup B_N = I$. Recall that customers’ bidding behavior changes as the number of in-stock retailers varies; thus, we again analyze two different scenarios corresponding to possible values of $A$.

4.1 One Retailer Runs Out of Stock

When one of the retailers runs out of stock, customers observe $A = (0,1)$ or $(1,0)$, and there is no competition. According to Proposition 1, all customers bid with the NYOP firm first, and consider buying at the posted price only if their bid is rejected.

**Proposition 4. (Sales Channel Realization; one in-stock retailer)** Suppose that retailer $i$ is out of stock and retailer $j$ is in stock, with posted price $p$ and reservation price $r$. Then there exists a unique $r_0 \in [p,p]$ such that

(i). if the reservation price is high ($r > r_0$), then all customers buy from the direct channel or leave empty handed, i.e., $B_0 = \emptyset$, $B_j = \{v : v_j \geq p\}$, $B_N = \{v : v_j < p\}$;

(ii). if the reservation price is low ($r \leq r_0$), then all customers buy from the NYOP firm or leave empty handed, i.e., $B_0 = \left\{v : v_j \geq r + \frac{G(r)}{g(r)}\right\}$, $B_j = \emptyset$, $B_N = \left\{v : v_j < r + \frac{G(r)}{g(r)}\right\}$.

Specifically, $r_0$ can be determined by $r_0 + \frac{G(r_0)}{g(r_0)} = p$.

Since only one retailer is in stock, the NYOP channel is practically selling regular goods, and customers bid according to Proposition 1. In this case, if the reservation price is high ($r > r_0$), all bids could not go through the NYOP channel as they do not meet the reservation price. Then, the customers with valuation above the posted price, $p$, will buy from the direct channel, while the remaining customers leave empty-handed. On the other hand, if the reservation price is low ($r \leq r_0$), a fraction of the bids will be accepted and no one will buy through the direct channel (since anyone who is able to afford the product at posted price will make a bid above the reservation price). From the retailer’s point of view, one channel will be idle under either kind of pricing schemes.
Recall from Proposition 1 (iii) that all customers post bids that aim to share at least half of the retailer’s margin. By Proposition 4 we can find that if the reservation price \( r \) is above \( r_0 = \frac{p_1 + p_2}{2} \), no one will win her bid from NYOP; otherwise, customers with valuation above \( r + \frac{G(r)}{g(r)} = 2r - \frac{p}{2} \) will win their bids, while the others will leave without any purchase.

### 4.2 Both Retailers Present

When both retailers have some inventory, i.e., \( A = (1, 1) \), the customers bidding below the reservation price \( r \) will be rejected and will join the direct channel. Let \( \Omega_i(\delta) = \Omega_i \cap \{ v : v_i - v_j \leq \delta \} \) denote the set of \( i \)-lovers with degree of preference less than \( \delta \), and \( \Omega_N(\delta) = \Omega_N \cap \{ v : \alpha_1 v_1 + \alpha_2 v_2 \geq \delta \} \) the set of budget customers with estimated opaque product value above \( \delta \). The final channel realization is characterized as follows.

**Proposition 5. (Sales Channel Realization; two in-stock retailers)** Suppose that both retailers are in stock with posted price \( (p_1, p_2) \), and that the reservation price with NYOP firm is \( r \). Then,

(i). \( i \)-direct buyers are all \( i \)-lovers whose degree of preference is higher than \( \hat{\delta}_i = \alpha_j^{-1} [p_i - r - G(r)/g(r)] \), \( \forall i = 1, 2, \ j \neq i \). That is, \( \mathcal{B}_j = \Omega_i / \Omega_i(\hat{\delta}_i) \);

(ii). NYOP buyers are all \( i \)-lovers whose degree of preference is lower than \( \hat{\delta}_i \), for \( i = 1, 2 \), and budget customers whose estimated value of the opaque product is above \( \hat{\delta}_N = r + G(r)/g(r) \). That is, \( \mathcal{B}_0 = \bigcup_{k=1,2,N} \Omega_k(\hat{\delta}_k) \).

\( \Omega_i(\hat{\delta}_i) \) is the set of \( i \)-lovers and \( \Omega_N(\hat{\delta}_N) \) the set of budget customers that will bid higher than \( r \), which comprise the NYOP buyers. On the other hand, \( i \)-lovers in \( \Omega_i(\hat{\delta}_i^*) / \Omega_i(\hat{\delta}_i) \) would not bid over \( r \), hence they will be rejected by the NYOP firm and buy from the direct channel in the end. These customers, together with the \( i \)-lovers that purchased directly from retailer \( i \) without attending the NYOP firm \( (\Omega_i / \Omega_i(\hat{\delta}_i^*)) \), form the \( i \)-direct buyers.
5. Equilibrium Pricing Decisions for the Retailers

We now analyze the first stage in Figure 1 and study how retailers determine their posted prices as well as the reservation price, taking into account the NYOP firm’s and customers’ strategies in later stages.

5.1 One Retailer Runs Out of Stock

When only one retailer, say \( j \), offers his products through the intermediary firm, then at the beginning of each time epoch he makes a decision on \( p \) and \( r \) without taking competitive factors into consideration. Denote \( p^*(x,t) \) and \( r^*(x,t) \) as the optimal pricing policy for the in-stock retailer \( j \) given inventory level \( x \) at time \( t \). Let us further denote by \( \Pi_j(x,t) \) the optimal expected revenue for retailer \( j \) if his decision follows the optimal policy throughout the time. The following summarizes the optimal strategy for the monopolistic in-stock retailer:

**Proposition 6. (Pricing strategy; one in-stock retailer)**

(i). A monopolistic retailer maximizes his expected revenue by using only direct channel.

(ii). Assume that customer valuations and estimations are uniformly distributed, i.e., \( v \sim U[0,1] \times [0,1] \) and \( \xi \sim U[0,p] \). Then at \( t = 1 \), \( \Pi_j(x,t) = \frac{1}{4} \) and \( p^*(x,t) = \frac{1}{2} \); and \( \forall t > 1 \),

\[
\Pi_j(x, t) = \Pi_j(x - e_j, t - 1) + \left( \frac{1 + \Pi_j(x, t - 1) - \Pi_j(x - e_j, t - 1)}{2} \right)^2,
\]

\[
p^*(x, t) = \frac{1 + \Pi_j(x, t - 1) - \Pi_j(x - e_j, t - 1)}{2}.
\]

In particular, suppose \( x = (x_i, x_j) = (0, 1) \). Then \( \lim_{t \to \infty} \Pi_j(x, t) = 1 \) and \( p^*(x,t) \) increases in \( t \).

If a monopolist retailer can sell his products directly at posted prices, or as opaque goods through the NYOP channel, or both, he will always choose to go with posted prices only. As
Proposition 4 implies, a combination of the two channels is not necessary because there will always be one channel that is idle. Hence, the problem is reduced to selecting the channel that brings in more revenue (direct channel with posted prices or NYOP). However, if the NYOP channel is chosen, the retailer would collect less than he could with a direct channel because the third-party NYOP firm will share a portion of the revenue.

Part (ii) characterizes the exact solution under some generic conditions. It can be shown that a monopolistic retailer can achieve his highest possible revenue given that the sales time is sufficient, i.e., \( \lim_{t \to \infty} \Pi_j(x, t) = 1 \) when \((x_i, x_j) = (0, 1)\). In addition, as time goes by, the optimal posted price \( p^* \) drops from 1 to 0.5. The NYOP channel does not bring in any additional revenue in an environment characterized by lack of competition.

5.2 Both Retailers Present

At the beginning of each time epoch \( t \), the retailers observe their inventory positions \( x = (x_1, x_2) \). Then they competitively determine their posted prices, \( p = (p_1, p_2) \). Denote by \( p^*(x, t) \) the equilibrium posted price for period \( t \), and by \( \Pi_i(x, t) \) the expected revenue for retailer \( i \) at the beginning of period \( t \), given that posted prices are the equilibrium ones. Obviously, \( \Pi_i(x, 0) = 0 \) and \( \Pi_i(x, t)|_{x_i=0} = 0 \).

We next analyze the problem in two steps. First, given posted prices \( p \) in period \( t \), what reservation price \( r \) will the retailers chose and who will be the opaque provider? This corresponds to the decisions in Stage 2 of Figure 1. Second, we study whether equilibrium posted prices can be achieved — Stage 1 decision making. More discussion regarding the equilibrium decisions will follow in §7 and §8.

5.2.1 Reservation Price

Let \( \Pi_i(p|x, t) \) denote the expected revenue for retailer \( i \) at time \( t \) if the posted price \( p \) applies at period \( t \) and equilibrium posted price \( p^* \) applies for \( t - 1, t - 2, ..., 1 \). Also, define the marginal value of inventory for each retailer at a given time epoch \( t \) as:
\[ \tilde{\Pi}_1(x, t) = \Pi_1(x_1, x_2 - 1, t) - \Pi_1(x_1 - 1, x_2, t) \]
\[ \tilde{\Pi}_2(x, t) = \Pi_2(x_1 - 1, x_2, t) - \Pi_2(x_1, x_2 - 1, t). \]

The marginal value of the inventory, \( \tilde{\Pi}_i \), measures the difference in retailer \( i \)'s expected revenue if he lets a customer go to his rival compared to his revenue if he serves the customer himself. In essence, it is the cost for retailer \( i \) with inventory level \( x \) for serving a customer at period \( t \).

At time \( t \), suppose the intermediary firm proposes a reservation price \( r \). Retailer \( j \) can choose not to be the opaque provider, match the reservation price \( r \), become the opaque provider by agreeing to a lower reservation price, \( r - \epsilon \), or even ask more by proposing a higher reservation price \( r + \epsilon \), depending on whether retailer \( i \) chooses to be the opaque provider at \( r \). It can be shown that

**Proposition 7. (reservation price; two in-stock retailers)** At time \( t \),

\( i \). the retailer with the lower marginal value of inventory will be the opaque provider;

\( ii \). if marginal value of inventory is the same for both retailers, then each serves as the opaque provider with an equal probability;

\( iii \). the reservation price represents the higher marginal value of inventory between the two retailers, i.e., \( r^*(x, t) = \max_{i=1,2} \tilde{\Pi}_i(x, t - 1) \).

The result resembles that of a reverse auction: the bidder with lower valuation of the marginal inventory becomes the opaque provider, and accepts a (reservation) price of his rival’s marginal valuation of inventory. In addition, although the retailers determine the reservation price after the posted prices are announced, Proposition 7 suggests that that this price relies little on \( p \).

Recall that the salvage value for the inventories are zero; the following corollary then follows immediately from (iii).
Corollary 1. At \( t = 1 \) and in the presence of competition \((x_i \cdot x_j > 0)\), the reservation price \( r^* = 0 \).

The corollary implies that the retailers will offer last-minute sales that lead to arbitrary bids through NYOP \((r^* = 0)\). The customers may take this fact into account when deciding their bid at \( t = 1 \). Still, it is possible for the retailers to post non-trivial tag prices at the same time, in order to capture the \( i \)-lovers through their direct channels. For example, assume that \( v \sim U[0,1] \times [0,1] \) and \( t = 1 \). The retailers would choose the posted price that maximize their revenue in the direct channel (as no revenue will come from the opaque channel). The last-period posted price for either retailer should be \( p_1 = p_2 = \arg \max_{0 \leq p \leq 0.5} p(1 - 2p)^2 = 1/6 \).

5.2.2 Posted prices

To verify the existence of pure-strategy Nash equilibrium (NE) \((p_1^*(x,t), p_2^*(x,t))\), we need to check if the payoff function is quasi-concave in \( p_i \). While this result seems analytically intractable for general distributions of \( v \), it can be proved that it holds for uniformly distributed valuations \( v \sim [0,1] \times [0,1] \). Numerical experiments suggests that NE exists in more general instances, such as normal distribution.

Proposition 8. (posted prices; two in-stock retailers) Consider uniformly distributed customer valuations, i.e., \( v \sim [0,1] \times [0,1] \). Then,

(i). pure-strategy NE in posted prices exists;

(ii). reservation price \( r^* \) is zero when both retailers oversupply, i.e., \( x_i \geq t \) for \( i = 1, 2 \).

Proposition 8 (i) legitimizes further study in §7 and §8; (ii) extends the result from Proposition 7 by establishing that zero reservation price occurs not only at the last moment, but also when both retailers oversupply at any time period.

\(^{15}\)In our model, this is the only moment in which customers do not need the prior.
6. Analysis of Some Special Games

Thus far, we have developed structural results for the dual-channel competitive framework. In this section, we apply some additional assumptions to the game in order to derive exact solutions and managerial implications: in §6.1 we reduce the complexity of customer valuation and analyze the impact of product differentiation on the retailers as well as the NYOP firm, while in §6.2, we shorten the time horizon by studying a two-period game. The robustness of these results will be shown in §7, which also discusses the impact of NYOP channel.

6.1 Market with Product Differentiation

This subsection will focus on two special sets of products (which yield two special sets of customer valuations) and derive some relevant managerial insights. Recall that the valuation of regular products \( \mathbf{v} \) may differ across the customers, yet the same belief \((\alpha_1, \alpha_2)\) holds about the outcome of the opaque product selection. We will say that the two underlying products are:

- **horizontally differentiated** if \( \alpha_i v_i + \alpha_j v_j = \bar{v} \) for any \( \mathbf{v} \) and some constant \( \bar{v} > 0; \ i, j \in \{1, 2\}; \)
- **vertically differentiated** if \( v_i - v_j = \bar{v} \) for any \( \mathbf{v} \) and some constant \( \bar{v} > 0; \ i, j \in \{1, 2\} \) and \( j \neq i. \)

The horizontal differentiation allows customers to have a common expectation regarding the opaque product, but potentially very different opinions about the regular ones; for example, given two three-star hotels, some customer might prefer Holiday Inn while others like Courtyard Marriott. To some extent, this is similar to the Hotelling’s model. Vertical differentiation, on the other hand, implies that one product \((i \text{ in this case})\) is commonly perceived as a better one; examples are the airline tickets, where for the same itinerary flights departing/arriving at regular hours are usually most preferred—yet customers cannot put such restrictions in the opaque channel. These special instances lead to following results.
Proposition 9. (horizontally vs. vertically differentiated products)

(i). If the products are vertically differentiated, then retailer $i$ sets a posted price such that all customers buy from its direct channel. The intermediary NYOP firm collects zero rents.

(ii). If the products are horizontally differentiated, then there exists $p^0 > 0$ such that (a) when the lower posted price, $p$, satisfies $p > p^0$, all customers are covered, and the intermediary NYOP firm earns positive revenue; (b) when $p \leq p^0$, some customers will not receive any of the products, and no customers will use NYOP.

The implication is that the intermediary firm should be careful when selecting regular products in order to construct opaque products in the NYOP channel. If it has been recognized that product candidates have a significant quality difference (a flight departing at one a.m. vs. a flight on the same route that leaves at noon) or one product has higher brand recognition (such that all customers may prefer one particular product, although the quality of the other is the same), then the intermediary firm may not benefit much from these differences. The main reason is that the retailer with “better quality/image” can obtain the deterministic difference in customer valuation by offering a posted price that customers “cannot refuse.” Under this posted price, the customers are convinced that the opaque product is not worth bidding on. On the other hand, if the products are horizontally differentiated (e.g., hotels in the same region and with the same star ranking), the valuation for the underlying product will be much more diversified. The retailers would then need the intermediary NYOP firm to shoulder part of this uncertainty such that their direct channels can target their own high-end customers. These results are consistent with the observations in Perkins (2006). Also, Priceline’s 2011 Annual Report to Stockholders (pp.55) states that

“Hotel room night reservations sold increased by 49.3% for the year ended December 31, 2009, ..., primarily due to ... an increase in the sale of Name Your Own Price room night reservations.... Rental car days sold increased by 12.4%, ..., primarily due to an increase in ... Name Your Own Price rental car days.
Airline tickets sold increased by 21.8% ..., due to an increase in the sale of price-
disclosed airline tickets, partially offset by a decline in Name Your Own Price
airline tickets."

It confirms that Priceline does benefit more from horizontally differentiated goods (hotel
rooms and rental cars) than from vertically differentiated ones (airline tickets).

6.2 A Two-Period Game

This subsection considers selling a general set of products, i.e., $v \sim U[0, 1] \times [0, 1]$, through
a two-period time horizon. The initial period, $t = 2$, represents regular sales season and the
last period, $t = 1$, approximates the last-minutes sales. We assume that when one retailer is
out of stock, customers estimate the reservation price as $\xi \sim U[0, \rho]$; thus $G_\xi(r) = r/\rho$; when
both retailers are in-stock, the estimation satisfies $\frac{16}{16} G_\xi(r) = \frac{(2 \rho - r^2)}{\rho^2}$. We also assume
that each retailer is expected to be the opaque product provider with the same chance; i.e.,
$\alpha_1 = \alpha_2 = 0.5$. For each period, we find the expected revenue for each retailer, reservation
price at the NYOP channel, opaque product provider, as well as equilibrium posted prices
using the structural results developed in SS4-6; details can be found in the Appendix, Tables
3–9. The results for the two-period game suggest the following:

Proposition 10. (expected revenue for the retailer) At any time epoch, the expected
revenue for retailer $i$, $\Pi_i$, (1) is non-decreasing in the remaining sales time $t$, (2) is non-
increasing in the rival’s (retailer $j$’s) inventory level $x_j$; (3) can be decreasing in $i$’s own
inventory level, $x_i$.

Both (1) and (2) are rather straightforward—(1) highlights the value of time in competitive
dynamic pricing, while (2) implies that carrying less inventory grants one’s rival operational
advantage in a competitive environment. However, does it also suggest that a retailer would
absolutely benefit from its own inventory, since it reduces the operational advantage to its
rivals? (3) shows that this logic does not necessarily hold—it captures a scenario that seldom

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16The prior comes from the assumption that each customer estimates the marginal value of inventory at
each retailer $\sim_{\text{i.i.d.}} U[0, \rho]$; and that the reservation price is the minimum of the two.
occurs in the lack-of-competition environment, in which one’s revenue might be decreasing with respect to his own inventory level, which can be described as follows.

In a two-period game, there will be a total of 2 arriving customers, each with unknown valuation. Since we assumed that $\mathbf{v}$ is uniformly distributed, there is practically no difference between the two products, and one can think of inventory level 1 as the “quota” level that splits the potential demand. When the rival stocks no more than this quota, $x_j \leq 1$, the game is less competitive, and the retailer’s expected revenue increases with his inventory level (see Table 6). On the other hand, when the rival stocks more than the quota, $x_j \geq 2$, but the retailer himself is within the quota, $x_i \leq 1$, the rival estimates that the retailer will behave less aggressively because it seems reasonable to expect that the retailer with less inventory will “sell less at a high price” instead of “price low and sell more.” Thus, $\Pi_i$ again increases in $x_i$ (Table 6). Finally, when both retailers stock over the quota ($x_i \geq 2, x_j \geq 2$), each retailer believes that his rival will take some competitive actions, and each indeed has the motivation to do the same, as he has the resource (high inventory levels) for achieving a greater market share. As a result, both posted prices and reservation prices become low as a result of the severe competition, which leads to the counterintuitive result that more inventory may lead to lower revenue (Table 6).

Table 9 yields another interesting observation upon who provides the product in the NYOP channel. The market again expects 2 arriving customers, and the “quota” for each retailer is set at 1.

**Proposition 11. (opaque provider)** In last-minute sales, i.e. for $t = 1$, each retailer is equally likely to be the opaque provider if both are in-stock. For regular season, i.e. for $t = 2$,

(i). the opaque provider is retailer 1 or 2 with equal probability if and only if (a) inventory levels are the same at two sites, $x_1 = x_2$; or (b) both parties over-supply, $x_1 \geq 2, x_2 \geq 2$;

(ii). the opaque provider is retailer $i$ if and only if (a) retailer $i$ carries more inventory, $x_i > x_j$, and retailer $j$ stocks less than its quota, $x_j < 1$; or (b) retailer $j$ carries more inventory, $x_i < x_j$, and retailer $i$ stocks no less than its quota, $x_i \geq 1$. 
When both firms oversupply (i.e., $x_1 \geq 2$ and $x_2 \geq 2$), both parties have zero reservation price, and the opaque provider is randomly drawn. In addition, whenever $x_1 = x_2$, the two firms have the same decisions about their reservation prices. While the final reservation price may be positive, each of the two firms has the same chance to be the opaque provider.

When the inventory level of one retailer is less than the quota (i.e., $x_j < 1$), then the opaque provider will be the one with more inventory. This is rather straightforward—when $x_j \leq 1$ and $x_j < x_i$, retailer $j$ is not motivated to sell more or to sell cheaply because of his low inventory level. However, if both inventories exceed the quota (i.e., $x_1 \geq 1, x_2 \geq 1$), the retailer with less inventory will supply the NYOP firm. Reversal of the previous claim makes this situation rather interesting—in this case, competition becomes more intense compared with the first case because neither retailer is expected to sell all of his inventory. Both retailers may prefer a less competitive environment, and the fastest way to get there is to grant the retailer with less inventory more opportunity to sell his units. The retailer with more inventory will not supply the NYOP firm until his rival’s inventory level hits 1.

We have verified that the same observation holds if we extends 2-period game to $n$-period game; Figure 3 illustrates the observation. [Insert Figure 3 here]

Note that Table 9 and Figure 3 are based on the assumption that $\alpha_1 = \alpha_2 = 0.5$. In the NYOP business model, opaque provider is determined before a customer submits her bid. Therefore, a discrepancy exists between a customer’s belief and the actual realization that is yet unannounced. The existence of such discrepancy is supported by data or factored in models in most of the existing literature—see, for instance, Hann and Terwiesch (2003), Spann et al. (2004), Terwiesch et al. (2005), and Cai et al. (2009). Nevertheless, it can be shown that under some instances this estimation is rational in the long run; that is, in expectation, half of the time opaque provider is indeed retailer 1, and the other half retailer 2.

**Proposition 12. (rational expectation of the opaque provider)** If the two retailers have the same initial capacity, i.e., $\bar{x}_1 = \bar{x}_2$, it is then rational for the customers to assume $\alpha_1 = \alpha_2 = 0.5$. 

7. Impact of NYOP Channel

We now look at numerical solutions for games with longer time horizon; our analysis confirms the robustness of results in §6.2 and delivers some additional insights. Our main goal is to study how the NYOP channel impacts posted prices as well as expected revenues on the supply side. In this section, customer valuation \( v \) is assumed to be uniformly distributed\(^{17} \) on \([0, 1] \times [0, 1] \). We assume equal capacity for the two retailers, hence it is rational to have \( \alpha_1 = \alpha_2 = 0.5 \). We will denote the dual-channel (direct and NYOP) case by “DC” and the single-channel (direct only) case by “SC” in this section. Our findings are summarized in Table 10. [Insert Table 10 here]

7.1 Expected Revenue

We discuss the expected revenue for a particular retailer depicted in two figures: Figure 4 assumes the competitor’s inventory level is given and examine how one’s revenue changes with the remaining number of selling periods and his own inventory level; Figure 5 looks at how the competitor’s inventory level would affect one’s expected revenue at a fixed time point. [Insert Figure 4 here] [Insert Figure 5 here]

- **Effect of Time.** Let us first look at Figure 4. Not surprisingly, the expected revenue grows with the length of the remaining time in both cases. For SC, the marginal revenue of one additional sales period, \( \Pi_1(x_1, 5, t) - \Pi_1(x_1, 5, t - 1) \), decreases with \( t \). For DC, the marginal revenue for a time period is decreasing only when some retailer does not oversupply (i.e., \( \min\{x_1, x_2\} < t \)). In other words, in presence of NYOP channel, it is possible that the time in regular sales season is more valuable than the last minute period. As Proposition 8 (ii) indicates, if both retailers oversupply (i.e., \( x_{1,2} \geq t \)), each retailer strives to sell his inventories, and the NYOP channel intensifies the competition, which leads to a zero reservation

\(^{17}\)We have also conducted the same analysis based on normal distribution. The results are similar to those obtained for uniform distribution and are omitted for brevity.
price. In these instances, one more sales period upfront (rather than later on) is contributing significantly to the expected revenue, as this unit has a non-trivial chance to be sold at some posted price. Note that this time-effect depends only on the fact that there is currently an over-supply, not on the specific inventory levels.

- **Effect of Inventory Levels.** In Figure 4, we can observe that while in SC the expected revenue grows with one’s own inventory, $x_1$, the same does not hold in DC, especially when $x_1 + x_2 \geq t$. Hence, while in SC “more is better,” in DC “more may be worse, especially when total supply exceeds total demand.” The diminishing margin of inventory holds under similar conditions for the two cases. Similar conclusions can be made for Figure 5, in which the marginal revenue of one’s own inventory can be negative. For example, if we consider DC, when $x_2 = 6 > 5$, $\Pi_1$ increases with $x_1$ as long as $x_1 \leq 4$ (we assume $t = 8$, so one can think of inventory level 4 as the “quota” level that splits the potential demand); it then decreases as $x_1$ changes from 5 to 8, and becomes constant for $x_1 > 8$. A similar impact has been discussed after Proposition 10 (3).

- **Comparison between DC and SC.** When comparing the two graphs in Figure 4, one may notice that the expected revenue is higher in SC than in DC. The difference in expected revenue is more significant as competition becomes more intense (i.e., $x_1$ becomes higher). The low revenue in DC generally comes from two sources. First, the NYOP channel provides a platform on which retailers compete to sell at a low price without damaging the integrity of their direct channel. As competition becomes more intense, the reservation price at the NYOP channel could become rather low (eventually becoming $r = 0$ in case of over-supply). Thus, there is a reasonable chance that some product quantity will be sold at lower prices in DC than SC. Second, the posted price competition becomes more intense as well. In the absence of the NYOP channel, retailers compete *publicly* through their posted prices only, and there is less uncertainty about whose product the customer will eventually buy. On the other hand, with the NYOP channel each retailer is aware that the other seller may “steal” his customer through NYOP. Hence, each retailer is competing with the part of the opaque
product that comes from his rival, as well as with other regular products in the market. Figure 6 also shows that posted prices in the presence of the NYOP channel usually have lower values than when this channel is absent. This provides experimental evidence that the NYOP channel intensifies the competition between the retailers.

7.2 Equilibrium Posted Price

Similarly to the expected revenue, we discuss the equilibrium posted price from two different perspectives as well. Figure 6 and Figure 7 depict equilibrium posted price given rival’s inventory level; Figure 8 illustrates how the NE posted price varies with the inventory level at a fixed time spot. [Insert Figure 6 here] [Insert Figure 7 here] [Insert Figure 8 here]

• Effect of Time and Inventory Level. Figure 6 shows that the posted price in SC decreases with the rival’s inventory level and elapsed time. In DC, we observe the same general trend with respect to posted prices. Nevertheless, we may observe significant differences between the two graphs. According to Proposition 8 (ii), reservation price will be zero if both retailers oversupply (i.e., $x_2 = 5 \geq t$ and $x_1 \geq t$). When this happens, the posted price also takes a very low value (around 0.167). This also explains why all of the lines with $x_1 \in \{5, 6, 7, 8, 9, 10\}$ coincide when $t \in [1, 5]$. Moreover, when a retailer has a greater inventory ($x_1 > x_2 = 5$) and his rival does not ($t > x_2 = 5$), his posted price stays at a moderate level. On one hand, he does not price too low because the rival is not oversupplying; on the other hand, it is not necessary to price too high since he has enough inventory. As one may be aware, many of the inconsistencies stem from oversupply. If we remove the oversupply effects by concentrating on smaller inventory levels, the graphs for SC and CD may look more alike (Figure 7).

At a given time period, say $t = 8$ (Figure 8), a retailer’s posted prices in DC are rather sensitive to inventory level when his rival stocks over the “quota” ($\frac{t}{2}$). When it is, indeed, the case that $x_2 > \frac{t}{2} = 4$, a retailer’s posted price first decreases with his inventory, $x_1$, until it hits $x_2$; then it begins to increase with $x_1$ after $x_1$ exceeds $x_2$. One of the possible
reasons for this can be found in our discussion in Proposition 11, which shows that when \( x_2 \geq \frac{t}{2} \), the opaque-product provider will be “retailer 2 \rightarrow retailer 1 \rightarrow retailer 2” as \( x_1 \) increases. This result suggests that a retailer’s posted price should be higher when he is not the opaque provider than when he is, because if his competitor (the opaque-product provider) is getting the low-end customers or the customers who do not differentiate much between the products, then it is better for the retailer to focus on his own loyal market (those customers who have strong preferences for him over his competitor). One effective way to identify such customers, and to improve the revenue, is through charging a higher price.

- **Comparison between DC and SC.** Although the expected revenues are overall higher in SC than in DC, the relationship between posted prices can go either way. For example, Figure 6 shows that when \( x_2 = 5 \), the posted prices in SC are generally higher when \( t \leq 5 \). However, as \( t \) becomes greater than 5 (retailer 2 is no longer oversupplying), posted prices in SC are lower than those in DC for some large values of \( x_1 \). In general, DC has higher posted price whenever one’s own inventory level is high, his rival’s inventory level is low, and/or the remaining time horizon is long. With these factors, the overstocking retailer will target his direct channel at high-end customers that would not use NYOP, while at the same time setting a very low reservation price to attract as many NYOP customers as possible.

### 7.3 The NYOP Paradox

Our findings in §7.1 imply that NYOP does not make either the retailers or the industry better off—the existence of an intermediary firm results from the competition between the retailers; however, with an NYOP channel the retailers are worse off in the intensified competition. This also matches the empirical evidence in Granados et al. (2010) that the opaque channel may not always expand the market. However, thus far one question remains unanswered: *if NYOP is not benefiting the retailers, why are they partnering with the intermediary firm?* We propose some possible explanations:

- **Information asymmetry.** As discussed in §2.2 (Retailer Participation), membership in an NYOP channel is generally not public information. If an in-stock retailer
(say, retailer 2) chooses not to join an NYOP channel, he is giving up the customers that might buy from him through the opaque channel (to his competitor, retailer 1). It is, then, in the best interest of retailer 1 to join the NYOP channel in this scenario, as the opaqueness still holds due to information asymmetry. He can then take advantage of retailer 2’s suboptimal channel decision. This scenario will become even more plausible in a more realistic setting with multiple retailers.

- **Competition vs. collusion.** Throughout our analysis, we assume that retailers are competitors and rule out any possibility of collusion. One may assume that, if the retailers were allowed to communicate and reach agreements that would remove the intermediaries from the market, they might generate higher revenues. These are, however, strong assumptions, which have not been observed in industry as of yet, and which may also have legal implications.

- **NYOP firm as a revenue maximizer.** If the sole purpose of NYOP intermediary is to maximize its own welfare, it would choose the mechanism that ignites severe competition among retailers. Indeed, during the early years of Priceline.com, it went through a lot of resistance from the airlines due to potential low bids in the opaque channel (Pederson (2004)). This is not surprising: even from the theoretical perspective, a revenue-maximizing strategy may lack long-term sustainability, as the retailers may decide to collude by not participating and thus driving the intermediary firm out of business. In particular, policies in Anderson (2009) suggest that Priceline.com may be more interested in expanding its retailer base than in maximizing its revenues with a selected, smaller number of retailers. This also implies a rich set of possible model extensions, and will be discussed in more details in §8.

8. **Concluding Remarks**

Advanced information technology never stops adding new dimensions to the way we sell and we buy. As a result, businesses are under an increased pressure to redesign their distribution
channels and pricing strategies. Our model framework blends dual channel management and competitive dynamic pricing together in understanding how a new sales format—Name-Your-Own-Price (NYOP)—may influence the industry. The results provide managerial insights for stakeholders (retailers, NYOP firms, customers) in a wide range of business from leisure travel (e.g., Priceline.com) to personal service (e.g., Urbanoffer.com). To the best of our knowledge, relevant studies have so far neglected to properly address the impact of NYOP on competing retailers.

While our paper deals with a stylized model in analyzing the impact of the NYOP channel, there are many directions in which this work can be extended (e.g., Granados et al. (2009)); we list a few of them below.

Opaque Product Line. The intermediary firm chooses the underlying products for the opaque goods. For a two-retailer-two-product problem, our results indicates that it is better to have horizontally differentiated products (e.g., hotels with different brand names but at the same star-level and nearby locations) than vertically differentiated ones (e.g., flights of the same route but one departs at one a.m. while the other departs at noon). As more retailers participate in the model, there is more room for the intermediary firm to increase the bundling assortment; the firm may also inform the customers about the underlying products that comprise the opaque product they are bidding for. There are many interesting approaches with regard to how such decisions could be made.

Contracting. As has been discussed earlier, the relationships between the intermediary firm and retailers can have many alternatives as well. The short-term goal for the intermediary firm needs not be restricted to maximizing its revenue; even if it was, the process of determining the opaque-product provider can take various alternatives. Our current setting assumes that the intermediary firm uses a descending auction to determine the reservation price. One realistic alternative may be to ask the sellers to privately submit their own reservation prices, but it may add a new level of competition and require some additional prior assumptions. Conversely, the problem could be simplified by letting the intermediary
firm set the reservation price and then randomly select a retailer that is willing to participate. In addition, revenue-sharing contacts in which the retailers receive a portion of the spread between the customer bid and the reservation price, \( b - r \), may be adopted.

**INFORMATION AVAILABILITY.** Information structure assumed in the model drives some of our results, and is applicable to some examples observed in practice. It is in the interest of the intermediary firm to explore the consequences of offering various degrees of information to its customers. One potential extension is to consider Hotwire.com (regular goods) and Priceline.com (opaque goods), and compare the optimal information-offering for both models.

We have also assumed that each retailer knows the capacity at both retailers; the problem can be modified by looking at instances with imperfect information. On one hand, players may not know demand distribution, and we may allow retailers/NYOP firms to learn from the bidding data. On the other hand, retailers may rely on their own mechanisms in estimating the remaining inventories of their competitors through their posted prices or their reservation prices at the intermediary firm.

**CUSTOMER BEHAVIOR.** In the presence of multiple sales channels and a finite number of sales periods, customers can strategically plan their purchasing/bidding timing across a long horizon. While our model assumes a static customer set, it would be of practical value to examine the problem in which strategic customers can make intertemporal decisions on channel selection and bidding, or in which returning customers can repeat their bids after the “frozen” period has elapsed.

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Appendix A: Figures

Figure 1: Game sequence in each period.

Figure 2: Customer’s Problem in Stage 3
Figure 3: Opaque Provider at $t = n$.

Figure 4: Expected revenue for retailer 1 as a function of the remaining time, $t$, and available inventory, $x_1$, when $x_2 = 5$. 
Figure 5: Expected revenue for retailer 1 as a function of available inventories, \((x_1, x_2)\), at \(t = 8\).

Figure 6: NE posted price for retailer 1 as a function of available inventories, \(x_1\) and \(t\), at \(x_2 = 5\).
Figure 7: NE posted price for retailer 1 as a function of the remaining time, $t$, and available inventory, $x_1$, when $x_2 = 1$.

Figure 8: NE posted price for retailer 1 as a function of the remaining time, $t$, and available inventory, $x_1$, when $t = 8$. 
### Table 1: Pricing Schemes for Regular and Opaque Goods

<table>
<thead>
<tr>
<th></th>
<th>Regular Goods</th>
<th>Opaque Goods</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Seller-Driven</strong></td>
<td>Expedia, Orbitz</td>
<td>Hotwire, BookIt</td>
</tr>
<tr>
<td><strong>Buyer-Driven</strong></td>
<td>eBay, Google</td>
<td>Priceline</td>
</tr>
<tr>
<td><strong>Auction:</strong></td>
<td>Unrevealed German NYOP firm</td>
<td>Urbanoffer</td>
</tr>
<tr>
<td><strong>NYOP:</strong></td>
<td>(Hann and Terwiesch 2003, Terwiesch et al. 2005)</td>
<td></td>
</tr>
</tbody>
</table>

### Table 2: Information Availability Among the Players

<table>
<thead>
<tr>
<th></th>
<th>Customers</th>
<th>Retailers</th>
<th>Intermediary Firm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Customer Valuation, $v$</td>
<td>Y</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>Posted Prices, $p$</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Initial Inventory Levels, $x$</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Real-time Inventory Availability, $A$</td>
<td>V</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Real-time Inventory Levels, $x$</td>
<td>N</td>
<td>Y</td>
<td>N</td>
</tr>
<tr>
<td>Reservation Price, $r$</td>
<td>N</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Opaque-Product Provider, $I$</td>
<td>N</td>
<td>Y</td>
<td>Y</td>
</tr>
</tbody>
</table>

### Appendix B: Tables
### Expected Revenue ($\Pi_1, \Pi_2$)

<table>
<thead>
<tr>
<th>On-hand Inventory</th>
<th>$x_2$</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>1</td>
<td>$\geq$ 2</td>
</tr>
<tr>
<td>0</td>
<td>(0,0)</td>
<td>(0, 1/4)</td>
<td>(0, 1/27)</td>
</tr>
<tr>
<td>$x_1$ 1</td>
<td>(1/4, 0)</td>
<td>(1/27, 1/27)</td>
<td>(1/27, 1/27)</td>
</tr>
<tr>
<td>$\geq$ 2</td>
<td>(1/4, 0)</td>
<td>(1/27, 1/27)</td>
<td>(1/27, 1/27)</td>
</tr>
</tbody>
</table>

Table 3: Expected Revenue for the Retailers at the Last Minute ($t = 1$)

### Posted Prices ($p_1, p_2$)

<table>
<thead>
<tr>
<th>On-hand Inventory</th>
<th>$x_2$</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>1</td>
<td>$\geq$ 2</td>
</tr>
<tr>
<td>0</td>
<td>(1,1)</td>
<td>(1, 1/2)</td>
<td>(1, 1/2)</td>
</tr>
<tr>
<td>$x_1$ 1</td>
<td>(1/2, 1)</td>
<td>(1/6, 1/6)</td>
<td>(1/6, 1/6)</td>
</tr>
<tr>
<td>$\geq$ 2</td>
<td>(1/2, 1)</td>
<td>(1/6, 1/6)</td>
<td>(1/6, 1/6)</td>
</tr>
</tbody>
</table>

Table 4: Equilibrium Posted Prices at the Last Minute ($t = 1$)
| Reservation Price $r$ | | $x_2$ |
|----------------------|------------------|
| On-hand Inventory    | 0                | 1    | $\geq 2$ |
| 0                    | 1                | 1/2  | 1/2      |
| $x_1$ 1              | 1/2              | 0    | 0        |
| $\geq 2$             | 1/2              | 0    | 0        |

Table 5: Reservation Price in the NYOP Channel at the Last Minute ($t = 1$)

| Expected Revenue ($\Pi_1, \Pi_2$) | | $x_2$ |
|-----------------------------------|------------------|
| On-hand Inventory | 0                | 1    | 2    | $\geq 3$ |
| 0                    | (0,0)            | (0, 25/64) | (0, 1/2) | (0, 1/2) |
| $x_1$ 1              | (25/64, 0)       | (0.2908, 0.2908) | (0.2107, 0.3029) | (0.2107, 0.3029) |
| 2                    | (1/2, 0)         | (0.3029, 0.2107) | (0.0741, 0.0741) | (0.0741, 0.0741) |
| $\geq 3$             | (1/2, 0)         | (0.3029, 0.2107) | (0.0741, 0.0741) | (0.0741, 0.0741) |

Table 6: Expected Revenue for the Retailers at the Regular Season ($t = 2$)
### Posted Prices \((p_1, p_2)\)

<table>
<thead>
<tr>
<th>On-hand Inventory</th>
<th>(x_2)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>(\geq 3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>(1,1)</td>
<td>(1, 5/8)</td>
<td>(1, 1/2)</td>
<td>(1, 1/2)</td>
<td></td>
</tr>
<tr>
<td>1(^1)</td>
<td>(5/8, 1)</td>
<td>(0.511, 0.511)</td>
<td>(0.456, 0.491)</td>
<td>(0.456, 0.491)</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>(1/2, 1)</td>
<td>(0.491, 0.456)</td>
<td>(0.167, 0.167)</td>
<td>(0.167, 0.167)</td>
<td></td>
</tr>
<tr>
<td>(\geq 3)</td>
<td>(1/2, 1)</td>
<td>(0.491, 0.456)</td>
<td>(0.167, 0.167)</td>
<td>(0.167, 0.167)</td>
<td></td>
</tr>
</tbody>
</table>

Table 7: Equilibrium Posted Prices at the Regular Season \((t = 2)\)

### Reservation Price \(r\)

<table>
<thead>
<tr>
<th>On-hand Inventory</th>
<th>(x_2)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>(\geq 3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>5/8</td>
<td>1/2</td>
<td>1/2</td>
<td></td>
</tr>
<tr>
<td>1(^1)</td>
<td>5/8</td>
<td>1/4</td>
<td>1/4-1/27</td>
<td>1/4-1/27</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1/2</td>
<td>1/4-1/27</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>(\geq 3)</td>
<td>1/2</td>
<td>1/4-1/27</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Table 8: Reservation Price in the NYOP Channel at the Regular Season \((t = 2)\)
Table 9: Opaque Provider in the NYOP Channel at the Regular Season ($t = 2$)

<table>
<thead>
<tr>
<th>On-hand Inventory</th>
<th>$x_2$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>$\geq 3$</th>
</tr>
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<tbody>
<tr>
<td>0</td>
<td></td>
<td>1 or 2</td>
<td>2</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>1</td>
<td>1 or 2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>1</td>
<td>2</td>
<td>1 or 2</td>
<td>1 and 2</td>
</tr>
<tr>
<td>$\geq 3$</td>
<td></td>
<td>1</td>
<td>2</td>
<td>1 or 2</td>
<td>1 or 2</td>
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Table 10: Impact of the NYOP Channel

<table>
<thead>
<tr>
<th></th>
<th>Single-Channel (Direct only)</th>
<th>Dual-Channel (Direct and NYOP)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Expected Revenue</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Impact of Time</td>
<td>Last-minute sales are more valuable than regular sales season</td>
<td>Regular season can be more valuable than last-minute sales</td>
</tr>
<tr>
<td>Impact of Inventory</td>
<td>More inventory does no harm</td>
<td>More inventory may reduce the expected revenue</td>
</tr>
<tr>
<td>Overall</td>
<td>Higher</td>
<td>Lower</td>
</tr>
<tr>
<td><strong>Equilibrium Posted Prices</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Impact of Time</td>
<td>Less sensitive to time; higher posted price than DC in last minute</td>
<td>More sensitive to time; higher posted price than SC in regular season</td>
</tr>
<tr>
<td>Impact of Inventory</td>
<td>Less sensitive to one’s inventory; higher posted price than DC when one’s own inventory is low, his rival’s inventory is high</td>
<td>More sensitive to one’s inventory; higher posted price than SC when one’s own inventory is high, his rival’s inventory is low</td>
</tr>
<tr>
<td>Overall</td>
<td>Posted price may be higher or lower in DC or SC depending on the leftover time and inventory levels at the two retailers</td>
<td></td>
</tr>
</tbody>
</table>
Appendix C: Proof for Main Content

Proof of Proposition 1

(i). The FOC for $V(v,b)$ is given by

$$g(b)[\min\{v,p\} - b] - G(b) = g(b)\left[\min\{v,p\} - b - \frac{G(b)}{g(b)}\right].$$

As $\min\{v,p\} - b$ decreases with $b$, $G(b)/g(b)$ increases with $b$, and $G(0)/g(0) = 0$, there exists a unique $b^* \in [0,\min\{v,p\}]$ that satisfies

$$\min\{v,p\} - b^* - \frac{G(b^*)}{g(b^*)} = 0.$$ 

It is easy to verify that the extreme solutions ($b = 0$ and $b = \min\{v,p\}$) are not optimal; therefore, $V(v,b)$ is maximized at FOC=0, i.e, when $b = b^*$.

(ii) Since $\min\{v,p\}$ is non-decreasing in both $v$ and $p$, and $b + G(b)/g(b)$ increases with $b$, $b^*$ is non-decreasing in $v$ and $p$.

(iii) When $\xi \sim U[p,p]$,

$$G(b)/g(b) = \begin{cases} 
0, & \text{if } b \leq p \\
b - p, & \text{if } p < b \leq p \\
p - p, & \text{if } b > p,
\end{cases}$$

hence the solution to FOC=0 is

$$b^* = \begin{cases} 
\min\{v,p\}/2, & \text{if } v \leq p \\
(p + \min\{v,p\})/2, & \text{if } p < b \leq p.
\end{cases}$$

Proof of Proposition 2:

For budget customers, NYOP is the only channel in which they might get some product, so they will always attend NYOP.
For customers with an external choice (i.e., \(i\)-lovers), denote the expected payoff for an \(i\)-lover with valuation \(v\) from the NYOP channel as \(V^N(v) = V(v, b^*(v))\), where \(b^*\) is her optimal bid. An \(i\)-lover will choose the NYOP channel if and only if it yields higher expected payoff than buying at supplier \(i\) directly:

\[
V^N(v) = G(b^*)(\alpha_i v_j + \alpha_i v_j - b^*) + \bar{G}(b^*)(v_i - p_i) > v_i - p_i.
\]

This requires

\[
b^* < (\alpha_i v_j + \alpha_i v_j) - (v_i - p_i) = \alpha_j(v_j - v_i) + p_i. \tag{C1}
\]

The \(V^N\) is concave under DRHR assumption so we only need to solve the FOC for \(b^*\):

\[
\frac{\partial V(v, b)}{\partial b} = g(b) [\alpha_j(v_j - v_i) - b + p_i] - G(b)
\]

The optimal bid then satisfies

\[
\alpha_j(v_j - v_i) + p_i = b^* + G(b^*)/g(b^*). \tag{C2}
\]

When \(\alpha_j(v_j - v_i) + p_i \leq 0\), it is optimal for the \(i\)-lover to submit \(b^* = 0\); otherwise, the bid is non-trivial, i.e., \(b^* > 0\). In the latter case, the NYOP participation condition (C1) will also hold by comparing it with (C2). Therefore, an \(i\)-lover participates in NYOP first if and only if \(\alpha_j(v_j - v_i) + p_i > 0\), which is equivalent to requiring that the degree of preference \(v_i - v_j \leq \delta^* = p_i/\alpha_j\).

\[\blacksquare\]

**Proof of Proposition 3:**

(i) For budget customers, the expected payoff from NYOP is \(V^N(v) = G(b^*)(\alpha_i v_j + \alpha_i v_j - b^*)\) and the FOC is given by

\[
\frac{\partial V(v, b)}{\partial b} = g(b)(\alpha_1 v_1 + \alpha_2 v_2 - b) - G(b)
\]
There exists a unique $b^* \in [0,p]$ that satisfies

$$\alpha_1 v_1 + \alpha_2 v_2 = b^* + G(b^*)/g(b^*). \tag{C3}$$

As $G(b)/g(b)$ increases in $b$, the optimal bid should increase with the expected valuation of the opaque product $\alpha_1 v_1 + \alpha_2 v_2$. Moreover, as $b + G(b)/g(b) = 0$ at $b = 0$, there must be $b^*(v) > 0$ in this case.

(ii) is straightforward by noting that the RHS of (C2) increases in $b$.

(iii) When $v \in \Omega_N$, by (C3) it is obvious that $b^* < \alpha_1 v_1 + \alpha_2 v_2$. Also, note that for the $i$-lovers, $\alpha_j(v_j - v_i) + p_i = \alpha_1 v_1 + \alpha_2 v_2 - (v_i - p_i)$, and $v_i - p_i > 0$. We should have $\alpha_j(v_j - v_i) + p_i < \alpha_1 v_1 + \alpha_2 v_2$. Thus, $b^* < \alpha_j(v_j - v_i) + p_i < \alpha_1 v_1 + \alpha_2 v_2$.

\[\] Proof of Proposition 4: By Proposition 1 the optimal bid satisfies $b^* + \frac{G(b^*)}{g(b^*)} = \min\{v_j, p\} \leq p = r_0 + \frac{G(r_0)}{g(r_0)}$. Hence, for all $v$ there is $b^*(v_j) < r_0$. Thus, if $r > r_0$, all bids will be rejected ($B_0 = \emptyset$) and customers who can afford the posted price ($v \geq p$) will attain the product through direct channel ($B_j = \{v : v_j > p\}$).

If $r \leq r_0$, then bid $b^*(v_j)$ for the customers with $v_j \geq r + \frac{G(r)}{g(r)}$ will exceed $r$ and be accepted by the NYOP firm (i.e., $B_0 = \left\{v : v_j \geq r + \frac{G(r)}{g(r)}\right\}$). The bids of customers with $v_j < r + \frac{G(r)}{g(r)} < p$ are rejected and they cannot afford the posted price $p$ either; thus, $B_N = \left\{v : v_j < r + \frac{G(r)}{g(r)}\right\}$ and no one buys from the direct channel (i.e., $B_j = \emptyset$).

\[\] Proof of Proposition 5: By Proposition 3, for $i$-lovers the optimal bid $b^*$ is determined through $v_j - v_i$; it exceeds $r$ if and only if $\alpha_j(v_j - v_i) + p_i > r + G(r)/g(r)$, which can be written as $[p_i - r - G(r)/g(r)]/\alpha_j > v_i - v_j$. This defines $\delta_i$. Similarly, for the budget customer $b^* > r$ if and only if $\alpha_1 v_1 + \alpha_2 v_2 > r + G(r)/g(r)$, which gives $\delta_N$.

\[\] Proof of Proposition 6:

(i) Recall that customers’ bidding behavior and final channel realization were characterized in Proposition 1 and Proposition 4, respectively. Assume that we know the optimal pricing policy as well as expected revenue contingent to any inventory levels up to period $t - 1$. For
period $t$ with $x_i = 0$, we can determine the best $(p, r)$ and the maximum expected revenue by solving:

$$
\Pi_j(x, t) = \max_r \left\{ \begin{array}{ll}
\bar{F}_j(p) \left[ p + \Pi_j(x - e_j, t - 1) \right] + F_j(p) \Pi_j(x, t - 1), & \text{if } r > r_0, \\
\bar{F}_j \left( r + \frac{G(r)}{g(r)} \right) \left[ r + \Pi_j(x - e_j, t - 1) \right] + F_j \left( r + \frac{G(r)}{g(r)} \right) \Pi_j(x, t - 1), & \text{if } r \leq r_0,
\end{array} \right.
$$

where $e_j$ is $(1, 0)$ if $j = 1$ and $(0, 1)$ otherwise. Specifically, $\Pi_j(x, 0) = 0$ for any $x$. Solving the problem, we find out the following.

If $t = 1$, denote $p^* = \arg \max F_j(p)p$. That is, $F_j(p^*)p^*$ is the best that the retailer can achieve with the direct channel only. If the retailer uses the NYOP channel as well ($r \leq r_0$), the objective becomes

$$
\max_{r \leq r_0} \bar{F}_j \left( r + \frac{G(r)}{g(r)} \right) = \max_{r \leq r_0} r \frac{G(r)}{g(r)} \bar{F}_j \left( r + \frac{G(r)}{g(r)} \right) \left( r + \frac{G(r)}{g(r)} \right) \\
\leq \max_{r \leq r_0} r \frac{G(r)}{g(r)} F_j(p^*)p^* \\
< F_j(p^*)p^*.
$$

Therefore, in the last time epoch it is always optimal not to let customers purchase from the NYOP channel when the retailer is alone.

If $t > 1$, let $r^*$ be the infimum of the range of values in $[0, r_0]$ that maximize

$$
\bar{F}_j \left( r + \frac{G(r)}{g(r)} \right) \left[ r + \Pi_j(x - e_j, t - 1) \right] + F_j \left( r + \frac{G(r)}{g(r)} \right) \Pi_j(x, t - 1).
$$

Let $p = r^* + \frac{G(r^*)}{g(r^*)} - \epsilon < r^* + \frac{G(r^*)}{g(r^*)}$; it is not hard to verify that:
\[ F_j(p) [p + \Pi_j(x - e_j, t - 1)] + F_j(p)\Pi_j(x, t - 1) \]
\[ = \bar{F}_j \left( r^* + \frac{G(r^*)}{g(r^*)} \right) [r^* + \Pi_j(x - e_j, t - 1)] \]
\[ + F_j \left( r^* + \frac{G(r^*)}{g(r^*)} \right) \Pi_j(x, t - 1) + \bar{F}_j \left( r^* + \frac{G(r^*)}{g(r^*)} \right) \frac{G(r^*)}{g(r^*)} \]
\[ + \left[ \bar{F}_j \left( r^* + \frac{G(r^*)}{g(r^*)} \right) - \bar{F}_j \left( r^* + \frac{G(r^*)}{g(r^*)} \right) \right] [r^* + \frac{G(r^*)}{g(r^*)} + \Pi_j(x - e_j, T - 1) - \Pi_j(x, t - 1)] \]
\[ - \epsilon \bar{F}_j \left( r^* + \frac{G(r^*)}{g(r^*)} - \epsilon \right). \]

When \( \epsilon \) is small enough, we have
\[ \left[ \bar{F}_j \left( r^* + \frac{G(r^*)}{g(r^*)} - \epsilon \right) - \bar{F}_j \left( r^* + \frac{G(r^*)}{g(r^*)} \right) \right] \left[ r^* + \frac{G(r^*)}{g(r^*)} + \Pi_j(x - e_j, t - 1) - \Pi_j(x, t - 1) \right] \]
\[ - \epsilon \bar{F}_j \left( r^* + \frac{G(r^*)}{g(r^*)} - \epsilon \right) \to 0. \]

Hence,
\[ \bar{F}_j(p) [p + \Pi_j(x - e_j, t - 1)] + F_j(p)\Pi_j(x, t - 1) \]
\[ \geq \bar{F}_j \left( r^* + \frac{G(r^*)}{g(r^*)} \right) [r^* + \Pi_j(x - e_j, t - 1)] + F_j \left( r^* + \frac{G(r^*)}{g(r^*)} \right) \Pi_j(x, t - 1). \]

It is, therefore, better not to use NYOP at any time \( t \).

(ii) At \( t = 0 \), \( p^* = \arg \max_p F_j(p)p = \arg \max_p (1 - p)p = 1/2, \Pi_j(x, 1) = F_j(p^*)p^* = 1/4. \) For \( t \geq 1 \),
\[ p^*(x, t) = \arg \max_p \bar{F}_j(p) [p + \Pi_j(x - e_j, t - 1)] + F_j(p)\Pi_j(x, t - 1) \]
\[ = \arg \max_p [1 - p] [p + \Pi_j(x - e_j, t - 1)] + p\Pi_j(x, t - 1) \]
\[ = \arg \max_p -p^2 + p[1 + \Pi_j(x, t - 1) - \Pi_j(x - e_j, t - 1)] + \Pi_j(x - e_j, t - 1) \]
\[ = \frac{1 + \Pi_j(x, t - 1) - \Pi_j(x - e_j, t - 1)}{2} \]
and

\[
\Pi_j^*(x, t) = \tilde{F}_j(p) \left[ p + \Pi_j(x - e_j, t - 1) \right] + F_j(p) \Pi_j(x, t - 1) \bigg|_{p=p^*(x, t)}
\]

\[
= \Pi_j(x - e_j, t - 1) + \left( \frac{1 + \Pi_j(x, t - 1) - \Pi_j(x - e_j, t - 1)}{2} \right)^2
\]

At \( x = (x_i, x_j) = (0, 1) \), the above becomes

\[
p^*(x, t) = \frac{1 + \Pi_j(x, t - 1)}{2}, \quad (C4)
\]

\[
\Pi_j^*(x, t) = \left( \frac{1 + \Pi_j(x, t - 1)}{2} \right)^2. \quad (C5)
\]

Applying the limit for \( t \) on each side of (C5), it immediately solves that \( \lim_{t \to \infty} \Pi_j^*(x, t) = 1 \).

Also, (C5) can be re-written as \( \Pi_j^*(x, t) - \Pi_j(x, t - 1) = \left( \frac{1 - \Pi_j(x, t - 1)}{2} \right)^2 \geq 0 \). Therefore \( \Pi_j^*(x, t) \) increases in \( t \). By (C4) we can conclude that \( p^*(x, t) \) increases with \( t \) too.

**Proof of Proposition 7:** At time \( t \), a revenue-maximizing NYOP firm would determine a minimum reservation price \( r \) such that at least one of the retailers is willing to be the opaque provider. For retailer 1, suppose an \( r \) greater than \( \tilde{\Pi}_1(x, t - 1) \) is proposed and

(a) retailer 2 is not an opaque provider; then, retailer 1 will choose to ask more by proposing a higher reservation price \( r + \epsilon \);

(b) retailer 2 is an opaque provider; then, retailer 1 can choose to

1. give up being the opaque provider. In this case, retailer 1’s expected revenue can be expressed by

\[
\Pi_1^>(r|p, x, t) = \left[ p_1 + \Pi_1(x_1 - 1, x_2, t - 1) \right] Pr\{v \in \mathcal{B}_1(r, p, x)\}
\]

\[
+ \Pi_1(x_1, x_2 - 1, t - 1) Pr\{v \in \mathcal{B}_0(r, p, x)\}
\]

\[
+ \Pi_1(x_1, x_2 - 1, t - 1) Pr\{v \in \mathcal{B}_2(r, p, x)\}
\]

\[
+ \Pi_1(x_1, x_2, t - 1) Pr\{v \in \mathcal{B}_N(r, p, x)\};
\]

2. match the reservation price \( r \). In this case, each retailer is the opaque provider with
equal probability.

\[ \Pi_1^{r^*}(p|x, t) = [p_1 + \Pi_1(x_1 - 1, x_2, t - 1)] Pr\{v \in B_1(r, p, x)\} \\
+ \left\{ \frac{1}{2} \left[ r + \Pi_1(x_1 - 1, x_2, t - 1) \right] + \frac{1}{2} \Pi_1(x_1, x_2 - 1, t - 1) \right\} Pr\{v \in B_0(r, p, x)\} \\
+ \Pi_1(x_1, x_2 - 1, t - 1) Pr\{v \in B_2(r, p, x)\} + \Pi_1(x_1, x_2, t - 1) Pr\{v \in B_N(r, p, x)\}; \]

3. become the opaque provider by agreeing to a lower reservation price \( r - \epsilon \). As \( \epsilon \to 0 \), the expected revenue is given by

\[ \Pi_1^{\leq r}(p|x, t) = [p_1 + \Pi_1(x_1 - 1, x_2, t - 1)] Pr\{v \in B_1(r, p, x)\} \\
+ [r + \Pi_1(x_1 - 1, x_2, t - 1)] Pr\{v \in B_0(r, p, x)\} \\
+ \Pi_1(x_1, x_2 - 1, t - 1) Pr\{v \in B_2(r, p, x)\} \\
+ \Pi_1(x_1, x_2, t - 1) Pr\{v \in B_N(r, p, x)\}. \]

The first, third and fourth item for each revenue function describe the expected income if the arriving customer at time \( t \) is a 1-direct buyer, 2-direct buyer, and empty hand, respectively. These three parts stay the same across each strategy. The second item represents the expected income if the next arrival is an NYOP buyer. It is the only part that varies according to the opaque strategy of retailer 1. Comparing the three strategies, it is not hard to verify that it is optimal for retailer \( i \) to become opaque provider whenever \( r > \tilde{\Pi}_i(x, t - 1) \), match when \( r = \tilde{\Pi}_i(x, t - 1) \), and give up when \( r < \tilde{\Pi}_i(x, t - 1) \).

Without loss of generality, assume that \( \tilde{\Pi}_1(x, t - 1) \leq \tilde{\Pi}_2(x, t - 1) \). Then, if the NYOP firm sets the reservation price \( r < \tilde{\Pi}_1(x, t - 1) \), neither retailer would become the opaque provider regardless of the other retailer’s decision on this issue. On the other hand, if \( r > \tilde{\Pi}_2(x, t - 1) \), retailer \( i \) would choose to become the opaque provider if his rival is also in, or match if his rival is out. When \( \tilde{\Pi}_1(x, t - 1) < r \leq \tilde{\Pi}_2(x, t - 1) \), retailer 2 cannot afford being the opaque provider, and retailer 1 will ask more up to \( \tilde{\Pi}_2(x, t - 1) \). Therefore, the minimum \( r \) that an NYOP firm can settle at is \( r^* = \tilde{\Pi}_2(x, t - 1) \). Retailer 1 will be the unique opaque provider unless \( \tilde{\Pi}_1(x, t - 1) = \tilde{\Pi}_2(x, t - 1) \), in which case both retailers are the opaque provider with equal probability.
Proof of Corollary 1: At \( t = 1 \), the marginal value of inventory \( \widetilde{\Pi}_i(x, t - 1) \) is zero for either retailer. By Proposition 7, the reservation price is \( r^* = \max_i \widetilde{\Pi}_i(x, t - 1) = 0 \). □

Proof of Proposition 8: (i). We need to prove that \( \Pi_i \) is quasi-concave in \( p_i \) for \( i = 1, 2 \).

For uniform distribution, it is sufficient to prove concavity for any \( v \in [0, 1] \), if \( v \) follows the distribution \( \alpha_1 v_1 + \alpha_2 v_2 \sim U[-\min\{v/\alpha_1, (1 - v)/\alpha_2\}, \min\{v/\alpha_1, (1 - v)/\alpha_2\}] \) (e.g., the two products are pseudo-horizontally-differentiated).

Without loss of generality, assume that \( \widetilde{\Pi}_2(x, t - 1) < \widetilde{\Pi}_1(x, t - 1) \); hence, retailer 2 is the opaque-product provider (other scenarios can be proved in a similar way). Then,

\[
\Pi_1(x, t) = 
\left[ p_1 + \Pi_1(x_1 - 1, x_2, t - 1) \right] Pr\{v \in B_1(r, p, x)\} + \Pi_1(x_1, x_2 - 1, t - 1)Pr\{v \in B_0(r, p, x)\}
+ \Pi_1(x_1, x_2 - 1, t - 1)Pr\{v \in B_2(r, p, x)\} + \Pi_1(x_1, x_2, t - 1)Pr\{v \in B_N(r, p, x)\}.
\]

- If \( p_1 \geq p_2 = p \), Proposition 5 implies that both \( B_2(r, p, x) \) and \( B_N(r, p, x) \) are unaffected by \( p_1 \), and \( \frac{\partial B_0(r, p, x)}{\partial p_1} = -\frac{\partial B_1(r, p, x)}{\partial p_1} \). So,

\[
\frac{\partial \Pi_1(x, t)}{\partial p_1} = 
Pr\{v \in B_1(r, p, x)\} + \left[ \widetilde{\Pi}_1(x_1, x_2, t - 1) - p_1 \right] \frac{\partial Pr\{v \in B_0(r, p, x)\}}{\partial p_1}
\]

\[
\frac{\partial^2 \Pi_1(x, t)}{\partial p_1^2} = 
-2 \frac{\partial Pr\{v \in B_0(r, p, x)\}}{\partial p_1} + \left[ \widetilde{\Pi}_1(x_1, x_2, t - 1) - p_1 \right] \frac{\partial^2 Pr\{v \in B_0(r, p, x)\}}{\partial p_1^2}.
\]

- If \( p_1 < p_2 \),

\[
\frac{\partial Pr\{v \in B_0(r, p, x)\}}{\partial p_1} = 
\frac{\partial Pr\{v \in B_1(r, p, x)\}}{\partial p_1} + \frac{\partial Pr\{v \in B_2(r, p, x)\}}{\partial p_1} + \frac{\partial Pr\{v \in B_N(r, p, x)\}}{\partial p_1}.
\]
thus

\[
\frac{\partial \Pi_1(x, t)}{\partial p_1} = \\
Pr\{v \in B_1(r, p, x)\} - \left[\tilde{\Pi}_1(x_1, x_2, t - 1) - p_1\right] \frac{\partial Pr\{v \in B_1(r, p, x)\}}{\partial p_1} \\
- \left[\Pi_1(x_1, x_2 - 1, t - 1) - \Pi_1(x_1, x_2, t - 1)\right] \frac{\partial Pr\{v \in B_N(r, p, x)\}}{\partial p_1}
\]

(C7a)

\[
\frac{\partial^2 \Pi_1(x, t)}{\partial p_1^2} = \\
2 \frac{\partial Pr\{v \in B_1(r, p, x)\}}{\partial p_1} - \left[\tilde{\Pi}_1(x_1, x_2, t - 1) - p_1\right] \frac{\partial^2 Pr\{v \in B_1(r, p, x)\}}{\partial p_1^2} \\
- \left[\Pi_1(x_1, x_2 - 1, t - 1) - \Pi_1(x_1, x_2, t - 1)\right] \frac{\partial^2 Pr\{v \in B_N(r, p, x)\}}{\partial p_1^2}.
\]

(C7b)

Proposition 5 implies that, given \(\alpha_1 v_1 + \alpha_2 v_2 = v\) for some particular \(v\), exactly one of \(B_0\) or \(B_N\) is 0.

\[\bullet\] \(Pr\{v \in B_N(r, p, x)\} = 0\).

For \(p_1 \geq p_2 = p\), it can be verified that \(Pr\{v \in B_0(r, p, x)\}\) is an affine function of \(p_1\) (i.e., \(Ap_1 + B\) with \(A \geq 0\)). Thus, \(\frac{\partial^2 Pr\{v \in B_0(r, p, x)\}}{\partial p_1^2} = 0\) and \(\frac{\partial^2 \Pi_1(x, t)}{\partial p_1^2} \leq 0\). By (C6), the payoff function is concave.

For \(p_1 < p_2\), it can be verified that \(Pr\{v \in B_1(r, p, x)\}\) takes the form \(-Ap_1 + B + C\delta_N(r | p_1)\) with \(A, C \geq 0\). Then,

\[
\frac{\partial^2 \Pi_1(x, t)}{\partial p_1^2} = 2 \frac{\partial Pr\{v \in B_1(r, p, x)\}}{\partial p_1} + (p_1 - r) \frac{\partial^2 Pr\{v \in B_1(r, p, x)\}}{\partial p_1^2} \\
= \left[-2A + 2C \frac{\partial (G/g)}{\partial p_1} + (p_1 - r)C \frac{\partial^2 (G/g)}{\partial p_1^2}\right] / v,
\]
Recall that in the proof in Proposition 9 (ii), \( \frac{G(r \mid p_1)}{rg(r \mid p_1)} = \frac{\dot{G}(r/p_1)}{\hat{g}(r/p_1)} := \delta(r/p_1) \). Thus

\[
2 \frac{\partial(G/g)}{\partial p_1} + (p_1 - r) \frac{\partial^2(G/g)}{\partial p_1^2} = 2 \frac{\partial r}{\partial p_1} \hat{\delta}(r/p_1) + (p_1 - r) \left[ \hat{\delta}''(r/p_1)(\frac{\partial r}{\partial p_1})^2 + \hat{\delta}'(r/p_1) \frac{\partial^2 r}{\partial p_1^2} \right]
\]

\[
= -\frac{2r^2}{p_1^2} \hat{\delta}'(r/p_1) + \frac{(p_1 - r)r^2}{p_1^2} \hat{\delta}''(r/p_1) 
\sim -2\hat{\delta}'(r/p_1) + (1 - r/p_1)\hat{\delta}''(r/p_1).
\]

(C8)

For all the DRHR distributions (truncated to [0, 1] if necessary, with distribution function \( \hat{G} \) and density \( \hat{g} \) listed in Table 1 of Bagnoli and Bergstrom (2005), it can be verified that \( \hat{\delta}(x) = \frac{\hat{G}(x)}{x\hat{g}(x)} \) satisfies \( \hat{\delta}' \geq 0 \) and \( \hat{\delta}'' \leq 0 \). Thus, (C8) \( \leq 0 \), which implies the concavity of \( \frac{\partial^2 \Pi_1(x, t)}{\partial p_1^2} \).

- \( Pr\{v \in B_0(r, p, x)\} = 0 \).

For \( p_1 \geq p_2 = p \), (C6b) \( \frac{\partial^2 \Pi_1(x, t)}{\partial p_1^2} = 0 \), so concavity holds.

For \( p_1 < p_2 \), \( Pr\{v \in B_1(r, p, x)\} \) is an affine function of \( p_1 \) with the form \(-Ap_1 + B\), where \( A \geq 0 \), and \( Pr\{v \in B_N(r, p, x)\} \) is an affine function of \( p_1 \) with the form \( Cp_1 + D \), where \( C \geq 0 \). Thus, \( \frac{\partial Pr\{v \in B_1(r, p, x)\}}{\partial p_1} \leq 0 \) and \( \frac{\partial^2 Pr\{v \in B_N(r, p, x)\}}{\partial p_1^2} = 0 \). By (C7b), \( \frac{\partial^2 \Pi_1(x, t)}{\partial p_1^2} \leq 0 \), and the payoff function is concave.

(ii). We can show that, for any \( t \), there exits a \( \pi_i \) such that for any \( x_1 \geq t \) and \( x_2 \geq t \),

\[
\Pi_1(x_1, x_2, t) = \Pi_2(x_1, x_2, t) = \pi_i,
\]

(C9)

which immediately leads to \( r^* = 0 \) for any \( x_1 \geq t \) and \( x_2 \geq t \). We show (C9) by induction.

First, it is easy to see that (C9) holds for \( t = 0 \). Now, suppose (C9) holds for all \( t < T \). We then have \( \Pi_1(x_1 - 1, x_2, T - 1) = \Pi_1(x_1 - 1, x_2 - 1, T - 1) = \Pi_2(x_1, x_2 - 1, T - 1) = \Pi_2(x_1 - 1, x_2 - 1, T - 1) \) for any \( x_1 \geq T \) and \( x_2 \geq T \). Hence, \( \Pi_i(x_1, x_2, T - 1) = 0 \) for \( i = 1, 2 \) and

\[
r^*(x_1, x_2, T) = 0, \quad \forall x_1 \geq T, x_2 \geq T.
\]

(C10)
(C10) implies that, when both retailers oversupply, the price of the opaque goods will remain zero until one retailer’s inventory becomes lower than the potential demand. The expected revenue function for retailer 1 is then

\[
\Pi_1(x, T) = \left[ p_1 + \Pi_1(x_1 - 1, x_2, T - 1) Pr\{v \in B_1(0, p, x)\} + \Pi_1(x_1 - 1, x_2, T - 1) Pr\{v \in B_0(0, p, x)\} \right.
\]

\[
+ \Pi_1(x_1, x_2 - 1, T - 1) Pr\{v \in B_2(0, p, x)\} + \Pi_1(x_1, x_2, T - 1) Pr\{v \in B_2(0, p, x)\} \right]
\]

\[
= p_1 Pr\{v \in B_1(0, p, x)\} + \pi_{T-1},
\]

which does not depend on \(x\) for \(x_1 \geq T\) and \(x_2 \geq T\). Similarly, \(\Pi_2(x, T) = p_2 Pr\{v \in B_2(0, p, x)\} + \pi_{T-1}\). When \(r = 0\), the expressions for \(B_1(0, p, x)\) and \(B_2(0, p, x)\) are the same.

It is then straightforward that in an equilibrium there should be \(p_1 = p_2\) and \(\Pi_1(x, T) = \Pi_2(x, T)\). Thus, (C9) also holds for \(t = T\). This completes the proof.

Proof of Proposition 9:

(i) Proposition 5 implies that, when products are vertically differentiated, then customers who eventually receive a unit will all buy either from retailer \(i\), or from retailer \(j\), or from the NYOP channel. We first argue that they would not buy directly from retailer \(j\), as there is always a strategy in which retailer \(i\) can set his posted prices as \(p_i = p_j + v_i - v_j - \varepsilon = p_j + \bar{v} - \varepsilon\) for some \(\varepsilon > 0\) such that all customers prefer to buy from retailer \(i\) at \(p_i\). With this in mind, retailer \(j\) would accept any reservation price \(r > 0\). Moreover, by setting \(p_i\) such that \(\delta_i(r) \leq \bar{v}\), retailer \(i\) has a strategy that may induce all customers to buy at his posted prices for all possible \(r\). This is feasible since \(\delta_i(r)\) decreases with \(r\) and increases with \(p_i\) for both \(p_i = p\) and \(p_j = p\). Given that it can win all customers at posted prices, retailer \(i\) does not have an incentive to bid a positive reservation price with the intermediary firm because retailer \(j\) would always react with a lower \(r\).

(ii) By Proposition 5, if \(\delta_N(r) < \bar{v}\), either (a) none of the customers will buy anything, or (b) some customers receive a unit of product from retailer 1, some from retailer 2, and the rest from the NYOP channel. We next prove that \(G/g\) is decreasing in \(p\), so that solving \(\delta_N(r) = r + G(r | p)/g(r | p) = \bar{v}\) gives the proper \(p^0\).

Lemma 1. \(G(r | p)/g(r | p)\) is decreasing and convex in \(p\).

Proof. Proof. Let \(\hat{G}\) and \(\hat{g}\) denote the respective distribution function and density of \(\xi\) when
the support is on $[0, 1]$, then $g(r | p)/G(r | p) = q \frac{\tilde{g}(rq)}{G(rq)}$, where $q = 1/p$. For all the DRHR distributions (truncated to $[0, 1]$ if necessary) listed in Table 1 of Bagnoli and Bergstrom (2005), it can be verified that $x \tilde{g}(x)/\tilde{G}(x)$ decreases in $x$. This implies that we should have $rg(r)/G(r) = rq \frac{\tilde{g}(rq)}{G(rq)}$ decreasing with $rq = r/p$. For any given $r$, this further implies that $G/g$ decreases with $p$.

Proof of Proposition 10:
By applying Proposition 6, we have the expected revenue, posted price, and reservation price in last minute $t = 1$ as given in Table 3, 4, and 5, respectively. By Propositions 7 and 8 and results in $t = 1$, the expected revenue, posted price, and reservation price in last minute $t = 1$ are given in Table 6, 7, and 8, respectively. Comparing Table 3 and 6 immediately yields (1), (2) and (3).

Proof of Proposition 11:
In the last minute, $t = 1$, marginal value of inventory is zero for either retailer. Then, by Proposition 7, retailer 1 and 2 are both opaque providers with equal probability. At regular sales season, $t = 2$, the opaque provider can be determined by examining Table 3.

Proof of Proposition 12:
Followed by the structural analysis in §3, 4, and 5, the sequence of customers with valuation $\Upsilon = \{v_T, v_{T-1}, ..., v_2, v_1\}$ determines the way inventory depletes and consequently (by Figure 3 or Table 9) the sequence of opaque providers $\{I_T, I_{T-1}, ..., I_2, I_1\}$. Specifically, as $\bar{x}_1 = \bar{x}_2$, we should have $I_T = \{1, 2\}$. That is, the initial opaque provider is retailer 1 or 2 with equal probability. Also, by Proposition 11, $I_1 = \{1, 2\}$.

For any $1 \leq t \leq T$ and $v_t = (v_{1,t}, v_{2,t})$, denote $\bar{v}_t = (v_{2,t}, v_{1,t})$, and let $\bar{I}_t = \{i\}$ if $I_t = \{j\}$, and $\bar{I}_t = \{1, 2\}$ if $I_t = \{1, 2\}$. Due to the symmetry in Figure 3 or Table 9, should the sequence of customers be $\bar{\Upsilon} = \{\bar{v}_T, \bar{v}_{T-1}, ..., \bar{v}_2, \bar{v}_1\}$, then the respective sequence of opaque providers will be $\bar{\{I_T, I_{T-1}, ..., I_2, I_1\}}$.

For $v \sim U[0, 1] \times [0, 1]$, all possible sequences of customers consist of (1): pairs of $\Upsilon$ and $\bar{\Upsilon}$ where $\Upsilon \neq \bar{\Upsilon}$; (2) $\{(0.5, 0.5), (0.5, 0.5), ..., (0.5, 0.5), (0.5, 0.5)\}$. It is, then, straightforward that in expectation each retailer is equally likely to be an opaque provider throughout the time.