Name-Your-Own-Price as a Competitive Secondary Channel in the Presence of Posted Prices

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Abstract

In this paper, we study how competitive suppliers with substitutable, non-replenishable goods may sell their products (1) as regular goods through a direct channel at posted prices, and/or (2) as opaque goods through a third-party channel, which allows for the name-your-own-price (NYOP) approach (e.g., Priceline.com). We model the third-party channel as an intermediary firm that collects the difference between the customers’ bids and reservation prices set by the suppliers. Different channel strategies and customers’ bidding strategies are discussed. We find out that high-end customers may demonstrate low-end behavior (that is, name their prices prior to attending the direct channel, making an even lower bid than the low-end customers). Moreover, the intermediary firm benefits more from horizontally differentiated goods than from vertically differentiated ones. We further analyze how suppliers should competitively determine channel prices for given initial inventory levels with the goal of maximizing the average expected profit. A dynamic programming approach is proposed in solving this problem. Our results suggest that the suppliers may not benefit from the existence of the NYOP channel. In particular, a monopolist would opt out of the NYOP channel and sell at posted prices only, which implies that NYOP is not appropriate for customer discrimination in the regular goods market. Numerical experiments show that suppliers are able to generate higher expected profits in the absence of the NYOP channel.

Keywords: Competition; Distribution channels; Dynamic pricing; e-Commerce; Name-Your-Own-Price; Nash equilibrium; Opaque goods; Priceline; Probabilistic selling;
1. Introduction

In recent years, we have been able to observe an increased variety of pricing schemes as e-business and online shopping have become vibrant parts of our lives. Current pricing schemes can be roughly divided into two categories: seller-driven pricing and buyer-driven pricing. Under seller-driven pricing, the seller sets the price of the goods, and the buyer simply makes a take-it-or-leave-it decision. This type of pricing is the one most commonly observed, and it has been referred to in literature as “posted-price” or “list-price.” Under buyer-driven pricing, the roles of the two parties are reversed: a buyer announces the price that she (hereinafter, buyers will be referred to with female pronouns, sellers with male pronouns) is willing to pay, while the seller decides whether and at what price to let the goods go. Auctions and name-your-own-price (NYOP) models belong to this pricing category; the former has been used by, for instance, eBay, Google, and compUSA, while the latter is being adopted by companies like Priceline.com and BookIt.com. While one may have an impression that buyers themselves drive the prices in these settings, it is the seller that designs the rules behind them, (e.g., the ending time, the number of available units, the number of times a buyer can participate, the minimum bid, the final sales price, the degree of information transparency, etc.).

At the same time, information availability about the products on sale has been changing as well. While in brick-and-mortar stores customers can touch and feel the things they are about to buy, in an on-line store the seller can provide detailed specifications of the product on sale. However, the seller may also decide to withhold some information from the customers. For instance, if a customer decides to purchase an air ticket on Priceline.com through “name-your-own-price,” she will not be able to learn all of the details of her trip (e.g., the airline name, departure time, etc. until the deal is finalized. Thus, the exact features of the product being bought are rather vague. Such products, whose characters are not fully revealed until after the purchase, have been referred to as opaque goods in industry (one may refer to Fay (2008) for a detailed description of opaque goods and Anderson (2008) for information on how Priceline.com works with its suppliers). As opposed to opaque goods, we will refer to the goods for which buyers know all features before purchase as regular goods. The following table summarizes how regular and opaque goods have been sold under different pricing schemes.

In this paper, we consider a problem with two competitive suppliers selling perishable and non-replenishable products. The products could be sold either as regular goods through their direct channel (e.g., stores or Web sites) at posted prices, or as opaque goods through a third-party intermediary firm that conducts NYOP service among the customers (e.g., Priceline.com). Thus, each supplier chooses its own set of pricing mechanisms (seller-driven pricing, buyer-driven pricing,

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1Starting in 2005, Priceline also has products with posted price (USA Today, Apr 11, 2005).
2The O/D dates and airports have to be confirmed, though.
or both at the same time) and the format of its products (regular, opaque, or both). At the beginning of each time epoch, the suppliers set their posted prices in their direct channels. Then, the suppliers inform the intermediary firm about their reservation prices, defining the lowest price at which they are willing to let a unit go. A customer arriving thereafter may decide to (1) buy at the posted price from her preferred supplier, or (2) go to the intermediary firm and name-her-own-price (NYOP). If a bid is submitted to the NYOP channel, the intermediary firm benefits from the difference between the customer’s offer and the lowest reservation price from the suppliers (Dolan 2000). In case the bid fails to meet the lowest reservation price, the customer is rejected by the intermediary firm, yet she still has a chance to buy the product at posted price. However, consistently with prior literature on this topic (e.g., Amaldoss and Jain 2008) and with Priceline’s policy (Dolan 2000), we assume that repeated bidding is prohibited.

While auctions have been studied extensively in the literature as one of the buyer-driven pricing schemes, NYOP has received more limited attention, despite the fact that it possesses some nice characteristics and is easy to implement. First, processing is fast—after a bid is submitted, the customer is normally informed about the outcome within minutes (or, in the worst case, within 24 hours). Hence, unlike auctions, which may take days or weeks before finalizing, NYOP does not discourage impatient customers from participating. Second, NYOP does not intensify the competition among customers. Whether a bid is accepted or not depends on the amount of the bid and the reservation prices determined by the suppliers—i.e., one does not have to worry about the behavior of other customers (like in an auction) and is more likely to enjoy the NYOP channel as an additional opportunity to get a good deal (since the customer can always buy from the direct channel if rejected). This also allows us to focus more on the competition between suppliers than on that among customers. Third, NYOP is simple for suppliers to implement. The inventory is depleted by at most one unit in every decision epoch, and suppliers can update their posted and reservation prices after that. When compared with the uncertain number of winners in an auction, NYOP gives firms more power in their inventory control, and allows for dynamic pricing.

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<tr>
<th></th>
<th>Regular Goods</th>
<th>Opaque Goods</th>
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<tr>
<td><strong>Seller-Driven</strong></td>
<td>Expedia, Orbitz</td>
<td>Hotwire, BookIt</td>
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<tr>
<td><strong>Buyer-Driven</strong></td>
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<tr>
<td>Auction:</td>
<td>eBay, Google</td>
<td></td>
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<tr>
<td>NYOP:</td>
<td>Unrevealed German NYOP firm</td>
<td>Priceline</td>
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<td>(Hann and Terwiesch 2003, Terwiesch et al. 2005)</td>
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Table 1: Pricing Schemes for Regular and Opaque Goods
in both channels. Finally, NYOP provides a platform of competition for opaque goods. Products sold through NYOP channel do not possess some important information (e.g., hotel brand) that customers may use to discriminate among products. The suppliers get the “brand shield” and “price shield” that the direct channels cannot offer (Dolan 2000)—the NYOP channel essentially provides a space in which suppliers compete head-to-head to win customers, who in turn give up the power to exercise their preferences.

Taking this into account, we want to study NYOP to see if suppliers benefit from adopting it as a secondary channel for increasing their sales. Our goal is to solve the optimal pricing/channel strategy for the suppliers in the presence of competition, and to study the impact that NYOP might have on an industry. Our analysis shows that an NYOP channel is less attractive for regular products—i.e., a monopolist would not use NYOP if it can sell in a direct channel at posted prices. However, in a duopoly market in which opaque products are available, both suppliers will try to utilize an NYOP channel. Interestingly, our numerical results suggest that the suppliers may see higher revenues in the absence of the NYOP channel. Thus, while the existence of the NYOP channel may be a result of the competition in the market, its presence further intensifies the competition (that is, posted prices and expected revenues are lower in the presence of NYOP than without it). We also characterize the purchasing/bidding strategies of the customers and the managerial implication for the intermediary NYOP firm.

The paper is organized as follows. In §2, we review the literature. In §3, we introduce our model. Customers’ purchasing/bidding behavior is analyzed in §4, and results for the purchasing channel are shown in §5. In §6, we propose a dynamic programming approach for solving the equilibrium pricing decision. Numerical results are provided in §7, and we conclude in §8.

2. Literature Review

Our work is related to two different streams of literature, which we analyze separately: the first stream captures the papers in the area of NYOP channels and opaque products, while the second deals with dynamic pricing strategies.

2.1 Name-Your-Own-Price and Opaque Products

The business model of NYOP and opaque goods has drawn increased attention in the recent operations management and marketing literature. Table 2 presents a selection of papers relevant to either NYOP or opaque products.

Many papers regarding NYOP consider regular products.\(^3\) They tend to be consumer-oriented

\(^3\)Some of the models may be applied to both regular and opaque products—we put them under the column “Regular Goods,” while the second column contains papers targeting only opaque products.
and emphasize consumer bidding behavior and/or restrictions that retailers may put on bidding. More specifically, most of the work assumes NYOP as the only sales channel (i.e., there is no parallel seller-driven (posted-price) channel that customers may turn to). Fay (2004) analyzes whether an NYOP retailer should encourage or discourage repeated bidding. Spann et al. (2004) analyze how a firm can learn customers’ willingness to pay and the transaction costs of repeated bidding. Hann and Terwiesch (2003) estimate the transaction cost per bid of the on-line customers in naming their own prices. Hinz and Spann (2008) model how social network may facilitate individuals in learning the reservation price of the seller. Some articles also shed light on the NYOP retailer’s policy. Terwiesch et al. (2005) analyze the repeated bidding behavior and suggest the threshold price for monopoly firms. Joint bidding for multiple items has been studied by Amaldoss and Jain (2008), while Wang et al. (2009) analyze a model for the airline industry in which only posted price is used in the first period, while both posted price and name-your-own-price might be adopted in the second period.

Problems involving opaque goods usually demand a very different model structure. As it usually requires two or more suppliers/products to make an opaque product, competition issues may arise (for monopolistic case see Jiang 2007; Fay and Xie 2008). Fay (2008) models selling opaque products at posted prices only. The suppliers have to precommit with the NYOP firm on the number of units to be sold as opaque products. The intermediary firm handles all of the pricing and product allocations with the customers. No bid/auction is involved in the model, and the intermediary firm

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<th>Regular Goods</th>
<th>Opaque Goods</th>
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<td><strong>Posted Prices</strong></td>
<td>Many</td>
<td>Jiang (2007)</td>
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<td>Fay (2008)</td>
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<td>Fay and Xie (2008)</td>
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<td>Jerath et al (2009)</td>
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<td>Hinz and Spann (2008)</td>
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Table 2: Literature of Regular vs. Opaque Products
adopts seller-driven pricing to set a firm price for the opaque products. Conditions under which the opaque good may bring down the price in the traditional channel and harm the profits are identified. Jerath et al (2009) consider the competition between two firms selling limited inventories in a two-period model. They allow the firms to simultaneously sell through an opaque channel instead of the direct channels for the second period, and analyze the impact of expected revenue in either case.

If NYOP is introduced in a model with opaque products, the notion of “NYOP firm” needs further clarification. In the presence of opaque goods, the NYOP system usually consists of an NYOP intermediary firm and a number of suppliers, in which the former acts as an agent that delegates the NYOP service for the latter. This is different from many models in the third quadrant of Table 2, which deal with only one “NYOP retailer”—a centralized supplier that also provides NYOP service. With opaque products, the supplier and NYOP service provider are usually separated. The suppliers determine their reservation prices, and the NYOP service provider selects the supplier that accepts the lowest payment. Fay (2009) studies the competition between Priceline.com and Hotwire.com. Both companies deal with opaque products, but the former uses a buyer-driven NYOP scheme while the latter uses a seller-driven, posted-price scheme.

2.2 Dynamic Pricing: Channels and Competitions

Most of the NYOP literature discussed so far did not put any restriction on the supply. In other words, the papers mentioned above assumed that the products are replenishable, so that the entire demand may be satisfied. In our model, we incorporate the notion of limited supply by looking at non-replenishable products. In maximizing the expected revenue, suppliers have to dynamically adjust their posted/reservation prices throughout the sales horizon. There has been a rich literature that applies dynamic pricing methods to improve the seller’s profit. One may refer to McGill and van Ryzin (1999) and Elmaghraby and Keskinocak (2003) for a general review of applications in perishable-yet-non-replenishable goods (such as hotels and airlines) and replenishable ones (in which retailers can make inventory decisions), respectively. However, most of the existing literature has been focusing on seller-driven pricing, while we are more interested in buyer-driven pricing.

As electronic markets became more convenient and easier to join, several review papers discussed the possibility of extending the models towards a broader aspect of pricing (e.g., Bitran and Caldentey 2003; Elmaghraby and Keskinocak 2003b; Pinker et al. 2003). The use of auctions as the unique sales channel has been discussed in Vulcano et al. (2002) and van Ryzin and Vulcano (2004). While the former looks at sales of a finite quantity of non-replenishable items, the latter considers replenishable products with an infinite sales horizon.

\[ \text{...maximizes the spread between the customer’s ‘named-price’ and the necessary payment to the airline partner...} \] (Dolan 2000).
Recognizing that many firms have multiple sales channels (in particular, a combination of a brick-and-mortar store and an online store), a number of articles study how a firm should set up its dual-channel strategy—namely, having seller-driven prices and buyer-driven prices at the same time. Etzion et al. (2006) model how to use auctions and posted prices at the same time, with infinite supply in a limited time. Caldentey and Vulcano (2007) study a similar problem but with limited supply. Jiang (2007) considers the market strategy a monopolistic airline should take if it can either sell some itineraries as regular products and/or pack them together and sell as opaque products. Huh and Janakiraman (2008) show that when products are replenishable, the optimality of \((s, S)\) inventory policies still holds under various pricing schemes, including buyer-driven ones like auctions and NYOP. Wang et al. (2009) consider a two-period model for the airline industry in which dual channel is allowed in the second period. They find out that decisions about adoption of an NYOP channel depend on the uncertainty of high-fare demand rather than the excess capacity.

The papers mentioned so far consider a single firm only. However, firms offering the same airline itineraries, nearby hotel rooms, same-size car rentals, or any other substitutable products may face even greater challenges in finding the right pricing policies and inventory control policies while competing with their rivals. The competition could even be the key driver for the selection of a certain pricing policy (Bitran and Caldentey 2003). In recent years, researchers have begun to study competition in revenue management. Netessine and Shumsky (2005) analyze the airline seat-allocation problem for both horizontal competition (firms offering the same single-leg itinerary) and vertical competition (firms offering different legs in a multi-leg itinerary) between two firms. Gallego and Hu (2007) model dynamic pricing under competition as a stochastic control problem in a continuous-time differential game. Lin and Sibdari (2008) study competitive dynamic pricing for multiple firms in which customers’ choice follows the multinomial logit model depending on the current prices of the market. Levin et al. (2009) present a model of oligopolistic dynamic pricing in which customers may strategically choose their purchase time.

In traditional revenue management (RM), the optimal pricing policy largely depends upon inventory availability. This increases the complexity of modeling competition in RM models, as rivals’ inventory levels may not be observable and customers’ preferences may be unknown. For analytical ease, many authors (see, e.g., Gallego and Hu 2007; Lin and Sibdari 2008; Levin et al. 2009) assume perfect information; that is, they assume that inventory levels and distributions of random factors are public knowledge. Still, there exists a number of papers that take different approaches to avoid such assumptions. In solving competitive pricing and the booking-limits problem, Perakis and Sood (2006) apply robust optimization to address the unknown distribution in demand. Zhang and Kallesen (2008) propose an MDP model for companies to estimate their rival’s prices through some “belief matrix.” Levina et al. (2009) allow a monopolist to learn the demand characteristics of its customers from the dynamic pricing process.
Due to the possible complexity, there is limited RM literature in the dual-channel multiple-firm quadrant. Jerath (2009) allow competing firms to participate in both direct and opaque selling channels, yet in a sequential manner. In particular, their focus is on how last-minute opaque selling (at seller-driven prices) and strategic customer behavior might affect the revenue under competition. However, we are interested in how firms would balance between the two channels – one seller-driven priced and the other buyer-driven priced – throughout the time.

Our model adds a new block to the current literature in the following aspects. (1) We look at dual channel–dynamic pricing strategy under a duopoly. Specifically, we allow the firms to adopt two channels at the same time. To our knowledge, this has not been extensively studied in the literature thus far. (2) We apply NYOP to opaque products by decentralizing the suppliers and the intermediary NYOP firm. The suppliers manage their own seller-driven pricing channels, while the intermediary firm adopts buyer-driven pricing. (3) Our focus is more supply-oriented, through the analysis of the suppliers’ equilibrium decisions under inventory constraints. Few NYOP papers have considered dual-channel strategy or limited supply, which is exactly the case in which NYOP is being applied in practice. We believe these distinctions make this problem/model a relevant and

![Table 3: Brief RM Literature Review with Single or Dual Channel](image_url)

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<th>Dual-Channel</th>
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<td>Wang et al. (2009)</td>
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interesting research topic.

3. The Model

Consider a model with two suppliers, one intermediary firm, and a sequence of customers. The product from supplier \( i \) is referred to as product \( i \), where \( i \in \{1, 2\} \). We assume that each supplier offers a distinct product and that supplier \( i \) has \( x_i \) units of initial inventory to sell over a finite period \( T \). The inventory is non-replenishable. We use a backward time index \( t = T, T - 1, \ldots, 1 \) to denote the current period, so a smaller number indicates that we are closer to the ending time. At each period, exactly one customer arrives, and she possesses random valuation \( v_i \) for product \( i \). We assume \( v_i \)'s are independently distributed on \([0, 1]\) with density \( f_i(\cdot) \) and cdf \( F_i(\cdot) \). Each customer demands at most one unit. This sequence of customers with constant arrival rate 1 can easily be extended to a sequence of homogeneous customers with a fractional arrival rate by adjusting the distributions of valuations accordingly.

At the beginning of each period, the suppliers publicly announce their posted prices, \( p = (p_1, p_2) \), at which customers can buy from their direct channels. After the posted prices are announced, the intermediary firm conducts an English auction with the suppliers to determine which one will fulfill the demand for the opaque product in this period. The suppliers take turns in submitting their desired reservation prices until someone is unwilling to decrease any further. The supplier with the lower reservation price, \( r \), is awarded the opportunity to supply the opaque product in this period. In the case of a tie, one supplier is randomly chosen with 50% probability. These processes are unobservable to the customers, who adopt their own priors in estimating the reservation price \( r \).

After the prices are set, a customer arrives with valuation \( v = (v_1, v_2) \) as her private information. Neither the suppliers nor the intermediary firm have any knowledge of \( v \) except its distribution function, \( F_{1,2}(\cdot) \). Based on the posted prices, \( p \), and her valuation, \( v \), the customer decides if she wants to buy directly from her preferred supplier or name her price (which we will refer to as “make a bid” for the reminder of the paper) \( b \) with the intermediary firm. In the second case, she may be matched up with either of the two suppliers if the bid is accepted. In case the bid is rejected, the customer can always return to her preferred supplier and buy the product at its posted price. However, making a bid is considered a commitment to buy, and thus a customer cannot decline a product assigned by the intermediary firm if she discovers \( ex-post \) that being awarded the alternative product or buying at the posted price would make her better off. We also assume that the customer is not allowed to make a second bid with the NYOP channel after the first bid is rejected. Repeated bidding would certainly generate more research questions that are worth analyzing. However, as our goal is to characterize the dual distribution channel by NYOP and posted prices, the effects of repeated bidding are not within the scope of this paper, and we simplify the scenario by applying
a one-time bidding restriction. This assumption aligns with the policy in Priceline that “only one offer was permitted in a seven-day period...” (Dolan 2000). Readers may refer to Fay (2004), Spann (2004), Terwiesch et al. (2005), and Hinz and Spann (2008) for problems involving repeated bidding/bid learning.

After receiving a bid, $b$, for the opaque product, the intermediary firm simply compares $b$ with the lower reservation price, $r$: if $b \geq r$, the bid is accepted. The intermediary firm collects $b$ from the customer, assigns her a unit of product from the supplier that requests payment $r$, and pays that supplier its requested amount. Otherwise, if $b < r$, the bid is rejected.

We assume that the intermediary firm requires that the reservation prices do not exceed the respective posted prices. One reason for this is that customers normally perceive the NYOP channel as a way to save (as they have restricted information compared with the posted price channel), and this is exactly how the intermediary firms market themselves. For example, Priceline.com consistently advertises on the Web site, “save 40% on flights, 50% on hotels, 30% on cars.” This assumption directly leads to

$$r \leq \min\{p_1, p_2\},$$

which will be used throughout the paper. Figure 1 illustrates the sequence of events for each period of the game.

4. The Customers

For each product, $i$, customers hold random valuation, $v_i$, which is defined on the support $[0, 1]$ with cumulative density function $F_i$. The customers cannot observe the final reservation price, $r$, but have perfect information about the posted prices, $p$. We allow the customers to assume that each supplier has its own reservation price, $r_i$, i.i.d. on $U[0, p]$, where $p = \min\{p_1, p_2\}$; the supplier with lower reservation price will provide the opaque good. This is equivalent to the assumption that $r$ is a random variable on $[0, p]$ with c.d.f. $G(b) = (2bp - b^2)/p^2$ and that each supplier has an equal chance to be an opaque good provider. Hiding the source of the opaque good provider is a must, and Fay (2008) also makes a “half-half” assumption on the choice of providers.

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5 The restrictions may have been updated, but the spirit is that such sites may not encourage repeated bidding within a short period.

6 We also considered the model in which the customers assume $r_i \sim U[0, p_i]$ for $i = 1, 2$. The results for channel selection and bidding behavior are very similar to the ones obtained here (e.g., Figure 3 preserves a similar pattern, but may not be symmetrically shaped). However, customers may overbid (i.e., $b \geq p$) for a better chance of getting the pricier product. As the difference between the results is not significant and overbidding rarely happens in real life, we omit this analysis from the paper.
Customers with valuation $v$ have three options to choose from: (1) buy directly from supplier 1 at $p_1$, (2) buy directly from supplier 2 at $p_2$, or (3) bid $b(v)$ with the intermediary firm. Even if a customer loses her bid with the intermediary firm, she can always go back and buy directly from one of the suppliers.

### 4.1 One Supplier Runs Out of Stock

When one of the suppliers runs out of stock, we assume that the posted price will be replaced by a “sold out” sign (which happens in real life). Thus, the product at the intermediary firm ceases to be an opaque product and becomes a regular one. It is straightforward that a customer will always use the NYOP channel under such scenario: given that there is now only one product, with posted price $p = p_j$ and reservation price $r = r_j \sim U[0, p]$, the customer has to determine her bid, $b$, which maximizes her ex-ante surplus, $V$, where

$$V(v, b) = Pr\{b \geq r\}(v - b) + Pr\{b < r\}\max\{v - p, 0\}$$

$$= \begin{cases} 
\frac{b}{p}(v - b) + \frac{p-b}{p}(v - p), & \text{if } v \geq p \\
\frac{b}{p}(v - b), & \text{if } v < p.
\end{cases}$$

This leads to our first result.
Proposition 1. If there is only one supplier in the market with posted price \( p \), the customer will name a price \( b = \min\{p, v\}/2 \) before attending the direct channel.

Thus, when granted a chance to get a “bargain” (at the NYOP channel), the customers would ask for at least a 50% discount from the tag price. Notice that the valuation support as well as the reservation price support are based on \([0, 1]\): if the customers estimate the reservation prices to be on \([p, p]\), the bid would be in the form \( b = \min\{p + p, v + p\}/2 \). In either case, if a customer believes that the price is likely to have large fluctuations (as in the case of perishable products such as airline seats or hotel rooms), she might submit a bid that is significantly below the posted price, given that she can always buy from the direct channel in case of a rejection.

4.2 Both Suppliers Present

When both suppliers have available inventory, the intermediary firm can sell opaque products. Customers in this scenario have to be aware that there is a chance that they may be assigned a less-favored product when bidding in the NYOP channel.

Denote \( \Omega_N = \{v : v_i - p_i < 0, i = 1, 2\} \) and \( \Omega_i = \{v : v_i - p_i \geq 0, v_i - p_i \geq v_j - p_j\} \). In the absence of the intermediary NYOP firm, customers in \( \Omega_N \) have no way to obtain either product, while customers in \( \Omega_i \) prefer buying from supplier \( i \) than from supplier \( j \). When an NYOP channel is available, all customers in region \( \Omega_N \) bid with NYOP, as this is the only chance that they might get any product. Customers in the other two regions have to decide whether to name their own price first (and buy at the posted price if the bid is not accepted) or to buy directly from their preferred supplier.

Consider a customer with valuation \( v \) who chooses to bid \( b \); her expected payoff is

\[
V(v, b) = Pr\{b \geq r_1, r_1 < r_2\}(v_1 - b) + Pr\{b \geq r_2, r_1 > r_2\}(v_2 - b)
+ Pr\{b < r_1, b < r_2\} \max\{v_1 - p_1, v_2 - p_2, 0\}
= \frac{b(2p - b)}{2p^2}(v_1 - b) + \frac{b(2p - b)}{2p^2}(v_2 - b) + (1 - \frac{b(2p - b)}{p^2}) \max\{v_1 - p_1, v_2 - p_2, 0\}
\]

\[
\frac{\partial V(v, b)}{\partial b} = \begin{cases} 
\frac{3b^2-(v_1+v_2+4p)b+p(v_1+v_2)}{p^2}, & \text{if } v \in \Omega_N \\
\frac{3b^2-(2p_i+v_j-v_i-4p)b+p(2p_i+v_j-v_i)}{p^2}, & \text{if } v \in \Omega_i, \ i = 1, 2.
\end{cases}
\]

The following proposition characterizes the optimal bid for each customer.
Proposition 2. Given the posted prices, \( p_1 \) and \( p_2 \), the optimal bid for a customer with a valuation \( \mathbf{v} = (v_1, v_2) \) is

\[
b(\mathbf{v}) = \begin{cases} 
  p - \frac{\sqrt{(v_1+v_2-2p)^2+12p^2-(v_1+v_2-2p)^2} + 12p^2}{6}, & \text{if } \mathbf{v} \in \Omega_N, \\
  p - \frac{\sqrt{(2p_i+v_j-v_i-2p_i)^2+12p_i^2-(2p_i+v_j-v_i-2p_i)^2} + 12p_i^2}{6}, & \text{if } \mathbf{v} \in \Omega_i, \text{ and } v_i - v_j < 2p_i \text{ for } i = 1, 2, \\
  0, & \text{if } \mathbf{v} \in \Omega_i, \text{ and } v_i - v_j \geq 2p_i \text{ for } i = 1, 2.
\end{cases}
\]

Moreover, (i) \( b(\mathbf{v}) < \min\{p_1, p_2\} \); (ii) \( b(\mathbf{v}) < (v_1 + v_2)/2 \).

It is interesting to note that if a customer cannot use the posted price channel (i.e., \( \mathbf{v} \in \Omega_N \)), her bid is related to her expected valuation of the opaque product, \((v_1 + v_2)/2\); on the other hand, if a customer has the option to use the direct channel (i.e., \( \mathbf{v} \in \Omega_i \) for some \( i \)), her bid is only related to the difference between her valuations of the two products, \( v_i - v_j \). Indeed, when NYOP is the only channel through which one can obtain some product, the customer’s bid has to reflect the expected value that she may receive. However, when a customer participates in the NYOP only to “gamble” and see if she can get her preferred product at a lower price, she needs to take into account how much more she values one product over the other, as NYOP comes with the risk of receiving the less-preferred product.

Corollary 1. If \( \mathbf{v} \in \Omega_i \) for some \( i \in \{1, 2\} \), \( b(\mathbf{v}) \) decreases with \( v_i - v_j \) and increases with \( p_i - p \).

Proof. It follows directly from (2). \( \square \)

The corollary above confirms two things. First, for those customers who have a direct channel as an alternative option, the bid decreases with the difference in valuation between the two products. Intuitively, if a customer has access to at least one of the posted-price markets yet chooses to use NYOP, her motivation is primarily getting a good “deal.” Even if the bid is declined by the intermediary firm, the customer can still exercise her exit option and buy directly from the supplier. Second, as \( p_i - p \) can only attain two values, 0 if \( p_i \leq p_j \) or \( p_i - p_j > 0 \) otherwise, customers who prefer the more expensive product without the NYOP channel would place a greater bid than their counterparts.

As we can see, when \( v_i - v_j \) is high, customers will place a relatively low bid in the NYOP system, which makes it unlikely that they will meet the reservation price. In fact, we can show that customers participate in NYOP only when \( v_i - v_j \) is below some threshold; otherwise, they buy directly from their preferred supplier to avoid the risk of receiving the less-preferred product. Denote by \( \Omega_i(\delta) = \Omega_i \cap \{v : v_i - v_j \leq \delta\} \) for \( i = 1, 2 \) the set of customers who prefer supplier \( i \) in the absence of the NYOP channel yet have the valuation difference less than \( \delta \). We then have the following result.
Proposition 3. For any $i \in \{1, 2\}$, there exists $\delta^*_i$ such that a customer with valuation $v \in \Omega_i(\delta^*_i)$ will use NYOP.

The bid $b(v)$ represents one’s valuation of the opaque product. Whenever $b(v)$ is nonzero, a customer will choose to name her own price first, before buying directly from a supplier. Figures 2 and 3 depict how the customers’ choice of the channel and bidding amount changes with posted prices and their personal valuations, respectively.

![Figure 2: Channel selection for customers with different valuations given posted prices.](image)

When the posted prices are equally small, say $p_1 = p_2 = 0.2$, Lemma 3 suggests that $\delta^*_1 = 0.4$. In this case, only customers who significantly discriminate between the two products buy from direct channels; customers who cannot afford either product at the posted price or who have similar valuations for the two products use the NYOP system. The maximum bid is less than 50% of the lowest posted price. As one of the posted prices increases, say $p_1 = 0.2$ and $p_2 = 0.4$, $\delta^*_1 = 0.4$ and $\delta^*_2 = 0.8$. We can see that more customers choose to bid at NYOP before direct purchase, and the maximum bid increases as well. When $p_1 = 0.2$ and $p_2 = 0.8$, $\delta^*_1$ stays the same while $\delta^*_2 = 1$. Customers who prefer supplier 1 still prefer to buy directly from him; however, those buying directly from supplier 2 abandon the posted price channel and use NYOP instead. The maximum bid can be as much as 75% of the lowest posted price.

It can also be noticed from Figure 3 that when $p_i$ is fixed, each customer’s bid is non-decreasing with $p_j$. For example, throughout the three cases, customers with $v_1 - v_2 \in \{0, 0.4\}$ consistently choose to buy from supplier 1 if NYOP does not work out. Their bids are, therefore, not affected by the posted price of the other supplier, $p_2$. The bids of customers with $v_1 - v_2 \in \{-0.8, 0\}$ increase when $p_2$ is 0.4 instead of 0.2. However, the underlying reasons for the increase differ across the customers. For example, for a customer with $v_1 - v_2 \in \{-0.2, 0\}$ the exit option (in case the bid for NYOP is not accepted) shifts from supplier 2 to supplier 1 as $p_2$ increases. Hence, the NYOP channel becomes more attractive to this type of customers, especially those who do not value supplier 1 too much. On the other hand, for a customer with $v_1 - v_2 \in \{-0.8, -0.2\}$ the exit option is consistently supplier 2, despite of the increase in $p_2$. However, her optimal payoff from the
Figure 3: Optimal bidding at NYOP for customers with different valuations.

direct channel decreases, which motivates her to place a higher bid and thus increase the chance of getting the product through the NYOP channel and mitigate the loss in her bottom-line payoff.

5. Final Sales Channel

In the previous section, customers made their initial channel selections based on their private valuations, posted prices, and their estimates of the reservation prices. The unobservable reservation price, $r$, may match some customers with a channel that is not their first choice. In this section, we discuss how the final sales channel is affected by the suppliers’ posted price decision, $p$, and reservation price decision, $r$.

Denote $r = \min\{r_1, r_2\}$; then, customers with bid $b(v) \geq r$ will be awarded one unit of the product at their named prices. Recall that customers’ bidding behavior changes as the number of in-stock suppliers varies. From the previous section, we know that $b(v, x) = \min\{v, p\}/2$ when $x_i = 0$ and $x_j > 0$, while $b(v, x)$ is given by (2) when $x_i x_j > 0$. Let us introduce the following notation:

- $B_0(r, p, x) = \{v : b(v, p, x) \geq r\}$—the set of customers whose bids are accepted at NYOP.
- $B_i(r, p, x) = \{v : b(v, p, x) < r, v_i - v_j \geq p_i - p_j, v_i \geq p_i\}$—the set of customers who will purchase from the direct channel of supplier $i$. This includes both the customers who buy at the posted price in the first place ($b = 0$) and those who lose their bid with the NYOP channel and go back to the direct channel afterwards ($0 < b < r$).
- $B_N(r, p, x) = \{v : b(v, p, x) < r, v_i < p_i, v_j < p_j\}$—the set of customers who will leave empty handed (or, equivalently, those whose bids were rejected at the NYOP channel and who cannot afford any of the posted prices).
5.1 One Supplier Runs Out of Stock: \( x_i = 0, x_j > 0 \)

When only one supplier is in stock, there is no competition, and \( p_j = p \) and \( r_j = r \).

**Proposition 4.** For \( x_i = 0 \) and \( x_j > 0 \), all customers bid with the NYOP intermediary firm first, and consider buying at the posted price only if their bid is rejected. The final sales channel is characterized by

(i) If \( 2r > p \),

\[
\mathcal{B}_0(r, p, x) = \emptyset, \quad \mathcal{B}_j(r, p, x) = \{v : v_j \geq p\}, \quad \mathcal{B}_N(r, p, x) = \{v : v_j < p\};
\]

(ii) If \( 2r \leq p \),

\[
\mathcal{B}_0(r, p, x) = \{v : v_j \geq 2r\}, \quad \mathcal{B}_j(r, p, x) = \emptyset, \quad \mathcal{B}_N(r, p, x) = \{v : v_j < 2r\}.
\]

Since only one supplier is in stock, the NYOP channel is practically selling regular goods, and customers bid according to Proposition 1. In this case, if the reservation price is high (\( 2r > p \)), all bids could not go through the NYOP channel as they will not meet the reservation price. Then, the customers with valuation above the posted price \( p \) will buy from the direct channel, while the remaining customers leave empty handed. On the other hand, if the reservation price is low (\( 2r \leq p \)), a fraction of the bids will be accepted and no one will buy through the direct channel (since anyone who is able to afford the product at posted price will make a bid above the reservation price). From the supplier’s point of view, there is always one channel that will be idle under either kind of pricing schemes.

5.2 Both Suppliers Present: \( x_i > 0, x_j > 0 \)

When both suppliers have inventory, the final channel realization is characterized as follows.

**Proposition 5.** For \( x_i > 0 \) and \( x_j > 0 \), the final sales channel is characterized by

\[
\mathcal{B}_0(r, p, x) = \bigcup_{i=1,2,N} \Omega_i[\delta_i(r)],
\]
\[
\mathcal{B}_i(r, p, x) = \Omega_i \setminus \Omega_i[\delta_i(r)],
\]

where

\[
\Omega_i(\delta) = \begin{cases} 
\Omega_i \cap \{v : v_i - v_j \geq \delta\}, & \text{if } i = 1, 2 \\
\Omega_i \cap \{v : v_i + v_j \geq \delta\}, & \text{if } i = N;
\end{cases}
\]

\[
\delta_i(r) = \begin{cases} 
2p_i - 4r - \frac{r^2}{p}, & \text{if } i = 1, 2 \\
4r + \frac{r^2}{p-r}, & \text{if } i = N.
\end{cases}
\]
\( \Omega_i[\delta_i(r)] \) is the set of customers that bid higher than \( r \) yet will buy from supplier \( i \) in the absence of the NYOP channel. \( \Omega_N[\delta_N(r)] \) are the customers naming a price higher than \( r \), but who cannot afford either of the two posted prices. The proof of this results follows immediately from (2), and is therefore omitted.

Figure 4 illustrates where the customers will get their products by following their channel/bidding strategies. Throughout the examples, we let \( \mathbf{p} = (0.2, 0.4) \). By varying the minimum reservation price from 0.04 through 0.08 to 0.1, we depict the sets of customers that will win the product from the NYOP channel, buy it from supplier 1/supplier 2, or leave empty handed. The dotted lines correspond to the second graph in Figure 2, showing the initial channel selection of the customers. Hence, when \( r = 0.04 \), customers with \( \mathbf{v} \) in the polygon bounded by \((0, 0.63), (0, 0.77), (0.23, 1), (0.37, 1)\), which we will denote as \( \mathbf{v} \in \mathcal{P}\{(0, 0.63), (0, 0.77), (0.23, 1), (0.37, 1)\} \), choose to name their own prices \( b(\mathbf{v}) \). By Lemma 2, their bids are all less than 0.04, and are therefore rejected. These customers eventually buy from supplier 2 after learning about the rejection. White areas represent the set of customers who will be awarded a unit of the products from the NYOP channel by bidding above \( r \), while black areas represent customers who can neither afford the posted prices nor bid above \( r \).

Notice that our discussion so far (customer behavior and final channel realization) does not use any particular assumption about the customer valuation distribution. Although we are more interested in two-dimensional general distributions of \( \mathbf{v} \), some simplified examples can provide helpful insights as well. In the proposition below, we use customer valuations to define the differentiation between two products.

**Proposition 6.**

(i) If the products are vertically differentiated (i.e., \( v_i - v_j = \vartheta \) for some constant \( \vartheta > 0 \)), then supplier \( i \) sets a posted price such that all customers buy from its direct channel. The intermediary firm collects zero rents.
(ii) If the products are horizontally differentiated (i.e., \(v_i + v_j = \bar{v}\) for some constant \(\bar{v} > 0\)), then there exists \(p^0 > 0\) such that (a) when the lower posted price, \(p\), satisfies \(p > p_0\), all customers are covered, and the intermediary firm earns positive profit; (b) when \(p \leq p_0\), some customers will not receive any of the products, and no customers will use NYOP.

The implication of the preceding result is that the intermediary firm should be careful when selecting regular products that will be used to construct opaque products in NYOP. If it has been recognized that product candidates have a significant quality difference (a flight departing at 1 am vs. a flight on the same route that leaves at noon) or one product has higher brand recognition (such that all customers may prefer the same product, although their quality may be equal), then the intermediary firm may not benefit much from these differences. The main reason is that the supplier with “better image” will get the deterministic difference through the direct channel by offering a posted price that the customers cannot refuse. Under this posted price, the customers hardly consider the opaque product worth bidding on. On the other hand, if the products are horizontally differentiated (e.g., hotels in the same region and with the same star ranking), the valuation for the opaque product will be much more diversified. The suppliers would then need the intermediary firm to shoulder part of this uncertainty so that their direct channel can target their own high-end customers. These results are consistent with the observations in Perkins (2006).

6. Optimal Pricing Decisions for the Suppliers

6.1 One Supplier Runs Out of Stock

When only one supplier, say \(j\), offers its products through the intermediary firm, then at the beginning of each time epoch it makes a decision on \(p\) and \(r\) without taking competitive factors into consideration. Let us denote by \(\Pi_j(p, r; x, t)\) the expected profit supplier \(j\) will receive when the posted and reservation prices for period \(t \leq T\) are \(p\) and \(r\), respectively. The customers’ bidding behavior and final channel realization were characterized in Proposition 1 and Proposition 4, respectively. Then, supplier \(j\)’s spot revenue is given by

\[
\pi_j(p, r) = \begin{cases} 
F_j(p)p, & \text{if } 2r > p \\
F_j(2r)r, & \text{if } 2r \leq p.
\end{cases}
\]

At \(t = 1\), supplier \(j\) will seek a pair \((p, r)\) that maximizes \(\pi_j(p, r; x)\). At \(t > 1\), we use dynamic programming to determine the best \((p, r)\):

\[
\Pi_j(p, r; x, t) = \max_{p \geq r \geq 0} \left\{ \begin{array}{ll}
F_j(p) [p + \Pi_j(x - e_j, t - 1)] + F_j(p)\Pi_j(x, t - 1), & \text{if } 2r > p \\
F_j(2r) [r + \Pi_j(x - e_j, t - 1)] + F_j(2r)\Pi_j(x, t - 1), & \text{if } 2r \leq p,
\end{array} \right.
\]

where \(e_j\) is a \(1 \times 2\) vector with the \(j\)th component equal to 1, and zeros everywhere else.
Theorem 1. A monopolistic supplier maximizes its expected revenue by using only posted prices in the direct channel.

In other words, if a monopolist supplier can sell its products at posted prices in the direct channel, as regular goods in the NYOP channel of the intermediary firm, or both, it will always choose to go with posted prices only. As Proposition 4 implies, a combination of the two channels is not necessary because there will always be one channel that is idle. Hence, the problem is reduced to selecting the channel that brings in more revenue (direct channel with posted prices or NYOP). A closer comparison of (3) and (4) suggests that the supplier has access to the same set of customers under either channel when \( 2r = p \). However, if the NYOP channel is chosen, the supplier would collect less than it could with a posted price \( r = p/2 \leq p \) because the third-party (intermediary firm) will share a portion of the revenue. However, this is not the entire reason: even if the intermediary firm is a part of the monopolist, so that the supplier receives the entire bid \( b \) in the NYOP channel (instead of \( r \leq b \) when the firms are decentralized), the revenue is still lower than what can be generated from a direct channel alone. If the customer valuations are uniformly distributed, we have the following analytical result.

Proposition 7. If \( v_j \sim U[0, 1] \), a centralized monopolistic supplier \( j \) that can provide NYOP service is better off with direct channel only.

This further confirms that the NYOP channel might not be a welcome addition to one’s distribution channels. The monopolist supplier loses in the presence of NYOP not just because the intermediary firm subtracts a part of the revenue—even if the entire bid from the customers goes to the supplier, NYOP is less attractive because customers use the opportunity to get a better deal. As an online channel, NYOP provides to the suppliers access to customers that they may otherwise lose (e.g., those belonging to \( \delta_N \)); however, it also cannibalizes the market that would buy from the direct channel when NYOP is not available.

Uniformly distributed valuation \( v_j \) allows us to provide an exact solution for the pricing strategy as well as the expected revenue as a function of the remaining time and available inventory. Denote \( \Pi_j(x, t) \) as the maximizer of \( \Pi_j(p, r; x, t) \) over all feasible pairs \((p, r)\). If \( v_j \sim U[0, 1] \), the supplier’s spot revenue can be expressed as

\[
\pi_j(p, r; x) = \begin{cases} 
  p - p^2, & \text{if } 2r > p \\
  r - 2r^2, & \text{if } 2r \leq p.
\end{cases}
\]

Then, for \( t = 1 \), \( x_i = 0 \), and \( x_j = 1 \), it can be verified that the optimal pair \((p, r)\) equals \((0.5, 0.5)\) and \( \Pi_j(x, 1) = 0.25 \).
Proposition 8. Assume \((v_i, v_j) \sim U[0, 1] \times [0, 1], x_i = 0\) and \(x_j > 0\). At \(t > 1\), optimal pair \((p, r)\) is given by

\[
p^* = r^* = \frac{1 + \Pi_j(x, t - 1) - \Pi_j(x - e_j, t - 1)}{2}
\]

and

\[
\Pi_j(x, t) = \Pi_j(x - e_j, t - 1) + \left(\frac{1 + \Pi_j(x, t - 1) - \Pi_j(x - e_j, t - 1)}{2}\right)^2,
\]

where \(e_j\) is a \(1 \times 2\) vector with the \(j\)th component equal to 1 and zeros everywhere else.

It is not hard to verify that \(\Pi_j(x, t)\) is nondecreasing with \(t\). It can also be shown that \(\lim_{t \to \infty} \Pi_j(x, t) = 1\) when \((x_i, x_j) = (0, 1)\): as time goes by, the optimal \(p^* = r^*\) drops from 1 to 0.5. The NYOP channel does not bring in any additional profit in an environment characterized by lack of competition.

6.2 Both Suppliers Present

At the beginning of each time epoch \(t\), the suppliers observe their inventory positions \(x = (x_1, x_2)\). For analytical convenience, we assume that the inventory position \(x_i\) is visible to both the supplier \(i\) itself and its competitor \(j\); similar assumptions have been made by many papers analyzing competitions in revenue management (e.g., Gallego and Hu 2007; Levin et al. 2009; Lin and Sibdari 2009). For imperfect information on parameters such as demand distributions or inventory levels, one may refer to Perakis and Sood (2006), Levin et al. (2008), Zhang and Kallesen (2009), etc. As our primary goal is to study how the presence of an NYOP channel may affect the strategic decisions of suppliers, this assumption falls within the scope of the paper.

After observing inventory level \(x\), the suppliers competitively determine their posted prices, \(p = (p_1, p_2)\). Denote by \(\Pi_i(p; x, t)\) the expected revenue for supplier \(i\) at the beginning of period \(t\) given posted prices \(p\), by \(p^*(x, t)\) the equilibrium posted price, if one exists, for period \(t\), and by \(\Pi_i(x, t) = \Pi_i(p^*(x, t); x, t)\) the expected revenue for supplier \(i\) at the beginning of period \(t\), given that posted prices correspond to equilibrium ones, \(p = p^*(x, t)\). Obviously, \(\Pi_i(x, 0) = 0\) and \(\Pi_i(x, t)|_{x_i=0} = 0\). When submitting the reservation price to the intermediary firm and assuming its competitor does not change its decision, a supplier has to decide whether it may accept a reservation price lower than the current one, \(r - \epsilon\) instead of \(r\), where \(\epsilon\) is the minimum decrement designed by the intermediary firm.
Suppose supplier 2 has reservation price \( r \); then, supplier 1’s problem is given by

\[
\Pi_1(x, t) = \max_{r \leq p} \left\{ \left[ p + \Pi_1(x_1 - 1, x_2, t - 1) \right] Pr\{v \in B_1(r, p, x)\} + \Pi_1(x_1, x_2 - 1, t - 1) Pr\{v \in B_0(r, p, x)\} + \Pi_1(x_1, x_2 - 1, t - 1) Pr\{v \in B_2(r, p, x)\} + \Pi_1(x_1, x_2, t - 1) Pr\{v \in B_N(r, p, x)\}, \right. \\
\left. + [r - \epsilon + \Pi_1(x_1 - 1, x_2, t - 1)] Pr\{v \in B_0(r - \epsilon, p, x)\} + \Pi_1(x_1, x_2, t - 1) Pr\{v \in B_N(r - \epsilon, p, x)\} \right\}. 
\]  

(7)

Then, at time \( t \), given posted prices \( p \), inventory level \( x \), and properly designed \( \epsilon \), there exists \( r^*(p, x, \epsilon, t) \in (0, p) \) and a supplier \( i \in \{1, 2\} \) that is willing to accept reservation price \( r^* \) when the other supplier, \(-i\), does not. Define

\[
\tilde{\Pi}_1(x, t) = \Pi_1(x_1, x_2 - 1, t) - \Pi_1(x_1 - 1, x_2, t), 
\]

(8a)

\[
\tilde{\Pi}_2(x, t) = \Pi_2(x_1 - 1, x_2, t) - \Pi_2(x_1, x_2 - 1, t); 
\]

(8b)

(8) provides the threshold prices below which some supplier would drop the auction. Obviously, the one with higher threshold, say \( \tilde{\Pi}_1 \), is less tolerant with this auction and will give up her candidacy as a opaque provider. Her rival supplier 2 will stay to serve the NYOP channel. Similar to the second-price auction, the NYOP firm rewards supplier 2 (if any) as high as \( \tilde{\Pi}_1 \), even though her actual threshold price is lower. We can now state the following result.

**Proposition 9. (Reservation Price Under Open Auction)**

\[
r^*(x, t) = \lim_{\epsilon \to 0} r^*(p, x, \epsilon, t) = \min\{p, \tilde{\Pi}_{-i}(x, t - 1)\}, 
\]

where \( i = \arg \max_{i=1,2} \tilde{\Pi}_i(x, t - 1) \). Specifically, at \( t = 1 \) and in the presence of competition \((x_1 > 0, x_j > 0)\), the reservation price is given by \( r^* = 0 \).

Notice that if \( v_1 \) and \( v_2 \) are identically distributed and \( x_1 = x_2 \), we have \( \tilde{\Pi}_1(x, t) = \tilde{\Pi}_2(x, t) \) and both suppliers stop at \( r^* \), unwilling to accept lower reservation prices. The intermediary firm can then allocate the customers with a half-half probability to the suppliers with the same reservation price, \( r^* \). One can verify that Proposition 9 still holds (after appropriate modification of (7)).

Although the suppliers determine their reservation prices after the posted prices are announced, Proposition 9 suggests that, besides being bounded above by it, \( r \) depends very little on the minimum posted price, \( p \). In addition, Proposition 9 implies that it may happen that suppliers
keep posting non-trivial prices in their direct channels even at the last minute, although the NYOP channel offers last-minute sales in which everyone can get a unit with an arbitrary bid.

To verify the existence of pure-strategy Nash equilibrium (NE) \((p^*_1(x, t), p^*_2(x, t))\), we need to check if the payoff function in \((7)\) is quasi-concave in \(p_i\). While this result seems analytically intractable for general distributions of \(v\), it can be proved that it holds for uniformly distributed valuations \((v_1, v_2) \sim [0, 1] \times [0, 1]\). Our numerical simulations also suggest that the same holds for i.i.d. normal distribution.

**Theorem 2.** If the customer valuations, \(v\), are uniformly distributed on \([0, 1] \times [0, 1]\), then

(i) Pure-strategy NE in posted prices exists;

(ii) Reservation price \(r^*\) is zero when both suppliers oversupply (i.e., \(x_i \geq t\) for \(i = 1, 2\)).

Theorem 2 (i) allows us to perform numerical analysis in the next section, and theorem 2 (ii) extends the result from Proposition 9, which states that oversupply at both suppliers at any time leads to zero reservation price. We will discuss our computation results in more detail in the next section.

7. Numerical Analysis

We next look at numerical solutions for reservation prices and equilibrium decisions in posted prices, given the current state \((x_1, x_2, t)\). In this section, customer valuation \((v_1, v_2)\) is assumed to be uniformly distributed on \([0, 1] \times [0, 1]\). Both dual-channel (posted prices and NYOP) and single-channel (posted price only) are examined. We will refer to the dual-channel case as “DC” and to the single-channel case as “SC” during the discussion.

7.1 Expected Revenue

• **VARYING TIME**

Figure 5 depicts how the supplier’s expected revenue varies with the remaining sales time \(t\) and its own inventory level \(x_i\), while its rival’s inventory level \(x_j\) is fixed.

**Effect of Time.** Not surprisingly, the expected revenue grows with the length of the remaining time in both cases. For SC, the marginal revenue of one additional sales period, \(\Pi_1(x_1, 5, t) - \Pi_1(x_1, 5, t - 1)\), decreases with \(t\). For DC, the marginal revenue of time is decreasing only when some supplier does not oversupply (i.e., \(\min\{x_1, x_2\} < t\)). As Theorem 2 (ii) indicates, if both suppliers oversupply (i.e., \(x_{1,2} \geq t\)), each supplier strives to sell its inventories, and the NYOP channel intensifies the competition, which leads to a zero reservation price. In these instances, one more sales period (or equivalently, one more unit of random-valued demand) is contributing
positively to the expected revenue, as this unit has a non-trivial chance to be sold at some posted price. This time effect depends only on the fact that there is currently an over-supply, not on the specific inventory levels.

**Effect of Inventory Levels.** While in SC the expected revenue grows with one’s own inventory, $x_1$, the same does not hold in DC, especially when $x_1 + x_2 \geq t$. Hence, while in SC “more is better,” in DC “more may be worse, especially when total supply exceeds total demand.” The diminishing margin of inventory holds under similar conditions for the two cases.

**Comparison.** When comparing the two graphs in Figure 5, one may notice that the expected revenue is higher in SC than in DC. The difference in expected revenue is more significant as competition becomes more intense (i.e., $x_1$ becomes higher). The low revenue in DC generally comes from two sources. First, the NYOP channel provides a platform on which suppliers compete to sell at a low price without damaging the integrity of their direct channel. As competition becomes more intense, the reservation price at the NYOP channel could become rather low (eventually becoming $r = 0$ in case of over-supply). Thus, there is a reasonable chance that some product quantity will be sold at lower prices in DC than SC. Second, the posted price competition becomes more intense as well. In the absence of the NYOP channel, suppliers compete publicly through their posted prices only, and there is less uncertainty about whose product the customer will eventually buy. On the other hand, with the NYOP channel each supplier is aware that the other seller may “steal” its customer through NYOP. Hence, each supplier is competing with the part of the opaque product that comes from its rival, as well as with other regular products in the market. Figure 7 also shows that posted
prices in the presence of the NYOP channel usually have lower values than when this channel is absent. This provides experimental evidence that the NYOP channel intensifies the competition between the suppliers.

- **Fixed Time** In Figure 6, we examine how the expected revenue varies with inventory levels, $x_1$ and $x_2$, at fixed time $t = 8$. Once again, we note that SC generates higher expected revenue than DC.

![Figure 6: Expected revenue for supplier 1 as a function of available inventories, $(x_1, x_2)$, at $t = 8$.](image)

Effects of Inventory Level. The marginal revenue of one’s own inventory is decreasing, but it remains non-negative in SC. On the other hand, the margin in DC is also decreasing but can become negative when the inventory level is within a certain range. Specifically, when $x_2 < [\frac{t}{2}] + 1 = 5$, the margin for $x_1$ is non-negative, but when $x_2 \geq [\frac{t}{2}] + 1 = 5$, the margin can be negative for $[\frac{t}{2}] + 1 = 5 \leq x_i \leq 8 = t$. For example, when $x_2 = 6 > 5$, $\Pi_1$ increases with $x_1$ as long as $x_1 \leq 4$, it decreases as $x_1$ changes from 5 to 8, and becomes zero for $x_1 > 8$; when $x_2 = 2 \leq 5$, $\Pi_1$ is non-decreasing with $x_1$ all the time. Because we assume $t = 8$, there will be a total of 8 arriving customers, each with unknown valuation. Since we assumed that $v$ is uniformly distributed, there is practically no difference between the two suppliers, and one can think of inventory level 4 as the “quota” level that splits the potential demand. When the rival stocks no more than this quota ($x_2 < 5$), the game is less competitive, and the suppliers’ expected revenue functions have similar shapes in DC and SC. When the rival stocks more than the quota ($x_2 \geq 5$) but the supplier itself is within the quota ($x_1 < 5$), the rival estimates that the supplier will behave less aggressively because it seems reasonable to expect that the supplier with less inventory will “sell less at a high price”
instead of “price low and sell more.” When both suppliers stock over the quota ($x_{1,2} \geq 5$), each supplier believes his rival will take some competitive actions, and each itself has the motivation to do the same, as it has the resource (high inventory levels) for achieving a greater market share. Both posted prices and reservation prices become low as a result of the severe competition, which leads to the counterintuitive result in which more inventory may lead to lower revenue.

### 7.2 Posted Price in Equilibrium

- **Varying Time**

Figure 7 is a counterpart of Figure 5, as it depicts equilibrium prices in the direct channel under the same conditions.

![Figure 7](image)

Figure 7: NE posted price for supplier 1 as a function of the remaining time, $t$, and available inventory, $x_1$, when $x_2 = 5$.

**Effect of Time and Inventory Level.** The posted price in SC decreases with the rival’s inventory level and elapsed time; the marginal value of time can be either decreasing (when the rival’s inventory level is low) or increasing (when the rival’s inventory level is high). In DC, we observe the same general trend with respect to posted prices. Nevertheless, we may observe significant differences between the two graphs. According to Theorem 2 (ii), reservation price will be zero (also shown in Figure 10) if both suppliers oversupply (i.e., $x_2 = 5 \geq t$ and $x_1 \geq t$). When this happens, the posted price also takes a very low value (around 0.167). This also explains why all of the lines with $x_1 \in \{5, 6, 7, 8, 9, 10\}$ coincide when $t \in [1, 5]$. Moreover, when a supplier has a greater inventory ($x_1 > x_2 = 5$) and its rival is not oversupplying ($t > x_2 = 5$), its posted price stays at a moderate level. On the one hand, it does not price too low because the rival is not oversupplying; on the
other hand, it is not necessary to price too high since it has enough inventory. As one may be aware, many of the inconsistencies stem from oversupply. If we remove the oversupply effects by concentrating on smaller inventory levels, the graphs for SC and CD may look more alike (Figure 8).

Figure 8: NE posted price for supplier 1 as a function of the remaining time, \( t \), and available inventory, \( x_1 \), when \( x_2 = 1 \).

Comparison. Although the expected revenues are overall higher in SC than in DC, the same is not true for posted prices. For example, Figure 7 shows that when \( x_2 = 5 \), the posted prices in SC are generally higher when \( t \leq 5 \). However, as \( t \) becomes greater than 5 (supplier 2 is no longer oversupplying), posted prices in SC are lower than those in DC for some large values of \( x_1 \). Thus, with enough periods to go, the overstocking supplier will target its direct channel at high-end customers that would not use NYOP, while at the same time setting a very low reservation price to attract as many NYOP customers as possible.

• Fixed Time

At a given time period, say \( t = 8 \), a supplier’s posted prices are rather sensitive to inventory level in DC when its rival stocks over the “quota” (\([\frac{t}{2}]\)). When it is, indeed, the case that \( x_2 > \left[\frac{t}{2}\right] = 4 \), a supplier’s posted price decreases (resp., increases) with its inventory, \( x_1 \), as \( x_1 \) approaches \( x_2 \) from below (resp. above).

7.3 Reservation Price

• Varying Time

Figure 10 is a counterpart of Figures 5 and 7, and it shows the corresponding reservation prices
when oversupply is likely to happen \((x_2 = 5)\); Figure 11 is a counterpart of 8 when oversupply is less likely to happen \((x_2 = 1)\). In both cases, reservation prices are zero when \(t \leq \min\{x_1, 5\}\), and they then increase with the remaining time. In Figure 10, when \(x_1 > 5\), the total supply exceeds potential demand, and the reservation prices are very small. The shape of ratio \(r/p\) is close to that of the reservation price, \(r\), when \(x_1 \leq 5\), and the discount is deeper as the remaining time decreases. However, for \(x_1 \geq 5\), \(r/p\) increases with time; this is again due to oversupply.

Figure 10: Reservation price at the intermediary firm as a function of the remaining time, \(t\), and supplier 1’s inventory level, \(x_1\), when \(x_2 = 5\).

When the inventory level is relatively small compared with potential demand \((x_2 = 1)\), the reservation prices show more consistency, as depicted in Figure 11. Both the reservation price, \(r\), and the degree of discount, \(r/p\), decrease with the remaining time and available inventory. With
the exception of the case in which the opaque product can be obtained for free (that is, when \( t = 1 \)),

the discount that one may get from NYOP is no less than 50% of the lower posted price, which matches some real-life observations.

Figure 11: Reservation price at the intermediary firm as a function of the remaining time, \( t \), and supplier 1’s inventory level, \( x_1 \), when \( x_2 = 1 \).

• **Fixed Time**

Finally, in Figure 12 we provide a 3-D plot of the reservation prices at some fixed time points. The reservation price decreases with the inventory levels and becomes zero after both parties oversupply.

Figure 12: Reservation prices at fixed time spot.

Specifically, in the last graph in Figure 12, the reservation price is zero when \( x_1 = x_2 > t/2 \).
This is potentially due to the fact that \( \widetilde{\Pi}_i \leq 0 \) (that is, \( \Pi_1(x_1, x_2 - 1, t) \leq \Pi_1(x_1 - 1, x_2, t) \)). Under the assumption of uniformly distributed customer valuations, each supplier is expected to have the same amount of units sold \((t/2)\), providing there is enough inventory, which is true in this case because \( x_2 > x_2 - 1 = x_1 - 1 \geq t/2 \). This implies that the expected number of units sold by supplier 1 will not change if its inventory level is \( x_1 \) or \( x_1 - 1 \). However, lower inventories at supplier 1 \((x_1 - 1 \text{ instead of } x_1 \text{ units})\) put less pricing pressure on its rival, supplier 2, who is then likely to sell at least \( t - x_1 + 1 \) units instead of \( t - x_1 \). For example, in the first graph of Figure 7 when \( t = 6 \), \( p_1(4, 5, t) \) is, indeed, higher than \( p_1(5, 5, t) \). Thus, a less competitive pricing environment that does not reduce expected sales enhances one’s expected profit, which intuitively explains why \( \Pi_1(x_1, x_2 - 1, t) \leq \Pi_1(x_1 - 1, x_2, t) \).

7.4 Opaque Product Provider

Besides analyzing the reservation price itself, which immediately determines the income of the intermediary firm, it is also interesting to consider which supplier would win the open auction and provide the opaque product. We refer to this supplier as the opaque product provider.

As demonstrated in Proposition 9, besides using open auctions, the opaque product provider can be determined by looking at the inventory level at a given time point; we illustrate this in Figure 13.

![Figure 13: Opaque product provider when \( t = n \).](image)

When both firms oversupply (i.e., \( x_1 \geq t \) and \( x_2 \geq t \)), both parties have zero reservation price, and the opaque product provider is randomly drawn. Additionally, whenever \( x_1 = x_2 \), the two firms have the same decisions about their reservation prices. While the final reservation price may be positive, each of the two firms has the same chance of being the opaque product provider. \( t = n \)
implies that the total number of upcoming customers is \( n \), and the “quota” for each supplier could be set at \( n/2 \). Figure 13 suggests that if the inventory level of one supplier is less than the quota (i.e., \( x_i \leq n/2 \) for some \( i \in \{1, 2\} \)), then the opaque product provider will be the one with more inventory; otherwise, if both inventories exceed the quota (i.e., \( x_{1,2} > n/2 \)), the supplier with less inventory will supply the intermediary firm. The first case is rather straightforward—when \( x_i \leq n/2 \) and \( x_i < x_j \), supplier \( i \) is not motivated to sell more or to sell cheaply because of its low inventory levels. The second case is more interesting in that it reverses the previous claim. In this case, competition becomes more intense compared with the first case because neither supplier is expected to sell all of its inventory. Both suppliers may prefer a less competitive environment, and the fastest way to get there is to grant the supplier with less inventory more opportunity to sell its units. The supplier with more inventory will not supply the intermediary firm until its rival’s inventory level hits \( n/2 \).

8. Discussion and Extensions

The Internet opened an entirely new world for consumers. At the same time, businesses are under an increased pressure to redesign their distribution channels and their pricing and inventory control strategies. Our model blends the elements of dual-channel and competitive-pricing models in trying to understand how a new sales format—name-your-own-price (NYOP)—may influence the industry. We model two suppliers who compete with each other and sell regular goods in their direct channels and opaque goods by contracting with an intermediary NYOP firm. The intermediary firm acts like an agent that hides brand and some other information about the products to ensure direct channel integrity. It benefits from the difference between customers’ willingness-to-pay for the opaque product and the lowest price at which a supplier would let its product go.

In discussing customers’ channel and bidding behavior, we find out that low-end customers tend to look at their average valuation of the products—their bids depend on how much they appreciate the “expected” product. However, high-end customers focus on differences in their valuation of the two products, and their bids decrease with the degree of differentiation. Thus, an intermediary NYOP firm should avoid bundling vertically differentiated products, but should concentrate on horizontally differentiated ones in choosing a set of potential opaque goods suppliers.

From the suppliers’ perspective, we provide a dynamic programming approach in determining the final reservation prices, equilibrium posted prices, and expected revenues. Our numerical results suggest that NYOP does not make the suppliers or the industry better off—the existence of an intermediary firm results from the competition between the suppliers. However, with an NYOP channel the suppliers are worse off in the intensified competition.

While our paper deals with a simple model in analyzing the impact of NYOP, there are many
directions in which this work can be extended (Granados et al 2009); we list a few of them below.

**Opaque Product Line**
The intermediary firm chooses the underlying products for the opaque goods. For a two-supplier-two-product problem, our results indicates that it is better to have horizontally differentiated products (e.g., hotels with different brand names but at the same star-level and nearby locations) than vertically differentiated ones (e.g., flights of the same route but one departs at 1 am while the other departs at noon). As more suppliers participate in the model, there is more room for the intermediary firm to increase the bundling assortment; the firm may also inform the customers about the underlying products that comprise the opaque product they are bidding for. There are many interesting approaches with regard to how such decisions could be made.

**Contracting**
The relationships between the intermediary firm and suppliers may have many alternatives. Our current setting assumes that the intermediary firm uses a descending auction to determine the reservation price. One realistic alternative may be to ask the sellers to privately submit their own reservation prices, but it may add a new level of competition and require some additional prior assumptions. Conversely, the problem could be simplified by letting the intermediary firm set the reservation price and then randomly select a supplier that is willing to participate. Additionally, the suppliers may receive a portion of the spread between the customer bid and the reservation price, $b - r$.

**Information Availability**
For analytical ease, we have assumed that each supplier knows the capacity available at both suppliers. We can modify the problem by looking at instances with imperfect information. On one hand, players may not know demand distribution, and we may allow suppliers/NYOP firms to learn from the bidding data. On the other hand, suppliers may rely on their own mechanisms in estimating the remaining inventories of their competitors through their posted prices or their reservation prices at the intermediary firm. As mentioned before, the intermediary firm could also vary the degree of information availability for the customers (how much should customers know about opaque products).

**Customer Behavior**
In the presence of multiple sales channels and a finite number of sales periods, customers can strategically plan their purchasing/bidding timing across a long horizon; Shen and Su (2007) review the recent literatures in strategic customer behavior in RM and auctions. While our model assumes a static customer set, it would be of practical value to examine the problem in which strategic cus-
tomers can make intertemporal decisions on channel selection and bidding, or in which returning customers can repeat their bids after the “frozen” period has elapsed.

References


Appendix

Proof of Proposition 2: If $v \in \Omega_N$, denote the roots of FOC as

$$b_1 = p - \frac{\sqrt{(v_1 + v_2 - 2p)^2 + 12p^2} - (v_1 + v_2 - 2p)}{6},$$

$$b_2 = p + \frac{\sqrt{(v_1 + v_2 - 2p)^2 + 12p^2} + (v_1 + v_2 - 2p)}{6},$$

where $b_1 \leq b_2$. Then, $V$ is maximized at $b = b_1$ or $b = p$. It is not hard to verify that $b_1 \leq p \leq b_2$. Hence, the optimal bid is $b = b_1 < \min\{p_1, p_2\}$. As $V = \frac{b(2p-b)(v_1+v_2-2b)}{2p^2} > 0$ and $b \leq p$, there should be $b \leq (v_1 + v_2)/2$, which proves (ii).

For $v \in \Omega_i$, $i = 1, 2$, the roots $b_1 \leq b_2$ are given by

$$b_1 = p - \frac{\sqrt{(2p_i + v_j - v_i - 2p)^2 + 12p^2} - (2p_i + v_j - v_i - 2p)}{6},$$

$$b_2 = p + \frac{\sqrt{(2p_i + v_j - v_i - 2p)^2 + 12p^2} + (2p_i + v_j - v_i - 2p)}{6}.$$

We also have $b_1 \leq p \leq b_2$ and $b = \max\{b_1, 0\}$. For $b > 0$, we need $2p_i + v_j - v_i + 4p \geq 0$ and $2p_i + v_j - v_i > 0$; that is, $v_i - v_j < 2p_i$. The rest of the proof follows similarly. \qed

Proof of Proposition 3: For any $v \in \Omega_i(\delta_i^*)$, let $\delta = \sup\{\delta_i : v \in \Omega_i(\delta_i)\}$. Denote $V^N = V(v, b(v))$ where $b(v)$ is defined by (2). Then, the customer will choose the NYOP channel if and only if $V^N \geq v_i - p_i$, where

$$V^N = \frac{b(2p-b)}{2p^2}(v_1 - b) + \frac{b(2p-b)}{2p^2}(v_2 - b) + \left(1 - \frac{b(2p-b)}{p^2}\right)(v_i - p_i).$$

We then have

$$v_i - p_i - V^N = \frac{b(2p-b)}{p^2}(v_i - p_i - \frac{v_1 + v_2}{2} + b)$$

$$= \frac{b(2p-b)}{p^2}(\frac{\delta}{2} - p_i + b)$$

$$= \frac{b(2p-b)}{p^2}\left(\frac{\delta}{2} - p_i + p - \sqrt{(2p_i - \delta - 2p)^2 + 12p^2} - (2p_i - \delta - 2p)\right).$$

(1) indicates that customers are aware that $r \leq p$, and hence they would not bid above the lower posted price. Thus, we have $\frac{b(2p-b)}{p^2} > 0$. In order to have $v_i - p_i - V^N \leq 0$, it is sufficient and necessary to have $\frac{A}{2} - \frac{\sqrt{A^2 + 12p^2} + A}{6} \leq 0$, or, equivalently, $A \leq 2p$, where $A = \delta - 2p_i + 2p$. Hence, $\delta_i^* = 2p_i$. \qed
Proof of Proposition 6: (i) Proposition 5 implies that when products are vertically differentiated, then customers who eventually receive a unit will all buy either from supplier $i$, or from supplier $j$, or from the NYOP channel. We first argue that they would not buy directly from supplier $j$, as there is always a strategy in which supplier $i$ can set its posted prices as $p_i = p_j + v_i - v_j - \epsilon = p_j + v - \epsilon$ for some $\epsilon > 0$ such that all customers prefer to buy from supplier $i$ at $p_i$. With this in mind, supplier $j$ would accept any reservation price $r > 0$. Moreover, by setting $p_i$ such that $\delta_i(r) = 2p_i - 4r - \frac{r^2}{p-r} \leq v$, supplier $i$ has a strategy that may induce all customers to buy at its posted prices for all possible $r$. This is feasible since $\delta_i(r)$ decreases with $r$ and increases with $p_i$ for both $p_i = p$ and $p_j = p$. Given that it can win all customers at posted prices, supplier $i$ does not have an incentive to bid a positive reservation price with the intermediary firm because supplier $j$ would always react with a lower $r$.

(ii) By Proposition 5, if $\delta_N(r) < v$, all customers will receive a unit of product from one of the suppliers or from the NYOP channel. Otherwise, those who do not buy from the direct channel will not bid with the intermediary either and will leave empty handed. Solving $\delta_N(r) = 4r + \frac{r^2}{p-r} < v$ gives $p^0 = \frac{rv-3r^2}{v-4r}$.

Proof of Theorem 1: For $t = 1$, we need to optimize $\pi_j(p,r;x)$. Denote $p^* = \arg\max \bar{F}_j(p)p$. Then, $F_j(p^*)p^*$ is the best that the supplier can achieve with the direct channel only. If the supplier uses the NYOP channel as well, the objective is then $\max \bar{F}_j(2r)r = \frac{1}{2}[\max \bar{F}_j(2r)2r] = \frac{1}{2}F_j(p^*)p^*$. Therefore, it is always optimal not to let customers purchase from the NYOP channel when the supplier is alone.

If $t > 1$, let $r$ be the infimum of the range of values that maximize $\bar{F}_j(2r)[r + \Pi_j(x - e_j, t-1)] + F_j(2r)\Pi_j(x, t-1)$. Then, $2r \leq p$ should hold. Let $p = 2r - \epsilon < 2r$; it is not hard to verify that

$$\bar{F}_j(p)[p + \Pi_j(x - e_j, t-1)] + F_j(p)\Pi_j(x, t-1)$$

$$= \bar{F}_j(2r)[r + \Pi_j(x - e_j, t-1)] + F_j(2r)\Pi_j(x, t-1)$$

$$+ \bar{F}_j(2r)r + [\bar{F}_j(2r-\epsilon) - \bar{F}_j(2r)](2r + \Pi_j(x - e_j, T-1) - \Pi_j(x, t-1)) - \epsilon \bar{F}_j(2r-\epsilon).$$

When $\epsilon$ is small enough, we have $[\bar{F}_j(2r-\epsilon) - \bar{F}_j(2r)](2r + \Pi_j(x - e_j, t-1) - \Pi_j(x, t-1)) - \epsilon \bar{F}_j(2r-\epsilon) \to 0$. Hence,

$$\bar{F}_j(p)[p + \Pi_j(x - e_j, t-1)] + F_j(p)\Pi_j(x, t-1) \geq \bar{F}_j(2r)[r + \Pi_j(x - e_j, t-1)] + F_j(2r)\Pi_j(x, t-1).$$

It is therefore better not to use NYOP at any time $t$. \qed
Proof of Proposition 7: For the centralized monopolistic supplier that provides NYOP service, the expected spot payoff is given by

$$\phi(p,r) = \begin{cases} \bar{F}_j(p)p, & \text{if } 2r > p \\ \bar{F}_j(p)\frac{p}{2} + \int_{2r}^{p} f_j(v)\frac{v}{2}dv, & \text{if } 2r \leq p. \end{cases}$$

For \( v_j \sim U[0,1] \),

$$\phi^* = \max_{p \geq r \geq 0} \phi(p,r) = \begin{cases} p - p^2, & \text{if } 2r > p \\ \frac{p}{2} - \frac{r^2}{4} - r^2, & \text{if } 2r \leq p. \end{cases}$$

At \( t = 1 \), if the centralized firm does not use NYOP channel, it will set the posted price at \( p^* = 0.5 \) with expected payoff \( 0.25 \); if it uses the NYOP channel, it sets the posted price at \( p^* = 1 \) and the reservation price at \( r = 0 \); hence, the expected payoff is again \( 0.25 \).

Denote \( \Phi(p,r;x,t) \) as the total profit of the monopolistic supplier and the intermediary firm, and let \( \Phi(x,t) \) be its maximizer over \( p \) and \( r \). Then, at \( t = 1 \) both single-channel and dual-channel strategy yield the same expected profit, \( \Phi(x,1) = \phi^* = 0.25 \).

For \( t > 1 \),

$$\Phi(x,t) = \max_{p \geq r \geq 0} \begin{cases} \bar{F}_j(p)p + \bar{F}_j(p)\Phi(x-e_j,t-1) + F_j(p)\Phi(x,t-1), & \text{if } 2r > p \\ \bar{F}_j(p)\frac{p}{2} + \int_{2r}^{p} f_j(v)\frac{v}{2}dv + \bar{F}_j(2r)\Phi(x-e_j,t-1) + F_j(2r)\Phi(x,t-1), & \text{if } 2r \leq p \end{cases}$$

$$= \max_{p \geq r \geq 0} \begin{cases} p - p^2 + (1-p)\Phi(x-e_j,t-1) + p\Phi(x,t-1), & \text{if } 2r > p \\ \frac{p}{2} - \frac{r^2}{4} - r^2 + (1-2r)\Phi(x-e_j,t-1) + 2r\Phi(x,t-1), & \text{if } 2r \leq p. \end{cases}$$

If the centralized firm decides to use single-channel strategies, it sets the posted price at \( p^* = \frac{1+\Phi(x,t-1)}{2} \) with expected payoff \( \Phi^S(x,t) = \Phi(x-e_j,t-1) + \left(\frac{1+\Phi(x,t-1)-\Phi(x-e_j,t-1)}{2}\right)^2 \); if it runs dual-channel strategy, it is optimal to set the posted price at \( p^* = 1 \) and the reservation price at \( r^* = \Phi(x,t-1) - \Phi(x-e_j,t-1) \), with expected payoff \( \Phi^D(x,t) = \Phi(x-e_j,t-1) + (\Phi(x,t-1) - \Phi(x-e_j,t-1))^2 + 0.25 \). It is easy to evaluate that

$$\Phi^S(x,t) - \Phi^D(x,t) = \frac{3}{4}(\Phi(x,t-1) - \Phi(x-e_j,t-1)) \left[ \frac{2}{3} - (\Phi(x,t-1) - \Phi(x-e_j,t-1)) \right]$$

$$\geq \frac{3}{4}(\Phi(x,t-1) - \Phi(x-e_j,t-1)) \left[ \frac{2}{3} - \phi^* \right]$$

$$= \frac{5}{16}(\Phi(x,t-1) - \Phi(x-e_j,t-1)) \geq 0;$$

hence, the centralized firm should always adopt a single-channel strategy without the NYOP channel. \( \square \)
Proof of Proposition 9: (7) implies that supplier $i$ would not accept a reservation price that is lower than $\Pi_i(x, t-1)$. On the other hand, a reservation price that is higher than the minimum posted price, $p$, will lead to $Pr\{v \in B_0(r, p, x)\} = 0$. To rule out the trivial cases, we let $r \leq p$.

Assume that the current reservation price satisfies $\max_{i=1,2} \Pi_i(x, t-1) \leq r \leq p = \min\{p_1, p_2\}$. We want to show that there is always an $\epsilon_0$ such that when $\epsilon \leq \epsilon_0$ at least one supplier is willing to accept a lower reservation price, $(r - \epsilon)$, if the other is not. Comparing the two terms in (7), it can be verified that for supplier 1 the gain from accepting $r - \epsilon$ is

\[
\left[ r - \epsilon - \Pi_1(x, t-1) \right] Pr\{v \in B_0(r, p, x)\} - [r - \epsilon + \Pi_1(x_1 - 1, x_2, t-1)] \Delta Pr\{v \in B_0(r, p, x)\} \\
- [p_1 + \Pi_1(x_1 - 1, x_2, t-1)] \Delta Pr\{v \in B_1(r, p, x)\} \\
- \Pi_1(x_1, x_2 - 1, t-1) \Delta Pr\{v \in B_2(r, p, x)\} - \Pi_1(x_1, x_2, t-1) \Delta Pr\{v \in B_N(r, p, x)\}
\]

(A1)

where $\Delta Pr\{v \in B_i(r, p, x)\} = Pr\{v \in B_i(r, p, x)\} - Pr\{v \in B_i(r - \epsilon, p, x)\}$ is positive for $i = 1, 2, N$, and negative for $i = 0$. As $\epsilon \to 0$,

\[
\lim_{\epsilon \to 0}(A1) = \lim_{\epsilon \to 0} \left\{ \left[ r - \epsilon - \Pi_1(x, t-1) \right] Pr\{v \in B_0(r, p, x)\} \\
- [r - \epsilon + \Pi_1(x_1 - 1, x_2, t-1)] \frac{\partial Pr\{v \in B_0(r, p, x)\}}{\partial r} \epsilon \\
- [p_1 + \Pi_1(x_1 - 1, x_2, t-1)] \frac{\partial Pr\{v \in B_1(r, p, x)\}}{\partial r} \epsilon \\
- \Pi_1(x_1, x_2 - 1, t-1) \frac{\partial Pr\{v \in B_2(r, p, x)\}}{\partial r} \epsilon - \Pi_1(x_1, x_2, t-1) \frac{\partial Pr\{v \in B_N(r, p, x)\}}{\partial r} \epsilon \right\} \\
= \left[ r - \Pi_1(x, t-1) \right] Pr\{v \in B_0(r, p, x)\} > 0.
\]

In fact, to guarantee the positivity of (A1), it is sufficient to have $\epsilon \leq \frac{[r - \Pi_1(x, t-1)]Pr\{v \in B_0(r, p, x)\}}{\max\{M, 0\}}$, where

\[
M = Pr\{v \in B_0(r, p, x) + [r + \Pi_1(x_1 - 1, x_2, t-1)] \frac{\partial Pr\{v \in B_0(r, p, x)\}}{\partial r} \\
+ \Pi_1(x_1, x_2 - 1, t-1) \frac{\partial Pr\{v \in B_2(r, p, x)\}}{\partial r} \\
+ \Pi_1(x_1, x_2, t-1) \frac{\partial Pr\{v \in B_N(r, p, x)\}}{\partial r},
\]

which implies that supplier 1 will accept the reservation price $r - \epsilon$ if supplier 2 will not. Thus, by properly designing $\epsilon$, the intermediary firm can at its best attain reservation price $\max_{i=1,2} \Pi_i(x, t-1)$ (if it is not exceeding $p$). If $\max_{i=1,2} \Pi_i(x, t-1) > p$, neither of the suppliers will sell through the intermediary firm, and we can simply let $r = p$.

At $t = 1$, $\tilde{\Pi}_1(x, t-1) = \tilde{\Pi}_1(x, 0) = \Pi_1(x_1, x_2 - 1, 0) - \Pi_1(x_1 - 1, x_2, 0) = 0$. Similarly, $\tilde{\Pi}_2(x, t-1) = 0$. Hence, we have $r^* = \min\{p, \max\{\tilde{\Pi}_1(x, t-1), \tilde{\Pi}_2(x, t-1)\}\} = 0$. $\square$

Proof of Theorem 2: (i). We need to prove that $\Pi_i$ is quasi-concave in $p_i$ for $i = 1, 2$. For uniform distribution, it is sufficient to prove concavity for any $v \in [0, 2]$, if $v$ follows the distribution
\( v_1 + v_2 = v \) and \( v_1 - v_2 \sim U[-\min\{v, 2-v\}, \min\{v, 2-v\}] \) (e.g., the two products are horizontally differentiated).

Let us first assume \( \iota = 1 \); hence supplier 2 is the opaque provider. Then,

\[
\Pi_1(x, t) = [p_1 + \Pi_1(x_1 - 1, x_2, t - 1)] Pr\{v \in B_1(r, p, x)\}
+ \Pi_1(x_1, x_2 - 1, t - 1) Pr\{v \in B_0(r, p, x)\}
+ \Pi_1(x_1, x_2 - 1, t - 1) Pr\{v \in B_2(r, p, x)\} + \Pi_1(x_1, x_2, t - 1) Pr\{v \in B_N(r, p, x)\}.
\]

- If \( p_1 \geq p_2 = p \), Proposition 5 implies that both \( B_2(r, p, x) \) and \( B_N(r, p, x) \) are unaffected by \( p_1 \), and \( \frac{\partial B_0(r, p, x)}{\partial p_1} = -\frac{\partial B_1(r, p, x)}{\partial p_1} > 0 \). So,

\[
\frac{\partial \Pi_1(x, t)}{\partial p_1} = Pr\{v \in B_1(r, p, x)\}
+ \left[\tilde{\Pi}_1(x_1, x_2, t - 1) - p_1\right] \frac{\partial Pr\{v \in B_0(r, p, x)\}}{\partial p_1} \tag{A2a}
\]

\[
\frac{\partial^2 \Pi_1(x, t)}{\partial p_1^2} = -2 \frac{\partial Pr\{v \in B_0(r, p, x)\}}{\partial p_1}
+ \left[\tilde{\Pi}_1(x_1, x_2, t - 1) - p_1\right] \frac{\partial^2 Pr\{v \in B_0(r, p, x)\}}{\partial p_1^2} \tag{A2b}
\]

- If \( p_1 < p_2 \), \( \frac{\partial Pr\{v \in B_0(r, p, x)\}}{\partial p_1} = \frac{\partial Pr\{v \in B_1(r, p, x)\}}{\partial p_1} + \frac{\partial Pr\{v \in B_2(r, p, x)\}}{\partial p_1} + \frac{\partial Pr\{v \in B_N(r, p, x)\}}{\partial p_1} \), thus

\[
\frac{\partial \Pi_1(x, t)}{\partial p_1} = Pr\{v \in B_1(r, p, x)\} - \left[\tilde{\Pi}_1(x_1, x_2, t - 1) - p_1\right] \frac{\partial Pr\{v \in B_1(r, p, x)\}}{\partial p_1}
- \left[\Pi_1(x_1, x_2 - 1, t - 1) - \Pi_1(x_1, x_2, t - 1)\right] \frac{\partial Pr\{v \in B_N(r, p, x)\}}{\partial p_1} \tag{A3a}
\]

\[
\frac{\partial^2 \Pi_1(x, t)}{\partial p_1^2} = 2 \frac{\partial Pr\{v \in B_1(r, p, x)\}}{\partial p_1} - \left[\tilde{\Pi}_1(x_1, x_2, t - 1) - p_1\right] \frac{\partial^2 Pr\{v \in B_1(r, p, x)\}}{\partial p_1^2}
- \left[\Pi_1(x_1, x_2 - 1, t - 1) - \Pi_1(x_1, x_2, t - 1)\right] \frac{\partial^2 Pr\{v \in B_N(r, p, x)\}}{\partial p_1^2}. \tag{A3b}
\]

Proposition 5 implies that, given \( v_1 + v_2 = v \) for some particular \( v \), exactly one of \( B_0 \) or \( B_N \) is 0.

- \( Pr\{v \in B_N(r, p, x)\} = 0 \).

For \( p_1 \geq p_2 = p \), \( Pr\{v \in B_1(r, p, x)\} = (v - 2p_1 + 4r + \frac{r^2}{p_1 - r})^+/v \), and \( Pr\{v \in B_0(r, p, x)\} = 2(p_1 + p_2 - 4r - \frac{r^2}{p_1 - r})^+/v \). It can be verified that \( \frac{\partial^2 Pr\{v \in B_0(r, p, x)\}}{\partial p_1^2} = 0 \), hence \( \frac{\partial^2 \Pi_1(x, t)}{\partial p_1^2} \leq 0 \). By (A2), the payoff function is concave.

For \( p_1 < p_2 \), we have \( Pr\{v \in B_1(r, p, x)\} = (v - 2p_1 + 4r + \frac{r^2}{p_1 - r})^+/v \), \( \frac{\partial Pr\{v \in B_1(r, p, x)\}}{\partial p_1} = \frac{2r^2}{v(p_1 - r)^2} \) > 0. Also, notice that as \( \iota = 1 \), there
should be \( r = \overline{\Pi}_1(x_1, x_2, t - 1) \geq \overline{\Pi}_2(x_1, x_2, t - 1) \). By (A3),
\[
\frac{\partial^2 \Pi_1(x, t)}{\partial p_1^2} = 2 \frac{\partial Pr\{v \in B_1(r, p, x)\}}{\partial p_1} + (p_1 - r) \frac{\partial^2 Pr\{v \in B_1(r, p, x)\}}{\partial p_1^2}
\]
\[
= \left[-4 - \frac{2r^2}{(p - r)^2} - \frac{2r^2(p - r)}{(p - r)^3}\right] / v
\]
\[
= -4/v \leq 0,
\]
so the profit function is again concave.

• \( Pr\{v \in B_0(r, p, x)\} = 0. \)

For \( p_1 \geq p_2 = p \), \( \frac{\partial^2 \Pi_1(x, t)}{\partial p_1^2} = 0 \), so the payoff function is concave by (A2).

For \( p_1 < p_2 \), \( Pr\{v \in B_1(r, p, x)\} = (v - p_1^+) / v \) and \( Pr\{v \in B_N(r, p, x)\} = v^{-1} \min\{v, p_2, p_1 + p_2 - v\} \).
Thus, \( \frac{\partial Pr\{v \in B_1(r, p, x)\}}{\partial p_1} = -1/v \leq 0 \) and \( \frac{\partial^2 Pr\{v \in B_N(r, p, x)\}}{\partial p_1^2} = 0 \). By (A3), \( \frac{\partial^2 \Pi_1(x, t)}{\partial p_1^2} \leq 0 \), and the payoff function is concave.

The case \( i = 2 \) can be proved similarly.

(ii). We can show that for any \( t \) there exits a \( \pi_t \) such that for any \( x_1 \geq t \) and \( x_2 \geq t \),
\[
\Pi_1(x_1, x_2, t) = \Pi_2(x_1, x_2, t) = \pi_t,
\]
which immediately leads to \( r^* = 0 \) for any \( x_1 \geq t \) and \( x_2 \geq t \). We show (A4) by induction. First, it is easy to see that (A4) holds for \( t = 0 \). Now, suppose (A4) holds for all \( t < T \). We then have
\[
\Pi_1(x_1 - 1, x_2, T - 1) = \Pi_1(x_1 - 1, x_2 - 1, T - 1) = \Pi_2(x_1, x_2 - 1, T - 1) = \Pi_2(x_1 - 1, x_2 - 1, T - 1)
\]
for any \( x_1 \geq T \) and \( x_2 \geq T \). Hence, \( \overline{\Pi}_i(x_1, x_2, T - 1) = 0 \) for \( i = 1, 2 \) and
\[
r^*(x_1, x_2, T) = 0, \quad \forall x_1 \geq T, x_2 \geq T.
\]
(A5) implies that when both suppliers oversupply, the price of the opaque goods will remain zero until one supplier’s inventory becomes lower than the potential demand. The expected profit function for supplier 1 is then
\[
\Pi_1(x, T) = [p_1 + \Pi_1(x_1 - 1, x_2, T - 1)] Pr\{v \in B_1(0, p, x)\} + \Pi_1(x - 1, x_2 - 1, T - 1) Pr\{v \in B_0(0, p, x)\}
\]
\[
+ \Pi_1(x_1 - 1, x_2 - 1, T - 1) Pr\{v \in B_2(0, p, x)\} + \Pi_1(x_1, x_2, T - 1) Pr\{v \in B_N(0, p, x)\}
\]
\[
= p_1 Pr\{v \in B_1(0, p, x)\} + \pi_{T-1},
\]
which does not depend on \( x \) for \( x_1 \geq T \) and \( x_2 \geq T \). Similarly, \( \Pi_2(x, T) = p_2 Pr\{v \in B_2(0, p, x)\} + \pi_{T-1} \). When \( r = 0 \), the expressions for \( B_1(0, p, x) \) and \( B_2(0, p, x) \) are the same. It is then straightforward that in equilibrium there should be \( p_1 = p_2 \) and \( \Pi_1(x, T) = \Pi_2(x, T) \). Thus, (A4) also holds for \( t = T \). This completes the proof. \( \square \)