Allocation of Greenhouse Gas Emissions in Supply Chains

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Abstract

Globalization, which exports production and jobs from rich countries to poor countries, also exports from rich countries to poor countries the greenhouse gas (GHG) emissions created from the production of the goods consumed by rich countries. But whose responsibility are the GHG emissions? Are they exclusively the responsibility of the producing countries, or exclusively the responsibility of the consuming countries? Or, perhaps, the responsibility for the GHG emissions should be shared by both the producers and the consumers? It is suggested in the literature that some level of double counting of carbon footprinting should be allowed in order to induce optimal effort levels for emission reduction by supply chain members. However, such double counting may reduce the credibility of carbon-offsetting initiatives. In this paper we formulate the GHG emission responsibility (GGER) problem as a cooperative game, referred to as the GGER game, and use cooperative game theory methodology to suggest allocations of GHG responsibility among the various parties in the supply chain. We prove that the GGER game is convex, and thus has a non-empty core, and we identify some easily implementable allocations as extreme core points. We further derive an explicit expression for the Shapley value of the GGER game, which is shown to have a very simple and intuitive interpretation. A Shapley value allocation of the GHG emission responsibilities induces supply chain members to invest in GHG emission reduction, and avoids the potential problem of double counting in supply chain carbon footprinting.
1. Introduction and Literature Review

In today’s global environment, both consumers and manufacturers are more conscious of their environmental footprint. According to Davenport (2013), some corporations are planning their future growth with the expectation that they will have to pay a price for carbon pollution. At the same time, as manufacturing is moving to less developed countries, it is not clear how should the responsibility for environmental pollution be allocated among different supply chain members. For instance, according to Porter (2013), about a fifth of China’s emissions are for products consumed outside its borders, and while Europe emitted only 3.6 billion metric tons of CO\textsubscript{2} in 2011, 4.8 billion tons of CO\textsubscript{2} were created to make the products Europeans consumed in that year. Davis and Caldeira (2010) find that in 2004, more than 30\% of consumption-based carbon emissions in some wealthy countries (Switzerland, Sweden, Austria, the United Kingdom, France) were imported, and suggest that sharing responsibility for emissions among producers and consumers could facilitate an international agreement on global climate policy. Lin et al. (2014) argue that US outsourcing of manufacturing to China might have reduced air quality in the western United States and led to an improvement in the east, due to the effects of changes in US emissions and atmospheric transport of Chinese pollution. Thus, providing appropriate incentives to Chinese manufacturers to reduce GHG emissions can impact US air quality as well.

Governments in many countries are either considering or already implementing mechanisms aimed towards curbing GHG emissions. The European Union Emissions Trading Scheme (EU ETS, http://ec.europa.eu/clima/policies/ets/index_en.htm) has started in 2005 as the first large GHG emissions trading scheme in the world. In 2014, the EU ETS covers more than 11,000 power stations and industrial plants in 31 countries, as well as airlines. The governments of the EU Member States agree on national emission caps which have to be approved by the EU commission. The countries then allocate allowances to their industrial operators, and track and validate the actual emissions compared to the relevant assigned allowances. According to the EU ETS website,

“After each year a company must surrender enough allowances to cover all its emissions, otherwise heavy fines are imposed. If a company reduces its emissions,
it can keep the spare allowances to cover its future needs or else sell them to another company that is short of allowances... By putting a price on carbon and thereby giving a financial value to each tonne of emissions saved, the EU ETS has placed climate change on the agenda of company boards and their financial departments across Europe.”

A recent World Bank report, “State and Trends on Carbon Pricing” (2014), states that 39 national and 23 sub-national jurisdictions (responsible for almost a quarter of the global GHG emissions) have implemented or are scheduled to implement carbon pricing instruments. These instruments include both emissions trading schemes (e.g., China now houses the second largest carbon market in the world; California’s cap-and-trade program was launched in December 2012) and taxes (new carbon taxes were introduced in Mexico and France in 2013). Paulson (2014) argues that the US should show its world leadership by introducing a tax on carbon emissions and by collaborating with China to combine innovation in developing and rolling out new, cleaner technologies. It is important to note that, regardless of the carbon pricing instrument implemented (cap-and-trade vs. taxes), proper evaluation of the GHG emissions attributed to a specific firm and/or country seems to be getting an increased importance in global economies.

It is also worth noting that, while different supply chain members can invest in reduction of their own carbon footprint, they may be also be convinced to assist in the reduction of GHG emissions by other members of their supply chain. Thus, it is important to provide appropriate incentives to supply chain collaborators in order to reduce overall GHG emissions. Identification of an appropriate scheme for the allocation of GHG emissions falls into this category. However, this topic has received limited attention in the supply chain literature so far. In the area of logistics, Cholette and Venkat (2009) study the impact of transportation and storage choice on carbon emissions in wine distribution, while Hoen et al. (2012) analyze the effect of environmental legislation on transportation choices in supply chains. Cachon (2013) studies the effect of retail store density on GHG gas emissions. Plambeck (2012, 2013) studies some challenges facing firms that try to reduce their GHG emissions and suggests some ways for dealing with them, while Sunar and Plambeck (2013) propose an allocation method of carbon emissions among co-products. Benjaafar et al. (2013) study simple supply
chain models and their extensions by the incorporation of carbon footprint parameters. Corbett and DeCroix (2001) and Corbett et al. (2005) study shared-savings contracts and their impact on the environment. Caro et al. (2013) study allocations of GHG emissions among supply chain members under two scenarios: in the first one, the social planner allocates the emissions and imposes a cost on them, while in the second one, a carbon leader offsets all supply chain emissions and contracts with individual firms to recover these costs.

It was suggested by Caro et al. (2013) that supply chains should allow for some level of double counting of carbon emissions in order to induce optimal effort levels for their reduction in supply chains. However, double counting has its negative implications as well. First, it may be more difficult to estimate the actual GHG emissions of a given supply chain (or that of a nation, or even worldwide). Moreover, double counting may have an adverse effect on carbon-offsetting, the relatively popular concept of compensating GHG emissions generated by one entity by reducing or preventing GHG emissions by another entity (carbon credits). Ideally, carbon offsets should be accounted for only once, but this is often not the case in real life. The various organizations that act as intermediaries between the two sides (the buyers and the sellers of carbon credits) may benefit from inflating carbon emission numbers (e.g., through double counting), reducing the credibility of carbon-offsetting initiatives. Hence, it is important to develop an appropriate method for allocating GHG emission responsibilities that avoids double counting.

Indeed, we propose and study in this paper a game theoretic model for the allocation of GHG emission responsibilities among all supply chain members involved in the production or consumption of a final good, and propose a simple allocation method that has the following properties:

**Property 1:** It allocates responsibilities for all GHG emitted by the supply chain members and avoids double counting.

**Property 2:** It is easy to implement in practice.

**Property 3:** No firm (or set of firms) is allocated a responsibility for GHG emissions that is larger than the GHG emissions for which they are directly or indirectly responsible for.
Property 4: Each firm is incentivized to reduce its GHG emissions and help in reducing GHG emissions of its supply chain partners.

In our analysis, we use tools from game theory and model the GHG emission as a co-operative game, to be referred to as GHG emission responsibility (GGER) game, in which the players correspond to the various suppliers and end consumers in the supply chain. We start by considering a simple case in which the supply chain can be modeled as a directed tree, with the leaves of the tree representing, e.g., material or energy suppliers, and the most downstream node being the end consumer.

For the directed tree graph case we identify a set of GHG allocations that belong to the core of the game, and thus satisfy Properties 1 and 3 above, and we further show that some simple “all-or-nothing”-type allocation rules can be represented as the extreme points of the core. However, while they are easy to implement, these rules do not satisfy Property 4 due to their extreme nature. We further demonstrate that the Shapley value of the GGER game belongs to the core and allocates equal shares of the GHG emission generated directly by each firm among all supply chain members downstream from it. As a result, the Shapley value satisfies Properties 1, 2 and 3. Moreover, the Shapley value of the GGER game provides the right incentives to the supply chain members. Specifically, the Shapley value allocation of the GHG emissions incentivizes supply chain members to invest in the reduction of their carbon footprint, and encourages them to collaborate with upstream supply chain partners to reduce their GHG emissions, and thus it satisfies Property 4 as well.

Gallego and Lenzen (2005) observe that in many real-life models outputs of one supply chain member can be inputs for both upstream and downstream supply chain partners (e.g., a nuclear power plant using fuel elements and providing electrical power to the fuel elements’ manufacturer), or that one supplier can deliver directly to its non-immediate downstream consumers (who are themselves producers—e.g., a power plant can deliver electricity to a steel manufacturer and an upholstery manufacturer). This creates feedback loops and can lead to double counting of emissions. In order to address this problem, we extend the GGER game model by considering tree graphs with feedback and direct non-intermediate downstream consumers. We provide appropriate modifications and demonstrate that, e.g., the Shapley value of the extended GGER game still satisfies Properties 1-4 identified above.
Finally, we consider a more general supply chain structure and show that we can still preserve our results.

Our basic GGER game model (defined on a directed tree) resembles Megiddo’s (1978) tree model, which is an extension of the classical airport game model (Littlechild and Owen, 1973.) In both models, as in the GGER game model, the players are represented by nodes in a directed tree (a directed path in the airport game). However, by contrast with the GGER model, players in these models need to connect to the root of the tree, and each player in these games is responsible for the cost of the directed path from this player to the root of the directed tree. By contrast, in the GGER game defined on a directed tree, each player is “responsible” for the cost (i.e., GHG pollution) created by the producers/suppliers in the player’s supply chain.

The plan of the paper is as follows. We first introduce our basic model in Section 2, and then describe the GHG emission responsibility (GGER) game in Section 3. Sections 4 and 5 describe our extensions: tree graphs with feedback and direct non-intermediate downstream consumers, and more general supply chains. We provide some concluding remarks in Section 6.

2. The Basic GGER Game Model and Some Definitions and Notation

Consider a supply chain consisting of several entities, such as suppliers, manufacturers, assemblers, etc. (hereafter referred to as players), which are cooperating in the production of a final product. The supply chain is represented by a directed graph, which initially will be assumed to be a directed tree $T$. The nodes of $T$, aside for the root node, represent the players, and the (directed) arc emanating from each node $i$ towards the root of the tree, denoted as node 0, represents the GHG emitted by player $i$ as a result of all her activities which can be solely attributed to the production of the final product. We assume that only one arc enters node 0, and this arc emanates from node 1, which can represent the end consumer or the most downstream manufacturer/assembler. On the other hand, the leaf nodes of the directed tree represent the most upstream suppliers in the supply chain. We
can also view node 1 as the consuming country of the final product being manufactured by
the supply chain, and all other nodes as the producing countries/regions.

Figure 1 below represents a supply chain, where node 0 therein represents the root of the
tree and all other nodes represent players. The numbers (weights) along the arcs (in italics)
represent the GHGs emitted by the players in order to produce the final product. For a node
\( i \in T \), we denote by \( e_i \) the directed arc which emanates from node \( i \) towards node 0 in \( T \),
and by \( a_i \) the weight of arc \( e_i \) in \( T \); that is, \( a_i \) is the pollution created directly by player \( i \).

![Figure 1: Supply chain with tree structure](image)

The total GHG emission in the process of producing the final product is the total sum
of the arc weights, and we consider here the issue of how to allocate the responsibility of
the total GHG emission among the players. Two obvious and somewhat extreme allocation
methods are as follows:

1. **Full Producer Responsibility**—Each member in the supply chain is responsible
for the emission it directly creates. According to this method, the consuming country bears no responsibility whatsoever for pollution created by the producing countries in the manufacturing process of the final product.

2. **Full Consumer Responsibility**—The most downstream manufacturer is responsible for all the pollution created by the supply chain. By contrast with the previous method, this allocation method is consistent with the scenario in which the GHG emission due to production in poor countries to satisfy demand in a rich consuming country remains the sole responsibility of the consuming country.

Both methods might be used in practice (e.g., in product labeling or in national GHG accounting; see Munksgaard and Pedersen, 2001).

In this paper we consider more formally responsibility allocation methods of the GHG emission among the parties involved. For that purpose we formulate the problem as a cooperative game in which the set of players, $N$, consists of all members of the supply chain (i.e., all nodes in the tree $T$ representing the supply chain). For each player $i$, let $c\{i\}$ represent the emission that this player is responsible for. We choose $c\{i\}$ to equal the emission created directly by player $i$ as well as the total emission created by all players supplying, directly or indirectly, to player $i$. For a given node $\ell$, let $T^{\ell} = (V(T^{\ell}), E(T^{\ell}))$ denote the subtree rooted at $\ell$, with node set $V(T^{\ell})$ and arc set $E(T^{\ell})$. Then,

$$c\{i\} = a(E(T^{i})) + a_i = \sum_{j \in T^i} a_j + a_i$$

where $a(E(T^{i}))$ denotes the sum of all arc pollutions in $E(T^{i})$.

For a subset of players $S$, we refer to player $i \in S$ as *maximal* in $S$ (with respect to the tree $T$) if there is no other player $j$ in $S$ which is on the directed path in $T$ from node $i$ to node 0. We denote the set of all maximal players in $S$ by $S^{\text{max}}$. For $S = \{2, 3, 5, 6, 8\}$ in Figure 1, we have $S^{\text{max}} = \{2, 3\}$. Let

$$c(S) = \sum_{i \in S^{\text{max}}} (a(E(T^{i})) + a_i).$$

Thus, $c(\{2, 3, 5, 6, 8\}) = c(\{2\}) + c(\{3\}) = (1 + 3 + 8) + (6 + 4 + 3) + 4 + 3 = 32$.

Note that for $Q \subset S$ and $j \in Q^{\text{max}}$ there is an $i \in S^{\text{max}}$, which possibly coincides with $j$, and which is on the path from node $j$ to node 0 in $T$. 

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We refer to \((N,c)\) as the GHG emission responsibility (GGER) game associated with the supply chain, where \(N\) is the set of players represented by the nodes of the tree \(T\), aside for the root node 0, and the characteristic function, \(c(S)\), is as defined above for all \(S \subseteq N\). In reference to Figure 1, we have, for example, that \(c(\{2\}) = 1 + 3 + 8 + 4 = 16\) and \(c(\{3, 4, 5\}) = (3 + 4 + 6 + 3) + 1 + 3 = 20\). We will employ in this paper solution concepts from cooperative game theory, such as the core and the Shapley value, as possible fair allocation schemes for the GGER game.

### 2.1 Game-theoretical concepts

The core, \(C((N,c))\), of a game, \((N,c)\), consists of all vectors \(x = (x_1, x_2, \ldots, x_n)\) which allocate the total cost, \(c(N)\), among all players in \(N\), such that no subset of players, \(S\), is allocated more than the cost, \(c(S)\), “associated” with it. That is, \(C((N,c)) = \{x \in \mathbb{R}^n : x(S) \leq c(S), \forall S \subset N, x(N) = c(N)\}\), where \(x(S) = \sum_{j \in S} x_j\). The core of a game could be empty, and if non-empty, it usually does not consist of a unique allocation vector. However, all allocations in the core are stable in the sense that no subset of players can be better off by seceding from the rest of the players and act on their own. Note that in the GGER game, all core allocations satisfy Properties 1 and 3.

Shapley value (Shapley, 1953), \(\Phi(c)\), of a cooperative game, \((N,c)\), is the unique allocation which satisfies the following axioms:

1. **Symmetry:** If players \(i\) and \(j\) are such that for each coalition \(S\) not containing \(i\) and \(j\),
   \[c(S \cup \{i\}) - c(S) = c(S \cup \{j\}) - c(S),\]
   then \(\Phi_i(c) = \Phi_j(c)\).

2. **Null Player:** If \(i\) is a null player, i.e., \(c(S \cup \{i\}) = c(S)\) for all \(S \subset N\), then \(\Phi_i(c) = 0\).

3. **Efficiency:** \(\sum_N \Phi_i(c) = c(N)\).

4. **Additivity:** \(\Phi(c_1 + c_2) = \Phi(c_1) + \Phi(c_2)\) for any pair of cooperative games \((N,c_1)\) and \((N,c_2)\).

An interpretation of Shapley value is given as follows. Consider all possible orderings of the players, and define a marginal contribution of player \(i\) with respect to a given ordering as his marginal worth to the coalition formed by the players before him in the order,
\[ c(\{1, 2, \ldots, i-1, i\}) - c(\{1, 2, \ldots, i-1\}) \]
where 1, 2, \ldots, i − 1 are the players preceding \( i \) in the given ordering. Shapley value is obtained by averaging the marginal contributions for all possible orderings. This average is given by

\[
\Phi_i(c) = \sum_{\{S:i \in S\}} \frac{(|S| - 1)!(n - |S|)!}{n!} (c(S) - c(S \setminus \{i\})).
\] (1)

It was shown by Shapley (1953) that (1) is the unique allocation rule which satisfies the above four axioms. A nice feature of the Shapley value stemming from this interpretation is that it takes into account players’ marginal contributions when determining their share of cost, thus providing a direct link between a player’s contribution and her corresponding allocation. As a result, the Shapley value might be perceived as more “justifiable” compared to some other allocation methods. Indeed, it was extensively considered as an allocation method in a variety of problems arising, e.g., in economics, management, and cost accounting. For example, it was considered as an allocation method of pollution reduction costs (Petrosjan and Zaccour, 2003), for generating airport landing fees (Littlechild and Owen, 1973), for allocation of transmission costs (Tan and Lie, 2002), for economic distributional analysis (Shorrocks, 2013), and in the non-atomic game formulation framework, e.g., to generate internal telephone billing rates (Billera et al., 1978). By contrast with the core, Shapley value always exists and it is unique, but it is not necessarily contained in the core.

The characteristic function of a game \((N, c)\) is said to be convex if \(c(S \cup \{i\}) - c(S) \leq c(R \cup \{i\}) - c(R)\) for all \(i \notin S\) and \(R \subseteq S \subseteq N\). The game \((N, c)\) is said to be convex if its characteristic function, \(c\), is convex. The core of a convex game is non-empty and its Shapley value is contained in the core (Shapley, 1971).

3. The GHG Emission Responsibility (GGER) Game

In this section we prove that the GGER game is convex, and thus has a non-empty core. We generate some interesting core allocation vectors of this game, and, in particular, we provide an explicit and intuitive characterization of the Shapley value of this game that can be generated very efficiently. Thus, since the Shapley value of convex games is contained in the core of the game, we are able to conclude that the Shapley value of the GGER game satisfies Properties 1-4.
We refer to a set $S$ which contains all players in $\cup (V(T^i) : i \in S^{\text{max}})$ as a complete set. Note that if $S$ is complete and $S^{\text{max}} \subset Q \subset S$, then, by definition of $c$, $c(S) = c(Q)$. Further, note that by definition of $c$, the GGER game is monotone. That is, for each $Q \subset S \subset N$, $c(Q) \leq c(S)$. It can be easily shown that the monotonicity of $(N,c)$ implies that any vector in the core of the GGER game, $\mathcal{C}((N,c))$, consists only of non-negative allocations, and, moreover, $\mathcal{C}((N,c))$ has a more compact representation as, $\mathcal{C}((N,c)) = \{x \in \mathbb{R}^n : x(S) \leq c(S) \text{ for all complete sets } S \subset N, x(N) = c(N)\}$.

For $i \in N$ and $S \subseteq N$, let
\[
a(E(T^i_S)) = \sum_{\ell \in V(T^i) \cap S^{\text{max}}} (a(E(T^\ell)) + a) ;
\]
that is, $a(E(T^i_S))$ is the pollution generated within the tree rooted in $i$ by all trees rooted in maximal nodes of $S$. By definition of $c$, we then have that for $S \subset N$ and $i \in N \setminus S$,
\[
c(S \cup \{i\}) - c(S) = \begin{cases} 
0, & \text{if } i \not\in (S \cup \{i\})^{\text{max}}, \\
(c\{i\}) - a(E(T^i_S)), & \text{otherwise}.
\end{cases}
\]
Clearly, for $Q \subseteq S$, it holds that
\[i \in (S \cup \{i\})^{\text{max}} \implies i \in (Q \cup \{i\})^{\text{max}},\]
and for $i \in (S \cup \{i\})^{\text{max}}$ we have
\[
c(S \cup \{i\}) - c(S) = c\{i\} - a(E(T^i_S))
\leq c\{i\} - a(E(T^i_Q)) = c(Q \cup \{i\}) - c(Q).
\]
Observe also that we might have $(Q \cup \{i\})^{\text{max}} \setminus (S \cup \{i\})^{\text{max}} \neq \emptyset$. Therefore,
\[
c(Q \cup \{i\}) - c(Q) - (c(S \cup \{i\}) - c(S)) = \begin{cases} 
0, & \text{if } i \not\in (Q \cup \{i\})^{\text{max}}, \\
c\{i\} - a(E(T^i_Q)), & i \in (Q \cup \{i\})^{\text{max}} \setminus (S \cup \{i\})^{\text{max}}, \\
a(E(T^i_S)) - a(E(T^i_Q)), & i \in (S \cup \{i\})^{\text{max}},
\end{cases}
\]
and we have the following result.

**Proposition 1** The GGER game $(N,c)$ is convex. That is, $c(S \cup \{i\}) - c(S) \leq c(R \cup \{i\}) - c(R)$ for all $R \subseteq S \subseteq N$ and $i \in N$.  


As previously mentioned, the convexity of the GGER game has some profound game theoretical implications. For example, it implies that the core is not empty and that the Shapley value is contained in the core. In fact, the Shapley value is the barycenter of the core (Shapley, 1971.)

Let us examine some core allocation vectors. In particular, consider the extreme points of the core. Let \( \pi = \{j_1, j_2, \ldots, j_n\} \) be an arbitrary permutation of the players in \( N \). Then, for a convex game, \( (N, c), x_\pi \equiv (x_{j_1} = c(\{j_1\}) - c(\emptyset), x_{j_2} = c(\{j_1, j_2\}) - c(\{j_1\}), x_{j_3} = c(\{j_1, j_2, j_3\}) - c(\{j_1, j_2\}), \ldots, x_{j_n} = c(\{j_1, j_2, \ldots, j_n\}) - c(\{j_1, j_2, \ldots, j_{n-1}\})) \) is an extreme point of the core. In fact, for convex games, \( x \) is an extreme point of the core if and only if \( x = x_\pi \) for some permutation \( \pi \) of the players in \( N \) (Shapley, 1971).

Let \( \pi^1 = \{j_1, j_2, \ldots, j_n\} \) be a permutation of the players in \( N \), such that if \( j_u < j_v \), then \( j_u \) is not on the path from \( j_v \) to node 0 in \( T \). Then, the extreme point of the core induced by \( \pi^1 \) corresponds to the full producer responsibility (that is, each member in the supply chain is responsible for the emission it directly created). Similarly, let \( \pi^2 = \{j_1, j_2, \ldots, j_n\} \) be any permutation of the players according to which \( j_1 = 1 \). That is, the first player in the permutation is the unique maximal player in \( N \). Then, the extreme point of the core induced by \( \pi^2 \) corresponds to the full consumer responsibility (that is, the most downstream manufacturer is responsible for all the pollution created by the supply chain). Thus, both allocations previously discussed are core allocations, and are stable in the sense that no subset of players can claim that they are allocated a GHG emission responsibility which strictly exceeds the emission that they directly or indirectly created. However, both these allocations are extreme, in the sense that they allocate all or nothing. Indeed, they are extreme points of the core.

For an arc \( e_j \) emanating from node \( j \) in \( T \), with an associated GHG emission weight \( a_j \), we denote by \( N^j \) the set of nodes (i.e., players) on the path from node \( j \) to node 0 in \( T \), including nodes \( j \) and 1. Thus, \( N^j \) consists of all players who directly or indirectly are responsible for \( a_j \). The next proposition demonstrates that any convex allocation of the responsibility for \( a_j \) among members of \( N^j \), for all \( j \in N \), is in the core of the GGER game \( (N, c) \).

**Proposition 2** The allocation vector \( y \) derived by allocating each \( a_j \), in a convex combina-
tion manner, among members in $N^j$, $j = 1, 2, \ldots, n$, is contained in $C((N, c))$.

Proof: By definition, $y(N) \equiv \sum_{j \in N} y_j = c(N)$. Let $S$ be an arbitrary subset of $N$. According to $y$, members in $S$ are allocated responsibilities only for (part) of the emissions that they either directly or indirectly created. Thus, $y(S) \equiv \sum_{j \in S} y_j \leq c(S)$.

Thus, it follows from Proposition 2 that there are many ways to allocate responsibilities for GHG emissions among the members in the supply chain in a manner that no subset of players is allocated more than it either directly or indirectly created. Which of these allocations should be recommended? In Theorem 1 below we prove that a core allocation which, for each arc pollution $a_j$, assigns the responsibility equally among all players that directly or indirectly created it is the Shapley value.

Theorem 1 The allocation according to which $a_j$ is allocated equally among members in $N^j$ for each $j \in N$ is the Shapley value of $(N, c)$.

Proof: For each $j \in N$ and $S \subseteq N$, let $(N, c^j)$ denote the game where

$$c^j(S) = \begin{cases} 0, & \text{if } S \cap N^j = \emptyset, \\ a_j, & \text{otherwise}. \end{cases}$$

One can easily verify that for each $S \subseteq N$, $c(S) = \sum_{j \in N} c^j(S)$. By its symmetry property, the Shapley value for the game $(N, c^j)$ is

$$\Phi_i = \begin{cases} \frac{a_j}{|N^j|}, & \text{if } i \in N^j, \\ 0, & \text{otherwise}, \end{cases}$$

and by its additivity property, $\Phi(c) = \sum_{i \in N} \Phi (c^i)$. Thus, the Shapley value of the GGER game $(N, c)$ allocates the cost of the pollution $a_j$ equally among all players who are directly or indirectly responsible for its creation.

This result has several nice practical implications. First, we have found an allocation method that is easy to implement, as pollution is equally allocated among all supply chain members who are directly or indirectly responsible for it, and thus satisfies Property 2. Second, the allocation belongs to the core, so no subset of supply chain members is allocated more than they directly or indirectly created (Property 3). Third, it allocates responsibilities for all GHG emitted by the supply chain members and avoids double counting (Property
1), and lastly, since pollution is allocated equally among those who are directly or indirectly responsible for it, the Shapley value allocation creates an incentive for all supply chain members to work towards reducing GHG emissions across the supply chain (Property 4).

Gallego and Lenzen (2005) have extended the traditional model with final-demand-driven industry by adding intermediate demand, and are primarily concerned with allocations of GHG emission responsibilities which satisfy Property 1. Their non game theoretic model has some nice features in common with our model. Specifically, they suggest that GHG emission responsibilities should be shared among all supply chain members who have directly or indirectly created these emissions. Thus, by Proposition 2, it follows that their allocations belong to the core of our GGER game, and thus satisfy Property 3. Note, however, that according to the Shapley value allocation for GGER games, emissions from a certain supply chain member, say \( i \), are equally shared among all supply chain members that use \( i \)'s output (that is, nodes on the path from \( i \) to the root node). By contrast, Gallego and Lenzen (2005) suggest that the parties further away from the source have to bear a proportionally smaller share of responsibility for emissions. Moreover, Gallego and Lenzen's allocations might be more complex to implement and are a bit arbitrary. For instance, consider a simple supply chain consisting of an electric power plant supplying electricity to a steel manufacturer, who supplies steel to a car manufacturer, who sells the product to the end consumer. Thus, there are four supply chain members who form a directed path supply chain from the power plant through the steel manufacturer to the car manufacturer to the end consumer, ending with the root node 0. Suppose that the GHG emission from the power plant is \( X \). The Shapley value will allocate one quarter of the emissions, \( X/4 \), to each of the four supply chain members. However, in the Gallego and Lenzen (2005), the power plant will be responsible for \((1 - \alpha)X \) fraction, while the rest, \( \alpha X \), will be shared downstream; the steel manufacturer will be responsible for \( \alpha(1 - \alpha)X \), while \( \alpha^2X \) will be shared downstream; finally, the car manufacturer will be responsible for \( \alpha^2(1 - \beta)X \), while the end consumer will be responsible for \( \alpha^2\beta X \), where \( \alpha, \beta \in [0, 1] \). If we assume \( \alpha = 1, \beta = 1 \), this allocation corresponds to the full consumer responsibility model; if we assume \( \alpha = 0 \), this allocation corresponds to the full producer responsibility model; if we let \( \alpha = \beta = 0.5 \), the power plant is responsible for half of the emissions, the steel manufacturer for a quarter, while the car manufacturer and the
end consumer are responsible for $1/8$ each. As mentioned above, for arbitrary non-negative $\alpha$ and $\beta$, Gallego and Lenzen (2005) allocation vector can be viewed as a core allocation in our GGER game. However, note further that there do not exist non-negative $\alpha$ and $\beta$ for which this vector can be completed to the Shapley value of this game.

4. Extensions to Tree Structures with Feedback and Direct Non-Intermediate Downstream Consumers

It was so far assumed that the structure of the supply chain is a directed tree. That is, we assumed that each player supplies only one other player, his immediate predecessor in the supply chain tree $T = (V(T), E(T))$. However, as mentioned earlier, it is possible that a producer in the supply chain supplies several players in the process of producing the final product (see, e.g., examples in Gallego and Lenzen, 2005; Lenzen, 2009). Accordingly, we extend the analysis in this section to supply chain structures represented by a directed graph $G = (V(T), E(G))$ which is not necessarily a directed tree. Specifically, we first consider supply chain structures where, aside for the underlying directed tree structure $T$, there exist other arcs in the supply chain of two possible forms:

1. **Feedback arcs**—arcs from nodes $i$ to nodes $j$, such that $i$ is on the unique path in $T$ from node $j$ to node 0;

2. **Direct arcs to non-intermediate downstream consumers**—arcs from leaf nodes in $T$ to other nodes in $T$ except for node 0, as long as there are no other arcs entering these leaf nodes.

Note that $E(G) = E(T) \cup A$, where $A$ is the set of feedback arcs and direct arcs to non-intermediate downstream consumers as described above. Figure 2 below describes an example of a graph $G$ which models the tree structure with feedback arcs and direct arcs to non-intermediate downstream consumers. In Figure 2, the set $A$ of additional arcs is $A = \{(9,4), (2,7), (1,10), (3,15)\}$, shown in broken line, and the set of leaf nodes is $\{8,9,12,13,14,15\}$. Player 9 has one non-intermediate downstream consumer, player 4, hence $(9,4)$ is a direct arc to a non-intermediate downstream consumer, while arcs $(1,10)$,
(2,7) and (3,15) represent the feedback arcs.

Figure 2: Supply chain with feedback and direct non-intermediate downstream consumer arcs

For each node $i \in T$ we denote by $R(i)$ the set all arcs in $A$ which enter node $i$,

$$R(i) = \{(j, i) : (j, i) \in A\}.$$ 

Thus, $R(4) = \{(9, 4)\}$, $R(7) = \{(2, 7)\}$, $R(10) = \{(1, 10)\}$, $R(15) = \{(3, 15)\}$. Note that player $i$ may now have more than one predecessor in $G$. Indeed, a leaf node $i$ in $G$ may have, in addition to its predecessor in $T$, another predecessor node which is on the path from $i$ to node 0 in $G$.

Denote by $(N, c_F)$ the GGER game associated with the supply chain given by the graph $G$. As in the previous definition, in the more general supply chain structure, $c_F(\{i\})$ designates
the total pollution created by player $i$ either directly or indirectly. Thus,

$$c_F(\{i\}) = c(\{i\}) + \sum_{j \in V(T^i)} a(R(j)),$$

where $a(R(j))$ is the pollution weight from all arcs in $R(j)$ for all nodes in the subtree rooted in $i$, $j \in V(T^i)$. Note that $c_F(\{i\})$ corresponds to the original GHG pollution generated by player $i$ (in the tree model) augmented by emissions from all feedback arcs and direct arcs to non-intermediate downstream consumers that end in the tree, $T^i$, rooted in $i$. The set of maximal players in $S \subseteq N$ (with respect to $T$) is defined as before, and $c_F(S)$ in the more general case is equal to

$$c_F(S) = \sum_{i \in S_{\text{max}}} c(\{i\}) + \sum_{j \in V(T^i), i \in S_{\text{max}}} a(R(j)).$$

For example in Figure 2, if $S = \{5, 6, 7, 8, 9\}$, then $S_{\text{max}} = \{5, 6\}$, for $j \in S$ we have

$$R(j) = \begin{cases} \emptyset, & j \in S, j \neq 7 \\ \{(2, 7)\}, & j = 7, \end{cases}$$

and $c_F(S) = c(\{5\}) + c(\{6\}) + a((2, 7))$. Clearly, the GGER game $(N, c_F)$ is still monotone.

For $S \subset N$ and $i \in N \setminus S$, $i \in (S \cup \{i\})_{\text{max}}$ let

$$a(R^i_S) \equiv \sum_{j \in V(T^i)} a(R(j)) - \sum_{j \in V(T^i), \ell \in S_{\text{max}} \cap V(T^i)} a(R(j)).$$

In other words, we are only considering feedback arcs and direct arcs to non-intermediate downstream consumers (elements of set $A$) that enter nodes of the tree rooted in $i$ which were not contained in any of the trees rooted in maximal nodes of $S$. Consider again, for example, Figure 2 with $S = \{5, 6, 7, 8, 9\}$, and let $i = 2$. Then, $V(T^i) = S \cup \{2, 4\}$, $S_{\text{max}} \cap V(T^i) = \{5, 6\}$, $R(2) = \emptyset$, $R(4) = \{(9, 4)\}$, and $a(R^i_S) = a(9, 4) + a(2, 7) - a(2, 7) = a(9, 4)$. Clearly, if $Q \subseteq S$, then there are no fewer elements from $A$ that enter trees rooted in maximal nodes of $S$ than those that enter trees rooted in maximal nodes of $Q$, hence $a(R^i_Q) \leq a(R^i_S)$. For $Q \subseteq S$, we also denote

$$a(R^i_{S \setminus Q}) \equiv a(R^i_Q) - a(R^i_S) = \sum_{j \in V(T^i), \ell \in S_{\text{max}} \cap V(T^i)} a(R(j)) - \sum_{j \in V(T^i), \ell \in Q_{\text{max}} \cap V(T^i)} a(R(j)).$$
Consider again Figure 2 with $S = \{5, 6, 7, 8, 9\}$ and $i = 2$, and let $Q = \{6, 8, 9\}$. Then, $a(R_Q^i) = a(9, 4) + a(2, 7)$, and $a(R_{S\setminus Q}^i) = a(2, 7)$.

To prove that $c_F$ is convex, recall that $a(E(T_S^i))$ is defined by (2) and note that by its definition, we have that for $S \subset N$ and $i \in N \setminus S$

$$c_F(S \cup \{i\}) - c_F(S) = \begin{cases} 0, & \text{if } i \notin (S \cup \{i\})^{\text{max}}, \\ c_F(\{i\}) - a(E(T_S^i)) + a(R_S^i), & \text{otherwise.} \end{cases}$$

For $Q \subseteq S$ and $i \in (S \cup \{i\})^{\text{max}}$ it is easy to see that

$$c_F(S \cup \{i\}) - c_F(S) = c_F(\{i\}) - a(E(T_S^i)) + a(R_S^i) \leq c_F(\{i\}) - a(E(T_Q^i)) + a(R_Q^i) = c_F(Q \cup \{i\}) - c_F(Q).$$

Observe also that

$$i \in (S \cup \{i\})^{\text{max}} \implies i \in (Q \cup \{i\})^{\text{max}},$$

hence we might have $(Q \cup \{i\})^{\text{max}} \setminus (S \cup \{i\})^{\text{max}} \neq \emptyset$. Therefore, we can write

$$c_F(Q \cup \{i\}) - c_F(Q) - (c_F(S \cup \{i\}) - c_F(S)) = \begin{cases} 0, & i \notin (Q \cup \{i\})^{\text{max}}, \\ c_F(\{i\}) - a(E(T_Q^i)) + a(R_Q^i), & i \in (Q \cup \{i\})^{\text{max}} \setminus (S \cup \{i\})^{\text{max}}, \\ a(E(T_S^i)) - a(E(T_Q^i)) + a(R_{S\setminus Q}^i), & i \in (S \cup \{i\})^{\text{max}}. \end{cases}$$

We have the following result.

**Proposition 3** The extended GGER game $(N, c_F)$ is convex.

Again, the convexity of $(N, c_F)$ implies that its core is not empty, and we can similarly proceed to construct some extreme-point core allocations in this game, as well as demonstrate that any convex allocation of the responsibility of an arc pollution among all players who are directly or indirectly responsible for it is in the core of $(N, c_F)$.

A slight modification in the procedure described in the proof of Theorem 1 to generate the Shapley value of $(N, c)$ is shown below to produce the Shapley value of $(N, c_F)$. First, we need to slightly modify the definition of the set of players who are directly or indirectly responsible for a pollution in an arc in $G$. Specifically, for an arc $e_j = (i, j) \in E(G)$, with arc-tail $i$ and arc-head $j$ and an associated GHG emission weight $a_{e_j}$, denote by $N_{e_j}$ the set of nodes (i.e., players) on the path, $P_j \in T$, from node $j$ to node 1, including nodes $j$ and 1,
and including node $i$, if $i$ is not on $P_j$. Note that $N^{e_j}$ consists of all players who are directly or indirectly responsible for the pollution $a_{e_j}$.

**Theorem 2** The allocation according to which, for each $e_j \in E(G)$, $a_{e_j}$ is allocated equally among members in $N^{e_j}$ is the Shapley value of $(N,c_F)$.

**Proof:** For each $e_j \in E(G)$ and $S \subseteq N$, let $(N,c^{e_j}_F)$ denote the game where

$$c^{e_j}_F(S) = \begin{cases} 0, & \text{if } S \cap N^{e_j} = \emptyset, \\ a_{e_j}, & \text{otherwise}. \end{cases}$$

One can easily verify that for each $S \subseteq N$, $c_F(S) = \sum_{e_j \in E(G)} c^{e_j}_F(S)$. By its symmetry property, the Shapley value for the game $(N,c^{e_j}_F)$ is

$$\Phi_i = \begin{cases} \frac{a_{e_j}}{|N^{e_j}|}, & \text{if } i \in N^{e_j}, \\ 0, & \text{otherwise}, \end{cases}$$

and by its additivity property, $\Phi(c_F) = \sum_{e_j \in E(G)} \Phi(c^{e_j}_F)$. Thus, Shapley value of the GGER game, $(N,c_F)$, allocates the cost of the pollution $a_{e_j}$ equally among all players who are directly or indirectly responsible for its creation.

5. **Extension to a General Supply Chain Structure**

We extend the analysis in this section to a more general supply chain structure. To elaborate on the challenge encountered in a general structure, consider the supply chain displayed in Figure 3 below, in which the supply chain is a directed path, $P$, from node 7, the leaf node, to node 0, the root of the path. Each player, $j$, directly created the pollution, $a_j, j = 1, \ldots, 7$, associated with edge $e_j$ emanating from node $j$ towards node 0. In addition, player 5 has directly created the pollution $a_{(5,2)}$ by supplying directly player 2 via arc $(5,2)$.

Player 5 is responsible for the pollution he directly created, $a_5$ and $a_{(5,2)}$, and is also indirectly responsible for $a_7$ and $a_6$. But, for which pollution is, say, player 4 responsible for? Clearly, player 4 is directly responsible for $a_4$ and is also indirectly responsible for $a_5$. However, is he also indirectly responsible for the entire pollution, $a_6$ and $a_7$, created by players 6 and 7? Indeed, player 4 may argue that some of these emissions were created in supply chain stages which were eventually used by player 5 to supply player 2 directly, and
player 4 should not be responsible for them. Thus, we suggest that both arc pollution $a_6$ and $a_7$ should be split to two parts—one part which was created to eventually supply player 4 via player 5, while the other part that was created so as to enable player 5 to supply player 2 directly. Specifically, we split the pollution $a_6$ (resp., $a_7$) to $a_6(P)$ and $a_6((5,2))$ (resp., $a_7(P)$ and $a_7((5,2))$, such that $a_6(P) + a_6((5,2)) = a_6$, $a_7(P) + a_7((5,2)) = a_7$, and, e.g., player 4 (resp., 3) is responsible, directly or indirectly, for $a_7(P), a_6(P), a_5$ and $a_4$ (resp., $a_7(P), a_6(P), a_5, a_4$ and $a_3$.)

We note that a number of environmentally cautious companies do attempt to evaluate their carbon footprints by considering all the “ingredients” that go into their final products, including the raw material, transportation, processing, etc. Timberland Co., a shoe company, found that despite their offshore manufacturing, transportation accounts for less than 5% of their carbon footprint, while leather is the biggest contributor (Ball, 2009). However,
Timberland’s leather suppliers argued that the GHG emissions from a cow should not be allocated to them, but should be entirely the responsibility of the beef producers. Their argument was that cows are grown mainly for meat, with leather as a byproduct, so that growing leather doesn’t yield emissions beyond those that would have occurred anyway. However, after realizing the huge amount of the environmental pollution generated through a cow’s lifetime and its impact on their end product, Timberland decided that they cannot ignore GHG emissions in that part of their supply chain. They considered information from the databases of “life-cycle analysis” consultants, determined that 7% of the financial value of a cow stems from its leather, and adopted guidelines requiring that the company should apply that percentage to compute the share of a cow’s total emissions attributable to leather. Likewise, Nike Inc. finds that about 56% of their emissions come from materials used to make their products (Nike, 2013). A conversation that one of the authors had with representatives from Nike reveals that Nike applies about 8% of a cow’s total GHG emissions to the leather used in their shoes. Similarly, Gallego and Lenzen (2005) consider a matrix of direct requirements whose elements are inter-industrial flows from an industry $i$ to an industry $j$ per gross output of sector $j$, based on the Leontief model from input-output theory.

Thus, as the examples above indicate, data for splitting arc pollution to its various relevant components can be obtained, and the suggested approach for allocating pollution responsibilities seems to be the proper one. Thus, in general, to extend the analysis we need to divide each arc pollution to several parts and to allocate the responsibility of each part to the correct set of players. To do so, consider a general supply chain graph, $G = (V(T), E(G))$, derived from the original directed tree $T = (V(T), E(T))$, by adding additional directed arcs between nodes in $V(T)$, corresponding to other processes/direct connections in the supply chain. A path $P$ in $G = (V(T), E(G))$ from node $i$ to node 0 is said to be simple if all nodes in $P$ are distinct, and is said to be pseudo-simple if all edges in $P$ are distinct. Two pseudo-paths from node $j$ to node 0 in $G$ are said to be distinct if their edge sets do not coincide. Clearly, a simple path is also pseudo-simple. Note that in our basic model (in which $G$ was a directed tree) the unique path from any node $j$ to 0 in $G$ is simple, and that in the extension with feedback and direct non-intermediate downstream consumers arcs, considered
in Section 4, there may exist some pseudo-simple paths.

For each arc \((j_1, j_2)\) in \(G\) denote by \(\mathcal{P}^{(j_1, j_2)} = \left\{ P_{(j_1, j_2)}^{(1)}, P_{(j_1, j_2)}^{(2)}, \ldots, P_{(j_1, j_2)}^{(n_{(j_1, j_2)})} \right\} \) the set of all distinct (simple and pseudo-simple) directed paths from node \(j_1\) to \(0\) in \(G\) in which \((j_1, j_2)\) is the first arc. We can associate the pollution \(a_{(j_1, j_2)}\), directly created by player \(j_1\), with all the pseudo-paths in \(\mathcal{P}^{(j_1, j_2)}\) in the sense that this pollution was created to provide the required inputs to the various pseudo-paths in the supply network which are required by player 1. We denote the part of the pollution of \(a_{(j_1, j_2)}\) which can be attributed to the pseudo-path \(P_{\ell}^{(j_1, j_2)}\) by \(a_{(j_1, j_2)} \left( P_{\ell}^{(j_1, j_2)} \right) \), where \(a_{(j_1, j_2)} \left( P_{\ell}^{(j_1, j_2)} \right) \geq 0\) and \(\sum_{\ell=1}^{n_{(j_1, j_2)}} a_{(j_1, j_2)} \left( P_{\ell}^{(j_1, j_2)} \right) = a_{(j_1, j_2)}\), and we assume that the values of \(a_{(j_1, j_2)} \left( P_{\ell}^{(j_1, j_2)} \right)\) are available for all arcs \((j_1, j_2)\) in \(E(G)\) and all corresponding pseudo-paths in \(\mathcal{P}^{(j_1, j_2)}\).

For each arc \((j_1, j_2)\) in \(E(G)\) and each path \(P_{\ell}^{(j_1, j_2)} \in \mathcal{P}^{(j_1, j_2)}\), let \(N \left( P_{\ell}^{(j_1, j_2)} \right)\) denote the set of players in \(P_{\ell}^{(j_1, j_2)}\). Consider player \(i\), and assume that \(i \in N \left( P_{\ell}^{(j_1, j_2)} \right)\) for some arc \((j_1, j_2)\) and some \(P_{\ell}^{(j_1, j_2)} \in \mathcal{P}^{(j_1, j_2)}\). Then, by the above discussion, \(a_{(j_1, j_2)} \left( P_{\ell}^{(j_1, j_2)} \right)\) can be attributed, perhaps not exclusively, to player \(i\). Furthermore, the total pollution that can be attributed to player \(i\) can be expressed as

\[
c_G(\{i\}) = \sum \left[ a_{(j_1, j_2)} \left( P_{\ell}^{(j_1, j_2)} \right) \right] : \text{ for all } (j_1, j_2) \in E(G), P_{\ell}^{(j_1, j_2)} \in \mathcal{P}^{(j_1, j_2)} \text{ such that } i \in N \left( P_{\ell}^{(j_1, j_2)} \right).
\]

Let \((N, c_G)\) denote the GGER game associated with a general supply chain structure \(G\). Then, in view of the above discussion, one can verify that for each \(S \subseteq N\),

\[
c_G(S) = \sum_{(j_1, j_2) \in E(G)} \sum_{P_{\ell}^{(j_1, j_2)} \in \mathcal{P}^{(j_1, j_2)}} c_G^{P_{\ell}^{(j_1, j_2)}}(S),
\]

where

\[
c_G^{P_{\ell}^{(j_1, j_2)}}(S) = \begin{cases} 0, & \text{if } S \cap N \left( P_{\ell}^{(j_1, j_2)} \right) = \emptyset, \\ a_{(j_1, j_2)} \left( P_{\ell}^{(j_1, j_2)} \right), & \text{otherwise}. \end{cases}
\]

Clearly, for any \((j_1, j_2) \in E(G)\) and \(P_{\ell}^{(j_1, j_2)} \in \mathcal{P}^{(j_1, j_2)}\),

\[
\left[ Q \subseteq S \text{ and } Q \cap N \left( P_{\ell}^{(j_1, j_2)} \right) \neq \emptyset \right] \implies S \cap N \left( P_{\ell}^{(j_1, j_2)} \right) \neq \emptyset,
\]
which implies that \((N,c_G)\) is monotone. For \(i \notin S\),

\[
c_G(S \cup \{i\}) - c_G(S) = \sum \left[ a_{(j_1,j_2)} \left( P_{\ell}^{(j_1,j_2)} \right) \right] : \text{for all } (j_1,j_2) \in E(G) \text{ and all paths } P_{\ell}^{(j_1,j_2)} \in \mathcal{P}^{(j_1,j_2)} \text{ such that } S \cap N \left( P_{\ell}^{(j_1,j_2)} \right) = \emptyset \text{ and } i \in N \left( P_{\ell}^{(j_1,j_2)} \right).
\]

Now, note that for each \((j_1,j_2) \in E(G)\) and \(P_{\ell}^{(j_1,j_2)} \in \mathcal{P}^{(j_1,j_2)}\),

\[
\left[ Q \subset S \subset N \text{ and } S \cap N \left( P_{\ell}^{(j_1,j_2)} \right) = \emptyset \right] \implies Q \cap N \left( P_{\ell}^{(j_1,j_2)} \right) = \emptyset.
\]

Thus, for \(Q \subset S \subset N\) and \(i \notin S\), \(c_G(S \cup \{i\}) - c_G(S) \leq c_G(Q \cup \{i\}) - c_G(Q)\), implying that \((N,c_G)\) is convex and thus, e.g., the Shapley value of the game \((N,c_G)\) is contained in its core.

**Theorem 3** The allocation according to which for each \((j_1,j_2) \in E(G)\) and \(P_{\ell}^{(j_1,j_2)} \in \mathcal{P}^{(j_1,j_2)}\),

\[
a_{(j_1,j_2)} \left( P_{\ell}^{(j_1,j_2)} \right)
\]

is allocated equally among members in \(N\) \(\left( P_{\ell}^{(j_1,j_2)} \right) \) is the Shapley value of \((N,c_G)\).

**Proof:** Recall that for each \((j_1,j_2) \in E(G), P_{\ell}^{(j_1,j_2)} \in \mathcal{P}^{(j_1,j_2)}\) and \(S \subseteq N\), \(\left( N, c_G^{P_{\ell}^{(j_1,j_2)}} \right)\) denotes the game where

\[
c_G^{P_{\ell}^{(j_1,j_2)}}(S) = \begin{cases} 0, & \text{if } S \cap N(P_{\ell}^{(j_1,j_2)}) = \emptyset, \\ a_{(j_1,j_2)} \left( P_{\ell}^{(j_1,j_2)} \right), & \text{otherwise}, \end{cases}
\]

and further recall that for each \(S \subseteq N\), \(c_G(S) = \sum_{(j_1,j_2) \in E(G)} \sum_{P_{\ell}^{(j_1,j_2)} \in \mathcal{P}(j_1,j_2)} c_G^{P_{\ell}^{(j_1,j_2)}}(S)\). By its symmetry property, the Shapley value for the game \(\left( N, c_G^{P_{\ell}^{(j_1,j_2)}} \right)\) is

\[
\Phi_i = \begin{cases} \frac{a_{(j_1,j_2)}(P_{\ell}^{(j_1,j_2)})}{|N(P_{\ell}^{(j_1,j_2)})|}, & \text{if } i \in N(P_{\ell}^{(j_1,j_2)}), \\ 0, & \text{otherwise}, \end{cases}
\]

and by its additivity property, \(\Phi(c_G) = \sum_{(j_1,j_2) \in E(G)} \sum_{P_{\ell}^{(j_1,j_2)} \in \mathcal{P}(j_1,j_2)} \Phi \left( c_G^{P_{\ell}^{(j_1,j_2)}} \right)\). Thus, for each \(P_{\ell}^{(j_1,j_2)} \in \mathcal{P}(j_1,j_2)\), the Shapley value of the GGER game \(\left( N, c_G \right)\) allocates the cost of the pollution \(a_{(j_1,j_2)}(P_{\ell}^{(j_1,j_2)})\) equally among all players in \(N(P_{\ell}^{(j_1,j_2)})\), who are directly or indirectly responsible for its creation.
6. Concluding Remarks

Our work was motivated by observing the increased impact of global supply chains on environmental pollution and the need to find appropriate ways to share pollution responsibility among supply chain members. At the outset, we restrict our attention to allocation methods that overcome double counting, and focus on identifying allocation rules that are easy to implement, do not over-allocate pollution to any set of supply chain members, and induce companies to invest in reduction of environmental impact created both by themselves and their supply chain partners. Starting with a simple tree representation of a supply chain, we first identify a set of core allocations (which do not over-allocate pollution to any set of supply chain members), and show that two simple allocation rules, the full producer responsibility and the full consumer responsibility, represent two extreme points of the core. However, as they might be difficult to justify because of their “all-or-nothing” character, we extend our analysis and show that the Shapley value in this instance corresponds to allocating the responsibility for pollution generated by one party equally among all of its downstream supply chain members, and that it possesses all the desired properties mentioned above. We then extend the analysis and demonstrate that our results are valid for more general supply chain structures.

References


