Day 5 Talk 1 - Lino,

Am singularity

\[(dM = 2)\]

\[x^2 + y^2 + \sum_{m \neq 2} w^m = 0, \quad m \neq 2.\]

\[\downarrow\]

\[p_w(z) = \sum_{m} w^m z^m + w_0.\]

\[g_w = x^2 + y^2 + p_w(z), \quad w = (w_0, \ldots, w_m) \text{ multipla-}\]

\[g_w = c \quad \text{w} \not\in \mathbb{C}^3, \quad \Omega \subset \mathbb{C}^3\]

\[c_1 = 0, \text{ so we can grade.}\]

\[\eta \ldots g = dz_1 \wedge dz_2 \wedge dz_3\]

\[\bar{w} \in \mathcal{W} \subseteq \mathbb{R}^{2m+2}(\delta)\]

\[X = \theta_{g_w}^{-1}(\theta) = E_{\bar{w}}\]

s.t. \(E_w \text{ smooth}\)

\[E = \exists (E_w, w) \text{ s.t. No Repeated}\]

\[w\]
\[ E = \{ (E_w, w) \} \]

\[ w \longrightarrow \text{Conf}^{m_{\text{mc}}} (D^2, D^2) \]

\[ w \longrightarrow \{ \text{2000s p} \} \]

\[ \pi_1 (\text{conf}^{m_{\text{mc}}}) = \pi_1 (W, \overline{w}) \]

\[ \therefore \text{Sym} (X) \]

So Braid group \( \text{Br}_{m+1} \) acts on \( X \) through symplectic automorphisms.

Then: \( \text{Br}_{m_{\text{mc}}} \xrightarrow{p} \pi_0 (\text{Sym} (X)) \) injective.

(For \( \text{D}_{m} \), this map exists but not known whether it's injective.)

\[ \text{Fuk} (X) : \quad X \xrightarrow{\pi} \{ x^2 + y^2 + p(z) = 0 \} \]
Fix $z_0$, $\pi^{-1}(z_0) = Q_z$.

If $z \in \Delta \Rightarrow Q_z \cong T^* S^1$.

**Special case of this:**

$x^2 + y^2 = 1.$

$$(u^2 + v^2) - (u_z^2 + v_z^2) = 1$$

$\langle u, v \rangle = 0.$

$(u, v) \in (R^2)^2 \quad \|u\|^2 - \|v\|^2 = 1$

$\langle u, v \rangle = 0.$

$$
\left( \frac{u}{\|u\|}, \frac{v}{\|v\|} \right) \in T^* S^1 = \left\{ \langle u, v \rangle, \|u\| = 1 \right\}
\left\langle u, v \right\rangle = 0.
$$

If $z \notin \Delta, Q_z = T^* S^1$, $\mathcal{L} \Sigma_z = S^1$.

If $z \in \Delta, Q_z = T^* S'/S'$, $\Sigma_z = \rho^1$. 
\[ L_y = \bigcup_{z \in \mathbb{R}} \exists z \]

**Ex:**

\[ \text{Diagram of } L_y \]

**Proof:**
1. \( L_y \) are log'n spheres
2. \( y \approx y' \Rightarrow L_y \sim L_{y'} \)

\[ \text{rank } H(\lambda_y, \lambda_y', \mathbb{Z}/2) = 2I(\delta, \delta') \]

**Remark:** About transverse intersections: \( \lambda_y, \lambda_y' \) ought to intersect cleanly at end points, use Morse-Bott theory for middle intersections.

\[ \sum_{\Delta \in \lambda_y \cap \lambda_y'} |\Delta| + \frac{1}{2} |\delta \cap \delta'| \]
Idea for counting intersections: first remove excess intersections of $X, Y$, use Morse-Bott type perturbations for rest.

$$X = \{ x^2 + y^2 + p(z) = 0 \}$$

Call $F_m < (V_1^\#, \ldots, V_m^\#) > \leq Fick(X)$ of gadings of the full subcat. gen. by these objects.

Pick gadings so that intersecstions are Am chain spheres
We can calculate $H^0(F_m)$ formally (Paul: total space deformation retracts into these spheres, i.e. $V S^2 \times$).

$H^F \ast(V, V_i, V_i^\#) \cong H^e(S^2)$ but

everything is exact.

$H^F \ast(V_j, V_j^\#) = \begin{cases} 0, & |i-j| > 2, \\ \mathbb{Z}, & j = i \pm 1. \end{cases}$

Products: Must should be zero.

Use Floor Poincaré duality: (which works here!)

(i.e. $HF^d(L, L') \cong HF^{n-d}(L', L) \to HF^\Delta(L, L)$)

so:

$\alpha_i \beta_i = k_i (\beta_{i-1} \alpha_{i-1}) \in \mathbb{Z}^\times.$
Now multiply generators by some constant so each $k_i = 1$.

Get $A_n$ quivers:

\[
\begin{array}{ccc}
\bullet & \rightarrow & \bullet \\
\downarrow & & \downarrow \\
\bullet & \rightarrow & \bullet \\
\end{array}
\]

- path algebra (cells are paths, products are composing paths)

modulo:

\[
\begin{align*}
(i | i+1 | i+2) &= 0 \\
(i | i-1 | i-2) &= 0 \\
(i | i-1 | i) - (i | i+1 | i)
\end{align*}
\]

Think of $A_n$ as an algebra over $\mathbb{C}$

paths of length 0 correspond to $1$, paths of length 1 correspond to integers, back and forth relations correspond to Poincaré duality argument.

1. $V_i^*$ split generate
2. this algebra is intrinsically formal.
in particular, any algebra that has homology.

this guy is formal, so we understand $F(X)$ completely.

$\Rightarrow \text{Tw } F(X) \in \text{mod } (A_n) \downarrow$ 

split closure

What is split closure? In a classical linear category, suppose you have an idempotent

$\Pi : X \rightarrow X$, $\Pi^2 = \Pi$.

image $\Pi$:

\[ \begin{array}{ccc}
X & \xrightarrow{k} & Z \\
\downarrow & & \downarrow \\
\text{image } \Pi & = & Z \\
\end{array} \]

\[ k \circ i = \text{id}_Z, \quad i \circ k = \Pi. \]

We call the category **split-closed** if $Z$ exists for any $\Pi$.

Can Aoo level, have to talk about idempotents up to homotopy (won't define here)

We'll use: Aoo category $A$ split-closed $\iff \Pi_0(A)$ split-closed.

(idea of $\Pi$ is take the minimal split-closed enlargement).
\[ A \rightarrow \mathbf{T} \mathbf{A} \]

minimal split-closed category containing \( A \)

"Kuroki completion"

Why do these things split generically?

Related to \( T_{\mathbf{v}^+}(L) = T_{\mathbf{v}^+}(L') \)

We use the following algebraic theorem: If you have a family of spherical objects \( Y_i \), s.t.

\[ A \times X, \quad T_{Y_i} T_{Y_2} \rightarrow T_{Y_m}(x) = X_{[\sigma]} \]

Then \( Y_1, \ldots, Y_m \) split generically.

Then \( y \neq 0 \),

So have to compute "global non-degeneracy" of the fibrations. In general, not identity.

In this case, if

\[ \phi = T_{Y_i} \rightarrow T_{Y_m}, \quad \not\phi \neq 0 \]

\[ \phi^{2m+2}(L) = L \left[ [\sigma] \right] \text{ for } L \text{-spht. Log's } \Sigma K \]

comes from fact that orig. poly was weighted homogeneous.
2) Intrinsic Formality: Recall,

Then: $H^2(A_m, A_m[z^2]) = 0, z \geq 3,$

$\implies$ if $A$ has $H(A) = A_m$, $A$ formal.

Now, just compute $H^2(A_m, A_m[z^2]).$

Doesn't use any big theorems, just write it down.
(Paul: secretly relies on some geometric intuition.)

This is much easier if $n > 2$, but in dim 1,
other things happen,
(The analogue of $Q_1 \rightarrow H^1$ is an iso. here).