Monotone Lagrangians
$L \in M, \mu : \pi_2(M, L) \to \mathbb{Z} \text{ relative } c,
\omega : \pi_2(M, L) \to \mathbb{R}$
$\int u^* \omega$

Monotone means $\mu = \lambda \omega$, $\lambda > 0$

Coeff ring:
$\Lambda = \{ \sum \alpha_k u^k \mid \sum \alpha_k u^k \to \infty, \alpha_k \in \mathbb{Z}, c \}$

Generalization: flat line bundles

Obj: $(L, u)$ $u$-flat $U(1)$ bundle on $L$

$\mathcal{F}(L_0, u_0), (L_1, u_1)) = \bigoplus_{p \in \text{Const}_L} \text{Hom}(u_0, u_1) \otimes \Lambda \otimes \theta$

Diagram:

$(h^{-1} (h \circ u, f) h_0 \circ u_0) : u_0 \to u_0$

rotation by $\theta$
idea of $S^2 = 0$. 

$\langle S^2, z \rangle$ — boundary points of $1$-dim'd moduli space $M(x, z)/\mathbb{R}$

$\mathcal{M}(x, z)$

$\mu(3) = 2$

In the $\mathcal{M}_3(x, z)$ — strips, disc bubbles, sphere bubbles.

$M$ is at least $2$

$x + z$ — at least one strip.

all bubbles will have to have $x = 1$ — no disks

$x = 2$:

\[ \begin{align*}
L_0 & \quad 2c_1 \quad \text{no sphere} \\
L_1 & \quad x = 2
\end{align*} \]

or $x = 2$

\[ \begin{align*}
L_0 & \quad 2c_1 \quad \text{no sphere} \\
L_1 & \quad x = 2
\end{align*} \]

$c_1 (\text{sphere}) = 1$

$2n + 2 - 6 = 2n - 4$, so no sphere bubbles (generic argument)
Define $m_0(L) = \sum_{\beta \in \mathbb{P}} \text{# disc of index } 2$

$\mu(\beta) = 2$ through a given point with $\left( \frac{\partial}{\partial \beta} \right)$

$C \cdot C$

$m_0$ is a complex # associate to $L_t \Delta$

$0 = \hat{\nabla}_M = \langle \delta^2 x, z \rangle + m_0(L_0) - m_0(L_1)$

**Point:** For each $x \in C$, we get a Fuk with objects Lagrangians with $m_0(L_0) = m_0(L_1) = \lambda$

(mo called "central charge" or really just a change)

**Then:** There is an action of $Q_1H(X)$ on $HF(L, L)$

Use: Morse- Bott picture.

CF geometric singular chains on $L$

$S = \mathcal{E} + S'$

$S = \mathcal{E} + \sum S_\beta, C$

$M_\beta = \text{moduli space of 2 point hol.}$ maps.

$m_0 = \text{genus}$

**See picture**
\[ \delta_\beta^1 c = \text{ev}_{-1\ast} (\text{ev}_1^\ast)(c) \]

\[ n + \mu(\beta) + 2 - 3 = n + \mu - 2 \]

\[ (2\mu + 2c_1) \]

\[ \dim (\delta^\prime_\rho c) = \dim_c C + (\mu - 1) \]

\[ (M_\beta, \xi) - \text{1 pointed, with} \]

\[ \text{ev}_1 \times ([M_\beta, \xi, 1]) \in C_\ast(L) \]

\[ m_\ast[L] \]

\[ m_2 \text{ 3 pointed discs} \]

\[ m_2(c_1, c_2) = \sum_\beta \text{ev}_1 \times (\text{ev}_1^\ast \times \text{ev}_2^\ast) c_1 \times c_2 \]

\[ Q \in C_\ast(M), \quad C \in C_\ast(L) \]

\[ Q \cap_\beta C \]
\[ M_{1,0,1}(\beta, L) \]

\[ \text{ev} \circ (\text{ev}_1 \times \text{ev}_1)[\mathcal{Q} \times \mathcal{C}] \]

\[ \mathcal{Q} \cap \mathcal{C} \quad \& \quad \mathcal{Q} \cap \mathcal{C} = \sum_{\rho} \alpha \cap \rho \cdot \mathcal{C} \]

\[ S(\mathcal{Q} \cap \mathcal{C}) = \pm (\mathcal{Q} \cap \mathcal{C}) \cap \mathcal{C} = \mathcal{Q} \cap \mathcal{C} \]

+ extra terms involving \( \mathcal{Q} \cap \mathcal{C} \)

\[ \mathcal{Q} \cap \mathcal{C} \]

\[ \text{Fano} : \quad -K_X \text{ is ample}. \]

\[ \Sigma : \mathbb{C}P^n, \quad c_1 = (n+1)P \]

\[ D = \text{union of coordinates} \mathbb{C}P^{n+1}; \]

\[ \mathbb{C}_1 = 0, \mathbb{C}_i = 0, \ldots, m. \]
On $X \setminus D$, we have a broken role form.

Thus: If $y_0$ is not an eigenvalue of $c_i(X): QH \to QH$ then $HF(L, L) = 0$.

i.e. this theorem is about "charge quantization" (i.e. most Lagrangians have $2 \pi$ $HF$).

Two ingredients:
- $[c_i(X)] \cap [L] = m_0 \cdot [L]$.
- $c_i - m_0 \cap [L]$.

(also known $Q \cdot n(Q_2 \cap C) = (Q_1 \times Q_2) \cap C$)

If $(c_i - m_0)^* \ast$ is invertible, then

$\chi \ast (c_i - m_0) \cap L = \alpha \ast 0 = 0$. 

$\chi \cap L$
ex: \[\mathbb{C}P^2\]  
\[
1 \quad 0 \quad p^2 \\
0 \quad 1 \quad 0 \\
p^2 \quad 0 \quad 0
\]
eigenvalues
\[
\lambda = 3p, \quad \text{char. } \chi^3 - 3. \\
\text{pol. } \chi^3 - 3.
\]
eig. values are \[3^{2/3}, \ 3^{3/\sqrt{12}} , \ 3 \left(3^{1/3}\right)^2 \frac{1}{\sqrt{3}}\]

Clifford torus: \[S^1 \times S^1 \times S^1 \subset \mathbb{C}P^2\]

different classes of \[T^2 \subset \mathbb{C}P^2\]

3 disks, which together give a sphere

Each disk has area \(1/3\).

Contributing \(3^{1/3}, 3^{1/3}, 3^{1/3}\), together \(3 \cdot 3^{1/3}\).

It corresponds to eigenvalue.

Charge local system on \(T^2\) so that holonomies are \(3^{1/3}\), get a different eigenvalue.
Conclusions:

Each eigenvalue gets a shifted torus and corresponding volume:

\[(1, 1, 1)\] unlikely result; then
\[(3, 3, 3)\] objects split growth
\[(3^2, 3^2, 3^2)\] relevant Fridays category.

(In fact case, \(\mu(B)\) given by \(B \cdot [0] \) or something, that's what helps.)

Paul: This doesn't use Fano case!

There are some issues w/ non-regularity.

Cf: Auroux, Mirror Symm & T-Duality