1. Wrapped Fukaya category of exact symplectic manifold $X$.

   Objects: not necessarily compact Lagrangians $L \subset X$

   Morphisms: spaces

   $$CW^*(L_1, L_2)$$

   $\infty$ category: $W(X)$

Hochschild cohomology:

$$CC^k(w, w) = \bigoplus_{L_1, L_2} \bigodot_{\text{Vec}} H^k(V_{L_1, L_2}, CW^*(L_1, L_2) \otimes CW^*(L_2, L_3), CW^*(L_3, L_3))$$

$+ \text{ differential terms}$

$\longrightarrow HH^k(w, w)$.

**Def:** An nc-vector field on $W$ is a cocycle of degree 1,

$$b \in CC^1(w, w),$$

$$b = b^0 + b^1 + b^2 + \cdots$$

Geometric triangle: Such a $b$ defines a vector field on the

"moduli space" of objects in $W"
\[ M(W) \]

\[ \mathcal{B}^i \text{ is the "Taylor expansion" of the vector field.} \]

\[ \text{For each } L \rightarrow f^o \in CW^i(L,L) \quad \text{is closed (cycle condition).} \]

\[ [b^o_L] \in HW^i(L,L) \text{ is the "value" of } b \text{ at } L. \]

"target space + moduli space of objects."

\[ i = \text{obj}. \]

When \( [b^o_L] = 0 \Rightarrow \text{fixed point.} \)

\[ \Phi \in \pi CW(L,L), \quad b^1 \in \pi HW(CW(L,L), CW(L,L)) \]

When \( [b^o_L] = 0 \), \( b^1_L \) gives an endomorphism \( CW^i(L,L) \)

but only if we choose a \( c_L \in CW^i(L,L) \) s.t. \( d^c_L = b^o_L \).

\[ \Rightarrow \Phi : b^1_L + \chi^2(c_L, \cdot) - \chi^2(\cdot, c_L) : CW^i(L,L) \rightarrow CW^i(L,L) \]

is actually a chain map, gives map on cohomology.

\[ \text{Proceeding is due to Eidel-Solomon.} \]

\[ [b^o_L] = 0 \Rightarrow L \text{ is } b^1 \text{-equivariant} \]

Choose \( c_L \rightarrow (L, c_L) \) $b^1$-equivariant Lagrangian.
\[ i = 1, 2 \]

\[ \left[ b_i^0 \right] = 0, \quad b_i^1, b_i^2 \text{ gives an endomorphism} \quad C^i \left( L_i, L_i \right) \]

Source of vc-vector fields:

Symplectic cohomology.

There's a map

\[ SC^i(X) \to C^i(w, w) \]

close-open map

\[ \text{joint w/ Y. Lekili and N. Sheridan} \]

Q: When does \( \mathfrak{g} \) = Lie algebra embed in \( SH^i(X) \)? And what representations do you get for equivalent lagrangians?

Q: If \( X \) has a mirror space \( X^\vee \):

Mirror symmetry for \( \mathcal{B} = \mathbb{G}/B \overset{\text{Borel}}{\cong} \text{(General k.Pietsch [-R. Macph])} \),

A semi-simple alg. gp. \( G/\mathfrak{g} \), \( B = G/B \) flag variety

\( e.g., \ G = SL_n, \mathcal{B} = FL_n = \sum_i CF_i \subset F_n \subset C^n \) with \( F_i = i \)

Intrinsic action of \( \mathfrak{g} = \text{Lie} \ G \) on \( \mathcal{B} \). induces \( \mathfrak{g} \rightarrow \text{vector fields} \ (\mathcal{B}) \).
Minors: \( \mathbf{R} \subset \mathbf{L} \subset \mathbf{B} \) = Langlands dual flag variety.

\[
\left\{ \text{complement of the opposite Schubert divisor} \right\}
\]

"Reduction variety" also a \( \mathbb{V} \) superpotential \( W : \mathbf{R} \to \mathbb{C} \) ("LG model")!

By mirror symmetry,

\[
\mathfrak{o}_g \to \operatorname{Vect}(\mathbf{B}) \xrightarrow{\sim} \mathbb{H}^\bullet(\mathbf{B}^\mathbf{\cdot}) \cong \mathbb{H}^\bullet(\mathbf{R}).
\]

**\( \text{SL}_2 \) case:** \( \mathbf{B} = \mathbb{P}^1 \), \( \mathbf{R} = \mathbb{C}^* = T^*S^1 \).

\[
\mathbb{H}^\bullet(\mathbf{R}) = \begin{cases} 
\mathbb{C} \left[ z, z^{-1} \right] & \ast = 0 \\
\mathbb{C} \left[ z, z^{-1} \right] z \ast = 1
\end{cases}
\]

\( \text{sl}_2 = \left\langle \partial_z, z \partial_z, z^2 \partial_z \right\rangle \).

Thus (levi-P.) these objects can be made \( \text{sl}_2 \)-equivariant (noting even in \( \mathbb{C} \mathbb{W}^1 \) to \( \mathbb{R}^2 \))

\( \lambda, \mathbf{c}_\lambda \equiv 0 \) (kind: need to push \( \mathbf{c}_\lambda \) for each vertex of \( \mathbb{A}^1 \))

and \( \mathbb{H}^\bullet(L_0 L_1) \) contains the rep of \( \text{sl}_2 \) of highest weight \( \lambda \), or an \( \text{sl}_2 \)-submodule.

Rule: \( L_0 \) and \( L_n \) are isomorphic to \( \mathbb{W} \) but this isomorphism does not respect theequivariant structures.
Rule: When making $L$ equivariant for $\phi \to C^1(W, W)$, must choose $\xi: \phi \to C^0(L, L)$.

The condition that the induced map $\phi \to \text{End} (C^0(L, L))$ is a Lie alg. map is potentially non-trivial.

(ned to check this in examples)

Current make progress on e.g., $sl_3$

Next example: $T^*P^1$ (T*B)

is the $SL_2$ case of the Springer resolution.

$T^*P^1 \to N = \left\{ (a, b, c, -a) \mid a^2 + bc = 0, \det = 0 \right\}$, quadratic case. (five $P^1$

(resolution)

Morse of $T^*P^1$ has been found by A. Noma, Australia.

is $(T^*S^2 \setminus D, W)$, $T^*S^2$

"smoothing"

Remove divisor: remove a fibre of multiple spheres.

superpotential.

\[ \begin{array}{c}
\text{"matching cycle"} \\
\end{array} \]
Claim: This is more of $T^2 \mathbb{P}^1$

Thm. (g) $\text{HW}(\mathbb{L}, \mathbb{L}) = \mathbb{C} [x, y^{\pm 1}, z]/xz = (1+y)^2$

Turning on $W$: $\text{look } y^{-1}$.

Final: global fans on $T^2 \mathbb{P}^1$ need to be constrained on $\mathbb{P}^1 \hookrightarrow \text{fans}$ on $\mathbb{N}$.

(See this to figure out how to embed $S(U)$.)

Can look at action of $SL_2(\mathbb{C})$ on $\text{HW}(\mathbb{L}, \mathbb{L})$

by isometries of hyperbolic space $\mathbb{H}^2$ (or projectivize) on $\mathbb{N}$.

Q: Are boundary cycles $T$?

(maybe for loop groups?)