Fabert - Fish - Colovers

User's Guide to Polyfolds

Classical $\mathbf{\varepsilon} \to \mathbf{\varepsilon} \text{ Ant.} \subset \mathbf{\varepsilon} \to \mathbf{\varepsilon} \text{ Polyfolds}$

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Classical $\text{same as Polyfolds}$

- Proving $\rightarrow$ charts
- Using indicator label/circular/effilence (easy)

Brain: Q: Can apply polyfolds to other than GW, SFT & formal?

Q: What need to prove, basic polyfolds?

- Basic

- Derived body phase a

These are not covered

Brain: smile also

Core: This is a ghost disk, so already there
Flow chart:

1. Composed + C*, + R *
2. Check for missing key phrases:
   - Basic + Edel + (to be collected in co-lecture)
   - Derived (if necessary)
   - Give reference to HWZ & Weltlin
3. Have an influence on definite doubtful chart & point out that operator is an X-Bi-Product reaction
4. Ask: symmetry?
   - No
   - Yes
      - Collect relevant facts with symmetry

Retracts given by identity + ap.

Body points: need to use what happens here. Normally let it be.
Feel: Cool. Convince you that in "non-degenerate" cases, putting your problem into polyfold framework is no more difficult than constructing a pre-glimm map (that exists).

Reason: Transversality comes for free.
Nice to pay is cheating definition.

Polyfold recipe:
1) Weak notion of sections. (To see how things can break)
2) Define polyfold/polydisk (w) inner - strictly defined.
   ...they make

Suffices to understand local models.
sc - B, r: U → U s.t. r|\partial U = r. Local models built on r(U)

\[ E_k = \sum_{k=1}^{\infty} \delta_k (R, R) \]

\[ \mathcal{R} = \left. \mathcal{R} \right|_0 \left( v, e, f \right) = \left( v, e, 0 \right) \]

\[ \mathcal{M} \left( v, m_1, m_2 \right) = \left( v, m_1 \Theta v m_2, m_1 \Theta v m_2 \right) \]

\[ \partial \left( v, m_1, m_2, e_1, e_2 \right) = \left( v, m_1 \Theta e_1, m_2 \Theta e_1, e_1 \Theta e_2, e_1 \Theta e_2 \right) \quad \text{sc - diff} \]

**Key Point:** Defining local models is no more difficult than pregluing.

**Q:** polyfold Fredholm?

**A:** Yes, if regularizing (elliptic regularity):

1. At each \( x \in X \), polyfold (base), \( (x, k) \) is Fredholm germ

\[ \mathcal{F}_x (m_1 \Theta v, m_2), \mathcal{F}_x (m_1 \Theta v, m_2) \]

linearize \( \mathcal{J}(x, m_1 \Theta v, m_2) \) at every \( h \) when \( m_1 \Theta v m_2 = 0 \) and show this is surjective.

**Hence:** Open issue: show that pregluing to these trajectories with stable breaking base depends on other by which we glue pieces.

\( \{ \mathcal{F}_{x} \} \) is compact.

\( \forall x \in \mathcal{F}_{x} \) is a fixed looking \( \mathbb{R} \) \( \mathbb{R} \) canon.
1) Maxi-Bott bundle, say $\mathcal{K}_V$
2) Embedding of classical Transversality $\mathcal{K} \subset \mathcal{D}$
   1) SFT of polarization spaces $S^1 \to V$ with $c_1(V) = \xi \in H^2(M)$

The idea: (Bougeois, ECH):
*Under moduli assumptions (in choice of differentials), SFT($V$) can be coupled from GW($\mathcal{K}$).

Now, let $V$:

2) (Bougeois): SFT($V$) $\cong$ MB-SFT($V$) (with cascades)

Problem: Transversality!

Need to prove 2 things:
   a) MB-SFT MB-SFT is well-defined
   b) MB-SFT $\cong$ SFT

Local models:
   a)

Ko: need $\nabla B_{\mathcal{D}}$ even in natural case

b) $\xrightarrow{}$

1) Perturb $\mathcal{M}_{\text{MB-SFT}}$, lose $S^1$-symmetry. So, need to go through.

Take $S^1 \times B_{\text{MB-SFT}}(V)$ $\times \pi_{\mathcal{D}}$ $\in \nabla B_{\mathcal{D}}$

2 ways of MB-SFT:
   1) with drift form
   2) with cascades: not yet done!

Problem: Need a stabilization step. Not trivial, be moduli spaces of perturbed instantons $S^1$-symmetry

Need distinct perturbations for polyfolds for defying MB-SFT (with cascades)

Q: Does it destroy $S^1$-symmetry?
A: No! After general perturbation, moduli space of holes may get perturbed.

\[ \text{unperturbed orbits} = \mathbb{P} \]

\[ \text{moduli space was already regular with } S^1 \text{- symmetry} \]

\[ \Rightarrow \text{still have } S^1 \text{- symmetry} \]

\[ \Rightarrow \text{evaluation } B \to \mathbb{P} \text{ independent of } S^1 \]

\[ \Rightarrow \text{still } S^1 \]

2) Embedding:

For gen - 1FT: need to prove Fredholm of gen onto of linearized.