Ellingsberg, Effect of Leg. Surgery IV

\[ \Theta \rightarrow Y_0 \]

\[ \bar{X}_0 = \left( \cup_{i} \right) \text{ boundary, } X_0 = \text{ open bun.} \]

\[ \Lambda \subset U_{A_i} \text{ sphere} \subset Y_0. \]

\[ Y = \bar{X}_0 \]

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**L H (\Lambda)** {\text{gen. by charts connecting manifold.}}

\[ U \]

\[ L H^{\text{comp}} (\Lambda) = \bigwedge H^{\text{comp}}_w \text{ grade cyc. permutation.} \]

\[ L H^{\text{comp}} (\Lambda) = \bigwedge H^{\text{comp}}_w \otimes \bigwedge H^{\text{comp}}_w \]

\[ w = C_i - C_w \]

\[ W = C_1 - C_w \]

\[ \text{cyclically o-positive.} \]

\[ \text{cyclically o-negative.} \]

\[ L H^{\text{comp}}_w = L H^{\text{comp}} \otimes R \]

\[ \Sigma R \xi_i, \xi_i \to \Lambda_i \]

\[ |2_k| = 0 \forall k. \]

\[ d \alpha = \bigwedge C_i, a \in C_i, a = \bigwedge C_i, da = \text{const.} \]

**Examples:**

\[ B^4 = \bar{X}_0, \Lambda \subset S^3 = Y_0 \text{ trivial knot.} \]

Doing surgery over \( \Lambda \), we have it's standard that \( X = T^2 \otimes S^1, Y = \text{ sphere bundle.} \) The latter should be non-trivial, e.g. \( H^*(X) \otimes \text{ (Abendroth-Schweig, etc.)} \).
Let's first note that \( Y = R^3 \), see what happens if we move to \( S^3 \).

R3, and check \( 0 = R^3 \), see what happens if we move to \( S^3 \).

...
Then add \( 12 \) more variables. How to move from \( \mathbb{R}^3 \) to \( S^3 \)?

- Take any non-constant periodic trajectory.
- Put another, so that it's on a non-constant trajectory.
- \( \Rightarrow \) index from non-\( \mathbb{R}^3 \) closed gets arbitrarily large non-constant orbit.

**Example #2:**

Take any Lagrangian, say, stabilization of eternal knot.

Take Whitehead double.

\[
\begin{array}{c}
\text{stabilization, determined } \mathcal{E}, \mathcal{H}, \mathcal{K}, \text{ etc.} \\
\text{can show for two knots that}
\end{array}
\]

\[
\mathcal{L} \mathcal{H}^{\mathcal{E}}(\Lambda_x) \neq \mathcal{L} \mathcal{H}^{\mathcal{E}}(\Lambda_y)
\]

for the knots \( \Lambda_x, \Lambda_y \).

(Next, to play some \( \mathbb{R}^3 \subset S^3 \) game.)

\( \Rightarrow \) manifolds not homeomorphic doesn't \( \Rightarrow \) \( \mathcal{E} \) are not contactomorphic.

In our case, we're ok.
thereafter:

under cobordism, augmenting map is an augmenting.

but don’t know to which one.

in particular, if we linearize guy has deg -2, and the
other kill algebra has all positive degrees, we are done.

exotic $T^k S^n$:

we know $\infty$ produces $T^2 S^2$.

Similarly, $0$ in higher degrees give us $T^k S^n$’s.

now, stabilize unknot:

no surgery on this knot.

(removes torus or lens product)

Then you get lens spaces, or something in different manifold.

get man Thurston-Bennequin, so can’t do this for $T^2 S^2$.

higher dimensions: $R^5 = J^1 (R^2) \approx J^1 (R^2)$.

take trivial knot, knot.

2-D flyby source.

stabilize: (add $5\'s$ work of extra loops).

each time we do this for $R^{2n-1}$ in odd, it doesn’t change. T-B invariant.
Local model:

Let $f$ be a function.

Graph of $f$.

How does $T(B)$ change? Decreases by $\Delta T(t)$ (domain of $t < 0$).

(Thm: $T(1-0, 1-0)$ is always, so this always decreases).

Stabilizes.

Claim: manifold set by surgery is $\mathbb{T}^3$.

But what's $S^4$ (this knot is a stabilization)?

Claim: it's always 0 if we stabilize.

(maybe $C^4$ sees more, like # stabilizations. Not sure yet.)

Stabilize in last picture:

$\partial c = 1$ in contact homology, 0 in other cases.

On $S^1$, we get $S^1$ next $\Rightarrow$ get two tools $c_{min} < c_{min}$.

And $c_{min} = 1$, $c_{max} = 1$. (Using 24 dimension of sphere, 0 in this

Get $c_{min}, c_{max}$, algebra.
Algebraic Lemma [Ng, Ekhola]: the case \( \Delta = \text{Lox, knot } - 1 \) are components.

\[
\widetilde{LH}(\Delta) = \left( \widetilde{LH}(\Delta), \hat{c}_1, \ldots, \hat{c}_k, x_0 \right)
\]

say

\[
\tilde{d}^n c = x c - x t + \delta_{H_0} c
\]

\[
\tilde{d} x = 0,
\]

is \( \sim L H_{\delta_0} \).

Ponchik: Each term have Lox, knot - 1 are components. Let \( \Delta_m = 1 \), then

\[
H(\bar{H}_{\delta_0}, \delta_{H_0}) = 0.
\]

\[c_0 \rightarrow c_0 \rightarrow c_0 \rightarrow c_0 \rightarrow \cdots\]

\[x \rightarrow x \rightarrow x \rightarrow x \rightarrow x \rightarrow \cdots\]

Use this lemma to prove ponchik. Use duality to prove Ponchik's Lemma: pencil.

Formalism by Rosloff to write down differential in this case.

Missing: description of the product.