INTRODUCTION TO TROPICAL ALGEBRAIC
GEOMETRY - EXERCISES

DIANE MACLAGAN

Lecture 1

(1) Download gfan from
http://www.math.tu-berlin.de/~jensen/software/gfan/gfan.html,
and play with it. Warning: Gfan uses the max-plus convention,
and requires homogeneous ideals.

(2) Write the arrangement $\mathbb{P}^2 \setminus \{x = 0, y = 0, y = x, y = -x\}$ as a
closed subvariety of a torus. Prove the general description for
how to do this described in the lecture.

(3) Compute $\text{trop}(Y)$ for the following $Y$, where $K = \mathbb{C}\{\{t\}\}$:
   (a) $Y = V(x^3 + 3x^2y^2 + y^3 - x + 2y) \subseteq T^2$;
   (b) $Y = V((t^2 - t^{5/2})y^2 + 5x^2 - 7xy + 8y - tx + t^3)$;
   (c) $Y = V(3x^2y + 2x^2 y^2 - xy^3 + 5xz - z^2)$;
   (d) $Y = V(x_1 + x_2 + x_3 + x_4, x_2 - 2x_3 + x_4)$.

(4) Show that if $Y = V(f) \subseteq (\mathbb{C}^*)^n$ is a hypersurface then $\text{trop}(Y)$
is the inner normal fan to the Newton polytope (convex hull of
exponent vectors) of $f$.

(5) Illustrate the fundamental theorem of tropical geometry (ie
check the sets coincide) for the following varieties.
   (a) $Y = V(x^2 + (t^2 + 1)x + t^2)$
   (b) $Y = V(x^2 + 2y - 3)$
   (c) $Y = V(x + y + z)$

(6) Compute $\text{in}_w(I)$ for the following $I$, and your choice of $w$:
   (a) $I = \langle x^2 + 3xy + 5y^3 - x - 2y + 1 \rangle \subseteq \mathbb{C}[x^{\pm 1}, y^{\pm 1}]$;
   (b) $I = \langle t^2 x^3 + x^2 y + xy^2 + t^2 y^3 + x^2 + t^{-1} xy + y^2 + x + y + t^2 \rangle \subseteq
\mathbb{C}\{\{t\}\}[x^{\pm 1}, y^{\pm 1}]$;
   (c) $I = \langle x + y + z, x + 2y \rangle \subseteq \mathbb{C}[x^{\pm 1}, y^{\pm 1}, z^{\pm 1}]$. 