Abstract III:

0) Family Floer homology

\[ Q \text{ smooth, closed, } \text{convexity radius } \frac{1}{2} \text{inj}(Q) \]

For Riemannian metric, \( L^\infty Q \)

\[ L^r Q \leq \text{space of piecewise geodesics, such that all segments have length } < \frac{1}{4}. \]

\( Q^r \) open subset (because \( 1 < \text{inj}(Q) \)).

Note: \( L^r Q \subset L^\infty Q \)

\[ r = \# \text{segments} \]

Explain maps

\[ HF^*_r (T^*Q) \quad \quad \quad H^*_n (L^r Q) \]

HF of any Hamiltonian + is mapping \( r \).

(i.e. \( H = r |p| \))

\[ HF^*_r (T^*Q) \rightarrow H^*_n (L^r Q) \]

Show that the compositions

\[ H_{n-r} (L^r Q) \rightarrow H_{n-r} (L^4r Q) \]

add \( \& \) constant segments, \( \& \)

\[ HF^*_r (T^*Q) \rightarrow HF^*_{4r} (T^*Q) \] continuous map.

Take direct limit \( r \rightarrow \infty \):

\[ H_{n-r} (L^\infty Q) \rightarrow SH^* (T^*Q) \text{ actually inverse. } \]

Advantages:
- No special choice of Hamiltonian
- Maps in both directions
I. From $H_\ast(2Q)$ to $H$:

Let $Q \in \mathcal{L}^2Q$

\[(q_2, \bar{q}_2)\).

Idea is to use $T^*_Q$ as Lagrangian boundary conditions. (from an small Hamiltonian?)

Let $\mathbf{A}$ be a function on $R$: 

\[ H = A(1p) \]

let $F$ be a line of slope $1$. 

\[ F = \text{line of slope } 1 \quad (x) \]

\[ \partial_x u = \partial_x u - X_h \]

Let $x$ be a $\text{Lie}_-1$ orbit of Hamiltonian $H$, $u$/slope $\approx r$.

Interpolate monotonically:

Slopes of Hamiltonians increase.

Asymptotic contact: dance on $h$ starting on $T^*_x Q$, ending on $T^*_x Q$.

For choice of Hamiltonian $(z)$: \( \exists A \) unique chord between successive tangent fibres.

Point: Conformal structure is fixed here.

\[ \mathcal{M}(x,y) \text{ is the model space of such maps.} \]

(Write more choices, etc... notice shouldn't be integral complex numbers)

\[ \text{put at cost of } A \]

(Write ODE maps).
Given a chain $C \in L^Q$, define
\[ M(x, e) = \bigwedge_{\gamma \in C} M(x, \gamma) \]
parameterized space over $C$.

Define a map
\[ F: C \times (L^Q) \to \mathcal{F}^x(H) \]
\[ F(C) = \sum_{x} \text{"ranks of moduli spaces"} M(x, e) \text{ which are rigid}. \]

Orientations:
\[ \text{det}(D_x) \otimes \text{det}(D_u) \sim \text{det}(D_x \otimes D_u) \]
\[ \text{det}(D_{\bar{z}_i, \bar{z}_j}) \text{ are normally trivial, so if you glue one up} \text{ an operator on the disc}. \]
So, \[ \text{det}(D_x) \otimes \text{det}(D_u) \cong \text{det}(D_{x^\times(TQ)}) \] \text{operator on } D^2 \text{ with log 3 conductors given by } D^\times(TQ). \]

da Silva, FOOO: If $Q$ orientable, then orientations of
\[ D_{x^\times(TQ)} \sim \text{spin structures} \]
parameterizations get orientable space of $C$ as well.
\[ \mathcal{M}(x) = \prod_{x \in \mathcal{L}(Q)} \mathcal{M}(x \setminus x). \]

\[ \mathcal{M}(x) = \text{Gromov-Hausdorff compactification.} \]

\[ (\text{exactness: no sphere or disc bubbling}) \]

\[ -\text{no step breaking or loft, b/c Hamiltonian chords are unique?} \]

Q: Why is it compact? \( L^4 Q \) is not compact (strictly speaking)

Length of geodesics could grow very long, reach \( \frac{1}{4} \) (go outside \( L^4 Q \))

(Otherwise: no problem, b/c we started w/ a compact chain \( C \).

We may a priori end in locally finite chains).

Main idea: Integrated maximum principle.

One if the chords escapes outside the unit-disc bundle

e.g. \[ h \]
If actions have correct sign, then $\tilde{\omega}(T^*Q \setminus D^*Q) = \varnothing$.

[See A.-S. Galot '07 on Virtue Functionality (wrapped HP).

This requires the # $Y$ for integrable minimum principle]

IV) $SH^x \rightarrow H_\ast (Q \cup Q) \rightarrow SH^x$

\[\text{ignore Y's issue} \]

\[\text{glue, r discs missing} \]

Let sequences of discs go to 0.

\[\text{discs attached to r points}
\text{ghost discs: have automorphisms}
\text{contraction map with some constants:}
\text{claim: that constant is 1!} \]

$\gamma^0Q \subset Q_r$
Point: The boundary conditions on discs sweep $T^*Q$.

$T^*Q \ni$ only constant solutions (6/c exact)

Small problem: $T^*Q < Q^*$ but not all of $Q^*$.

Solution: (1) Define the map over all of $Q^*$ & note by maximum principle, all solutions stay in $L^*Q$, so no constant.

There is no constant $\Rightarrow$ the other boundary part in the annuli space of annuli is contracting...

Idea: Can replace studying "A structure" by studying instead families of Lagrangians/objects.

In the other direction, (ignoring $4v$)

Bean, Cohn, Fedosov, A, Gonatra

Only solution is $\bar{q}_i = q_i^*$.

This should have some analogy in (t+1)th, where $G(H) \subset QH$ & $H_0(\mathcal{W})$ analog of some sheaf (algebraic)