Abnormal $T^Q$:

\[ \nabla \cdot (Q \cdot \nabla) \eta = -pdg \quad \omega = da \]

Even floor has: $H: T^Q \to \mathbb{R}$.

Consider clean up $g$, by thee -- 1 orbit of $H$.

If $GF^+(M)$ differentiated "good" hole discs.

Warning: Acting $H$, $d^2 \neq 0$.

Then even in $H$, flow $f: M \to \mathbb{R}$ Maxe - Maxe.

\[ m = s^2 \quad f: \]

\[ 0 \leq m = s^2 - b(1 - d^2) \neq 0 \]

For technical reasons, for a metric on $Q$, consider $H$ with agree with $b \cdot |p|$ for $|p| \to 0$, where $b \in \mathbb{R}$.

\[ T^S \]

e.g. \[ \forall m \in \{ T^S \} \]

\[ \phi_m \left( T^S \right) \]

Obv. Identify $T^Q = \mathbb{R}$ with metric on $H$. Flow of $b \cdot |p|$ agree with $b$-geodesic flow in unit sphere bundle.

For an explicit model of $H$, building $H$.

\[ \text{HMC:} \eta \left( \text{flow of } b \cdot |p| \right) \]

\[ \text{HMC:} \eta \left( \text{flow of } b \cdot |p| \right) = \{ \text{geodesics on } \mathbb{S} \} \]

\[ \text{HMC:} \eta \left( \text{flow of } b \cdot |p| \right) \to \text{HMC:} \eta \left( \text{flow of } b \cdot |p| \right) \]
\[ \lim_{\text{up to} \infty} H^x \rightarrow (1, \Phi H) \quad \lim_{\text{up to} \infty} H_\Phi \rightarrow (2, \Phi) \]

\[ \text{Stokes on these homology groups.} \]

\( SH^* \rightarrow \mathbb{Z}/\mathbb{Z} \quad \text{graded abelian group.} \]

What's the grading? \( H \rightarrow \Phi H \quad \mathbb{H} = \mathbb{Z}/\mathbb{Z} \).

Take \( \Delta \in \mathbb{R} \times \mathbb{R} \), \( \mathbb{T} \times \mathbb{R} \).

Unoriented, grading of \( x \) is the intersection grading of \( \Delta \). \[ \Delta = \int_{\Delta} \Phi H \] up to shift by \( \mathbb{Z} \).

(If \( \mathbb{Q} \) is orientable, then this lifts to \( \mathbb{Z} \) grading,

\[ \delta (T^* \mathbb{Q}) = 0 \].

Need a laminarization of \( \pi^*(T^* \mathbb{M}) \) for every loop \( x : S^1 \rightarrow T^* \mathbb{Q} \).

(Use laminarize to get a path of symplectomorphisms, strict/malcev cycle)

Pick an orientation of \( \mathbb{Q} \). This determines an orientation of

\[ (2 \times \mathbb{Q})^* (T^* \mathbb{Q}) \]

(orient to base)

Lagrangian sub-bundle of \( \pi^*(\mathbb{T}^* \mathbb{Q}) \).

Claim: \( S^3 \mapsto \mathbb{C}^* \) \( \text{monodromy of } \pi^*(\mathbb{T}^* \mathbb{Q}) \text{ up to homotopy mapping} \)

this loop of Lag's + a loop of Maslov index 0.

(Thus fixes the choice of laminarization).
\[ \{ \pm 1 \} \cong \mathbb{Z}/2 \Rightarrow U(1) \cong S^1. \]

(since orientations did not matter in the end).

**Algebraic structures**

\[
\begin{array}{c}
\text{asympotic markers: extra data.} \\
\end{array}
\]

\[
\begin{align*}
\Sigma \setminus \{p_0\} & \to M \\
\end{align*}
\]

\[
\begin{align*}
&\to \text{HF}(H^3) \to (\text{HF}(H^2))_0^2 \\
\end{align*}
\]

How to force convergence among punctures? Aligned at punctures don't have exact points, but orbits do.

Use asymptotic markers.

\[
\begin{array}{c}
\text{Moduli of Riemann surfaces with choices defines operad} \\
\text{on } \text{SH}^*(T^\infty Q). \\
\end{array}
\]

We'll discuss genus 0, one loop case.

Work at level of cohomology.

**Generators:** Two maps "generate" all such operations:

1. pair of pants product
2. BV operator.

\[
\begin{align*}
\Delta &= \frac{2a}{g+1} \\
\end{align*}
\]

Relation: \( \Delta(abc) = (\Delta a)bc + \cdots \)

\[
\begin{align*}
\Delta(ab)c + \cdots \\
\end{align*}
\]

\[\text{draws by } 3, \text{ loops by } 1.\]

\[
\begin{align*}
\frac{2a}{g+1} &= J(\frac{2a}{g+1}, X) \\
\end{align*}
\]
(1) $S^1$ acts on $LQ$

\[ H^*(LQ) \rightarrow H^*(LQ) \otimes H_1(S^1) \]
\[ \alpha \rightarrow \alpha \otimes [S^1] \]

\[ H^* + 1(LQ) \rightarrow \Delta \]

(2) Chas–Sullivan product: If $Q$ is oriented,

\[ H_p(LQ) \otimes H_q(LQ) \rightarrow H_{p+q-n}(LQ) \]

Defined by fiber product over $Q$:

i.e. ev: $LQ \rightarrow Q$

restrict to generic locus where loops agree at $t=0$

concatenate loop, (if transverse)

$\rightarrow$ BV structure.

**Guess:** These are isomorphic BV algebras.

**Reality:** Only true if $Q$ admits a spin structure (due to Krzysztof). Generating further repetitions.

Sendai confirmed this for $T^2, CP^2$.

Correct statement involves twisted versions of loop homology and symplectic cohomology.
Q not spin.

\( \omega_2(TQ) \in \text{H}^2(Q; \mathbb{Z}_2) \) doesn't vanish.

\( \text{Stiefel-Whitney class} \).  

\[ \text{consider real space of rank 1} \]

\[ \text{How to define } \sigma \text{ on homology?} \]

\[ \text{have } \text{H}_3(LQ, \mathbb{Z}_2) \xrightarrow{f_*} \text{H}_2(Q; \mathbb{Z}_2) \]

\[ \text{obvious map: take the cycle represented by the loop space.} \]

\[ f^* \text{ dual.} \]

\( \sigma_{TQ} \) the associated local system to \( f^*\omega_2(TQ) \)

\[ \text{to a loop } x \in LQ \text{ associate monodromy } \Rightarrow \text{if } \omega_2 \text{ (associated to } f^* x \text{) evaluates finally} \]

\[ -1 \text{ otherwise.} \]

**Assertion:**

\[ \text{SH}^{-w}(T^*Q, \sigma_{TQ}) = \text{H}_{-w}(LQ, \mathbb{Z}) \]

as BV algebras.

This has a generalization to non-orientable manifolds \( Q \).

**More general result:** for a local system \( \gamma \) on loop space,

\[ \text{SH}^{-w}(T^*Q, \gamma \otimes \sigma_{TQ}) = \text{H}_{-w}(LQ, \mathbb{Z}) \]

as BV algebras, if \( \gamma \) in image of \( \text{H}_2(Q; \mathbb{Z}_2) \rightarrow \text{H}_1(LQ, \mathbb{Z}_2) \).

So take \( \gamma = \sigma_{TQ} \) to get a formula for \( \text{SH}(T^*Q, \mathbb{Z}) \).

**Remark:** this is twisted version of \( \text{QH}^{-w}(M) \) in case of \( M \) not flat.

**(Note:** we only show up for discs, not spheres.)