Day 1, Nadler — Overview

Theme: Unity of Quantum Geometry of Symplectic Manifolds

Warmup: Cohomology of a manifold $X$

Ex.: $X = S^2$.

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3 viewpoints:

Topological

$C^*(X) = \{ \text{sing. cochains} \}$

Algebraic

$\Omega^*(X)$

"fulfills" terms

Analytic

Morse complex = $\mathcal{M}^*(X)$

Let:

\[ s_i = \text{cost.} \]

\[ x(i) \]

\[ (0) \]

\[ x(1) \]

\[ (1) \]

\[ 1 \rightarrow \] cell

\[ \rightarrow \]

\[ \rightarrow \]

\[ \rightarrow \]

\[ \text{cell} \]

\[ H^*(S^1) = 0 \] k.

pos: uses as little structure as possible

use smooth manifold

use metric, ODE, etc., but get very finite object.

"expected in exact structure, e.g., metric"

1) "Quantum" is already here: linearity (e.g., superposition)

8) Cohomology should mean the chain complex.

picture to keep in mind:

$S^3 \text{ that } S^2$. classical easier problems with easier problems abstract.

$S^2$
usual spectral sequence:

\[ H^*(S^1) \xrightarrow{k} H^*(S^2) \xrightarrow{k} \cdots \]

connecting map: extra information - comes from chains, not cohomology.

\[ H^*(S^2) \]

**Symplectic fields:**

\[ H, \omega \]

\( \omega \) : closed, non-deg. 2-form.

**Darboux:** locally, \( \mathbb{R}^2 \), \( \omega = \sum_i x_i \, dy_i \).

"quantum" means noncommutative deformation.

For coordinates \( x_i, y_i \):

\[ x_i \cdot y_j - y_j \cdot x_i = \delta_{ij} \]

Want to study: submanifolds & other geometric objects:

quantum means they make sense after this noncommutative definition.

\( \rightarrow \) Uncertainty principle: NCM must be coisotropic.

In particular, smallest submanifolds are \( \mathbb{C} \)M lagrangians.

(\( \mathbb{C} \)M cut out by quantum braids. i.e. braids kill off \( x_i, y_i \)).

Ex. \( \mathbb{R}^2 \):

\( \{(0,0)\} \) cut out by \( x = 0 = y \).

In noncommutative world, \( x \cdot y - y \cdot x = 1 \), impossible.

But \( y = 0 \) works!

(need one-sided ideal... etc.)
need a left ideal \( \langle y \rangle \)

not necessarily interested in \( \phi_y \) of funs. on \( \mathfrak{g} \).
interested in module, \( \Phi_y \) of funs.
left quotient \( \langle y \rangle \)

Associativity:

\( \mathfrak{g} \) would like:

\( (M, \omega) \rightarrow \) quantum category whose \( \text{objects} \) are \( \omega \)-vanes general co-isotopic sub-

\( \text{morphisms} \): quantum interactions.

Basic case: \( M = T^* X \) e.g. \( X = S^1 \)

\[ \text{Toric}
\]

Const. sheaves

on \( X \)

"Lag'n theory"

\( U \)

(coisotropic sheaves)

\( X \) complex.

\( D \)-modules on \( X \)

"Coisotropic theory"

\( \text{Fix} \) sub-

\( \text{Analytically} \)

Inf. Fukaya category

"Lag. theory."

Replacement: wrapped Fukaya category.

Two outside are equivalent, \( \text{middle is richer but contains "Lag." subcategory.} \)

"Yes and I will intersect if at some point in the future we ever cross paths."

Why success here? \( \mathfrak{g} \) a dilatation:

1) attracting dilatation. (exact structure)

2) polarized: \( \text{Lag'n foliation by fibers of } T^* X \rightarrow X \)

Towards end of week: try to remove \( \Phi_y \).
Ex: 1) $M \text{ Kähler } \subset \mathbb{CP}^n$

$V \quad H = \mathbb{CP}^n.$

$M \setminus (M \setminus H)$

exact (we have trivialized the Kähler class)

so $e$ contracting dilatation.

2) $\leq$ Kleinian surface singularities

$\xrightarrow{\text{Day 4}}$

$\subseteq$ symplectic resolution.

$\xrightarrow{\text{?}}$ orthogonal bundle if this is true!

**Day 4. Torsion examples:**

1) $\mathbb{R}^2 \xrightarrow{\phi} M = T^* (S^1)^n \longrightarrow (S^1)^n$

$\xrightarrow{\text{(IR)}^n}$

2) $M = T^* \mathcal{O}_Y = \mathcal{O}_X \times \mathcal{O}_Y \longrightarrow \mathcal{O}_X$

Fourier transform.

Gen. principle: Quanta geometry of $\rightarrow$ classical geometry.

$H_{\text{nucl.}}$

$H_{\text{phys.}}$

Is this plausible? Can relate them via this picture?

**Today:** Smooth geometry is wild.

1) $\rightleftharpoons$

Time geometry (finite type)

b) chain complexes $\rightarrow$ homological algebra

"reasonable way"

3) Topology of subspace of $X \rightarrow$ constructible sheaves.

(?) Topological quantization of $\mathbb{R}$ $\rightarrow X.$