Ahoz and: Simple Happy of nearby Log'ing (joint w/ T. Kragh)

Ahoz'd Conj: \( \text{if } Q \text{ is closed w/ } L \stackrel{\sim}{\rightarrow} Q \text{ closed exact} \)

Log', the L is the notopic to zero section

FSS, Nadler, & Kragh: Under these assumptions,
the projecton \( L \stackrel{f}{\rightarrow} Q \) is a homotopy equivalence.

Recenly, Kragh has found a proof which does not require using "Asa category."

Guillemin has a proof using microlocal sheaf theoretic methods

Today's result: says that \( L \times [0,1] \cong Q \times [0,1] \) homeo

Hilbert cube

This is equivalent to \( L \& Q \) being "simply homotopy equivalent."
also equivalent to

vanishing of Whitehead torsion of map \( f \).

Whitehead torsion

Fix a group \( G \), & consider
\[ R = \mathbb{Z}[G], \]

Consider pairs \( (C, B) \) where \( C \) is a ch. split over \( R \) & \( B \) is a basis.

Mod out by the following equiv. reln:

1. Change basis: any basis elt. by multiplier \( s \cdot y \equiv y \) (\( g \in G \)).

2. Apply an upper-triangular change of basis w/ 1's along diagonal.

3. Allow adding a summand of form \( R \rightarrow \mathbb{R} \).
The collection of acyclic complexes \( (\mathbb{S}_s, s) \) and relations form
the Whitehead group \( \text{Wh}(G) \).

Intuition: Say \( M \xrightarrow{f} \mathbb{R} \) Morse fan, under the lift
\( \tilde{f} : \tilde{M} \rightarrow \mathbb{R} \) to universal cover.

\[ CM \times (\tilde{f}) \text{ naturally a } \pi_1 M \text{ module (not acyclic, but operators clear).} \]

1. Choice of lift of crit. pt. \( m \in M \).
2. "Elementary transformations" \( (\cdot, \cdot) \)
   - Oper. appears when constants map proof of invariance.
   - Clearly map proof.
   - Clearly map.

3. Bousfield

What is the Whitehead torsion of a map \( L \rightarrow Q \)?

Let's approximate this by a cellular map.

(Translating...

Then \( \mathbb{C} \xrightarrow{f} \mathbb{C} \mathbb{Q} \) cellular chains

Induces us on homology.

\[ \text{Cone}(f) \mathbb{C} (\tilde{Q}) \oplus \mathbb{C} (\mathbb{Q}) \mathbb{L} \]

\( (d, f) \) is an acyclic cplx.

with a preferred cellular basis from \( \rightarrow [f] \in \text{Wh}(\pi, Q). \) That \( g \) can be defined...
(Mc Sullivan): Assume $L \subset M$ satisfies $HF^*(L; \mathbb{Z}, L) = 0$. Then, the algebraic torsion $\text{tor}_W$ of $\text{CF}^*(L; \mathbb{Z}, L)$ is well-defined & invariant under homotopy equivalences.

$HF^*(L) \sim HF^*(L, \mathbb{Z}, L)$

$\text{CF}^*(L, L; \mathbb{Z}(\pi, L)) \cong \bigoplus \mathbb{Z}[\pi_1 L]^\times \times \text{Ext} \mathbb{Z}[\pi_1 L]^\times$

Keep track of http://class.

Extend this def to $HF^*(L, Q; \mathbb{Z}(\pi, L))$, torsion associated to the pair is invariant.

Def: $L \& Q$ are Lagrangian in a symplectic $M$, then we say that $L \sim Q$ if they are Floer-theoretically equivalent. Thus $u \& v \in HF^*(L; Q; \mathbb{Z}) \Rightarrow HF^*(Q; Q)$

$s.t. \quad u \circ v \in HF^*(Q; Q)$

This makes $L \& Q$ are homotopic.

Proof (sketch): Suppose $L \sim Q$. These assumptions imply that $HF^*(L; L) \Rightarrow HF^*(L; Q) \Rightarrow HF^*(Q; Q)$ are all weak. I also consider superscript $u \& v$.

$u \& v$ are uniquely defined up to sign.

Remark: in exact setting, can use action.

Modified, & proof is much shorter than bifurcation analysis.

Lemma: (A Vaug) If $L, Q \subset M$ are Floer-theoretically equivalent then a map $\text{CF}^*(L, L; \mathbb{Z}(\pi, L)) \to \text{CF}^*(Q, Q; \mathbb{Z}(\pi, M))$ is a quasi-isomorphism.

Torsion is independent of homotopy equivalences.
We know that there are Floer-theoretical Fukaya categories.

So, we obtain a map \( \mathcal{C}^*(L; \mathbb{Z}_\pi, \mathcal{Q}) \to \mathcal{C}^*(Q, \mathbb{Z}_\pi, \mathcal{Q}) \)
which is acyclic \& has trivial torsion.

(From Floer theory).

1) Show that this agrees w/ classical torsion.
2) Show that it vanishes.

Invariance says that the torsion of the map above agrees w/ \( \mathcal{C}^*(L, L'; \mathbb{Z}_\pi, \mathcal{Q}) \to \mathcal{C}^*(Q, Q', \mathbb{Z}_\pi, \mathcal{Q}) \) whenever \( L' \sim L \& Q' \sim Q \).

To define an analogy, pick a Morse from \( F: Q \to \mathbb{R} \) \& let \( Q + tF \to \text{graph of } DF \).

Define \( L^2 = \text{tr}(DF) \) be image of \( L \) under “flow over additive” \( tF \).

\[ L^2 = Q + tDF \quad \Rightarrow \quad Q \]

This uses trace class \& value of \( F \).

The action filtration on \( \mathcal{C}^*(L, L + tDF) \) breaks up into subfiltered pieces (below)
by \( \text{Crit } F \); i.e. Actions of integers \& “concentrate” near the values of \( F \).

Exercise: 

The map \( \mathcal{C}^*(L, L + tDF) \to \mathcal{C}^*(Q, Q + tDF) \) induces an “\( \mathcal{Q} \otimes \mathbb{R} \)”
series of s.s. assoc. to the above filtration (i.e. \( \mathcal{Q} \otimes \mathbb{R} \) have small action).

If we work over \( \mathbb{Z}/2\mathbb{Z} \), then the homology of each assoc. graded is free \( \mathbb{Z}/2\mathbb{Z} \).

To compute the torsion of the Floer theorems map, just need to compute the “diagonal term.”

If there was a large part, the disc would have one \( \Delta \) above everything
with a disc \& maximal area to be maximized has large energy \( E \) \( \mathcal{Q} \otimes \mathbb{R} \) (but energy bounded).

Pretend statement: To map an assoc. graded is obtained for map
of \( \mathcal{Q} \) spheres over \( \mathbb{Z} \), by knowing \( \mathbb{Q} \otimes \mathbb{Z}_\pi, \mathcal{Q} \). (\( \Rightarrow \) no twisted torsion).
Part (1) 

\[ L : T^*Q \to \mathbb{R} \]

\[ \text{a map} \]

\[ h = \frac{\mathbf{p}^2}{2} + \Phi, \quad \text{this is } L \text{ in } \phi. \]

\[ \text{use } \Phi \text{ to } \frac{\mathbf{p}^2}{2} \text{ instead.} \]

\[ \text{say } h = \frac{\mathbf{p}^2}{2} \, \mathbf{q} \]

\[ \text{cautious } b/c \quad q \wedge v = 1 \]

This has some basis as original map. But, \( L \) is exact, so disc constant.

So only more flowing outside \( \Rightarrow \) classical form.