The \( \dim_{\mathbb{R}} = 2 \) case:

- \( X \) open Riemann surface
- \( Q \) ribbon graph giving a skeleton of \( X \)
  (i.e. graph + cyclic ordering of edges adjacent to each vertex)

\( \rightsquigarrow B \) cosheaf on \( Q \):

- stalk at a smooth point = category with one object, morphisms \( k, \text{id} \).
- at a vertex of valence \( m = \begin{array}{c}
\end{array} \end{array} \) \( m_0(x_1, \ldots, x_m) = \text{id} \)

\( \Rightarrow \) the disc giving \( m_0 \).

The criterion of yesterday's talk holds:

\( H_\ast(Q, HH_\ast(B)) \longrightarrow H_\ast(X, \Omega X) \)

\( [Q] \longrightarrow [X] \)

Higher dimensions: first example

- A compact smooth manifold, choose a triangulation of \( Q \)
- get \( B = \text{contrat cosheaf whose stalk is } \ast \text{id} \) \((= \text{HF}^\ast(T^*_Q, T^*_Q)) \)
  in "unwrapped" (Nadler-Zaslow) sense.

This gives yet another proof that

\( \text{Junk}_{\text{compact}}(T^*_Q) \hookrightarrow \text{mod-}C^\ast(Q) \) (assuming \( \tau_q Q = 0 \))
Next example: A locally modelled on \( \bigtimes \times (n-1)\)-disk

ie. \( Q_i \) compact smooth manifold w/ boundary
\( P \) compact smooth \((n-1)\)-mfd (not neces. connected)
\( \partial Q_i \to P \) diff., on each compact
\( Q = (U Q_i)/\sim \) ~ identifies pts of \( \partial Q_i \) with same image in \( P \).

Choose also a locally contract cyclic ordering of pre-images of pts of \( P \).

\[ \Rightarrow \] local coord's on \( Q \)

Again stalks = \( \oplus \text{id} \) at smooth pts, \( \oplus \mu^k \text{id} \) at singular pts.

This is a skeleton for \( X = \left( \bigcup T^* Q_i \right) \cup \left\{ T^* P \times \bigstar \right\} \)/\( \sim \)

(“boundary plumbing” of \( T^* Q_i \)'s at \( \partial Q_i \)).

Valency depends on valency at compact of \( P \)

Ex: \( P = 2 \text{pt's}, \ Q_i = \text{intervals} \)

At \( p \in P \), the Lagrangian we take are \( T_p^* P \times \bigstar \)

By projecting ‘locally’ we compute the stalks at \( p \in P \) to be as above
\& see \( \text{Assoc. structure} \) is as in the \( \text{Id} \) case.

This category satisfies the generation criterion of yesterday's talk

\( \Rightarrow \) can compute \( HF^*(\kappa, \kappa) \) from the associated modules.

Goals: understand how usual plantings can be understood in terms of this picture.
Setup: $N_1, N_2$ closed smooth manifolds

$$B \hookrightarrow N_1 \preceq N_2$$

embedding with isomorphic normal bundles $\cong V$

$$V_c \cong T^*B \ (\cong \text{real of } B \text{ inside } T^*N_i)$$

can be an involution exchanging factors in $V_c \cong V \oplus V$

$\implies$ plumbing $= \text{ glue } T^*N_1 \text{ to } T^*N_2 \text{ along } V_c \cong T^*B$

$$= T^*N_1 \boxplus T^*N_2$$

Example:

$$\begin{array}{cc}
\text{pt} & T^*\mathbb{R} \\
\mathbb{S}^1 & T^*\mathbb{R} \\
\end{array}$$

For codim $B = 1$ this clearly looks like above (make 4-valent vertices)

diagram $B$ along $B$

Problem: If codim $B > 1$, then there's no nice family of lines in $T^*N_1 \boxplus T^*N_2$ parameterized by $N_1 \cup_B N_2$.

Example: $N_1 = N_2 = D^2 \supset B = \text{origin}$

Idea (Kotschik?): Replace $\bigcirc$ by $\bigstar$

i.e. plumbing has a skeleton with 3 "smooth components"

$$\begin{array}{cc}
\{N_1 - \text{open nbd of } B\} & \{N_2 - \text{open nbd of } B\} \\
\wedge \Downarrow \text{cyclic ordering} & \\
\{\text{disc bundle of normal bundle } V \cup_B \} \\
& \{\text{open nbd of } B \text{ in } N_i\}
\end{array}$$
\textbf{Applying (A-Smith):}

\[ N_1 = N_2 = S^n, \quad B = \text{pt}, \quad X = T^k S^n \times T^k S^n \]
\[ = A_2 - \text{ACE space} \]

in \( X \), all exact layers of \( \mu \) index 0 are Fukaya-isomorphic to an iterated Dehn twist of \( N_2 \) (mod \( N_1, N_2 \)). In particular they're all homology spheres.