M. Atiyah, Family $\mathcal{F}$ (II)

Last time's Associates to $L \cong X$ or identification of $\mathcal{F}$ with $L$, and to $L'$, a module

over this.

Each sufficiently small norm on $H_1(L)$ is ab of $H_1(L)$, so relies $H_0(L)$, w/ 

associated not $k$-ll $P \in H^1(L, \mathbb{R})$, we can find a completion

$H_0(L \leq \hat{H}_0(L) \leq H_0(L)$

so that the module is defined over this smaller ring.

Consider $L$ which is aspherical

$\Rightarrow \chi_i(L) = 0 \quad (i > 1)$.

Then, $H_0(L)$ is or supported in degree 0.

to $\mathcal{F}$, of course deform $\geq$ of the cost of rigid hol. does 

passing through the base point vanishes.

(Maslov 2)

Because $H_0$ vanishes,

$\Rightarrow$ The deform is necessarily toral.

$\Rightarrow$ The space of "bending of charts" is toral.

(Zolot and Veeb's)

The class of index 0 define a deformation of the product on $\hat{H}_0(L)$, those of 

index $\geq 2$ introduce $\text{mor}_2$, $\rightarrow$. Beyond $\text{mor}_2$, get no contradiction.

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Specialize to $L = T^n$. Recall that

$H_0(L) \cong \Lambda [\frac{\mathbb{Z}}{2\mathbb{Z}}], \ldots, \frac{\mathbb{Z}}{2\mathbb{Z}}]$ (or $\Lambda [H_0(L, \mathbb{Z})]$).

The norm field

When we complete, get $\text{extension}$ Laurent series which converge on the unitary element (i.e.,

$v \leq \left(\begin{array}{c} b_0 \\ b_1 \end{array}\right) = 0 \forall i$)

$v \ni \left\{ \begin{array}{c} \mathbb{Z} \\ \mathbb{Z} \end{array} \right\}$

$\Rightarrow \nabla < \mathbb{Z}, \mathbb{Z} >$

So, $\mathbb{Z} < \mathbb{Z}, \mathbb{Z} >$ but alg. structure possibly defined too.
Consider the closed ideal generated by $m$, $\text{ideal}(m)$, and take the radical of this, and take its closure.

Take critical loci of $m$ as an analytic subvariety of $\mathbb{H}_2(L, U_L)$.

Local system $\Delta$, ideal $\mathfrak{m}$, module $\Delta[\mathbb{Z}_2, \mathbb{Z}_2]$.

Hence this gives rise to a local system $L$, $E_L$.

$\Rightarrow (L, E_L)$ is a subobject of the Frobenius category of $X$.

Ans: The subcategory of $\Theta(X)$ generated by "unobstructed local systems $L"$ embeds fully faithfully in modules over $HF^*(L, E_L, L, E_L)$.

(Remark: It may not split-exact, just split-sense in this sense.)

This is an algebra conjecture: does $\text{Crit} W$ generate $MF(W)$ when $W$ arbitrary bad?

Today: Local $\Rightarrow$ Global.

Assume that $X$ is a Lagrangian torus fibration with no singularities.
Write $X_2$ for fibre at $q \in \Omega$. 

play the role of $L$ in prev. discussion/yesterday

Let $L \subset X$ some other Lag in $L'$ (l' from yesterday).

Goal: Reconstruct $H^\ast(L, L)$ from $H^F(L)$.

\[ H^F(L, X_2) \quad \text{put } \Omega X_2 \]

Recall that $\Omega$ is an integral affine manifold, meaning that for $q \in \Omega$, $\mathcal{P}$

\[ \mathcal{P} \leq H^1(X_2, \mathbb{R}) \]

convex polygon

any such $\mathcal{P}$ or

\[ \text{int}_+ \text{ ball for a norm on } H^\ast(X_2, \mathbb{R}), \quad \text{6. hence guess} \]

\[ H^\ast \Omega X_2 = \Gamma_p \]

The name will be an analytic space

\[ Y = Y_p \subset H^1(X_2, \Delta^\times) = (\mathbb{C}^\times)^n. \quad \text{Below this is} \]

\[ y \downarrow y_p \text{ via (P)} \downarrow \text{val} \]

\[ \mathbb{C} \quad \mathbb{P} \leq H^1(X_2, \mathbb{R}) \]

\[ \Gamma_p = \text{Ring of functions on } Y_p \text{ are} \]

analytic forms on $(\Delta^\times)^n$ which converge on $Y_p$.
Last time, claim that \( \mu \) is \( \mathcal{O} \)-torsion, \( \Rightarrow \) \( \mathcal{O} \)-torsion under all choices.

\( \Rightarrow \) coherent sheaf over \( \mathcal{Y} \).

If we could give them together, then we would get a coherent sheaf on \( \mathcal{Y} \).

We need to work at the clear level to get gluing to work.

The module over \( \mathcal{Y} \) arises from \( \mathcal{O}^*_{\mathcal{Y}}(\mathcal{X}_2, \mathcal{L}) = \bigoplus_{x \in \mathcal{L} \times \mathcal{X}_2} \mathcal{O} \). 

perfect \( \Sigma \) gives rise to a finite complex of free \( \mathcal{P} \) module, hence \( H^0 \) is a coherent sheaf.

In order for this complex to be defined, we need \( \mathcal{L} \times \mathcal{X}_2 \), but it is implausible that this holds for all \( \mathcal{L} \).

In addition to transversality, need that \( \mathcal{P} \) is "small enough" that the isoperimetric inequality \( \Rightarrow \) convergence.

\( (\text{Area}(\mathcal{D}))^{\frac{1}{2}} \propto \mathcal{O} ) \) depends on \( \mathcal{X}_2 \) and \( \mathcal{L} \).

Redraw: Replace this estimate by existence of a diffeomorphism \( \psi_2 : \mathcal{X}_2 \to \mathcal{P} \), \( \forall \mathcal{L} \in \mathcal{P} \).

1. \( \psi_2^* (\mathcal{L}) = \mathcal{L} \)
2. \( \psi_2^* (\mathcal{X}_2) = \mathcal{X}_2 \).
3. Preserves tautness of almost complex structure.

Note: \( \psi_2 \) sends arcs \( J \) on \( \mathcal{X}_2 \) to \( J \) on \( \mathcal{X}_2' \), all that changes...
Choose a path $\gamma \in \Omega$, a Lagrangian $L_\gamma$ which is then isotopic to $L$.

Then, get $M_2$: complex of modules over $\Gamma_{P_2}$ where $P_2$ is a small nbhd of $\gamma \in \Omega$.

For $\{P_i\}$ each small, then

$$M_2 \otimes \Gamma_{P_2} \sim \to M_2 \otimes \Gamma_{P_2}\Gamma_i$$

Twisted sheaf short after taking cohomology

On chain level, need higher cohomology

The covering dimension of basis is $n$, only need to refine $P_i$ finitely many times.

(If base is not $S^1$, $\gamma$ have a loop in base, want to know doesn't affect Spin structure.)

What is the twisting sheaf?

Lines on $H^2(Q, \text{Aff}) \to H^2(Y, \mathcal{O}^*)$