M. Gross, "MS and the SYZ Conjecture"

arXiv: 1312.4220

Shawinger, Yay, Zaslow 1996.

Calabi-Yau manifolds: means \( K_X \simeq \mathbb{C} \). No. of holes in cycles = periods of \( \lambda^+ \) on cycles.

\[ \mathbb{D} \text{Coh}(X) \simeq \mathbb{D} \mathcal{F}(X^\vee) \]

\[ H^4(X, \Lambda^+ T_X) \cong \mathbb{Q} H^4(X) \]

\[ \mathfrak{g} \text{aff} = \text{GL}(n, \mathbb{Z}) \times \mathbb{R}^n \]

\( \Lambda^+ = \{ \xi \in T^* B \mid \langle \xi, \omega \rangle \in \mathbb{Z} \} \)

\( \mathfrak{h} = \text{complexification } T^* B / \Lambda^+ \)

Problem: Not many spaces look like this. Special varieties are Kodaira-Tian-Donaldson.

\[ \mathfrak{h} \]
SYZ: "KS is Tduality"

Leung-Yau-Zaslow: Explains how to try to implement this in the context; think of KS as a Fourier transform in this case (not clear outside first case).

HMS:

coherent sheaves

e.g. hol. vector bls on 6. side \( X = T \mathbb{B} / \Delta \)

\[ \uparrow \]

Lagr. submanifolds on sympl. side \( X^* = T^* \mathbb{B} / \mathbb{B} \). + extra data.

\[ \text{we'll use this as a sanity check.} \]

Actually... singular fibers.

B "singular affine manifold?"

e.g. K3 surface \( \frac{3}{3} \sum_{i=0}^{3} x_i^{4} = 0 \) \( \mathbb{B} \subset \mathbb{C}P^3 \).

Use \( \text{hyperbolic} \) gl. to turn elliptic fibration into slct fibration (for not).

Get:

\[ \text{le3} \]

\[ \downarrow \]

\[ \text{S}^2 \]

\[ \text{semitoric on integrable syst.} \]

\[ \text{(good sympl. local models)}. \]

\[ \frac{24}{3} \text{ sing. pt's}. \]

\[ \text{How to build a opt. model? (tricky to glue together pieces/vigidity)} \]

Reconstruction problem.

Also, how to unk down a KS fibration? Essentially impossible to do.
Can still unk down base thing.
Idea: $X \to$ regular affine $\mathbb{B}$

exercise: affine $\mathbb{A}$

$X^v \sim \mathbb{B}^r$?

Tropical geometry (degenerate case: stick to some limit) "stretch the neck."

log geometry

= study algebraic degenerations of certain types.

e.g. $\mathcal{F} + \sum x_i y_i + x_0 x_1 x_2 x_3 = 0$ $\forall i$ still $k_3$

$+ = 0 \implies \text{get}$

log tilde; draw amoebas

large rank, converge to

$\rightarrow$ continuous $\&$ PL

get

real picture

$\sim$ piecewise linear graphs, -1 calculus.

Then, reconstruct using log geometry.

Gross-Siebert: in an alg. framework, can carry out this process using

contraction/tropical/log geometry.
\[ x_0 x_1 x_2 x_3 = 0. \]

(1) Local model: \( \mathbb{C}^4 \rightarrow \mathbb{C}^2 \)

\[ (t, x, y) \quad \mapsto \quad (x, y) \]

\[ xy = 0 \]

\[ |z| = \text{const.} \quad \text{or} \quad |y| = \text{const.} \]

(2) s.t. minor symmetry

(3) Active unfoldings of singularities

(4) Tropical geometry

In the paper of "Tropical geometry" by Lim, need to add a polynomial. In the paper of "Tropical geometry" by Lim, need to add a polynomial. In the paper of "Tropical geometry" by Lim, need to add a polynomial. In the paper of "Tropical geometry" by Lim, need to add a polynomial. In the paper of "Tropical geometry" by Lim, need to add a polynomial. In the paper of "Tropical geometry" by Lim, need to add a polynomial.

Problem: it is not always non-zero as we smooth.

\[ xy = t \]

\[ |x| = \text{const.} \quad \text{and} \quad |y| = \text{const.} \]

\[ \text{deg}_x y \]

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