Lekili - Poutz:

\[ \Sigma \]

Morse (not self-indexing, *)

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Qultz Flora: Han.

\[ \text{HF}^*(L_0, L_1, \ldots, L_{n-1}, \eta) = \text{HF}(L_0 \times L_1 \times \ldots \times L_{n-1}) \]

\[ = \text{HF}^*(L_0 \cdot L_2, L_2 \cdot L_3) \]

\[ = \text{HF}^*(\Gamma_0, \Gamma_1) = H\hat{F}(\mathcal{V}) \]

Restricted to \( \text{Sym}^k (\Sigma_1 \cdot L_2) \), for \( \Sigma \) a surface with \( k \) to get other theories compactly!

\[ L_{01} \cdot L_{12} \subset pt. \times \text{Sym}^2 (\Sigma_2) \approx \Pi_\beta = \beta_1 \times \beta_2 . \]

\[ = \{(pt, y) \mid y \in \text{Sym}^2 (\Sigma_1) \mid (pt, x) \in L_{01}, (xy) \in L_{12}\} . \]
This is functorial, e.g. take $Y^3$ with $2Y^3$. 

$F_2 \otimes \bar{Y}$

$(0, 1, \ldots, L_{n-2}, n)$ generalized Lagrangian submanifold of $\text{Sym}^3(F/\mathbb{Z})$.

General story (rigorous, dim 4) for extended (2+1+1) top. field theory (4-mfd $\to$ number)

closed 3-mfd $Y \to$ abelian group $\text{Hochschild-Woodward}$

closed surface $F \to$ category $\mathcal{C}(F)$ ($= \text{ext. equiv. category} \text{Sym}^3(F/\mathbb{Z})$)

This is essentially the story for SWF & fft, /-torek for dim 4 numbers

5.d., if have $\gamma^3, \forall \gamma \in F \Rightarrow \text{object of } \mathcal{C}(F)$  

$$\gamma \mapsto \text{thickening (Lekili-Pardon)}$$

Thus (Lekili-Pardon): $\text{thickening (up to quasi-iso.) is indep. of Morse fan}$.

$\gamma^3 \equiv F_\gamma (-F_\gamma)$ (Ax).

2. $Y \cong F_\gamma \gamma (-F_\gamma) \Rightarrow \text{factor } \mathcal{C}(F_\gamma) \to \mathcal{C}(F_\gamma)$

$\mathfrak{M}_a$ (= gen. Lagr. correspondence).

(Actually factor $a \mod F_\mathbb{Z}^\# \to \mod F_\mathbb{Z}^\#$).

e.g. a bimodule.

--- Behaves naturally w.r.t. gluing ---

--- e.g. ---
\[ Y_1 \cup_w Y_2 \]
\[ \partial Y_1 = F = -2 Y_2. \]
\[ \pi Y_1 : pt \to \text{Sym}^9(F-2) \]
\[ \pi Y_2 : \text{Sym}^9(F-2) \to pt. \]
\[ [\leftrightarrow pt. \to \text{Sym}^9(-F)] \]

\[ H^F(\pi_{Y_2} \circ \pi_{Y_2}) \]
\[ = H^F(\pi_{Y_1}, T_{-Y_2}) \]
\[ \text{in } H^F(\text{Sym}^9(F-2)) \]
\[ = \text{hom}_{E(F)}(\pi_{Y_1}, \pi_{-Y_2}). \]

(Strictly, \( F^9(\text{Sym}^9(F-2)) \) embeds into modules over a simple algebra, so, to the one written down is bordered Floer homology).

Q: What do Floer categories have to say about bordered HFL?
Bordered HF homology (2008)
Lipschitz - Osvárt - Thurston (~270 pgs.).

Simple example:

\[ \widehat{HF}(S^1 \times S^2 \# S^1 \times S^2) \]
\[ \cong S^1 \times S^2 - B^3 \cup_2 S^1 \times S^2 - B^3 \]

\[ \widehat{HF}(S^1 \times S^2) \otimes \widehat{HF}(S^1 \times S^2) \]

True (Obs): \[ \widehat{HF}(Y_1 \# Y_2) \cong \widehat{HF}(Y_1) \otimes_{\mathbb{F}_2} \widehat{HF}(Y_2) \]

More generally:

\[ W(M) \]
\[ W(Y_1 \# Y_2) \]

LCE: rejoin \( S^1 \)?

Object in 2-category.

Want: \( CF(Y_1) \otimes CF(Y_2) \cong CF(Y_1 \cup_Y Y_2) \)

where \( \partial Y_1 = \emptyset = \partial Y_2 \),

But doesn't \( \partial \)-rejoin work?

\[ \widehat{CF}(Y_1) \otimes \widehat{CF}(Y_2) \cong \widehat{CF}(Y_1 \cup_Y Y_2) \]

In algebra, where \( \partial Y_1 = \emptyset = \partial Y_2 \)

\[ \Sigma Y \]
Cut simple closed contractible subsets. Hom surface.

**Definition:** A pointed matched circle \( \mathbb{Z} \) consists of

\[ \tilde{S}^1, 4k \text{ pts. } \alpha = \{ \alpha_1, \ldots, \alpha_k \} \]

a basepoint \( z \in \tilde{S}^1 \).

\( M \) as \( S \) which is a free-\( pt \). free modulo:

- such that
- attaching any arc between paired points
- gives a connected \( S \) boundary

\[ \alpha \times \mathbb{Z} \times \mathbb{R} \]

not contractible.

\[ \text{product not contractible!} \]