Discussion:

(Alg.) K-Theory:
- Periodic cyclic homology + general homological objects
- Relationships between various approaches / geometric interpretation of cyclic relations

Problem

Periodic Cyclic Homology / K-theory

A unital A-no category
Tw A triangulated closure of A in mod-A

Q: are there higher K-groups?

\[ \text{Ko}(A) = \frac{\mathbb{Z} \text{ objects of Tw } A}{(\text{CAJ} + [\mathbb{C}] - \text{CAJ})} \text{ if } \mathfrak{a} \text{ f: } A \to B. \]

Easy things:
\[ \text{Ko}(A) \to \text{HH}_0(A) = \mathbb{C} + \text{Co}(A) \]
\[ \text{CAJ} \to e_A e_{\text{hom}}(A, A) \]
\[ b + b \]

Urschel: ch: Ko(A) \to \oplus \text{HC}_2n(A) comes - Chen-Chas.チェック

\[ \text{PFM}_n(A) \]
\[ p^2 = p \to \sum_{i=0}^{\infty} \text{Pin}_i \otimes \otimes \text{Pin}_i \text{ Pin}_i \]

N.B. pairs to drag module.
HKR: $A = k[X]$, then $\mathcal{H}_G = \mathcal{R} \otimes_{\mathcal{O}^G} \mathcal{H}_G$

$1 + C \xrightarrow{S} 1 + C$

Poincaré cyclic homology = \lim \mathbb{Z} map

$\mathcal{H}^n \quad$ ch factors through poincaré cyclic homology.

(Rem: negative cyclic associated w/ fixed points of $S$ action more philosophically)

Q: "Split this anywhere? Take homotopy, then derive map to hyper cyclic group?"

Paul: Alg. yes: $\text{Sh}(X) \rightarrow \mathcal{H}^n$, chern classes $\otimes (p,q)$ classes

($p,q$ Hodge cohomology)

For $X$ affine, $\mathcal{C}^n$ is always $H^n(X)$.

ch still makes sense here.

Rudi: K theory fun while relating to properties is on you.

(this is a Quillen $\mathbb{Z}_2$ for commutative rings)

but our rings are more commutative.

need ray of finite global dimension, but we have those.

Claim: $K_0$(subset perfect objects) = $K_0$ (projective one)

If we took free module, only got $\mathcal{H}_0$.\n
Problem: how to use it for anything interesting?

This morning: If $A$ is the wrapped category, then it has a finite set of split generators. This tells you nothing about the size of $K_0$.

Oh no! Every smooth projective variety of dimension $d$ has a split generator.

$K_0$ (Elliptic curve) is uncountable. (3rd of Jacobian) too big.

Side note: Simple applications:

$M = \mathbb{P}^1$ is the central bundle.

$L \subset M$ (non-split exact.

Application: $H^*(M) \to H^*(L)$ factors.

$H^*(\mathbb{P}^1)$ so it's homologically contained in $K_0$.

Mystery: It already follows by $SH^*$ result, under some bundle attachment by Verdier specialization.

Assuming our degree (L) is $SH^*$.

$M_1 \to M_2$.

Say $L_{12} \to M_1 \to \mathbb{H}_2 \to L_{12}$ (how does this behave w.r.t. $\text{Ch}$?).

Induces $F_{12}: \text{Fuk}(M_1) \to \text{Fuk}(M_2)$ (really $F^*$, but it's ok.). $L_2$ of split generator.
\[ K_0 (\text{Fuk}(M_1)) \rightarrow K_0 (\text{Fuk}(M_2)) \]

\[ \downarrow c \]

\[ \downarrow c \]

\[ \text{do they cancel?} \]

\[ \text{what about} \]

\[ \text{But interests} \ c_4 \rightarrow H_{0 \text{a}} ? ? \]

\[ \text{Compute} \]

\[ \text{Free} \]

\[ \text{Projective} \]

\[ \text{not just free.} \]

\[ \text{Need to compare } \text{Tw} W \text{ and } Tw^* W \text{ to get this ch.} \]

\[ \text{We have} \]

\[ \text{Interej chen change} \]

\[ \text{requires knowing } \gamma (a) \]

\[ \text{This is the only diagram, obviously misleading.} \]

\[ \text{Maybe at some } \mathfrak{p}, \text{we'd have growth, not split generation} \text{ but it's not.} \]

\[ \text{Periodic cycle homology: (P. Seidel)} \]

\[ H^*_c (A) \text{ is } \mathbb{K} [\mathfrak{u}] \text{-module, } H^*_c \mathbb{R} = \mathbb{R}. \]
Framework to build the \( \text{C}^* \) complex (same as equiv. cohomology)

\[
C^* \quad \text{(Hochschild cochain complex, e.g. Hochschild on \( Map(L,S \)) action)}
\]

\[
d_0 : C^k \rightarrow C^{k+1}
\]

\[
d_1 : C^k \rightarrow C^{k-1} \quad (\text{inn. } B \quad \text{invariant cohomology, which w/ v.c. that defines } S \text{ action})
\]

\[
d_2 : C^k \rightarrow C^{k-2}
\]

in more general, want

\[
d_2 : C^k \rightarrow C^{k-3}
\]

In singular homology, Poincaré duality: \( d_2^2 = 0 \)

\[
\text{(a sequence)} \quad \text{(to prevent stupid things)}
\]

only pos. pos.

\[
\mathbb{Z} \xrightarrow{d_0} \mathbb{Z} \xrightarrow{d_1} \mathbb{Z}
\]

\[
d_0 = d_0 + u d_1 + u^2 d_2 + \cdots
\]

\[
H(C[[u]], d_0) = H C^*
\]

why does polynomial not make sense?

If some guy asks quasi-iso or do, should on all \( d_i \), otherwise pull...

here, there is no equal to its own inverse but it's a filter

(\( \text{nic } H^* \), keep is quasi-iso if \( H^0 \) is...)

\[
\mathbb{K}(u)/\mathbb{K}[u] \cong \mathbb{K}^{[u^{-1}]} \quad \text{negative cycle homology}
\]

\[
\mathbb{K}^{[u^{-1}]} \quad \text{du} \cdot \mathbb{K}^N
\]

exact in \( H^* \), take \( H^S \) apply to one cycle

or: Poincaré duality, \( CP^* \), \( CP^{n+1} \). Take submanifolds, pass to \( CP^* \) on rest, \( CP^{n+1} \) cycle, push forward to get \( CP^* \).
3. \( \text{IK}(\text{u}) \)

\[
\text{C}(\text{u}) = \left( \text{C}[\text{C}(\text{u})] \right)[\text{u}^{-1}]
\]

\[\mathcal{H} \otimes \mathcal{K}\]

Obvious E.S., not exciting.

Remark: If non-unital, this doesn't work, might want to use torus bicomplex.

- E-E don't care with any of Reeb chords but maybe they're not missing unfair.

\[\mathcal{H} \otimes \mathcal{K} \text{ should exist.} \]

Example: \( A = C_N(\mathcal{L}) \)

\[\mathcal{H} \otimes \mathcal{K}(A, A) = \mathcal{H} \otimes \mathcal{K}(\mathcal{L}).\]

\[\mathcal{H} \otimes \mathcal{K}(A, A) \text{ depends only on } \mathcal{H} \otimes \mathcal{K}(\mathcal{L}).\]

Surprisingly: \( \mathcal{H} \otimes \mathcal{K}(A) \text{ depends only on } \pi_1(N) \text{, vastly simplified.} \)

\[\text{(Jones)} \]

The reason: Hodge theory.

\[\mathcal{H} \otimes \mathcal{K}(A) \text{ depends only on } \pi_1(N) \text{, vastly simplified.} \]

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One may family induces a connection on \( \text{IK}\) of \( \mathcal{H} \otimes \mathcal{K}\).

So \( \mathcal{H} \otimes \mathcal{K}\) of \( \text{IK}\) of \( \mathcal{H} \otimes \mathcal{K}\) does not matter.

\[\mathcal{H} \otimes \mathcal{K}(A/\mathcal{J}) = \mathcal{H} \otimes \mathcal{K}(A/\mathcal{J})\]

\[\text{HP}_k(A) = \text{HP}_k(A/\mathcal{J}).\]
A formal definition of $A_0$ \( L \mathbb{K}^{+1} \times \mathbb{K} \).

\[ HP(A_0) = HP(A_0) \mathbb{K}^{+1} \]

Warning: base change (e.g., base $t$, pass to generic fibre, doesn't go through here).

If complete, $\mathbb{K}^{+1}$ doesn't go through, occ. here.

[Simple call: come, smthn, make a file, not sure, see w/ writing $t$]

Ok, in symp. Geo.

check over $A_0$: all generating have deg $0$ night.

\[ A_{\delta s}, \quad H^k(A) = 0 \quad \text{for } k < 0 \]

then $HP(A) = HP(H^0(A)), \quad$ (Seeks, $\text{thinkless}$).

Let $T^4 N$

restrict $T^1 N \to T^h L$

\[ G(x\mathbb{R}) \; \leftrightarrow \; (\mathbb{T} \times \mathbb{R}) \]

$HP(\mathbb{R} \times L) \; \leftrightarrow \; HP(\mathbb{R} \times N)$

 localization w.r.t. symplectic homology if $(\mathbb{R}_g \times) \text{ depends on}$

for sp. uses $\quad$ (if C.S. exist be $\mathbb{K}((i, 1))$

for $K(i, 1)$ get $HP$ (p.s.m.)
Pentz: SWF is expressly a S-equivalence, resembling Hc.

HM r a.n. when y is invisible,

depends on \( H^1(\text{Mor}) \) one 3 for given by

fold up product.

\[ \text{Geometry} \]

\( \text{Is } \text{Mayer} / \text{Fliess} \text{ have Sym}^3(\mathbb{E}) \), some hypers-ley.

\( u = \text{intersection of hypersurfaces} \).

But no one can compute HF\( ^{\infty} \) analyze of HA for

\( \text{not } \text{homology spheres} \).

Q: Is filtered situa is sympl. + p, is there a story like this one?

Q: [Seduk] Is this something you can do more generally for other manifolds?

\[ \text{Do } \text{this power of } \text{ kill it } \text{ really get iterates of monodromy} \]

\( p : E \to \mathbb{C} \)

\[ \text{fibre } \text{M, monodromy } \phi : M \to \text{M} \]

\[ \Delta \text{ Lefschetz thm: } \]

\( \text{vanishing cycle} \)

Spectral sequence,

\[ K \overset{\oplus}{\to} H^F(\mathbb{A}^k(L), L) \Rightarrow HW^r(A, \Delta) \]

\[ \text{KZ1} \]

\( \text{lots of higher differentials} \)

\[ \Delta \text{ dga. } \text{ one dga } A \]

\[ \text{differential} \text{ is even } k \text{. lamin. } \]

\[ \text{are } H_P \text{'s the same? } \text{ if deformation invariance?} \]

\[ \Delta \text{ is fibre of fibre } + \text{e} \]
If localized $\Sigma$ equiv. $\mathcal{SH}$ is that easy to compute??

It's fairly easy, put perturbation on $\mathcal{SH}$.

Strange to believe.

be sure: check these properties are being applied smoothly.

Q: If HP preserved under this $\Rightarrow$.

Always: never mind, need to meet $\ast$ again.

Lenny: find things where chains all have index 0 or rigde.

These do HP computing, then away, away, away.

Another thing: existing short exact sequence of alg. $\Rightarrow$ (un-ended sequence).

Short sequence is $K$-theory.

$$0 \to I \to A \to A/I \to 0.$$  (no units).

$$HP_0(I) \to HP_0(A) \to HP_0(A/I) \vee$$

            \text{not the first and} \text{ HC.}

        \text{not the first and HC.}$$

Look at $HP(\text{pot. Lagh})$. Is that like a lagh, free theory coupled to $K$-theory.

There are cases in which this term of $\mathbb{Z}$ is $K_o(\mathbb{Z})$

Schenker theory is coupled to $K_1$, Dirac operator.

but $K_1$(loop space) $= K_0$.

Koshelev-Ma blank construction in Lagh Free theory.
Sidel: in SWF, if take dual, can maybe forget k, class.

Also, how about categorical localization? What happens?

Variety, scheme, localize.

\[ \text{Mod}(Y) \leftarrow \text{Mod}(X) \rightarrow \text{Mod}(X \setminus Y) \] sounds like a coreum.

Localize w.r.t. ideal??

How does HP deal w/ (localization)?

Which: k- for non-commutative rings; localization is delicate.

Example: Affine line.

\[ p \leftarrow A^1 \rightarrow (A^1 \setminus \phi) \]

\[ HP(A^1) = C \{ (u) \} \]

\[ \text{is the zero as the set of a point, actually} \]

\[ C \rightarrow 0 \rightarrow C \] as a sheaf.

Talk: \( \Theta \), minimal resolution, two pieces of free sheaf, \( \text{or } \text{k-theory} \).

\[ C \rightarrow 0 \rightarrow H^\ast (\mathbf{S}) \] exact sequence.

\[ HP \text{ arise to compute/ketter how to use k-theory??} \]

\[ H^2 (\mathbf{A}_M) \text{ is square } \Theta \text{ extension} \text{ of A by M.} \text{ intrinsic definition by} \]

But: How about resolving (point, \( \text{ultrah} \)).
RhHom \( \mathbb{A} \times \mathbb{A} \) should be denoted equiv to \( \mathbb{A} \times \mathbb{A} \) by perfect idea: replace algebras by commutative ones (like this) and work with them.

Do we know when \( C^*(\mathbb{A}, \mathbb{A}) \) is formal? Results by some. Yet? Kotschik? Don't know.

\[ A : \bigoplus_{i \in I} A_i, \quad A_0 = k, \quad \text{it becomes a module.} \]

\[ \text{Ext}^1(k, k) \rightarrow \text{RhHom}(k, k) \]

\[ \text{If } \mathbb{A} \text{ is Koszul, then Koszul dual: do it again, get back } \mathbb{A}. \]

If assuming nothing, do it twice, when do we get back original algebra?

- module space of non-funnel stacking
- Koszul vs. funnel, vs. strong non-funnel
- Koszul has higher homogeneity property, monomorphism, vs. product quadratic.
- Some Feufer transform.
- Aside that...

Define \( \mathbb{A} \) by adding central cell, defines Koszul.

Higher central cell: is the same way to work to so the difference.

But no closed formula.

Our complex is not formal if it is Koszul.

How do formal things work under Koszul dual?