Let \( R_{m,n} := \mathbb{C} \left[ \alpha, \beta \right] / (t^n + a_{n-2} t^{n-2} + \cdots + a_0) \)

\[ = (t^n + b_{n-2} t^{n-2} + \cdots + b_0)^m. \]

This means, use level in \( t \)-wise generators after expading in \( t \).

An \( R_{m,n} = \binom{a+n+1}{n} \) cdot \frac{1}{m+n+1} \)

\( \subseteq \mathcal{O}(\mathbb{C}^1 \rightarrow \mathbb{C}^1) \) \( C_{m,n} := \{ x^m = y^n \} \) \( (m,n) = 1 \)

Say \( t \) is a local param on \( \mathbb{P}^1 \).

\( x(t) = t^n + a_{n-2} t^{n-2} + \cdots + a_0 \)

\( y(t) = t^n + b_{n-2} t^{n-2} + \cdots + b_0 \)

\( \text{s.t.} \ (x(t))^{m/n} = (y(t))^{m/n} \iff x(t) = C y(t)^{m/n} \iff [t^{mn}] (y(t))^{m/n} = 0 \)

\[ \text{reg on } a_i, b_i; \]

\( \subseteq \mathcal{O}(\mathbb{P}^1) \) \( \subseteq \mathbb{C}^2 \) \( \text{relations} \) \( \text{formal power series} \).
Let's analyze the given mathematical content step by step.

1. **Very Singular Lines:**
   - The text mentions a section of Grassmannians, which are spaces of linear subspaces of a given dimension. The text notes that this section is very singular.

2. **Examples:**
   - The text provides examples for specific dimensions:
     - For $m=3$, $n=2$, $Sp_{m,n} = P^1$ (projective line).
     - For $m=4$, $n=3$, $Sp_{m,n} = Cone(P^1 \times P^1)$.

3. **Homological Results:**
   - The text includes homological results involving cohomology groups, denoted as $H^*(Sp_{m,n}, \mathcal{O})$.
   - It mentions the use of maximal ideals $\mathfrak{m}$ in $\mathcal{O}$ and the determinant $\det(xI-I)$ for certain cases.

4. **Vautical Structures:**
   - The text introduces structures defined by the determinants, with $V_n = \wedge/\mathfrak{m}^n$.
   - A rule is given: $Sp_{m,n} \leftarrow V_n = \wedge/\mathfrak{m}^n$.

5. **Generality:**
   - A general statement is made about $\mathfrak{g} \in \mathfrak{g}(\mathbb{C})$ and its Lie algebra properties.
   - The text introduces a field $F = \mathbb{C}[t, t^{-1}]$.

The provided text contains advanced mathematical concepts and proofs, requiring a deep understanding of algebraic geometry, homological algebra, and Lie algebra theory.
After some fix

\[ S_{g} = \{ g \in G(F) / G(0) \mid \text{Ad}(g^{-1}) x \in g(0) \} \]

\[ S_{p,m} = S_{p,1} \text{ where } D_{m,n} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}^{m \times n}. \]

3) \[ C_{g} = \{ \det(x) \neq 0 \} \]

\[ C_{g}^{2} X \]

\[ C_{g} = \{ \gamma \in X \mid t^{-n} = 0 \} \]

Conj: Assume \( C_{g} \) is of geometric genus 0, i.e., have \( P^{1} \to C_{g} \)
and has only one singularity.

Then, \( H^{*}(S_{g}, X) \cong \operatorname{Gr} \left( O \left[ [P^{1} \to C_{g}] \right] \right) \quad \text{(closest to affine symmetry)} \text{ conjecture.} \)

(see) Study space fiber, flag, vertex, \( \Lambda \left[ [P^{1} \to X] \right] \)
(see \((1.4)\) for example of \((1.8)\).

\[ \chi(S_{g}) = \text{dim} O([P^{1} \to C_{g}]). \]

And here is a conjecture about quantum cohomology too!

\((\text{Affine analogue of the Floppy string need to work equivalently})\)
\[ \text{Then } [0, \text{Yan}]: H^0(S_{\nu, n}) \text{ has a section of natural cherednik algebra.} \]

\[ \text{Hilb}_C^N(C) = \left\{ I \subset \mathcal{O}_C \mid \text{dim } \mathcal{O}_C/I = CN \right\} \]

\[ C \subseteq \mathbb{C}^2 \]

\[ C \text{ smooth } \Rightarrow \text{Hilb}_C^N(C) = \text{Sym}_C^N \]

In general, have a map \( \text{Hilb}_C^N(C) \rightarrow \text{Sym}_C^N \).

\[ x \mapsto \text{Hilb}_C^N(C) \rightarrow \text{Sym}_C^N \]

\[ \text{Hilb}_C^N(C, z) := \text{Hilb}_C^N(C \setminus \{z\}, z) \]

\[ \{z \in C \mid \text{fibers of this map} \} \]

\[ \text{Examples: } C = C_{\nu, n} = \{ x^\nu = y^\nu \} \quad z = (0, 0). \]

\[ \begin{array}{c|c|c|c}
\text{m/n} & \text{Hilb}_C^N(C_{\nu, n}, z) & N \gg 0 \\
\hline
2/1 & \text{pt.} & \times \\
2, 3 & \mathbb{P}^1 & \times \\
3, 4 & \text{cone } (\mathbb{P} \times \mathbb{P}) & \times \\
\end{array} \]

\[ \text{Then [Laumon]: } C \text{ is unibranch at } z, \text{ then } \]

\[ \Rightarrow \text{Hilb}_C^N(C, z) = \text{Sym}_C^N \quad \forall N > 0. \]
Ex. \( x^2 = y^3 \) \( x = t^3 \), \( y = t^2 \), \( z = (q, t) \).

\[ \mathfrak{O}_2(C_{2,3}) = \mathbb{C}[t^2, t^3] \mathbb{C}[t] \]

\[ \mathfrak{O}_2(C_{2,3}) = \mathbb{C}[\left[ t^2, t^3 \right]] \]

\[ \text{Spec}(\mathbb{C}[\left[ t^2, t^3 \right]]) = \mathbb{P}^2 \]

Problems: Classify all ideals in \( \mathfrak{O}_2 \)

Problem 0: 
\[ \mathfrak{O}_2 \subset \mathbb{C}[t] \]

Solve: \( \mathfrak{I} = \langle f \rangle \)

\( \mathfrak{I} = f = t^N + a_{N+1} t^{N+1} + a_{N+2} t^{N+2} + \cdots + a_{N+3} t^{N+3} \)

\( \Rightarrow t^2 f \in \mathfrak{I} \)

\( t^2 f \in \mathfrak{I} \)

\( \Rightarrow f = \bar{f} + t^{N+1} \in \mathfrak{I} \)

\( \Rightarrow \) All principal ideals are \( \langle t^N + a t^{N+1} \rangle = \mathfrak{I}_a \)

\( \mathfrak{I}_a \neq \mathfrak{I}_b \) if \( a \neq b \).
\[ I = (f, g) \]
\[ f = t^N + a \cdot t^{N+1} \]
\[ g = t^M + b \cdot t^{M+1} \]
\[ m = n. \]
\[ \Rightarrow n = M < N+2 \]
\[ \Rightarrow M = N+1. \]
\[ \Rightarrow I = (t^N + t^{N+1}) \]
(\text{and no 3-gen ideal.})
\[ \Rightarrow \text{Hilb}^N(\langle t, z \rangle) = P^1. \]

Remark: only the \textbf{if} \textbf{N} \textbf{\Rightarrow} \textbf{I}.
\[ \text{e.g. } (t^N + q \cdot t^{N+1}) \text{ does not work for } N = 1, N = 0. \]
\[ 6/4 \not \in \mathbb{E}. \]

Thus (Hausk, Yan, Berger, Hirschhorn, Strong):\
\[ \sum_{i, N=0}^{\infty} \text{dim} \text{ H}^i(\text{Hilb}^N(C, z)) \cdot t \cdot q^N. \]
\[ = \frac{1}{1-(q^2)} \sum_{i, j, k \geq 0} \text{Gr}^j_{i+k} \text{H}^i(SpC) \cdot q^j \cdot t \cdot q^k. \]

\[ \text{extra grading "perversity grading"} \]
\[ \text{(coming from "monodromy"/}) \]
\[ \text{JC decomposition theorem}. \]
Thm: \( \{0, \ldots, n\} \) has a basis of rat'l Chevalley algbera \( \hbar_{S_n} (S_n) \) (proves perverse grading)

Prop: \( \overline{H_c} (S_n) = C \langle x_1, \ldots, x_n, y_1, \ldots, y_n \rangle \rtimes S_n \)

\( [x_i, x_j] = [y_i, y_j] = 0 \)

\[ S_n \otimes \sigma \cdot g x_i = x_{\sigma(i)} g \quad \sigma y_i = y_{\sigma(i)} g \]

\[ [x_i, y_j] = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases} \]

\( \overline{H_c} (S_n) \subset \overline{H_c} (\mathfrak{s}_n) \) version of \( \overline{H_c} (k) \) which is like \( GL_n \)

\[ \text{Ex:} \quad \overline{H_c} (S_n) = \text{Diff}(\mathbb{C}^n) \rtimes S_n \]

\[ \overline{H_o} (S_n) = \text{Diff}(\mathbb{C}^n) \rtimes S_n \]

\( n = 2: \quad \overline{H_c} (S_2) = \langle x, y, s \rangle \mid sx = -xs \mid [x, y] = [x, s] = 1 + cs \]

\( \text{Diff}' \) operates on some \( \alpha \) (fixed).

\( \text{Hc}(S_n) = \text{Diff}(\mathbb{C}^n / S_n) \sim \mathbb{C}^n \rtimes S_n \sim \mathbb{C}^n / S_n \)

\( \text{Sym}(\mathbb{C}^2) \sim \mathbb{C}^2 \)
\[ H_1 \left( \text{End}_n(C^2) \right) = \Omega \left( C^2, 0, 0 \right) \]