§0.1.        The many and the none....................................................................................... 3
§0.2.        one and zero or more vs. one or more and zero or more............................ 5
§0.3.        Morphological preliminaries........................................................................... 7
§1.           Knowledge of singular and plural................................................................. 9
§1.1.        Inference in the object language................................................................. 9
§1.2.        The language of the partitive construction................................................... 14
§1.2.1. A primitive partitive relation ‘of(ξ,ς)’ .................................................................. 16
§1.2.2.     Definite description......................................................................................... 17
§1.3. Constitution and Comprehension....................................................................... 17
§1.3.0.     Excursus on ‘.pl’ and ‘.∅’ ................................................................................ 18
§1.3.1.     Restricted Comprehension.............................................................................. 22
§1.4.        Semantics......................................................................................................... 24
§1.4.1.     Against singularism ......................................................................................... 26
§1.4.2.     The semantic type of singular, plural and mass (in)definite descriptions...... 27
§1.4.3.     Monadic second-order logic and the language of the partitive construction.... 28
§2.           Essential plurals in natural language............................................................. 32
§2.1.        Cardinality predicates and relations............................................................ 32
§2.2.        Eventish......................................................................................................... 35
§2.2.1.     Plural reference and event quantification ...................................................... 36
§2.2.1.1.  Geach-Kaplan reciprocal sentences ................................................................. 36
§2.2.1.2.  Separation within simple clauses...................................................................... 38
§2.2.2.     Plural event quantification.............................................................................. 39
§2.2.3.     Coordination and thematic relations............................................................. 41
§2.2.4.     Clause structure and relations between events............................................. 44
§2.3.        Russelling Eventish....................................................................................... 46
REFERENCES...................................................................................................................... 51
NOTES................................................................................................................................ 55

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Zweig.
Plurals are essential where what is said of what the plural refers to in (1) cannot be said of any one of what it refers to, (2).

(1) The oven fires clustered.
The custards clumped.
The pastries flowed into the customers’ hands.
The chocolates are twelve.

(2) *The oven fires each clustered.
*The custards each clumped.
*The pastries each flowed into the customers’ hands.
*The chocolates are each twelve.

*An oven fire clustered.
*A custard clumped.
*A pastry flowed into the customer’s hands.
*A chocolate is twelve.

Extension of the logical language to deliver plural reference and the logical relations that constitute knowledge of the singular and plural acquires empirical bite just in case it conforms with increasing precision to the syntax of the natural language and affords explanation of what speakers know about the distribution and meaning of plural expressions in their language.

As for the syntax of natural language, this discussion, being none too precise, is guided throughout by just two considerations and their immediate consequences, discussed at greater length in §0. The first, morpheme univocality, is that a morpheme despite its various syntactic and morphological contexts has a single meaning that supports all its occurrences: bare nouns, (fire, custard, pastry, chocolate, etc.), the article the, quantifiers (some, any, all, most), partitive of, and any others that transgress the boundaries of singular, plural and mass terms are never on these grounds to be treated as ambiguous among two or three homophones. Similarly, the morpheme cluster, occurring in different parts of speech, in both verb and noun in (3), is univocal too (see Parsons 1990).

(3) The fires clustered in two clusters.

The second consideration is the conservation of lexical classes: any morphemes that belong by all grammatical reckoning to the same lexical class, zero and two, as an example with portent, do so and therefore share the same logical syntax.

Assuming what has just been observed, that the language presents phrases Φ[ξ], such as clustered[ξ], complex or simple, for which a plural idiom is essential, §1 develops an apparatus for plural reference—plural (and mass) definite and indefinite descriptions and the partitive construction. Much of what constitutes a speaker’s knowledge of singular and plural is reflected in inferences within the language of the partitive construction relating singular and plural expressions. A discussion of nominal syntax and morphology and the axioms supporting inference within the object language precedes statement of its semantics and what is to be said about plural reference itself.
Section §2 goes on to survey the language that makes plurals essential: the inventory of primitive plural vocabulary, the composition of singular or plural expressions into complex phrases $\Phi[\xi]$ that in turn demand plural $\xi$, and the combinatorial interactions between plural quantification and the other phrases that lie within simple sentences. These interactions conclude in §2.2.4 and §2.3 in a revision of basic clause structure.

§0.1. The many and the none

The logical syntax of (4)-(10) does not diverge from that of (11)-(17) under any plausible parse of the natural language\(^1\). In so far as sentences such as (4)-(10) are true, (4), given the definite description $[\text{the } \xi : \text{nonselfidentical custards}[\xi]]^2$,3, entails (18).

(4) The nonselfidentical custards are zero in number.  
   The moons of Venus are zero in number.\(^4\)  
(5) The nonselfidentical custard is as abundant as the nonselfidentical flan.  
(6) The nonselfidentical custard is as perfectly round as the nonselfidentical flan.  
(7) The at most one person still alive with a face like that descends from a tribe of Brooklyn.  
(8) The hairs on Humpty Dumpty’s head are sparse.  
(9) The hair on Humpty Dumpty’s head is sparse.  
(10) The zero or more solutions to this equation are all unidentified prime numbers.  
(11) The custards are twelve in number.  
   The moons of Jupiter are more than 60 in number.  
(12) The custard is as abundant as the flan.  
(13) The custard is as perfectly round as the flan.  
(14) The person with that face descends from a tribe of Brooklyn.  
(15) The hairs on Rapunzel’s head are luxuriant.  
(16) The hair on Rapunzel’s head is luxuriant.  
(17) The three solutions to this equation are all unidentified prime numbers.  
(18) $\exists \xi \text{zero}[\xi]$  

Given the truth of (4)-(7), nothing in the meaning of the article the or the plural, mass or singular morphemes proper entails a nonzero measure in either number or amount of what the description refers to, and neither does the existential quantification in (18) or the evaluation of the variable $\xi$ of plural reference. Plural expressions refer fluently to the many and the none.

What has been observed in definite descriptions holds as well of the variables and morphology engaged in distributive quantification if (22)-(26) parse along the lines of (19)-(21):

(19) $\text{[Any } \xi : \text{nonselfidentical custards}[\xi]]\ldots\text{zero}[\xi]\ldots$  
(20) $\text{[Any } \xi : \text{nonselfidentical custard}[\xi]]\ldots\text{zero}[\xi]\ldots$  
(21) $\text{[Any } \xi : \text{(sg.)of the nonselfidentical custard(s)}[\xi]]\ldots\text{zero}[\xi]\ldots$  
(22) a. Any nonselfidentical custards are zero in number.  
    b. F Any nonselfidentical custards are one or more in number.
(23)  a. Any of the nonselfidentical custards are zero in number.
    b. F Any of the nonselfidentical custards are one or more in number.

(24)  a. Any nonselfidentical custard is zero desserts.
    b. F Any nonselfidentical custard is one or more desserts.

(25)  a. Any of the nonselfidentical custards is zero desserts.
    b. F Any of the nonselfidentical custards is one dessert.

(26)  a. Any of the nonselfidentical custard is zero desserts.
    b. F Any of the nonselfidentical custard is one or more desserts.

Here too, given that (22)a.-(26)a. are true and (22)b.-(26)b. are not, nothing in the
evaluation of the variable \( \xi \), whether taken to be plural, (22)-(23), singular (24)-(25), or
mass (25)/(26) entails a nonzero measure nor does the meaning of the singular, plural or
mass morpheme or the partitive construction entail that what satisfies the restriction has a
nonzero measure.

Both definite descriptions ((27) and (28)) and distributive quantifiers ((29) and
(30)) talk glibly about the none as what no one can be ((27), (29)) and as what no several
can be ((28), (30)) using essential plurals:

(27) The custards each of which is not identical to itself are zero in number.
    [The \( \xi : \text{custards}[\xi] \) & [Each \( \varsigma : \text{of}[\varsigma, \xi] \) → \( \varsigma = \xi \) … zero \( \xi \)…
(28) The custards that outnumber themselves are zero in number.
    [The \( \xi : \text{custards}[\xi] \) & outnumber themselves \( \xi \)] … zero \( \xi \)…

(29) Any custards each of which is not identical to itself are zero in number.
    (F Any custards each of which is not identical to itself are one or more in number.)
    [Any \( \xi : \text{custards}[\xi] \) & [Each \( \varsigma : \text{of}[\varsigma, \xi] \) → \( \varsigma = \xi \) … \( \xi \)…
(30) Any custards that outnumber themselves are zero in number.
    (F Any custards that outnumber themselves are one or more in number.)
    [Any \( \xi : \text{custards}[\xi] \) & outnumber themselves \( \xi \)] … \( \xi \)…

If the definite descriptions parse as shown, (28) entails (31), and yet it should not of
course imply that the zero outnumber themselves:

(31) \( \exists \xi (\text{custards}[\xi] \& \text{outnumber themselves}[\xi] \& \text{zero}[\xi]) \)

Something intervenes between the devices that refer to the none and the descriptive
content that describes it not. Let it be stipulated that restriction to a quantifier subjects
descriptive content to the operator defined in (33):

(32) \[ \text{the } \xi : |\Phi[\xi]|, \text{ any } \xi : |\Phi[\xi]| \]
(33) \[ |\Phi[\xi]| \leftrightarrow \exists x \text{ is one of } \xi \rightarrow \Phi[\xi] \]

Although ‘\( \text{custards}[\xi] \& \text{outnumber themselves}[\xi] \)’ fails to be true of any \( \xi \),
‘\( |\text{custards}[\xi] \& \text{outnumber themselves}[\xi] | \)’ is true of the none.
Elsewhere, as in (34)-(38), an appearance to the contrary—that what satisfies the restriction has a nonzero measure—betrays a separate, unspoken measure, any (one (or more)) (of the) custards, so that custards zero in number do not falsify (34)b.-(38)b. 6

(34)  a. #Any custards are zero or more in number.
     b. Any custards are one or more in number.

(35)  a. #Any of the custards are zero or more in number.
     b. Any of the custards are one or more in number.

(36)  a. #Any custard is zero or one dessert.
     b. Any custard is one dessert.

(37)  a. #Any of the custards is zero or one desserts.
     b. Any of the custards is one dessert.

(38)  a. #Any of the custard is zero or more desserts.
     b. Any of the custard is one or more desserts.

It would of course defeat the point of a measure phrase if it fell within the scope of the operator defined in (33), which recommends its position in (39), a position corroborated in §1.2.2.

(39)  [the $\xi$ : (\(\mu P\)) |\(\Phi[\xi]\)], [any $\xi$ : (\(\mu P\)) |\(\Phi[\xi]\)]

§0.2. one and zero or more vs. one or more and zero or more

Nothing in this reference to the none should however be mistaken to imply that the primitive ‘custard’ is as promiscuous as the phrasal ‘\(\text{custards}[\xi]\)’ in being true of it. One might after all cherish a concept of custard that fails to be true of the none no matter how meager the diet. To put it concretely in more familiar language, the phrase that is an immediate constituent of the definite description may be a second-order description, itself true of the none, projected as in (40) from a first-order property that is not:

(40)  \[\text{They are custards} \leftrightarrow \forall x(Xx \rightarrow \text{custard}(x))\]
     ‘They are custards ↔ Any one of them is custard.’

A primitive concept of custard, one that is first-order and thus true only of what there is, enters a description of the none such as ‘\(\text{nonselfidentical custards}[\xi]\)’ only by a logical construction. If the primitive concept is first-order and singular, ‘\(\text{custards}\[\xi]\)’ is derived as in (40) or to the same effect through the intervention of a partitive relation discussed further below:

(41)  \[\text{custards}[\xi] \leftrightarrow \forall x(x \text{ is one of } \xi \rightarrow \text{custard}(x))\]

Either way, (40) or (41), singular reference is primitive and plural reference to the many and the none is derived. Alternatively, while reference to the none remains a second-
order construction (42), it could be that the primitive, first-order concept itself subsumes plural reference, to the many as well as the one, despite certain knowledge that many concepts such as custard are distributive, as in (43):

(42) \(|\text{custards}[\xi] \leftrightarrow \exists x \text{ is one of } \xi \rightarrow \text{custard}(\xi)|
\)
(43) \(\text{custard}(\xi) \leftrightarrow \forall x(x \text{ is one of } \xi \rightarrow \text{custard}(x))\)
    \(\text{custard}(\xi) \rightarrow \exists x \text{ is one of } \xi\)

It just so happens, as everyone knows who knows what custard is, that things are custards just in case each of them is. The primitive, first-order concept ‘custard(\xi)’ privileges singular reference no more than it privileges dual or triple reference, all of which rather require supplement to the primitive concept:

(44) \(\text{custard}.sg[\xi] \leftrightarrow_{df} \text{custard}(\xi) \& \text{sg}[\xi]\)
    \(\ldots\)
    \(\text{custard}.n[\xi] \leftrightarrow_{df} \text{custard}(\xi) \& \text{n}[\xi]\)

A contest between (41) and (43) looks to be a sterile one unless appreciated in full generality to characterize the primitive conceptual vocabulary. On the larger scale, indifference to the contrast between singular and plural reference may matter. Singular reference is concept-dependent: often a fire is many scattered fires, but never is a custard several scattered custards. Moreover, a judgment that there is one fire where there are two, or that there is one musical passage where there are two, reflects arcane knowledge of the subject and may itself be context-dependent and interest-relative:

(45) \(\Gamma_{\text{fire}[x,y]} \vdash (\text{fire}(x) \& \text{fire}(y) \& x \neq y) \rightarrow \exists z(z \neq x \& z \neq y) \& \text{fire}(z)\)
(46) \(\Gamma_{\text{custard}[x,y]} \vdash (\text{custard}(x) \& \text{custard}(y) \& x \neq y) \rightarrow \exists z(z \neq x \& z \neq y) \& \text{custard}(z)\)

Yet, as arcane as it may be to know what constitutes a fire, a custard or a musical passage, once singular reference is fixed, so it seems is plural reference. Perhaps plural reference is so easily extended to arbitrary concepts because there is indeed so little to it beyond a partitive relation ‘is one of’ or some other logical construction and thus so little to know to extend it. If, on the other hand, plural reference is intrinsic to primitive concepts in general, the arcane knowledge attested about a concept’s singular reference is merely one of its many aspects:

(47) \(\Gamma_{\text{fire}[\xi,\zeta]} \vdash (\text{fire}(\xi) \& \text{sg}[\xi] \& \text{fire}(\zeta) \& \text{sg}[\zeta] \& \xi \neq \zeta) \rightarrow \exists z(z \neq \xi \& z \neq \zeta \& \text{fire}(z) \& \text{sg}[z])\)
(48) \(\Gamma_{\text{custard}[\xi,\zeta]} \vdash (\text{custard}(\xi) \& \text{sg}[\xi] \& \text{custard}(\zeta) \& \text{sg}[\zeta] \& \xi \neq \zeta) \rightarrow \exists z(z \neq \xi \& z \neq \zeta \& \text{custard}(z) \& \text{sg}[z])\)

Among the primitive concepts, there could be those that fail one or the other direction of the biconditional in (43):

(49) \(\text{custard}'(\xi) \rightarrow \forall x(x \text{ is one of } \xi \rightarrow \text{custard}'(x)) \&
    \neg \forall \xi(\forall x(x \text{ is one of } \xi \rightarrow \text{custard}'(x)) \rightarrow \text{custard}'(\xi))\)
(50) \(\forall \xi(\forall x(x \text{ is one of } \xi \rightarrow \text{custard}''(x)) \rightarrow \text{custard}''(\xi)) \&
    \neg \forall \xi(\text{custard}''(\xi) \rightarrow \forall x(x \text{ is one of } \xi \rightarrow \text{custard}''(x)))\)
And, among these, one could imagine that things each of which is a custard' are some custard's only under conditions that are as arcane but still different from those that assemble a single custard'. Or worse, that the conditions constituting three custard's from three each of which is a custard' are yet again different from those constituting two custard's from two each of which is a custard'. The world of singular and plural reference is then again remade anew with fire and musical passages. If speakers are not as diffident in their grasp of plural reference as these remarks imply they should be, it implies that some further taxonomy or classification informs them—perhaps, those primitive concepts expressed by a primitive morpheme eligible to be a noun are with few exceptions distributive,7 and, in marked contrast, the concepts of primitive plural reference expressed by natural language determiners, cardinality predicates and many a verb are known not to be. It could then be allowed that the asymmetry between singular and plural reference, the extensibility and near logicality of the latter—that the concept-, context- and interest- dependence of singular reference does not find its match in plural reference— argues only for more theory rather than for first-order primitives that are only singular.8 If so, it remains largely an empirical question the extent to which plural reference reaches into the primitive vocabulary, requiring investigation piecemeal through the language.9

§0.3. Morphological preliminaries

Singular, count nominals, plural nominals and mass nominals share a common vocabulary of primitive lexical items, custard, fire, pastry, chocolate. The denotation of any of these must be consistent with its occurrences in singular, plural and mass expressions, which are themselves related by analytic truths such as (52)-(55):

\[(52) \quad \text{Any/all custard is custard.}\]
\[(53) \quad \text{Any a/Every custard is custard.}\]
\[(54) \quad \text{Any custards are custard.}\]
\[(55) \quad \text{Any a/Every custard is one or more custards.}\]

\[(56) \quad \text{That ((one) custard) is (a) custard.}\]

The same primitive ‘custard(\(\xi\))’, denoting whatever it denotes, occurs throughout, modified by singular(.sg)-, plural(.pl)- or mass(.∅)- term morphology. The sentences of (56), varying the occurrence of count and mass terms, can nevertheless be true of the very same thing.

All that will be said later about the reference of count terms and plural terms and about their inferential behavior is extensible to novel vocabulary whenever a new item is dressed in the appropriate nominal morphology. Generalization to novel vocabulary itself demands a parse of the natural language that factors out the bare nouns and makes explicit their morphological modification. The analytical truths (52)-(55) relating count
and mass terms then suggest that the bare ‘custard’ that pluralizes is the same lexical item expressing the same concept as the bare ‘custard’ that massifies.\textsuperscript{10}

As with nouns, the lexical quantifiers, articles and partitive of that occur across singular, plural and mass terms are not themselves specified as singular, plural or mass:

\begin{align*}
(57) & \quad \text{Any (one) custard is dessert. (cf. Any tart is pastry.)} \\
(58) & \quad \text{Any (three) custards are dessert.} \\
(59) & \quad \text{Any (one) of the custards is dessert.} \\
(60) & \quad \text{Any (three) of the custards are dessert.} \\
(61) & \quad \text{Any of the custard is dessert. (cf. Any of the sugar dissolves in water.)}
\end{align*}

It is thus left to the quantifier’s restriction to enforce singular, plural, or mass reference as in \([\text{any } \xi : \text{custard.sg}[\xi]])\), \([\text{any } \xi : \text{custard.pl}[\xi]])\), or \([\text{any } \xi : \text{custard.Ø}[\xi]])\). With little else to tell (62) and (63) apart, the partitive construction is itself taken to sort out singular \(\xi\) in (62) from nonsingular \(\xi\) in (63). Both (62) and (63) contain the same lexical item, the partitive of, which in (62) is further modified by (unspoken) singular morphology:

\begin{align*}
(62) & \quad \text{Any of the custards is a custard.} \\
& \quad [\text{any } \xi : \text{sg.of the custards}[\xi]] \text{ is a custard}[\xi] \\
(63) & \quad F \text{ Any of the custards are a custard.} \\
& \quad [\text{any } \xi : \text{of the custards}[\xi]] \text{ are a custard}[\xi]
\end{align*}

Continuing flat-footed to treat the morphology the same wherever it is encountered, the singular quantification in mass terms, both simple and partitive, translates as:

\begin{align*}
(64) & \quad \text{Any of the pastry is one or more pastries.} \\
& \quad [\text{any } \xi : \text{sg.of the pastry}[\xi]] \text{ is one or more pastries}[\xi] \\
(65) & \quad F \text{ Any of the pastry is a pastry.} \\
& \quad [\text{any } \xi : \text{sg.of the pastry}[\xi]] \text{ is a pastry}[\xi] \\
(66) & \quad \text{Any/all pastry is one or more pastries.} \\
& \quad [\text{any } \xi : \text{pastry.sg}[\xi]] \text{ is one or more pastries}[\xi] \\
(67) & \quad F \text{ Any/all pastry is a pastry. (cf. F Any/all patisserie is a pastry.)} \\
& \quad [\text{any } \xi : \text{sg.pastries}[\xi]] \text{ is a pastry}[\xi] \\
(68) & \quad \text{None of the pastry is more than some pastries.} \\
& \quad [\text{None } \xi : \text{sg.of the pastry}[\xi]] \text{ is more than some pastries}[\xi] \\
(69) & \quad F \text{ None of the pastry is more than one pastry.} \\
& \quad [\text{None } \xi : \text{sg.of the pastry}[\xi]] \text{ is more than one pastry}[\xi] \\
(70) & \quad \text{No pastry is more than some pastries.} \\
& \quad [\text{No } \xi : \text{sg.of the pastry}[\xi]] \text{ is more than some pastries}[\xi] \\
(71) & \quad F \text{ No pastry is more than one pastry. (cf. F No patisserie is more than one pastry.)} \\
& \quad [\text{No } \xi : \text{sg.of the pastry}[\xi]] \text{ is more than one pastry}[\xi]
\end{align*}

If the least of pastry is a single pastry, one might have anticipated that (65) and (69) would be true. Apparently, however, any quantity of pastry is pastry, in the singular\textsuperscript{11}, and therefore one or perhaps more pastries rather than a single one. But, if what be pastry is further restricted to what are pastries, anything singular is just a pastry, and thus (72) and (73) contrast minimally with (65) and (69):
Any of the pastries is a pastry.
\[\text{any } \xi : \text{sg.of the pastries}[\xi] \text{ is a pastry}[\xi]\]

None of the pastries is more than one pastry.
\[\text{None } \xi : \text{sg.of the pastries}[\xi] \text{ is more than one pastry}[\xi]\]

Consonant with these remarks, both mass terms and singular, count terms are taken to share the same singular morphology and are now to be parsed accordingly:

\[\text{custard.pl.sg}[\xi]. \quad \text{[Every } \xi: \text{custard.pl.sg}[\xi], \text{ every custard.}]
\[\text{custard.pl}[\xi]. \quad \text{[All } \xi: \text{custard.pl}[\xi], \text{ all custards.}]
\[\text{custard.∅.sg}[\xi]. \quad \text{[All } \xi: \text{custard.∅.sg}[\xi], \text{ all custard.}]

Singular morphology therefore does not imply a countable domain, which is indicated rather by the presence of the plural morpheme so-called, nor does it imply non-zero measure, as remarked above in (4), (6), (7), (9), (24) and (26) (although it will imply measurement no greater than one).

§1. Knowledge of singular and plural.

§1.1. Inference in the object language.

To know of an \(F\) is to know of one or more \(F\)s, and often, what is true of every one of the \(F\)s is true of every \(F\), and conversely, what is true of every \(F\) is true of every one of the \(F\)s. At least so much is distilled from distributivity inferences like (75)-(84). Extensibility, as reflected in a speaker’s willingness to extend these inferences to arbitrary substitutions for \(F\) (under conditions noted below), including novel vocabulary, and even to affirm them for \(F\) of uncertain meaning, is bedrock for a speaker’s knowledge of singular and plural.

\textit{Distributivity}

\[\text{(75)} \quad \text{a.} \quad \text{A fire expired. } \vdash (\text{Some}) \text{ one of the fires expired.}
\[\text{b.} \quad (\text{Some}) \text{ one of the fires expired. } \vdash \text{A fire expired.}
\[\text{(76)} \quad \text{a.} \quad \text{Every one of the custards hides a jewel. } \vdash \text{Every custard hides a jewel.}
\[\text{b.} \quad \text{Every custard hides a jewel. } \vdash \text{Every one of the custards hides a jewel.}
\[\text{(77)} \quad \text{a.} \quad \text{None of the puff pastries is more puff than pastry. } \vdash \text{No puff pastry is more puff than pastry.}
\[\text{b.} \quad \text{No puff pastry is more puff than pastry. } \vdash \text{None of the puff pastries is more puff than pastry.}
\[\text{(78)} \quad \text{Many a fire of the fires from heaven scorches the earth. } \vdash \text{Many a fire from heaven scorches the earth.}
\text{Many a fire from heaven scorches the earth. } \vdash \text{Many a fire of the fires from heaven scorches the earth.}
\[\text{(79)} \quad \text{Many fires of the fires from heaven scorch the earth. } \vdash \text{Many fires from heaven scorch the earth.}
\text{Many fires from heaven scorch the earth. } \vdash \text{Many fires of the fires from heaven scorch the earth.}
**Plural Partitivity**

(80)  
a. Some fires expired. $\vdash$ Some of the fires expired.  
b. Some of the fires expired. $\vdash$ Some fires expired.  

(81)  
a. Any of the custards hide a jewel. $\vdash$ Any custards hide a jewel.  
b. Any custards hide a jewel. $\vdash$ Any of the custards hide a jewel.  

(82)  
a. None of the puff pastries are more puff than pastry. $\vdash$ No puff pastries are more puff than pastry.  
b. No puff pastries are more puff than pastry. $\vdash$ None of the puff pastries are more puff than pastry.  

(83)  
Most any of the butter pastries is all butter.  
Most any butter pastry is all butter.  

(84)  
Most of the butter pastries are all butter.  
Most butter pastries are all butter.  

Note that the patterns in (85) instantiated by (75)a.- (77)a. and (80)a.- (82)a. are valid just in case (86) holds, and those in (87) instantiated by (75)b.- (77)b. and (80)b.- (82)b., just in case (88) holds:

(86) \[ \Phi[\xi] \rightarrow \Phi'[\xi] \]

(87) \[ [\text{Some } \xi : \Phi'[\xi] \Psi[\xi]] \vdash [\text{Some } \xi : \Phi[\xi] \Psi[\xi]] \]
\[ [\text{Every } \xi : \Phi[\xi] \Psi[\xi]] \vdash [\text{Every } \xi : \Phi'[\xi] \Psi[\xi]] \]
\[ [\text{Any } \xi : \Phi'[\xi] \Psi[\xi]] \vdash [\text{Any } \xi : \Phi[\xi] \Psi[\xi]] \]
\[ [\text{No } \xi : \Phi'[\xi] \Psi[\xi]] \vdash [\text{No } \xi : \Phi[\xi] \Psi[\xi]] \]

(88) \[ \Phi'[\xi] \rightarrow \Phi[\xi] \]

The speaker reasoning all directions in (75)-(77) thus requires (89):

(89) \[ \Phi[\xi] \leftrightarrow \Phi'[\xi] \]

With nonmonotonic quantifiers, *many* in (78)-(79) and *most* in (83)-(84), a single inference in either direction of its own demands the biconditional (89). Accepting the distributivity inferences of (75)-(84) thus commits speakers to the instances of (89) relating singular and plural as in (90), and plural to plural partitive as in (91), which become (92) and (93), assuming a common syntax and semantics for the partitive of and suppressing grammatical variation. Constitution principles are those like (92) and (93) that constitute *via* a partitive construction the plural reference of the plural definite description from the denotation of the singular and plural predicates.

(90) \[ \text{one of the custards}[\xi] \leftrightarrow \text{custard}[\xi] \]
\[ \text{a fire of the fires from heaven}[\xi] \leftrightarrow \text{fire from heaven}[\xi] \]
\[ \text{any of the butter pastries}[\xi] \leftrightarrow \text{any butter pastry}[\xi] \]
An account of plurals in natural language promises an analysis of the constitution principles (92) and (93) that derive the elementary inferences of distributivity and plural partitivity in (75)-(84). Given that the constitution principles are as numerous as substitutions for $F$ are unbounded, these principles must derive from some generalization about the object language. The correct generalizations are not (94) and (95) that any plural definite description instantiates valid constitution principles:

$$\forall \Phi (\text{of the } \Phi.\text{pl} [\xi] \leftrightarrow \Phi [\xi])$$

The generalizations must rather be restricted as warranted by (96)-(112) to those descriptions that do not themselves contain essential tokens of plural terms:

(Some) one of the custards that blanketed a buffet was served in a cup with a cherry on top.
$$\forall \Phi (\text{of the custards that blanketed a buffet} [\xi] \rightarrow \text{custard that blanketed a buffet} [\xi])$$

A custard that blanketed a buffet was served in a cup with a cherry on top.
$$\forall \Phi (\text{of the custard that blanketed a buffet} [\xi] \rightarrow \text{custard that blanketed a buffet} [\xi])$$

Any custards that blanketed a buffet were served in a cup with a cherry on top.
$$\forall \Phi (\text{of the custards that blanketed a buffet} [\xi] \rightarrow \text{custards that blanketed a buffet} [\xi])$$

A custard that is one of the custards that blanket a buffet, a description that is essentially about some custards, is not itself a custard that blankets a buffet. Similarly, some of the custards that blanket a buffet need not themselves blanket a buffet.

Plying a more subtle observation about plural definite description, the converse inferences are also seen to fail. All of what the description the custards that blanket...
*exactly one buffet* refers to, if it refers at all, is some custards that blanket exactly one buffet. With two buffets blanketed in custards, definite reference fails: to refer to the custards on just a single buffet, one and not the other, fails to include the custards on the other buffet that do themselves blanket exactly one buffet; yet, to refer to all the custards is not to refer to custards that blanket exactly one. A failure of definite reference undermines converses of (100)-(103):

(104) Thirty custards that blanketed exactly one buffet were served in a cup with a cherry on top.

(105) Every thirty of the custards that blanketed exactly one buffet were served in a cup with a cherry on top.

(106) None of the custards that blanketed exactly one buffet were served in a cup with a cherry on top.

(107) No custards that blanketed exactly one buffet were served in a cup with a cherry on top.

The converses of (96)-(99) are also unsound, provided one is careful not to equivocate on the meaning of the definite description:

(108) A custard that blanketed exactly one buffet was served with a cherry on top.

(109) Every custard that blanketed exactly one buffet was served with a cherry on top.

(110) None of the custards that blanketed exactly one buffet was served with a cherry on top.

(111) No custard that blanketed exactly one buffet was served with a cherry on top.

(112) Every custard that blanketed exactly one buffet was served with a cherry on top.

In imagining a premise made true by a giant custard draped across the buffet, one should not then take the plural definite description to tacitly contain a distributive quantifier, *the custards that each blanketed exactly one buffet*, for which the constitution principle (112) and associated inferences are indeed valid. Note that in formulating a generalization to cover the valid constitution principles, it is to be explained how the insertion of ‘each’ manages to classify the description that contains it among those in which plural terms do not occur essentially, resulting in the contrast between (111) and (112).

Joining the distributivity of count terms are analogue inferences of dissectivity among mass terms, (113)-(117), analogous constitution principles in (118) and analogue restrictions on their generalization (119)-(126).

**Dissectivity**

(113) a. (Some) fire expired. ⊨ Some of the fire expired.

b. Some of the fire expired. ⊨ (Some) fire expired.


b. Any custard hides a jewel. ⊨ Any of the custard hides a jewel.
(115)  a. None of the puff pastry is more puff than pastry.  \(\vdash\) No puff pastry is more puff than pastry.
    b. No puff pastry is more puff than pastry.  \(\vdash\) None of the puff pastry is more puff than pastry.

(116)  Much fire of the fire from heaven scorches the earth.  \(\vdash\) Much fire from heaven scorches the earth.  
               Much fire of the fire from heaven scorches the earth.  \(\vdash\) Much fire of the fire from heaven scorches the earth.

(117)  Most of the butter pastry is all butter.  \(\vdash\) Most butter pastry is all butter.  
               Most butter pastry is all butter.  \(\vdash\) Most of the butter pastry is all butter.

Constitution for Mass Terms

(118)  \[
\begin{align*}
  &\text{of the fire (from heaven)}[\xi] \leftrightarrow \text{fire (from heaven)}[\xi] \\
  &\text{of the custard}[\xi] \leftrightarrow \text{custard}[\xi] \\
  &\text{of the puff pastry}[\xi] \leftrightarrow \text{puff pastry}[\xi] \\
\end{align*}
\]

(119)  Some of the puff pastry that blanketed a buffet was more puff than pastry.  
               \(\not\vdash\) (Some) puff pastry that blanketed a buffet was more puff than pastry.

(120)  Any fire that dotted Carmel had a cool spot.  
               \(\not\vdash\) Any of the fire that dotted Carmel had a cool spot.

(121)  No custard that fills a hundred pastries is without a jewel hidden somewhere inside.  
               \(\not\vdash\) None of the custard that fills a hundred pastries is without a jewel hidden somewhere inside.

(122)  \[
\begin{align*}
  &\not\vdash \text{of the puff pastry that blanketed a buffet}[\xi] \rightarrow \text{puff pastry that blanketed a buffet}[\xi] \\
  &\not\vdash \text{of the fire that dotted Carmel}[\xi] \rightarrow \text{fire that dotted Carmel}[\xi] \\
  &\not\vdash \text{of the custard that fills a hundred pastries}[\xi] \rightarrow \text{custard that fills a hundred pastries}[\xi] \\
\end{align*}
\]

(123)  (Some) puff pastry that blanketed exactly one buffet was more puff than pastry.  
               \(\not\vdash\) Some of the puff pastry that blanketed exactly one buffet was more puff than pastry.

(124)  Any of the fire that dotted exactly one estate in Carmel had a cool spot.  
               \(\not\vdash\) Any fire that dotted exactly one estate in Carmel had a cool spot.

(125)  None of the custard that fills a hundred pastries but no more is without a jewel hidden somewhere inside.  
               \(\not\vdash\) No custard that fills a hundred pastries but no more is without a jewel hidden somewhere inside.

(126)  \[
\begin{align*}
  &\not\vdash \text{puff pastry that blanketed exactly one buffet}[\xi] \rightarrow \text{of the puff pastry that blanketed exactly one buffet}[\xi] \\
  &\not\vdash \text{fire that dotted exactly one estate in Carmel}[\xi] \rightarrow \text{of the fire that dotted exactly one estate in Carmel}[\xi] \\
  &\not\vdash \text{custard that fills a hundred pastries but no more}[\xi] \rightarrow \text{of the custard that fills a hundred pastries but no more}[\xi] \\
\end{align*}
\]

Since distributivity, plural partitivity, dissectivity, constitution and (52)-(56) are all extensible to novel \(F\), it compels a parse that factors out the recurrent \(F\) and makes explicit its morphological modification. In particular, the constitution principles become (127)-(129), instantiating NP with the same \(F\) throughout.

(127)  \[
\begin{align*}
  &\text{of the NP.pl}[\xi] \leftrightarrow \text{NP.pl.sg}[\xi] \\
  &\text{of the fire.pl (from heaven)}[\xi] \leftrightarrow \text{fire.pl.sg(from heaven)}[\xi] \\
  &\text{of the custard.pl}[\xi] \leftrightarrow \text{custard.pl.sg}[\xi] \\
  &\text{of the butter pastry.pl}[\xi] \leftrightarrow \text{butter pastry.pl.sg}[\xi] \\
\end{align*}
\]
in accepting (127)-(129) affirms instances of (130)-(132):

contain expression of a relation to whatever the definite description denotes. A speaker
in the object language, the natural language on display in (75)-(84) and (113)-(117),

which rely on speakers knowing (135):

singular first argument are related by valid inferences of the form in (133) and (134),

The partitive relations thus exposed, the nonsingular relation and its restriction to a
singular first argument are related by valid inferences of the form in (133) and (134),

which rely on speakers knowing (135):

Any of the custards is one or more of the flans. ⊤ Any of the custards are one or more of the flans.
Some of the custards are none of the flans. ⊥ Not every one of the custards is one of the flans.
(Some) n of the custards are not n of the flans. ⊥ Not every one of the custards is one of the flans.

\[ \forall \xi \forall \gamma \left( \forall \xi (sg.of[\xi, \gamma] \rightarrow sg.of[\xi, \gamma]) \rightarrow \forall \xi (o(\lfloor \xi, \gamma \rfloor) \rightarrow o(\lfloor \xi, \gamma \rfloor)) \right) \]
Similarly, the converse inferences rely on the converse to (135) in (138):

\( \forall \xi \forall \gamma \left( (\forall \xi (\text{sg.of}[\xi,\xi] \rightarrow \text{of}[\xi,\gamma])) \rightarrow \forall \xi (\text{sg.of}[\xi,\xi] \rightarrow \text{sg.of}[\xi,\gamma]) \right) \)

Speakers also know the nonsingular partitive relation to be reflexive (140) in affirming all instances of (139):

\( \forall \xi \text{of}[\xi,\xi] \) (reflexivity)

With reflexivity, (135) and (138) entail (141) and, in turn, the transitivity of the nonsingular partitive relation:

\( \forall \xi \forall \gamma \left( (\forall \xi (\text{sg.of}[\xi,\xi] \leftrightarrow \text{of}[\xi,\gamma])) \leftrightarrow \text{of}[\xi,\gamma] \right) \)

\( \forall \xi \forall \gamma \left( (\forall \xi (\text{of}[\xi,\xi] \rightarrow \text{of}[\xi,\gamma])) \leftrightarrow \text{of}[\xi,\gamma] \right) \) (transitivity)

The nonsingular partitive relation is also known to be antisymmetric (144), as reflected in speakers’ acceptance of all instances of (143):

\( \forall \xi \forall \gamma \left( (\text{of}[\xi,\xi] \& \text{of}[\xi,\xi]) \rightarrow \xi = \gamma \right) \) (antisymmetry)

Antisymmetry and (141) derive a principle of extensionality for the singular partitive relation:

\( \forall \xi \forall \gamma \left( (\forall \xi (\text{sg.of}[\xi,\xi] \leftrightarrow \text{sg.of}[\xi,\gamma])) \leftrightarrow \xi = \gamma \right) \) (extensionality)

Extensionality holds of the partitive construction occurring in the distributive quantifiers of (75)-(83), whatever may be meant by ‘sg.of[\xi,\xi]’. But, unpacking it as in (146) will afford a common syntax and semantics for the singular morpheme here and elsewhere:

\( \text{sg.of}[\xi,\xi] \) for (sg[\xi] & of[\xi,\xi])

\( \text{sg.}\Phi[\xi,\xi] \) for (sg[\xi] & \Phi[\xi,\xi])

As far as a speaker’s inferential behavior goes as reflected in constitution principles and inferences of distributivity, plural partitivity and dissectivity, the morphological and syntactic analysis has bottomed out. It could leave speakers in command of a primitive constitution relation around which the partitive constructions are constructed, of[\xi,\xi] for ‘of[\xi,\xi]’.

Instead, joining the
literature, the singular morpheme is defined in terms of the nonsingular partitive construction.

§1.2.1. A primitive partitive relation ‘of(ξ,ς)’ is a partial order with zero:

(148) \( \forall \xi \ of(\xi,\xi) \) (reflexivity)
(149) \( \forall \xi \forall \zeta \ ((of(\xi,\zeta) & of(\zeta,\xi)) \rightarrow \xi = \zeta) \) (antisymmetry)
(150) \( \forall \zeta \forall \gamma \ (\forall \xi (of(\xi,\zeta) \rightarrow of(\xi,\gamma))) \leftrightarrow of(\zeta,\gamma)) \) (transitivity)
(151) \( \exists \xi \forall \zeta \ (of(\xi,\zeta) & (of(\zeta,\xi) \rightarrow \xi = \zeta)) \) (zero)

Some zero or more \( \xi \) be singular just in case nothing else nonzero is of them:

(152) \( sg[\xi] \leftrightarrow \forall \zeta \forall \gamma (-of(\zeta,\gamma) \rightarrow (of(\zeta,\xi) \rightarrow of(\xi,\zeta))) \)

Recall §0.1 that zero is singular—(6), (7), (24), (25) (and, also nonsingular (4), (5), (10), (22)-(24), (26)). A domain restricted to the nonzero as in (34)-(38) is restriction to the existent:

(153) \( E[\xi] \leftrightarrow \exists \zeta -of(\xi,\zeta) \)

As above, the singular and nonsingular partitive relations are related by extensionality, which can be formulated with equivalent results either as (154) or (155):

(extensionality)
(154) \( \forall \zeta \forall \gamma \ (\forall \xi ((sg[\xi] & of(\xi,\zeta)) \leftrightarrow (sg[\xi] & of(\xi,\gamma))) \leftrightarrow \zeta = \gamma). \) That is, 
\( \forall \zeta \forall \gamma \ (\forall \xi (sg.of[\xi,\zeta] \leftrightarrow sg.of[\xi,\gamma]) \leftrightarrow \zeta = \gamma) \)

(155) \( \forall \zeta \forall \gamma \ (\forall \xi ((sg[\xi] & E[\xi] & of(\xi,\zeta)) \leftrightarrow (sg[\xi] & E[\xi] & of(\xi,\gamma))) \leftrightarrow \zeta = \gamma). \) That is, 
\( \forall \zeta \forall \gamma \ (\forall \xi (sg.E.of[\xi,\zeta] \leftrightarrow sg.E.of[\xi,\gamma]) \leftrightarrow \zeta = \gamma) \)

As (155) invites, a partitive construction can be viewed as overt expression of the predication relation that holds between a singular, first-order predicate and anything it denotes, translating as in (156):

(156) ‘\( Xx \)’ for ‘sg.E.of[\( x,X \)]’ where uppercase and lowercase variables belong to the same sort in the language of the partitive construction.

In a first-order language where any primitive noun is singular, including those underlying mass terms such as fire, any \( X \) be fire if and only if \( \exists xXx \& \forall x(Xx \rightarrow fire(x)) \). Any quantity of fire, on this view, is a first-order object of fire. Although it may cheat an intuition that the concept fire is pre-individuative, the view is common to all accounts searching out a common semantics for mass and count terms. In the language of the partitive construction, absent the assumption of a first-order syntax with singular variables, it is declared outright that if there exist anything at all—some fire, for example—(at least) some of it is singular:

(157) \( \forall \zeta (E[\xi] \rightarrow \exists \zeta (sg[\zeta] & E[\zeta] & of(\zeta,\xi))) \)
§1.2.2. Definite description.

The partitive relation becomes the basis for a translation of definite descriptions in which the same lexical item, the article *the*, appears in singular, plural and mass definite descriptions, as desired. Define first the iota operator as in (158)²³:

\[
(158) \quad [\iota \xi : \Phi[\xi]] \Psi[\xi] \leftrightarrow [\exists \xi : \Phi[\xi] \& \forall \zeta (\Phi[\zeta] \rightarrow \text{of}(\zeta, \xi)) \& \forall \gamma (\forall \zeta (\Phi[\zeta] \rightarrow \text{of}(\zeta, \gamma)) \rightarrow \text{of}(\xi, \gamma))] \Psi[\xi]
\]

Many definite descriptions, *the* \(\Phi\), are adequately translated without further comment assuming ‘the’ to be the pronunciation of the iota operator. In *the custard(s) that blanketed exactly one buffet*, the condition that \(\Phi[\xi]\), that the custard(s) referred to be custard that blanketed exactly one buffet, induces reference to fail if the custard(s) referred to blanketed two buffets. Yet, the condition that \(\forall \zeta (\Phi[\zeta] \rightarrow \text{of}(\zeta, \xi))\) requires that any custard(s) that blanketed exactly one buffet be some of what is referred to, and thus if two buffets are blanketed in custard, reference again fails if any of it is excluded.²⁴

The definite description in (159) containing a prenominal measure phrase fails to refer, as above, if there are two buffets blanketed in custards— even if one buffet holds 365 and the other 248. In this respect, the prenominal measure phrase agrees with the nonrestrictive modifier in (161) rather than the restriction in (160), and the translation is (163) rather than (162):²⁵

\[
(159) \quad \text{The 365 custards that blanketed exactly one buffet were served in a cup with a cherry on top.}
\]
\[
(160) \quad \text{The custards that blanketed exactly one buffet and numbered 365 were served in a cup with a cherry on top.}
\]
\[
(161) \quad \text{The custards that blanketed exactly one buffet, which numbered 365, were served in a cup with a cherry on top.}
\]
\[
(162) \quad * \quad [\text{the } \xi : \mu P[\xi]] \Psi[\xi] \quad \text{for} \quad [\iota \xi : \mu P[\xi] \& \Phi[\xi]] \Psi[\xi]
\]
\[
(163) \quad \text{[the } \xi : \mu P[\xi]] \Psi[\xi] \quad \text{for} \quad [\iota \xi : \Phi[\xi] \& [\iota \xi : \Phi[\xi]] \mu P[\xi]] \Psi[\xi]
\]

§1.3. Constitution and Comprehension.

With definite descriptions in place, it can be verified that the constitution principles (130)-(132) are equivalent to (164)-(166)²⁶, which take on the form of conditional comprehension principles:

**Conditional Singular-Plural Comprehension**

\[
(164) \quad \exists \xi \text{NP.sg}[\xi] \rightarrow \exists \xi \forall \zeta (\text{sg.of}[\xi, \zeta] \leftrightarrow \text{NP.sg}[\xi])
\]

**Conditional Plural-Plural Comprehension**

\[
(165) \quad \exists \xi \text{NP.pl}[\xi] \rightarrow \exists \xi \forall \zeta (\text{of}[\xi, \zeta] \leftrightarrow \text{NP.pl}[\xi])
\]

**Conditional Mass-term Comprehension**

\[
(166) \quad \exists \xi \text{NP.}\emptyset[\xi] \rightarrow \exists \xi \forall \zeta (\text{of}[\xi, \zeta] \leftrightarrow \text{NP.}\emptyset[\xi])
\]
The cited inferences of distributivity, plural partitivity and dissectivity are nothing stronger; but, the zero that (151) stipulates and the nonself-identical custard(s) and the like refer to warrant unconditional comprehension principles. If, as supposed in §0.1 (v. (33)), the quantifiers’ restrictions—plural ‘(nonselfidentical[ξ] & custard.pl[ξ])’, singular ‘(nonselfidentical[ξ] & custard.pl.sg[ξ])’ (v. (6), (7) (24)) and mass ‘(nonselfidentical[ξ] & custard.∅[ξ])’—are true of zero ξ, the conditions of (165)-(166) are satisfied so that:

**Singular-Plural Comprehension Principles**
(167)  \( \exists \varsigma \forall \xi (\text{sg.of}[\xi, \varsigma] \leftrightarrow \text{NP.sg}[\xi]) \)

**Plural-Plural Comprehension Principles**
(168)  \( \exists \varsigma \forall \xi (\text{of}[\xi, \varsigma] \leftrightarrow \text{NP.pl}[\xi]) \)

**Mass-term Comprehension Principles**
(169)  \( \exists \varsigma \forall \xi (\text{sg.of}[\xi, \varsigma] \leftrightarrow \text{NP.∅}[\xi]) \)

These comprehension principles and their related constitution principles, as numerous as the valid substitutions for NP, are to be derived from a generalization to a single restricted comprehension axiom:

**Restricted Comprehension Axiom**
(170)  \( \vdash (\\forall \Phi[\xi]) \exists \varsigma \forall \xi (\text{sg.E.of}[\xi, \varsigma] \leftrightarrow \text{sg.E.Φ}[\xi]) \)

As soon as a universal quantifier over phrases of the object language is introduced, the axiom’s formulation is constrained by Russell’s paradox, taken up in §1.3.1., to which the reader should turn to if unconcerned by the details deriving the comprehension and constitution principles from (170). Axiom (170) has abstracted away from the nominal morphology, from the plural and mass morphemes in particular, and these need to be reintroduced as discussed in §1.3.0 in any derivation of the comprehension and constitution principles.

§1.3.0.  Excursus on ‘.pl’ and ‘.∅’.

Given (4), (10), (22) and (23), nothing in the meaning of the plural morpheme proper entails a nonzero number. The plural morpheme does not count what the pluralized nominal denotes; but, what a pluralized nominal denotes meets conditions necessary for counting, 27 the morpheme itself encoding the count/mass distinction 28 . Recognizing as much provides a probe into the internal structure of pluralized nominals such as some fires and two fires and also of the singular count a fire. If some fires is to be composed from some-ζ, fire[ζ], and pl[γ], it will not be ‘[some ζ : fire[ζ] & pl[ζ]]’ as invited when mistaking the plural morpheme for ‘more than one’.

(171)  Some fires dissipated.
(172)  Clint extinguished some fires.
Imagine (171) or (172) true of a hill in Carmel covered in chaparral with some scattered fires, two of which expire as described. Now, without change to what fire expires, set the intervening chaparral ablaze in one continuous terrain of fire from which the expiring fire is subtracted. Some of the fire dissipated (or, Clint extinguished some of it), the same fire, by hypothesis, as earlier imagined; but, this latter scene no longer supports the plural reference in (171) or (172). What fire expires has not changed; what else is fire has. Probing further, a continuous terrain of fire burns with variable intensity from cool orange to hot blue. Two hot spots from within that fire expire so that (173) and (174) are indeed true; and yet plural reference still fails in (175)-(176):

(173) Some islands of blue fire dissipated.
(174) Clint extinguished some islands of blue fire.

(175) Some blue fires dissipated.
Some fires that were blue-hot dissipated.
(176) Clint extinguished some blue fires.
Clint extinguished some fires that were blue-hot.

What blue fire expires be what islands of blue fire expire and count as two when so described. But, in this scene, what falls under the description fire is not what falls under the description island of blue fire, and the former is too much to count (as other than one big fire). Plural reference in (175)-(176) is not rescued even if the islands of blue fire are all the blue fire there is. What must meet the conditions for counting is all that falls under the noun to which the plural morpheme attaches, fire, (or, perhaps the noun and its complement, as in island of blue fire, excluding modifiers, pre-nominal and relative clause, from that description. This scene of a single fire in variegated hue contrasts (177)-(178), which describe it accurately, and the infelicitous (179)-(180):

(177) Some fire that was (two) islands of blue flame dissipated.
(178) Clint extinguished some fire that was (two) islands of blue flame.

(179) Some fires that were (two) islands of blue flame dissipated.
(180) Clint extinguished some fires that were (two) islands of blue flame.

The logical form of some fires is better represented by (181), and two fires by (182), where the plural morpheme, whatever proves to be the correct analysis of the count/mass distinction, imposes conditions for counting on all that is fire:

(181) [some \( \xi \) : fire[\( \xi \)] & [\( \iota \xi \) : fire[\( \xi \)]pl[\( \xi \)]]
(182) [two \( \xi \) : fire[\( \xi \)] & [\( \iota \xi \) : fire[\( \xi \)]pl[\( \xi \)]]; or, [\( \exists \xi \) : two[\( \xi \)] & fire[\( \xi \)] & [\( \iota \xi \) : fire[\( \xi \)]pl[\( \xi \)]]]

What has been observed about modification translates some blue fires as (183) and some fires that \( \Phi \) as (185), arguing against (184) and (186):

(183) [some \( \xi \) : blue[\( \xi \)] & fire[\( \xi \)] & [\( \iota \xi \) : fire[\( \xi \)]pl[\( \xi \)]]
(184) [some \( \xi \) : blue[\( \xi \)] & fire[\( \xi \)] & [\( \iota \xi \) : blue[\( \xi \)] & fire[\( \xi \)]pl[\( \xi \)]]
(185) [some \( \xi \) : fire[\( \xi \)] & [\( \iota \xi \) : fire[\( \xi \)]pl[\( \xi \)] & \( \Phi \)[\( \xi \)]]
(186) [some \( \xi \) : fire[\( \xi \)] & [\( \iota \xi \) : fire[\( \xi \)] & \( \Phi \)[\( \xi \)]pl[\( \xi \)] & \( \Phi \)[\( \xi \)]]
The singular *a fire* is also a count expression, and what is observed above repeats *mutatis mutandis*, imagining the expiration of only a single island of blue fire. The logical form of *a fire* therefore cannot be ‘*[\(a \varsigma : \text{sg}[\varsigma] & \text{fire}[\varsigma]\)]*’. It must similarly distinguish a fire, the one that will be said to expire, from the countable fires from which it is plucked. Singular count morphology incorporates the plural morpheme so-called, that is, the count morpheme, and some other morpheme to distinguish the singular from the non-singular:

(187)  \[a \varsigma : \text{sg}[\varsigma] & \text{fire}[\varsigma] & [i\varsigma : \text{fire}[\varsigma]]\text{pl}[\varsigma]\]

In summary, it is correct, as (181) and (187) allow, that *some fires* and *a fire* are quantifiers restricted by formulas in a single free variable, (188) and (189), unpacked according to (190), and error to unpack them as in (191) and (192):

(188)  \[\text{some } \varsigma : \text{fire.pl}[\varsigma]\]
(189)  \[a \varsigma : \text{sg}[\varsigma] & \text{fire}[\varsigma]\]
(190)  \[\text{NP.pl}[\varsigma] \text{ for } \text{NP}[\varsigma] & [i\varsigma : \text{NP}[\varsigma]]\text{pl}[\varsigma]\]
(191)  \[*\text{some } \varsigma : \text{fire}[\varsigma] & \text{pl}[\varsigma]\]
(192)  \[*a \varsigma : \text{sg}[\varsigma] & \text{fire}[\varsigma] & \text{pl}[\varsigma]\]

Factoring out \(F\), a noun with occurrences in singular, plural and mass terms, requires an expansion, as in (181)-(187), that applies the plural (count) morpheme to a term distinct from that associated with the quantifier or expressions of cardinality such as *two*. The mass-term morpheme, opposite the plural morpheme, is assumed to play by the same rules.\(\text{32}\) Whatever conditions are required for a domain to be measurable rather than countable, these too are imposed on all that the modified nominal describes:

(193)  \[\text{some } \varsigma : \text{fire.}\&[\varsigma]\]
(194)  \[\text{some } \varsigma : \text{fire}[\varsigma] & [i\varsigma : \text{fire}[\varsigma]]\&[\varsigma]\]
(195)  \[\text{NP.}\&[\varsigma] \text{ for } \text{NP}[\varsigma] & [i\varsigma : \text{NP}[\varsigma]]\&[\varsigma]\]

The revised internal structure of nominals prompts revision of the constitution principles. Accepting an inference (75)-(83) of distributivity commits the speaker to the corresponding constitution principle that now takes the form expanded in (196). Similarly, any inference of plural partitivity (80)-(84) or dissectivity (113)-(117) implies knowledge of a principle expanded as in (198) or (200).

**Singular-plural constitution principles**

(196)  \[\text{the } \varsigma : \text{NP.pl}[\varsigma] \text{ sg.of } [\varsigma,\varsigma] \leftrightarrow \text{NP.pl.sg}[\varsigma]
\text{the } \varsigma : \text{NP}[\varsigma] & [i\varsigma : \text{NP}[\varsigma]]\text{pl}[\varsigma])\text{(sg}[\varsigma] & \text{of}(\varsigma,\varsigma)) \leftrightarrow (\text{sg}[\varsigma] & \text{NP}[\varsigma] & [i\varsigma : \text{NP}[\varsigma]]\text{pl}[\varsigma])\]
(197)  \[\not\vdash \forall\text{NP}(\forall\Phi) \text{ ([the } \varsigma : \text{NP.pl } \Phi[\varsigma]\text{sg.of } [\varsigma,\varsigma] \leftrightarrow (\text{sg}[\varsigma] & \text{NP}[\varsigma] & [i\varsigma : \text{NP}[\varsigma]]\text{pl}[\varsigma] & \Phi[\varsigma]))\]

**Plural-plural constitution principles**

(198)  \[\text{the } \varsigma : \text{NP.pl}[\varsigma]\text{of}(\varsigma,\varsigma) \leftrightarrow \text{NP.pl}[\varsigma]
\text{the } \varsigma : \text{NP}[\varsigma] & [i\varsigma : \text{NP}[\varsigma]]\text{pl}[\varsigma])\text{of}(\varsigma,\varsigma) \leftrightarrow (\text{NP}[\varsigma] & [i\varsigma : \text{NP}[\varsigma]]\text{pl}[\varsigma])\]
Constitution principles for mass terms

Constitution principles for mass terms

As before (cf. (94)), the generalization from which these principles derive cannot be so broad as to apply in the manner of (197), (199) and (201) to any formula $\Phi$ of the object language free in a single variable. The descriptions containing essential plural terms in (96)-(112) and their mass term counterparts in (119)-(126) remain counterexamples to such a generalization.

As remarked above in §1.3, constitution principles are entailed by unconditional comprehension principles, to which they are in fact equivalent in a theory stipulating (151) that there are the zero. It is unconditional comprehension (202)-(204) that is generalized to derive its valid instances and valid instances of the constitution principles.

Singular-plural comprehension principles

Plural-plural comprehension principles

Mass-term comprehension principles

Since the plural- and mass- term morphemes occur within sentences without free variables, they can be factored out rewriting (202)-(204) as the equivalent (205)-(207):

Singular-plural comprehension principles

Plural-plural comprehension principles

Mass-term comprehension principles

But, these principles expose a flaw. Consider a substitution for $\text{NP}$, such as $\text{fire}$, for which the constitution principles and the inferences of distributivity, plural partitivity and dissectivity have all been previously judged sound, now evaluated in a context where what is fire fails the conditions to be countable, that is, $\neg [i\xi : \text{fire}([\xi])]_{\text{pl}}$. A denial of reflexivity, $\neg of([\xi],[\xi])$, then follows from the second conjunct of (206), and from the second
conjunct of (205), it follows that anything singular violates reflexivity, which is equally incoherent. One response is to revisit the data concerning the constitution principles and their associated inferences. Perhaps in judging them sound, speakers presuppose from the use of a count term that whatever is under discussion meets the condition to be counted, so that accepting them commits speakers only to the first conjuncts of (205)-(207):

**Singular-plural comprehension principles**

(208) \[ [\xi : \text{NP}[\xi]] \text{pl}[\xi] \rightarrow \exists \xi \forall \xi (\text{sg.of}[\xi, \xi] \leftrightarrow (\text{sg}[\xi] \& \text{NP}[\xi])) \]

**Plural-plural comprehension principles**

(209) \[ [\xi : \text{NP}[\xi]] \text{pl}[\xi] \rightarrow \exists \xi \forall \xi (\text{of}[\xi, \xi] \leftrightarrow \text{NP}[\xi]) \]

**Mass-term comprehension principles**

(210) \[ [\xi : \text{NP}[\xi]] \text{∅}[\xi] \rightarrow \exists \xi \forall \xi (\text{of}[\xi, \xi] \leftrightarrow (\text{sg}[\xi] \& \text{NP}[\xi])) \]

It now enters as an empirical hypothesis that speakers know that any NP, or at least any NP for which there is a valid constitution principle, is such that under any conditions of evaluation, \[ [\xi : \text{NP}[\xi]] \text{pl}[\xi] \vee [\xi : \text{NP}[\xi]] \text{∅}[\xi], \] that is, \( \text{NP} \) either meets the conditions for counting or for non-count measurement. Then, to know a singular-plural constitution principle or a mass term constitution principle for some \( \text{NP} \) is to know tout court a comprehension principle of the form in (211):

**Singular comprehension principles**

(211) \[ \exists \xi \forall \xi (\text{sg.of}[\xi, \xi] \leftrightarrow (\text{sg}[\xi] \& \text{NP}[\xi])) \]

As it will turn out, those NPs for which there is a valid instance of (211) are just those for which (211) entails a plural comprehension principle (cf. (209)):

**Plural comprehension principles**

(212) \[ \exists \xi \forall \xi (\text{of}[\xi, \xi] \leftrightarrow \text{NP}[\xi]) \]

A generalization of (211) to a comprehension axiom summarizes the speaker’s inferential behavior if its instantiations manage to be restricted to just those nominal descriptions for which distributivity, plural partitivity and dissection are observed to hold.

§1.3.1. Restricted Comprehension.

Russell’s paradox constrains generalization, excluding unrestricted comprehension axioms, such as (213)-(216), instantiating as shown any formula \( \Phi \) free in a single variable. Substituting the negation of the reflexive of the constitution relation, whatever that may be, results in contradiction:\(^{33}\)

(213) \( \not\vdash (\forall \Phi[\xi]) \exists \xi \forall \xi (\text{of}[\xi, \xi] \leftrightarrow \Phi[\xi]). \) Contradiction substituting ‘\( \text{of}[\xi, \xi] \)’ for \( \Phi[\xi]. \)

(214) \( \not\vdash (\forall \Phi[\xi]) \exists \xi \forall \xi (\text{sg.of}[\xi, \xi] \leftrightarrow \Phi[\xi]). \) Contradiction substituting ‘\( \text{sg.of}[\xi, \xi] \)’ for \( \Phi[\xi]. \)

(215) \( \not\vdash (\forall \Phi[\xi]) \exists \xi \forall \xi (\text{E.of}[\xi, \xi] \leftrightarrow \Phi[\xi]). \) Contradiction substituting ‘\( \text{E.of}[\xi, \xi] \)’ for \( \Phi[\xi]. \)

(216) \( \not\vdash (\forall \Phi[\xi]) \exists \xi \forall \xi (\text{sg.E.of}[\xi, \xi] \leftrightarrow \Phi[\xi]). \) Contradiction substituting ‘\( \text{sg.E.of}[\xi, \xi] \)’ for \( \Phi[\xi]. \)
In contrast, restricted comprehension (217) escapes. Substitution of the Russell predicate \( \neg \text{sg.E.of}[\xi, \xi] \) in (218) benignly entails that there is what is zero in number or more than one.

**Restricted Comprehension Axiom**

\[
(217) \quad \vdash (\forall \Phi[\xi]) \exists \varsigma \forall \xi (\text{sg.E.of}[\xi, \varsigma] \leftrightarrow \text{sg.E.}[\xi])
\]

\[
(218) \quad (217) \vdash \exists \varsigma \forall \xi (\text{sg.E.of}[\xi, \varsigma] \leftrightarrow \text{sg.E.}(\neg \text{sg.E.of}[\xi, \xi]))
\]

As presented, the restriction includes restriction to the existent. Since \( \Phi \) may be any formula of the object language in one free variable, consider the substitution of \( \neg \xi = \xi \) into an axiom unrestricted to the existent:

\[
(219) \quad (217) \vdash (\forall \Phi[\xi]) \exists \varsigma \forall \xi (\text{sg.of}[\xi, \varsigma] \leftrightarrow \text{sg.of}[\xi])
\]

\[
(220) \quad (219) \vdash \exists \varsigma \forall \xi (\text{sg.of}[\xi, \varsigma] \leftrightarrow (\text{sg}[\xi] & \neg \xi = \xi))
\]

It will contradict that the zero ((151)) are singular and of everything. The same substitution into an axiom restricted to the existent entails harmlessly that the zero are not existent:

\[
(221) \quad (217) \vdash (\exists \varsigma \forall \xi (\text{sg.E.of}[\xi, \varsigma] \leftrightarrow (\text{sg}[\xi] & \text{E}[\xi] & \neg \xi = \xi))
\]

The restricted comprehension axiom (217) reprises in the language of the partitive construction second-order comprehension (222), the validity of which is deductively equivalent to second-order logic itself (Boolos 1985b).

**Second-Order Comprehension Axiom**

\[
(222) \quad \vdash (\forall \Phi) \exists X \forall x (Xx \leftrightarrow \Phi[x]), \text{where } \Phi \text{ is any formula free in only the first-order variable } x.
\]

Apart from a reconciliation with Russell’s paradox, restricted comprehension serves sound empirical purpose too. Recall that the inferences of distributivity, plural partivity, and dissection, their constitution principles and the comprehension axiom from which they all derive must be restricted as warranted by (96)-(112) to those descriptions that do not themselves contain essential tokens of plural terms. For arbitrary \( \Phi \) in any one free variable, its restriction to \( \text{sg.E.}[\xi] \) forms a description that contains no essential plural terms or their mass term analogues.
In affirming the constitution principle (223) and denying (224), a speaker discriminates between formally identical plural definite descriptions, neither of which contains restriction to the singular:

(223) \( \text{of the glazed custards}\{\xi\} \leftrightarrow \text{a glazed custard}\{\xi\} \)
(224) \( \text{of the clustered custards}\{\xi\} \leftrightarrow \text{a clustered custard}\{\xi\} \)

Restricted comprehension (217) merely guarantees that there are zero or more things each of which is as the singular description on the right-hand side of (223) or (224) describes it, without securing, as it should not in light of (224), that these things are the referents of the corresponding plural description unrestricted to the singular. What distinguishes glazed custards from clustered custards is that the former is distributive. It applies to some things just in case it applies to each of them:

(225) Distributive(\( \Phi \)) \( \leftrightarrow \forall \xi(\Phi[\xi] \leftrightarrow \forall \zeta(\text{sg.E. of}\{\zeta, \xi\} \rightarrow \Phi[\zeta])) \)

A definite description finds its reference in the things that satisfy a possibly non-distributive description and includes all such things. It of course does not follow that just any thing among what has just been referred to also satisfies that description. An affirmed constitution principle is the speaker’s knowledge that the description is distributive and subject to comprehension. Classification of a complex \( \Phi \) as distributive is itself a deduction, assisted in part by considerations such as (226) and (227):

(226) Distributive(\( \Phi \)) \& Distributive(\( \Psi \)) \( \rightarrow \) Distributive(\( \Phi[\xi] \& \Psi[\xi] \))
(227) Distributive([\( \forall \zeta\colon \text{sg.E. of}\{\zeta, \xi\}\)]\( \Phi[\xi] \))

§1.4. Semantics.

For the semantics of an object language with plural expressions, it cannot be wrong for the metalanguage to include all the resources of its object language, the use of plural expressions in particular, its crucial clauses (228)-(231) quantifying in the plural over things and speaking in the plural of assignments of an object to a variable (Boolos 1985a and others):

(228) \( \Sigma(<\alpha, \xi>) \leftrightarrow_{\text{def}} (\text{sg}[\alpha] \& E[\alpha] \& \text{sg}[\xi] \& \text{variable}(\xi) \& \text{of}(<\alpha, \xi>, \Sigma)) \)
(229) \( \Sigma \approx \Sigma' \leftrightarrow_{\text{def}} \forall \zeta(\zeta \neq \xi \rightarrow \forall X(\Sigma(<X, \zeta>) \leftrightarrow \Sigma'(<X, \zeta>))) \)
(230) \( \Sigma \text{ satisfy } [\exists \xi: \Phi] \Psi' \leftrightarrow \\
\{\exists X: \exists \Sigma' (\Sigma \approx \Sigma' \& \forall Y(\Sigma'(<Y, \xi>) \leftrightarrow \text{sg.E.of}[Y,X]) \& \Sigma' \text{ satisfy } \Phi)\} \)
(231) \( \Sigma \text{ satisfy } [\forall \xi, \zeta] \Psi' \leftrightarrow \\
\exists X \exists Y(\forall Z(\Sigma(<Z, \xi>) \leftrightarrow \text{sg.E.of}[Z,X]) \& \forall Z(\Sigma(<Z, \zeta>) \leftrightarrow \text{sg.E.of}[Z,Y]) \& \text{of}(X,Y)) \)

How could one spurn a semantics that aims at disquotational truths such as (232) and reproach the theorist’s use of a plural ‘some custards’ on the right-hand side to interpret the plural some custards on the left-hand side taken from her own language?

(232) Some custards are dessert is true \( \leftrightarrow \) Some custards are dessert.
Yet, along the way, regimentation in terms of a partitive relation ‘of’ and the nominal morphology as defined above spins off some empirical claims about the natural language in use that are not as innocent as the disquotations aimed at. Any subject who assents to (233) commits herself to (234):

(233)  Two custards are dessert.
(234)  $\exists \xi \neg \text{sg}[\xi]

Acquiring a free-standing word as in (235) with the meaning of the negated singular morpheme, she can be expected to assent to (236) and (237), which are, as fits her usage, indeed true under the proposed analysis.

(235)  $\text{non-singular}[\xi] \leftrightarrow_{M} \neg \text{sg}[\xi]
(236)  Two custards are non-singular.
(237)  Some things are non-singular.
        There are non-singulars.
(238)  *Something is non-singular.
        *There is a non-singular.

As soon, however, as she slips from (237) to (238), she utters a falsehood. It is to suffer a linguistic illusion, so the analysis claims, to believe that (237) entails (238) or for the theorist to think that (237) and (238) are indifferent paraphrases of (234). Likewise, the literal meaning of (239) does not contradict (234) or deny the existence of what the two custards or any other plural expression refers to, and illusion to hear it otherwise.

(239)  Nothing is (a) non-singular.
        There is no non-singular.

The margins of this illusion are exposed when juxtaposed to alternative paraphrases for ‘$\neg \text{sg}[\xi]$’ and for (234). Introduce to the subject a verb: to non-singularize is to be non-singular, so that (240) is fair translation of (234).

(240)  There are the things that non-singularize.
        The things that non-singularize exist.
(241)  *Any of the things that non-singularize non-singularizes.

With some prompting, the subject recognizes that (241) is false, like any of (96)-(112) or like (243) that would apply an essentially plural description to a singular object:

(242)  There are the things that are more than one.
        The things that are more than one exist.
(243)  *Any of the things that are more than one is more than one.
(244)  Everything is one of the things that are more than one.

There is, in effect, no difference in meaning between be more than one and non-singularize or be non-singular. And, yet, although the noun in (246) is declared synonymous and (246) supposed therefore to be as transparently false as (241) or (243) is, (246) comes across as true as “any of the-whatevers is a whatever”: 
There are the non-singualrs.
The non-singualrs exist.
Any of the non-singualrs is a non-singular.

Unlike verbs and relative clauses, frequent vehicles for expressing the essentially plural, the speaker who accepts (246) easily has assumed that nouns are for the most part distributive expressions ((225)).

The natural language has proven to be more precise—or, precious—than common usage is always aware of, but without obvious hindrance to its expressive power. The theorist speaker finds words, if not one way then another, to accurately speak her mind that (234) \( \exists \xi \neg \text{sg} [\xi] \). Moreover, despite the regimentation of singular and plural reference and the restriction on comprehension, there is implied no confinement on what she may quantify over. If she uses everything, a singular quantifier, she may use it intending without contextual restriction to quantify over absolutely everything there is and everything she thinks there is. Of course it is not the case that any two things that there are is something that there is, affording a contrast between (247) and (248), although any of them is indeed something that there is and any of them is among the things she quantifies over.

Some things that there are are somehow related.
Something that there is somehow related.

It seems benign for the analysis to deny that any things that there are is something that there is, at least to the extent that it does not violate any deeply held pre-theoretic judgment to the contrary, even without instruction from Russell’s paradox. A more controversial implication of the analysis is its dark view of any alleged identity between the one and the many:

The cards are one deck.
The trees are one copse.

No doubt (249) and (250) are true sentences of English, but the analysis then requires that the copula occur here not as an expression of identity or predication but as a nonlogical relation expressing coincidence.

§1.4.1. Against singularism.

Singularism is the mistaken view that the semantics of any language can be developed in a metalanguage with only singular reference so that an object language that contains both singular and plural expression is projected in the metalanguage onto that fragment of itself that contains only singular expression. In showing that all plural talk is mere disguise reducible to a formally more restricted singular idiom, singularism would have been a significant discovery, unlike the banal achievement (§1.4.) of giving a semantics for plural expressions in a language that uses them. An obstacle for the singularist view (adapted from Rayo & Yablo 2001: 75), typical of those canvassed in the literature, is how to keep the disquotational truths of (251) from becoming the self-
defeating contradictions of (252) when the object language is interpreted without benefit of plural expressions in the metalanguage:

(251) “Some things are too many to be a thing” is true iff some things are too many to be a thing.
“Some things are not a thing” is true iff some things are not a thing.

(252) “Some things are too many to be a thing” is true iff a thing [viz., the thing that those some things are] is too many to be a thing.
“Some things are not a thing” is true iff a thing is not a thing.

In this setting, if the theory that entails (234) is not to entail the contradiction that a thing is non-singular (cf. (238)) or that a thing non-singularizes, (234) had better not find its way into the object language. The theorist’s conviction that (234), which formerly may have provoked some mistranslation and linguistic illusion, lapses here into the ineffable. As in the language with a homophonic semantics §1.4., it cannot be asserted within a language for which a singularist semantics has been defined that there is a nonsingular thing, a plural object, but singularism has no other conveyance for (234).

§1.4.2. The semantic type of singular, plural and mass (in)definite descriptions.

A plural or mass definite description is used as an expression of direct or demonstrative reference to the same extent as a singular counterpart, and thus no classification of their semantic types should divorce them. That direct or demonstrative reference defies paraphrase in words is sometimes taken as evidence that expressions of direct or demonstrative reference are terms or arguments rather than predicates. Yet, one in (253) is unmistakably a predicate, both syntactically and semantically (the red and the blue being only a few individuals among the ones), and hardly less immune to paraphrase than its demonstrative antecedent:

(253) These are indescribable, and the three red ones and the one blue one will taste that same indescribable taste everybody remembers.

If the demonstrative contributes nothing more than its reference to (253)’s meaning, then so too does the predicate one contribute nothing more than what it denotes. Arguments that the name Aristotle bears direct reference to Aristotle apply, as Burge (1973) points out, with equal force to the relation between any of the Aristotles and Aristotle as it occurs in the three Aristotles or those Aristotles, where it is plainly a predicate. As with the simple name, nothing ‘predicational’ is felt to intervene between such an austere predicate and what it denotes. An impression of direct reference offers no insight into the logical form of the phrase bearing it— Fido-Fido or Fido-[x : Fido(x)]. Burge’s remarks about the logical syntax of direct reference are next joined by a Fregean point. If in the constant presence of mutually occlusive objects, demonstrative reference to these or those goes unresolved and their number unknown unless resolved under some concept, it must be that whenever definite reference succeeds, speaker and hearer have grasped just such a concept, even if words should fail to paraphrase and nothing ‘predicational’ seems under introspection to be involved. Just imagine failures, where the hearer is at a loss for the concept intended, as when gesturing to an assortment, the speaker says (254) or (255):
These have been arranged in an attractive pattern for you and you alone. As many as I could arrange, I arranged in an attractive pattern just for you.

Every use of a demonstrative, these, those, they and so on, is these NP, those NP, the-NP, and every use of a quantifier all, most, some, as many as Φ is all NP, most NP, some NP, as many NP as Φ, even if NP is unspoken, where any demonstration (to a point in space) or any counting must be supplemented with a concept in order to settle reference on some intended objects.

A related observation is that quantifying-in elicits the predicative de re, assuming the restriction in restricted quantification to be a predicate, Φ in (256):

(256) \[ Q : Φ \]
(257) a. \[ [the ξ : ...[Q : ξ] Ψ...][ Φ ] \]
     b. \[ [the ξ : ...][Q : ξ] Ψ...[ Φ ] \]

An honest casino is invaded one night without the house’s knowledge by underage gamblers. Sentences (258)-(261) can make true de re reports that ascribe neither knowledge of a felony nor knowledge of a game that is anything but fair, certainly with winners and losers but without anyone’s winnings certain in advance:

(258) The casino operator knew that some underage gamblers must win and some lose. (adapted from Bricker 1989)
     The casino operator knew that some patrons that night that unknown to him were underage must win and some lose.
(259) Them, with the false IDs around their necks, the casino operator knew that some must win and some lose.
(260) The underage gamblers were such that the casino operator knew that some must win and some lose.
(261) The gamblers such that the casino operator knew that some must win and some lose were underage.

(262) \[ [ιξ : underage gamblers[ξ]] the casino operator knew that must [some ζ : ξ[ζ]] ζ win and [some ζ : ξ[ζ]] ζ lose. \]

As in (262), the logical form for these sentences quantifies in the restrictions to the quantifier some, a predicate construed de re, and in (259)-(261), the antecedent for this predicate is a plural expression. If plural definite descriptions are univocal in their semantic type, and plural definite descriptions sometimes quantify in the restrictions to quantifiers, then the variables of plural quantification, ξ in (257) and (262), always belong to the same type, the type of predicates.46

§1.4.3. Monadic second-order logic and the language of the partitive construction.

If the logical form of natural language quantification presents as in (263) two predicates Φ and Ψ, one might expect, absent special pleading, that speakers evaluate quantifying in the one and the other in roughly the same way, especially if the quantifier is symmetric, as some is in (264) and (265):
As above, sentence (266) quantifies in the restriction to *some*, and thus its logical form realizes (264). If now (268) is taken to realize (265) (as in effect when stipulating that (268) paraphrases second-order logic ‘∀X∃xXx’) quantifying in *some’s* matrix predicate, we should arrive at a pair of sentences that speakers should judge rather alike:

(266) There are some things such that some are things.
(267) There are the things such that some are things.

(268) There is something that some things are. (*cf. *∃X∃xXx*)
(269) There are the things that some things are.

Yet, as in Williamson (2003), the resemblance fades as soon as these sentences are embedded in modal contexts:

(270) Something that possibly some things are, some things are.
(271) The things that possibly some things are, some things are.

(272) Some things such that possibly some are things, some things are (them).
(273) The things such that possibly some are things, some things are (them).

(274) Some things such that possibly some are things, some are things.
(275) The things such that possibly some are things, some (of them) are things.

Quantifying into the matrix predicate of *some things*, sentence (271) is plainly false in that the possibility of some things being wealthy, beautiful and wise does not entail that some things are. In contrast, a necessary truth of metaphysical identity results, (273) or (275), if instead one quantifies in the restriction to *some*. If one sticks to the logical translation proposed, the report is allegedly of a contrast in truth between (276) (translating (273)) and (277) (translating (275)):

(276) [the ζ : ◊[some ξ : things[ξ]] ζ[ξ]][some ξ : things[ξ]] ζ[ξ]
(277) [the ζ : ◊[some ξ : ζ[ξ]] things[ξ]][some ξ : ζ[ξ]] things[ξ]

Something has been lost in translation since (276) and (277) cannot contrast in truth. A dilemma has been exposed: if, uncontroversially, natural language quantification is as in (263), then which of (266)/(267) or (268)/(269), if either, realizes (monadic) second-order quantification, quantifying in a predicate, Φ or Ψ respectively?

Without faulting the parse in (263) or denying the implied symmetry between (264) and (265), translation should scorn less the grammar of sentences (268) and (269). These sentences, after all, quantify in the complement to a tensed copula. They therefore do not realize (265), which may in fact go unattested in natural language, and rather quantify in not Ψ but a phrasal position properly contained within Ψ. The dilemma is resolved: quantifying in a quantifier’s restriction and also plural quantification, in so far as it
belongs to the same type as argued in §1.4.2, realizes quantifying in a monadic predicate, i.e., monadic second-order quantification. In contrast, sentences such as (268) and (269), quantifying in the complement of a tensed copula, exemplify either quantifying in at least a dyadic relation (between objects and times or states (and perhaps worlds)) or substitutional quantification over verb-phrasal complements, the latter suggestion cognizant of the fact that speakers seem to have under consideration when asserting (278) a list of alternative descriptions of what the psychiatrist might be:

(278) There is something that the old psychiatrist never was and may never yet be to anyone that consults him—attentive and interested more in her problems than his fee.

Substitutional quantification is undefeated by the obvious truth of (279) and the fact that the takings of arbitrary objects outnumber the phrases of any natural language:

(279) Take any objects you like, there is something that they and only they are. (Rayo & Yablo 2001)

For, among those phrases are those that embed anaphoric expressions, already attested in (278), so that a verifying continuation of (279) is (280):

(280) Take any objects you like, there is something that they and only they are—namely, “some of them”.

Some discussions genuflecting to Frege treat (monadic) second-order logic as if it were given to be anything more than a calculus in search of an interpretation, as if it were given that if ‘∃X∃xXx’ (or (265)) means anything at all, it means what (268) means. Speakers’ reliable intuitions about the meaning of (268) are then mistaken as insight into the meaning of ‘∃X∃xXx’ and second-order logic in general, even though ‘∃X∃xXx’ is lame as an analysis of (268) and introspection about (268) is not introspection about any such logical form. Shunting aside (268)-(271) as either substitutional or polyadic second-order quantification, it is rather quantifying in a quantifier’s restriction (264) that realizes in natural language monadic second-order quantification. Then, since plural quantifiers quantify in quantifier restrictions, plural quantification is monadic second-order quantification.

In quantifying in monadic predicates, of may be deployed in the interpretation of their predication relation (v. (156); Boolos 1984, 1985ab; Higginbotham 1998, 2000; Hossack 2000):

(281) \[ \Sigma \text{satisfy } V_{yi} \iff \\
∃X∃Y(∀Z(Σ(<Z,Y>) \iff \text{sg.E.of}[Z,X]) \& ∀Z(Σ(<Z,Y>) \iff \text{sg.E.of}[Z,Y]) \& \text{sg.E.of}[X,Y]) \]

Thus, the partitive construction and predication are just two sides, pronounced and unpronounced, of the same coin. Occurrences in the object language of ‘V_{yi}’ (v. (258)-(261), (272)-(275)) could all be replaced (v. (156)) with ‘sg.E.of[Vi,Y]’ provided that the partitive goes unpronounced here. Correlatively, it could be denied that of tokens a relation in the object language and is rather the pronunciation of the concatenation indicating predication.
The language of the partitive construction has resumed in a syntax of monosortal variables the essential features of monadic second-order logic. Like predicative variables, variables in the language of the partitive construction stand in indifferently for count predicates and mass predicates. Like predicates, they may denote nothing at all, and thus the language of the partitive construction supports an analogue of unconditional second-order comprehension:

Unconditional, Restricted Comprehension Axiom

(217)  \( \vdash (\forall \Phi) \exists \xi \forall \varsigma (\text{sg.E.of}[\xi, \varsigma] \leftrightarrow \text{sg.E.}\Phi[\xi]) \), where \( \Phi \) is any formula free in only \( \xi \).

(Unconditional) Second-Order Comprehension Axiom

(222)  \( \vdash (\forall \Phi) \exists X \forall x (Xx \leftrightarrow \Phi[x]) \), where \( \Phi \) is any formula free in only the first-order variable \( x \).

Furthermore, the semantic type of the direct object of the partitive construction is the same as that which restricts a quantifier, that is, the semantic type of predicates, one presumes. Boolos’ (1985a) semantics for monadic second-order logic extended here to the language of the partitive construction shows, conforming to his nominalism, that there is reference to the none just in case there is no reference to anything that there is, without reference to a concept or to a null object. A language with only one sort of variable is threatened by Russell’s paradox, prompting a comprehension axiom that is restricted, which in this case is not without empirical justification (§1.3.1.) and implies no restriction on the expressive power of the language provided its semantics is not singularist (§1.4.1). The choice posed between a language with one sort of variable and another with two seems to me to be decided in part by one’s view of a single morpheme, ‘sg’. If one holds that it has a meaning, \( \text{sg}[\xi] \), true or false of some \( \xi \), it is hard to imagine how it could not apply to \( \xi \) that could not also be the direct objects of the partitive relation, as supposed, for example, by the definition (152), thus allowing that ‘\( \xi \)’ is a variable of plural reference:

(152)  \( \text{sg}[\xi] \leftrightarrow_{df} \forall \varsigma \forall \gamma (\neg \text{of}(\varsigma, \gamma) \rightarrow (\text{of}(\varsigma, \xi) \rightarrow \text{of}(\xi, \varsigma))) \)

But, if singular variables do not occur in conjunction with the singular morpheme, where else would they? It seems that two-sorted variables are better suited to the claim that the singular morpheme does not mean anything at all and is rather the auditory analogue of graphical lowercase indicating a first-order variable, a suggestion which semanticists in the business of finding meaning everywhere might recoil from.\(^{50}\) Note that if the language of the partitive construction has only the one sort of variable and its type is that of predicates, it is then a language without any first-order variables. To be is to be some zero or more things satisfying ‘\( \text{E}[\xi] \& \Phi[\xi] \)’ for some formula \( \Phi \).
§2. Essential plurals in natural language.

Essential plurals occur as the second argument to *of* but not as argument to the primitive concept *custard*, which is distributive, and they occur again in many complex phrases as above in (1), (3), (4)-(17) and (96)-(112). What is the inventory of primitive vocabulary with essentially plural arguments, from which derive their essential occurrences in complex phrases?

In the limit, *of* is the only one, (or, there aren’t any if monadic second-order logic is both object and meta-language. v. n. 49). Appearances to the contrary would be rescued by what has been called changing-the-subject (Oliver & Smiley 2001) and the surrogate method (Rayo 2002), which exposes a hidden parameter to explain away any illusion to the contrary.

§2.1. Cardinality predicates and relations.

One might have thought, for example, that the predicate *zero*[$\xi$] expressing a primitive ‘zero($\xi$)’ is an example par excellence of the primitive second-order (v. n. 4). But, *zero*[$\xi$] can be taken to pronounce a relation, *card*[$0, \xi$], to a first-order object, the number 0, and that relation may in turn be analyzed without recourse to any primitive relation to plurals or to concepts other than that expressed by *of* (283)-(284) or equivalently analyzed entirely within the analogue (monadic) second-order logic (285)-(286):

(283) \[\text{injective}[0] \iff \forall \xi \forall \zeta \forall \nu ((\text{sg.E.of}[<\nu, \zeta>, 0] \& \text{sg.E.of}[<\nu, \xi>, 0]) \rightarrow \xi = \zeta) \& ((\text{sg.E.of}[<\xi, \nu>, 0] \& \text{sg.E.of}[<\zeta, \nu>, 0]) \rightarrow \xi = \zeta)\]

(284) \[\text{card}[\zeta, \xi] \iff \exists \theta: \text{injective}[\theta][\forall \zeta (\text{sg.E.of}[\zeta, \xi] \iff \exists \nu (\text{sg.E.of}[\nu, \xi] \& \text{sg.E.of}[<\nu, \zeta>, 0]))\]

(285) \[\text{injective}[0] \iff \forall x \forall y \forall z ((\theta(<x, y>) \& \theta(<y, z>)) \rightarrow y = z) \& ((\theta(<y, x>) \& \theta(<z, x>)) \rightarrow y = z)\]

(286) \[\text{card}[n, X] \iff \exists \theta: \text{injective}[\theta][\forall x (X x \leftrightarrow \exists \nu (\nu < n \& \theta(<\nu, x>)))\]

A relation $\theta$ is injective, that is, one-to-one, just in case anything it relates it relates to exactly one thing, and anything that is related by it is related by it to exactly one thing. Some $X$s and Some $Y$s have the same cardinality, *card*[X, Y] just in case there is an injective relation between them. Cardinal equivalence as defined in (283)/(285) provides the resources to express in (288) an equivalent of (287) if one is feckless enough to ignore the syntax of (287)’s comparative construction:

(287) The ordinals are as many as the cardinals.

(288) \[\forall X: \text{ordinals}[X][\exists Y: \text{of}(Y, X)\& \forall Z: \text{cardinals}[Z]\& \text{card}[Y, Z]\]

The special case of equivalence to a number is then defined in (284)/(286): some $X$s have the same cardinality as some number $n$, *card*[n, X], just in case they have the same
cardinality as the numbers less than \( n \). With the cardinal predicate construed as a relation, (289) comes out as true without further comment:

(289) The moons of Venus are zero in number.
[The \( \xi \) : moons of Venus(\( \xi \))] \( \text{card}[0, \xi] \)

If not relational, \( \text{zero}[\xi] \) is indeed a primitive second-order property, a comment on the concept the moons of Venus, as Frege thought. In particular, its meaning cannot otherwise be constructed from plural, arithmetic predicates that are first-order, those that hold only of what there is or are (v. §0.2):

(290) \( \text{First-Order}(\Phi) \leftrightarrow_{df} \forall \xi(\Phi[\xi] \rightarrow E[\xi]) \)

The first-order arithmetic predicates would be those that count more than zero, whether primitive properties, ‘1(\( \xi \)’), ‘2(\( \xi \)’), …, or relations ‘\( \text{card}[1, \xi] \)’, ‘\( \text{card}[2, \xi] \)’, ….

The point to be observed is that the meaning of \( \text{zero}[\xi] \) cannot be, as if it were ‘Nonumber’, that no first-order arithmetic predicate holds of \( \xi \):

(291) \( \text{The cardinals are zero in number.} \)
[The \( \xi \) : cardinals(\( \xi \))] \( \text{card}[0, \xi] \)

(292) * [The \( \xi \) : cardinals(\( \xi \))] \( \neg \exists \text{n card}[n, \xi] \)

* [The \( \xi \) : cardinals(\( \xi \))] \( \neg \exists N \text{N}[\xi] \)

For, on such a construal, sentence (291), which is plainly false and comes out as such interpreted according to (283)/(285), would be rendered true as in (292) since the cardinals indeed have no cardinality. In desperation, it could be claimed that zero in number is a colorful way of denying existence, without genuine arithmetic content or understanding of what ‘zero’ contributes, while one in number, two in number and so on are still alleged to express first-order properties.51 If however the pretense of linguistic analysis is not abandoned, a common semantics for zero in number, one in number, two in number and so on requires either a relational analysis to numbers or a full Fregean embrace of the higher-order.

With numbers and relational cardinality in hand, whatever useful remarks are to be made about natural language quantifiers in terms of cardinality can be made within the language of the partitive construction (or within monadic second-order logic). Let a term-forming operator apply in (293) to \( \Phi \) to refer to the cardinality of the \( \Phi \)s:

(293) \( \kappa(\Phi) = n \leftrightarrow_{df} [\exists \xi : \Phi[\xi]] \text{card}[n, \xi] \)

A result from the study of generalized quantifiers52 is that any quantifier \( Q \) of natural language expresses a relation among four numbers if such are determined by the quantifier’s restriction \( \Phi \) and matrix \( \Psi \) (294), and any two quantifiers agreeing on the relation among numbers expressed are the same quantifier (295): 53

(294) \( \exists n_1 \exists n_2 \exists n_3 \exists n_4 (n_1 = \kappa(\Phi \land \neg \Psi) \land n_2 = \kappa(\neg \Phi \land \Psi) \land n_3 = \kappa(\Phi \land \Psi) \land n_4 = \kappa(\neg \Phi \land \neg \Psi)) \rightarrow [Q : \Phi] \Psi \leftrightarrow Q(\kappa(\Phi \land \neg \Psi), \kappa(\neg \Phi \land \Psi), \kappa(\Phi \land \Psi), \kappa(\neg \Phi \land \neg \Psi)) \)
A semantics for natural language that relies too much on these remarks, as in (296) and (297), fails (298), the cardinals and the ordinals having no cardinality (v. Rayo 2002), and (299), the natural numbers, the primes and the composites all having the same.

(298) Most cardinals are ordinals.
Many cardinals are ordinals.
Few cardinals are named.

(299) Most natural numbers are composite.
Few natural numbers are prime.

The lapse is however not in the analysis of cardinal predicates or in terms referring to cardinals—all of which remains within the language of the partitive construction—but in the reduction of generalized quantifiers to relations among them. Moreover, taking a natural language quantifier literally to be a quadratic relation infects even the innocent, finitistic sentences such as (300), since the things that are neither loved by Borges nor cardinals named the Aleph, which are too many for any cardinal, become a term for the relation among cardinals that ‘some’ is alleged to express.

(300) Borges loved some cardinal named the Aleph.

Absent reduction to a relation among cardinals, the vocabulary of (298)-(299), many, most, few, etc., may just involve primitive, second-order concepts that go beyond the resources of the language of the partitive construction and second-order logic. But, this conclusion presumes that one has struck bottom and exposed the primitive vocabulary, with no structure concealing further parameters to be discovered in the syntax of sentences (298)-(299). Yet, many, much, few and little are always understood to be evaluated with respect to some standard of comparison, which becomes the focus of explicit quantification and restriction in comparative constructions (Hackl 2001, 2003ab; Schwarzschild 2002, 2005; Schwarzschild & Wilkinson 2002; and references cited therein):

(301) Randy observed fewer nebulae than there are planets in the solar system.
Robin observed as many nebulae as Randy (did/ observed).

(302) Randy observed fewer nebulae than planets.
Robin observed as many nebulae as planets.

(303) Randy observed fewer nebulae than nine.
Randy observed fewer than nine nebulae.

Robin observed as many nebulae as nine.
Robin observed as many as nine nebulae.
Robin is more observant than Randy (is).
Robin is as observant as Randy (is).

Canonical translations for (301)-(303) turn out to be something like (305)-(307) with concurrent quantification over both planets and extents (or degrees):

(305) To an extent, less than that to which there are planets in the solar system Randy observed so few nebulae.
(306) To an extent, less than that if Randy observed planets Randy observed so few nebulae.
(307) To an extent, less than that if Randy observe nine nebulae Randy observed so few nebulae.

A univocal morpheme few recurs in its comparative and superlative forms, few-er and few-est and so do little, many and much recur in theirs, although obscured by suppletion (cf. good, better < *good+er, best < *good+est): little, less, least; many, more, most; much, more, most. In the comparative construction, these morphemes combine with a comparative morphology, er-than or as-as, with its own syntax and semantics of sufficient generality both to count nebulae when occurring in a nominal construction as in (301)-(303) and to measure powers of observation as in the adjectival construction in (304), resorting therefore to quantification over extents or degrees (v. Schwarzschild 2002, 2005 and Schwarzschild & Wilkinson 2002 for extensive discussion). The than- or as- phrase describing extent (or degree) proves (Hackl 2001) always to be a clause even if much of it is unspoken as in (302)-(303),54 which defeats speculation that the likes of (303) may be insightfully represented with a primitive quantifier fewer-than-nine (“(<9x)”) as if it were a single lexical item like all combining with nebulae in all nebulae. When few, little, many, much occur in comparative constructions, these morphemes express a relation to extents or degrees drawn from a theory of measurement and its special vocabulary (Schwarzschild 2002, 2005 and Schwarzschild & Wilkinson 2002), and they do so wherever else they occur, including (298)-(299). The primitive vocabulary of (298)-(299) is thus also relational measuring extent or degree. The uncovered parameter undermines interest in the earlier observation that if most, many and few occurred in (298)-(299) as primitive generalized quantifiers, they would be higher-order, and there has been no suggestion that a semantics for (298)-(299) could not be formulated within the language of the partitive construction and second-order logic drawing on the additional vocabulary supplied by a theory of measurement.55

§2.2. Eventish.

Whatever is said about determiners and quantifiers, it plays no role in the occurrence of essential plurals in (1),(3),(4)-(17) and (96)-(112), which look instead to the vocabulary of simple clauses, verbs, prepositions and verbal morphology. If clause structure as antecedently established in Parsons 1990 and much linguistic research since reflects a neo-Davidsonian decomposition demanded by the proper treatment of variable polyadicity, adverbial modification, nominalization, causativization, tense and aspect and so on, the primitive vocabulary of simple clauses comprises subatomic thematic relations and event concepts. These, for reasons unrelated to plurals and plural reference, happen to provide a hidden parameter referring to events. It may be that this parameter is sufficient to dissolve the appearance of primitive plural reference beyond of. If not,
primitive plural reference beyond of amounts to discovering it among one of the thematic relations or event concepts that neo-Davidsonian analysis takes to comprise the primitive vocabulary.

If of is the only primitive with an essentially plural argument, all of the predicates in (1) must exploit the hidden parameter, so that cluster is not ‘cluster(ξ)’ but rather \( \text{cluster}[e,ξ] \), expressing some complex relation that all and only the ξs each bear to the revealed e, as for example in (308) (or (309), its analogue in monadic second-order logic (Higginbotham & Schein 1989, Schein 1993)):

\[
\begin{align*}
(308) & \quad \text{cluster}[e,ξ] \leftrightarrow \text{cluster}(e) \& ∀ζ(\text{sg.E.of}[ζ,ξ] \leftrightarrow \text{Theme}(e,ζ)) \\
(309) & \quad \text{cluster}[e,X] \leftrightarrow \text{cluster}(e) \& ∀x(\text{Xx} \leftrightarrow \text{Theme}(e,x))
\end{align*}
\]

As soon as such an analysis is advanced for (1), the observation that the same morpheme cluster occurs twice in (310) and that (310) entails (311) leads straightaway to plural reference with respect to the unspoken parameter in (311), plural reference to some clusters or some clusterings, which is as essential to (311)’s meaning as plural reference to the fires.

\[
\begin{align*}
(310) & \quad \text{The fires clustered in two clusters (and not in one).} \\
(311) & \quad \text{The fires clustered.}
\end{align*}
\]

If indeed of is the only primitive with an essentially plural argument, it intervenes in logical form with every instance of plural reference so that (311)’s predicate is the complex relation in (314) (or, (317)). Every one of the fires is in one of the clusters and anything in any of the clusters is one of the fires:

\[
\begin{align*}
(312) & \quad \text{cluster}[e] \leftrightarrow ∀δ(\text{sg.E.of}[δ,e] \rightarrow \text{cluster}(δ)) \text{ (cf. (40))} \\
(313) & \quad \text{Theme}[e,ξ] \leftrightarrow ∀ζ(\text{sg.E.of}[ζ,ξ] \leftrightarrow ∃δ(\text{sg.E.of}[δ,e] \& \text{Theme}(δ,ζ))) \\
(314) & \quad [\text{VPcluster}][e,ξ] \leftrightarrow \text{cluster}[e] \& \text{Theme}[e,ξ] \\
(315) & \quad \text{cluster}[E] \leftrightarrow ∀e(\text{Ee} \rightarrow \text{cluster}(e)) \\
(316) & \quad \text{Theme}[E,X] \leftrightarrow ∀x(\text{Xx} \leftrightarrow ∃e(\text{Ee} \& \text{Theme}(e,x))) \text{ (Pietroski 2003: 282)} \\
(317) & \quad [\text{VPcluster}][E,X] \leftrightarrow \text{cluster}[E] \& \text{Theme}[E,X]
\end{align*}
\]

§2.2.1. Plural reference and event quantification.

Plural reference and plural quantification interact with the clause structure that the neo-Davidsonian analysis presents them with and thus corroborate that the decomposition is writ across the syntax.56

§2.2.1.1. Geach-Kaplan reciprocal sentences.

In a discussion that surveys various complex phrases demanding essential plurals, Boolos (1984) proves that the Geach-Kaplan sentence in (318) requires a formalization
with essentially plural reference to some critics, and the formalization considered is (319):

(318) Some critics admire only each other.
(319) $\exists X (\exists x Xx \land \forall x \forall y (Axy \rightarrow (x \neq y \land Xy)))$

As a comment on the logical structure of natural language sentences, it applies with equal force to the reciprocal construction in (320) the translation of which should, parallel to (319), solve for ‘$Axy$’. Substituting ‘suffocate($x,y$)’ as in (321) however mistranslates the sentence, which is true under conditions where no cockroach can be said to have suffocated any other.

(320) Some cockroaches suffocated only each other.
(321) $\star \exists X (\exists x Xx \land \forall x \forall y (\text{suffocate}(x,y) \rightarrow (x \neq y \land Xy)))$

Imagine some cockroaches in a bottle with a diminished supply of air at the cusp of catastrophe. They would have all made it had not another cockroach joined them, making them one too many to survive. At issue, ignoring the contribution of only to the proof that (318) is nonfirstorderizable, is that (322) does not mean the same as any of either (323) or (324):

(322) Some cockroaches suffocated each other.
(323) Some cockroaches each suffocated the others.
(324) Some cockroaches suffocated, each suffocating the others.

Neither (320) nor (322) is accurately translated when each (‘$\forall x$’ in (321)) includes in its scope the verb (Langendoen 1978). What each cockroach does is something less than a suffocation, rather an acting against as in (325), where altogether the cockroaches acting against each other results in or amounts to their suffocation:

(325) Some cockroaches suffocated (themselves), each acting against (some of) the other(s).

The reciprocal construction in (322) proves to be a reduced version of the adverbial clause in (325) (Schein 2001/2003), conforming to the syntax and semantics of adverbial modification. As of this writing, the only game in town (since Parsons 1990) treats this adverbial modification as a relation between the events (or states) described by the adverbial clause and those described by the modified, matrix clause:

(326) [Some $X : \text{cockroaches}][\forall E : \text{each of them}_X \text{acting against (some of) the other(s)}][\exists E' : \text{R}[E, E']] \text{suffocated}[E'] (\text{themselves}).
In contrast to (324), the reduced adverbial clause in (320)/(322) omits the verb describing suffocations and includes only the subatomic vocabulary of thematic relations, substituting in effect ‘Agent(e,x) & Patient(e,y)’ for ‘Ax’ in (319). Only a few considerations lead up to this conclusion: ‘each’ in each other is a universal distributive quantifier (as the discussion of the Geach-Kaplan sentence assumes), its scope cannot in the case of (320)/(322) include the verb, and yet, whatever is its scope forms a phrase with each that modifies suffocate. Note that the reduced adverbial clause translating the reciprocal construction separates thematic relations from the verb, corroborating that these are constituents separate from the verb in accord with the neo-Davidsonian analysis of variable polyadicity.

§2.2.1.2. Separation within simple clauses.

Apart from the reciprocal construction of the Geach-Kaplan sentence, which turns out to hide a second, adverbial clause, within simple clauses, separation is discovered in the interaction of plurals and quantifiers (Schein 1993: chapter 4). In (328), the terms decomposing the verb phrase, Theme[e,X] and cover(e'), apply to different events, and they are separated by elements from elsewhere in the sentence: the quantifiers two workbenches and each include within their scope cover(e') but not Theme[e,X].

(327) Three hundred quilt patches covered over two workbenches each with two bedspreads.

(328) \[\exists e(\exists X: 300 \text{ quilt patches}) \text{ Theme}[e,X] \& [\exists Y: \text{two workbenches}] \text{ [Each } y : Yy] \\]
\[[\exists e': e' \leq e] (\text{cover}(e') \& \text{Goal}[e',y] \& [\exists Z: \text{two bedspreads}] \text{ with}[e',Z])\]

The separation of Theme[e,X] and cover(e') is essential to the extent that sentences like (327) have interpretations that can be represented only by the likes of (328). A tedious argument shows that no other logical syntax will do (Schein 1993: chapter 4), but it is easy enough to imagine conditions for the truth of (327) that are congenial to (328). Imagine that four bedspreads, draped as described, are made altogether from a total of three hundred quilt patches. The three hundred patches together cover the workbenches but do not all go into the bedspreads on any one bench. Moreover, some of the individual patches have themselves been torn between this or that bedspread. There is in this case a large event, e in (328), where exactly three hundred patches covered workbenches with bedspreads, and nothing more precise can be said about how the patches were disposed of, just that this large event comprises two smaller events, e' in (328), in each of which a workbench is covered by patches making up two bedspreads. The sentence (327) can be taken to assert that two workbenches were each covered over with two bedspreads while leaving vague the distribution of the quilt patches. It is this combination of distributivity between two workbenches each and two bedspreads with the vague distribution of the quilt patches that makes the separation of thematic relations in (328) essential in this and many like examples.
§2.2.2. Plural event quantification.

The Davidsonian analysis prefixes to every clause an existential quantifier over events, which must itself be plural quantification, as remarked in §2.2, if the same morpheme cluster occurs twice in (310), and (310) entails (311):

(310) The fires clustered in two clusters (and not in one).
(311) The fires clustered.
(329) $\exists E ([\text{The } X: \text{fires}[X]] \text{Theme}[E,X] \& \text{cluster}[E])$

Likewise, when the current Broadway season finds twenty composers divided among seven, rival and cutthroat productions, what they do is hardly a collaboration but several, which verify plural quantification over events in (330):\footnote{61}

(330) Twenty composers collaborated on seven shows. (Gillon 1987)
(331) $\exists E ([\exists X: 20[X] \text{composers}[X]] \text{Agent}[E,X] \& \text{collaborate}[E] \& [\exists Y: 7[Y] \text{shows}[Y] \text{on}[E,Y])$

Plural quantification over events is evident in the logic as well. First consider (332) on the reading indicated:

(332) Twenty truckers loaded up one or more trucks.
‘Whenever there was a loading up of one or more trucks, 20 truckers were the loaders’.
$[\forall e: \text{load up of one or more trucks}[e] ] [\exists X: 20[X] \text{truckers}[X]] \text{Agent}[e,X]$

There is no felt implication that it was the same 20 truckers in every event, and thus the relevant domain of events is not closed under fusion. Otherwise, the fusion of all loadings up of one or more trucks would itself be a loading up of one or more trucks, and they could each involve twenty truckers only if they were the same twenty.

(333) These 10 truckers loaded up one or more trucks.
Those 10 truckers loaded up one or more trucks.
The 20 truckers loaded up one or more trucks.

On the other hand, (333) is valid; and unlike the universal, distributive quantifier in (332), the sentences in (333) must not lead with a singular ‘there was an event of 10 truckers...’ and ‘there was another event of 10 truckers’. Even if there is one loading by these 10 truckers and another by those 10 truckers, there is no certainty that the domain contains their fusion, a single event of loading by the twenty truckers. Rather, these sentences start off in the plural, ‘there were some events...’, and the inference in (333) follows as a matter of logic: There were loadings by these 10 truckers and loadings by those 10 truckers, and so there were loadings by the 20 truckers. (Schein 1993: 107ff.)

In light of such elementary examples, there is little to the Davidsonian design unless its foundation is plural quantification over events (Schein 1993, 2002, 2001/2003).

Plural quantification over events displays characteristics of plural, count quantification.
The vegetables are too heavy for the laboratory scale and too light for the bathroom scale. (Schwarzschild 1991, 1996)

The vegetables weigh one kilogram.

As Gillon 1990 and Schwarzschild 1991, 1996 point out, sentences (334) and (335) weigh the vegetables individually or as a single collection but in no other configurations unless the context individuates them, as when it becomes understood that the vegetables have been divided among several trials each of which is to weigh the contents of a basket of vegetables. In such a context, (334) acquires the additional interpretation that the vegetables in each trial, the contents of a basket, are too heavy for one scale and too light for the other, and (335), that the vegetables divide among trials that turn out each to have been a weighing one kilogram. Schwarzschild (1996: 82f., 92f.) also offers a spatial analogue. Speakers do not hesitate to judge (336) true of (337), parsing the scene into a running parallel that relates the rectangles’ horizontals and another that relates their verticals, the logical structure of (336) and (330) being the same in this respect.

The sides of R1 run parallel to the sides of R2. (Scha 1984)

The double lines run parallel to the single lines.

Yet, the same logical structure fails to provide (338) with an interpretation true in (339) or (340), where speakers would sooner go blind than parse these scenes into the runnings parallel necessary to make the sentence true:

The double lines run parallel to the single lines.
If the truth of sentences such as (336) and (338) (as well as (334) and (335)) depend on
the conditions under which events or states are individuated, it does these sentences no
harm for them to contain a term in which this dependence is explicit. It is rather further
evidence of plural quantification over what are individuated.

§2.2.3. Coordination and thematic relations

Plural quantification over events is endemic, and the verbs \textit{cluster}[$E$] and
\textit{collaborate}[$E$] that have appeared in the sentences of §2.2.2 apply distributively to the
events $E$. In support of the sense and logic of these sentences, it is necessary and
sufficient that their thematic relations be cumulative (341), as they would be in
consequence of their first-order, distributive definition in (313)/(316):

\begin{equation}
\text{Cumulative}(\theta) \iff \\
\forall e (Ee \leftrightarrow (E_1 e \lor E_2 e)) \land \forall x (X x \leftrightarrow (X_1 x \lor X_2 x)) \to ((\theta(E_1, X_1) \land \theta(E_2, X_2)) \to \theta(E, X))
\end{equation}

\begin{align}
\text{cluster}[E] & \leftrightarrow \forall e (E e \to \text{cluster}(e)) \\
\text{Theme}[E, X] & \leftrightarrow \forall x (X x \leftrightarrow \exists e (E e \land \text{Theme}(e, x)))
\end{align}

These verbs and thematic relations manage well enough without primitive plural
arguments (except as argument to partitive \textit{of}); but, cumulativity in these sentences could
also be served if the expressions tokened in (329) and (331) are themselves the result of a
distributivity operator applied to yet more primitive thematic relations (or verbs) for
which plural reference is for some reason held to be essential (v. Landman 1995, 2000): 62

\begin{align}
\text{Theme}[E, X] \leftrightarrow & \forall X'(\text{sg.E.of}[X', X] \leftrightarrow \exists E'(\text{sg.E.of}[E', E] \land \text{sg}[X'] \land \text{Theme}(E', X'))) \\
\text{cf. (313), (316)}
\end{align}

In this section, an empirical consideration is introduced in favor of a strengthened
argument against essentially plural thematic relations: everywhere they occur, they
occur distributively, and thus there can be no context that shows a plural to be essential
for thematic relations.

To this end, thematic relations under coordination looks to constrain their
meaning. It has long been known (Perlmutter & Ross 1970, Jackendoff 1977, McCawley
1981) that apparent reduction in a coordination acquires essentially plural interpretations
for $\Psi$ in (343) that are absent from the unreduced counterpart. The first sentence of (344) describes a collective flooding and blanketing of a thousand fields of grain that the second sentence does not. Similarly, the reduced coordination in (345) allows it to comment on attitudes towards a collective proposition about student and professor.

\[ \Phi_1 \text{ and } \Phi_2 \Psi \Leftrightarrow \Phi_1 \Psi \text{ and } \Phi_2 \Psi \]

(344) The surging waters flooded and the hailstones blanketed a thousand fields of barley and rye (between them). $\Leftrightarrow$ The surging waters flooded a thousand fields of barley and rye (between them) and the hailstones blanketed a thousand fields of barley and rye (between them).

(345) Not many a student proposed and not many a professor (of his) accepted that they should collaborate even more than they already have. $\Leftrightarrow$
Not many a student proposed accepted that they should collaborate even more than they already have, and not many a professor (of his) accepted that they should collaborate even more than they already have.

Despite reduction, it is uncontroversial that \textit{and} in (344) and (345) is a sentential connective, with each conjunct containing a tensed verb, and it has long been an open problem how the phrases $\Psi$ acquire their collectivizing force in this setting of sentential coordination.\textsuperscript{63} Nevertheless, this setting extends to the following, where adverbs within each conjunct (and iterated in (348)) indicate sentential coordination, even though the reduction has cut deeper eliding the verb:

(346) Robin by this morning and Hillary by last night have drunk more bordeaux between them than the region produces in a week.

(347) Saul (while) hooting and David (while) hollering are drowning out the lecture heckling each other.

(348) Marvin this afternoon from Great Neck and Bernice this evening from Syosset are schlepping the hors d’oeuvres to Leonard’s in a rented Mercedes.

But, if there must be unspoken predicates for these adverbs to modify, there is then no reason to deny that they also occur unmodified in (349)-(351) (the adverbs themselves never being obligatory) within clauses conjoined by that same sentential connective \textit{and}.

(349) Robin and Hillary have drunk more bordeaux between them than the region produces in a week.

(350) Saul and David are drowning out the lecture heckling each other.

(351) Marvin and Bernice are schlepping the hors d’oeuvres to Leonard’s in a rented Mercedes.

\textit{Whatever} derives the collectivizing interpretations for (346)-(348) and for (352)-(354) below (and for reduced sentential coordinations in general as in (344) and (345)) cannot help but derive it for (349)-(351) and (355)-(357) just the same. Sentences such as (349)-(351) are thus poor excuse to plead an ambiguity, defining \textit{ad hoc} an \textit{and} joining two terms to form another referring plurally, an orphan left helpless when sentential coordination is explicit or anywhere else a coordination other than of terms collectivizes, such as when generalized quantifiers seem to conjoin in (355)-(357).

(352) No philosopher today and no linguist yesterday have drunk more bordeaux between them than the region produces in a week.

(353) No linguist (while) hooting and no philosopher (while) hollering are drowning out the lecture heckling each other.

(354) No caterer this afternoon from Great Neck and no florist this evening from Syosset are bringing (as scheduled) the hors d’oeuvres and centerpieces we ordered from them months ago.
No philosopher and no linguist have drunk more bordeaux between them than the region produces in a week.

No linguist and no philosopher are drowning out the lecture heckling each other.

No caterer and no florist are bringing (as scheduled) the hors d’oeuvres and centerpieces we ordered from them months ago.

If adverbs, today, yesterday, from Great Neck, etc., describe events, the unspoken predicates in (346)-(348) and (352)-(354) express how the subjects of each conjunct participate in the events described, that is, a thematic relation. It is safe to assume that the unspoken content will be the same thematic relation in both conjuncts:

\[ \exists E (\text{Agent}[E,X] \& \text{yesterday}[E]) \text{ and } \exists E (\text{Agent}[E,Y] \& \text{today}[E]) \& \ldots \]

Since what is today is not yesterday and what is this afternoon from Great Neck is not this evening from Syosset, the events described by each conjunct are distinct events that only the subject participates in. Yet, the point of these sentences is to know what the subjects did severally by their joint effect. There are some events among which their agents are distributed each in her own as the adverbs impose, and these amount to drinking more bordeaux than the region produces, to drowning out a lecture or to catering the banquet. Distributivity with respect to thematic relation is apparently no impediment to the collectivizing interpretation of larger phrases.

With this in mind, consider a plausible empirical generalization that there is nothing that can be said of the \( n \) Bs that could not also be said of \( b_1 \) and… and \( b_n \), replacing the plural term with a coordination of names for the individuals referred to (whenever there are such names).

\[ \text{The } n \text{ Bs } \Phi \Rightarrow b_1 \text{ and… and } b_n \Phi. \]

\[ \text{The Beatles harmonized. } \Rightarrow \text{John, Paul, George and Ringo harmonized.} \]

\[ [\text{The } X: n[X] \text{ Bs}[X]] \Rightarrow \exists E \theta[E,X] \& \Phi \Rightarrow [\exists x: b_1[x]] \exists E \theta[E,x] \text{ and… and } [\exists x: b_n[x]] \exists E \theta[E,x] \& \Phi \]

That is as much as to say that there is nothing that can be said of the \( n \) Bs that cannot be said of some events in which they participate distributively as in (360). Given this generalization, it is necessary that the thematic relations in use are always distributive, and if so, there can never be need to recognize a primitive thematic relation with an (essentially) plural argument relating the \( n \) Bs to events.

The empirical generalization is however breached if it is held to be a fact of semantics that the advance scout whose reconnaissance suffices to make (361) true and felicitous fails to do the same for (362) although the named are F Troop’s soldiers (Gillon 1987, 1990; Lasersohn 1988, 1989, 1990, 1995; Schwarzschild 1991, 1996).

\[ \text{The soldiers of F Troop spotted the Indians. (Gillon 1987)} \]

\[ \text{Capt. Parmenter, Sgt. O’Rourke, Cpl. Agarn, Trooper Dobbs,… and Trooper Duddleson spotted the Indians.} \]
If so, the event or events that the soldiers of F Troop participate in jointly (363) amount to spotting the Indians; but, the events that the soldiers participate in individually do not (364):

(363) \[\text{[The X: soldiers of F Troop][x]} \exists E \text{Experiencer}[E,x] \land \Phi\]

(364) \[\langle x: \text{Parmenter}[x]\rangle \exists E \text{Experiencer}[E,x] \land \ldots \land \langle x: \text{Duddleson}[x]\rangle \exists E \text{Experiencer}[E,x] \land \Phi\]

This is not a finding that a primitive ‘Experiencer\((E,x)\)’ occurs in (361)/(363) and joins of in requiring essentially plural arguments. It is enough for (361)/(363) that the event that amounts to spotting the Indians is some event \(e_{F \text{ Troop}}\) that \[\text{[The X: soldiers of F Troop][x]} \forall x (Xx \leftrightarrow \text{Experiencer}(e_{F \text{ Troop}}, x))\], where the thematic relation continues to be first-order and distributive, and this event \(e_{F \text{ Troop}}\) is other than any that a trooper experiences alone. The loss of the empirical generalization does however leave the door open to argument or whim in favor of one more primitive with plural arguments, although all the known semantic requirements for thematic relations have fit within the language of the partitive construction and second-order logic.

The empirical generalization is propped up, and with it the positive argument that there can be no context showing essentially plural thematic relations, if the contrast between (361) and (362) is ruled pragmatic rather than semantic. Under the circumstances described, the speaker has no grounds to draw attention in (362) to individual events that are very far from salient. Discussing examples from Dowty 1987 similar to (365)-(367), Schwarzschild (1996: 96f.) observes the contrast now under discussion—a greater reluctance to accept (366) as true and felicitous when some of those named were too silent or soft than to accept (365) under the same conditions:

(365) After the last game of the 1992 season, the sportscasters at the press conference peppered the Managing General Partner of the Texas Rangers with questions he couldn’t answer.

(366) After the last game of the 1992 season, Angell, Barber, Cosell, Garagiola, Kahn, Remy and Rizzuto peppered the Managing General Partner of the Texas Rangers with questions he couldn’t answer.

(367) After the last game of the 1992 season, the sportscasters from ABC, CBS and NBC at the press conference peppered the Managing General Partner of the Texas Rangers with questions he couldn’t answer.

But, (367), as Schwarzschild points out, elicits the same reluctance if a network is unrepresented among the hardball questions, and yet (367) does not differ from (365) in its logical syntax or semantics in any relevant respect, and the empirical generalization is otherwise spared. If so, if indeed \(b_1, \ldots, b_n\) can always stand in for \(the n Bs\) and the former is a coordination of sentences that apply the thematic relation to their subjects, then of necessity the thematic relation is distributive.

§2.2.4. Clause structure and relations between events

The individuals’ actions in (368) and (369) are all different—none of the hooting, hollering, rasping, warbling, twanging and bellowing is the same event as any of the
others—and none of the individuals’ actions is sufficient to drown out the lecture or harmonize.

(368) Saul hooting and David hollering drowned out the lecture.
(369) John rasping C, Paul warbling E♭, George twanging G and Ringo bellowing B♭ harmonized in c minor.

Some relation, mereological, topological or causal, mediates between the individuals’ actions and the larger events to convey that they amounted to a drowning out or harmonizing:

(370) \( \exists E (\text{Agent}[E,X] & \text{hooting}[E]) \text{ and } \exists E (\text{Agent}[E,Y] & \text{hollering}[E]) \text{ and } R[E,E'] \text{ and } \text{drown out the lecture}[E'] \)

The logical form of (370) is further revision to the citation (neo-) Davidsonian forms. The thematic relations and verbs are no longer applied to the same event or events (cf. §2.2.1.2.). Logical form applies them to distinct events, and the sentence is held together with the introduction of further relations between sentences, ‘R[E,E’]’ in (370). Alongside coordinate structures like (368) and (369), plural quantification over events and the underlying distributivity of thematic relations combine on their own to present an empirical problem the solution of which corroborates the revision in the logical syntax of simple clauses.

According to Broadway tradition, the casts of rival shows retire after every performance to Patsy Grimaldi’s where they hold separate court, darting poisonous glances and feigning indifference at other tables. The composers who have collaborated on a show share a ritual of toasts and pizza, so that tonight at Patsy Grimaldi’s with 17 composers from rival shows present and the kitchen turning out 23 pizzas (Schein 1993: 126ff., Schein 2002):

(371) a. 17 composers share 23 pizzas.
   b. 17 composers share at the show’s table 23 pizzas.
   c. 17 composers share, every composer breaking pizza with every other composer, 23 pizzas.

The composers are divided among several tables at which every composer shares with every other and across which there is no such conviviality. That is, (371) is true only to the extent that several sharings verify it, as many as there are tables.

(372) \( \text{Agent}[E,X] \text{ & } \text{share}[E] \text{ & } \text{Patient}[E,Y] \)

As the composers are neatly divided among these, a distributive thematic relation describes accurately their participation in the sharings. The pizza however is ordered and served by the slice, and it happens that this evening none of the twenty-three pizzas is consumed at any one of the tables. Even so, (371) remains true and indifferent to these pizza particulars, but (372) is false interpreting ‘Patient[E,Y]’ distributively ((cf. (313), (316)), (342)) to imply that any a pizza is consumed at an event of sharing. The truth of (371) looks to argue that ‘Patient[E,Y]’ expresses a relation between plural arguments, the sharings and the pizzas, that cannot be reduced to a first-order, distributive relation. But,
the argument above resumes—The twenty-three pizzas may be referred to severally and their participation qualified by adverbs:

(373)  a. 17 composers share the first pizza first, the second pizza second,…and the 23rd pizza 23rd.
     b. 17 composers have shared at their (own) show’s table the first pizza first, the second pizza second,…and the 23rd pizza 23rd.
     c. 17 composers share, every composer breaking pizza with every other composer, the first pizza first, the second pizza second,…and the 23rd pizza 23rd.

If Grimaldi’s sells off one pizza before starting in on the next, (373) is true *ceteris paribus*, even though distributivity has been imposed on the thematic relation relating pizzas to events. The thematic relation relates pizzas to events distributively, but pizzas cannot be related to the sharings distributively. Revising therefore the syntax of simple clauses, the events distributively related to the pizzas are not the sharings, which have been distributively related to the composers:

(375)  Overlap[E₁,E₂] ↔ₚ ∀e₁(E₁e₁↔∃e₂(E₂e₂ & Overlap(e₁,e₂)))

Rather, what the agents do *E₁* and what happens to patients *E₂* is mediated by some other relation (Schein 1993, 2002, in preparation), mereological coincidence or overlap in this case, which is itself distributive ((375)) and projected from the primitive, first-order mereological relation. 67

These observations may be revisited at every position for plural predication (376) prompting the revision in (377) to basic clause structure that applies thematic relations to their own events and holds the sentence together with further relations such as ‘Cause*[E,E’]’ and ‘Overlap*[E,E’]’:

(376)  Lenny toasting, Kurt praising, …, and George dishing it out share the first pizza first,…, and the 23rd pizza 23rd.

§2.3. Russelling Eventish 68

According to the neo-Davidsonian analysis, event quantification and thematic relations to the events quantified over occur in the logical form of every sentence in natural language, simple or complex. For at least some events and some thematic relations, events do not bear thematic relations to themselves:

(378)  The fires expand.
(379)  The fires cluster.

When (378) and (379) are true, some events, a clustering or clusterings and an expansion or expansions, are such that their only themes are the fires, and thus these events are not their own themes.

(380)  The events that are not their own themes expand (in their own space and time).
The events that are not their own themes cluster.

Given some such events, (380) is trivially true; and, reflecting on how they are gotten from truths like (378) and (379), it is also clear that there are enough of them for (381), even for a single, dense cluster of them. Yet, according to the neo-Davidsonian analysis, the sentences are true just in case there are some events, some expansions in (380) and a clustering in (381), that the events that are not their own themes are themes of. Russell’s paradox threatens (Oliver & Smiley 2001, Rayo 2002, Yi 1999: 186 n. 34): If the expansions are among their own themes, then they must be among the events the subject refers to, that is, among events that are not their own themes. On the other hand, if the expansions are not events that are their own themes, they are among the events the subject refers to and hence their own themes. Implying a clustering in (381) similarly tilts the neo-Davidsonian analysis towards contradiction. With a commitment to sentence-verifying events and thematic relations, paradox will threaten as soon as the object language contains plural definite descriptions deploying thematic relations or their equivalent to refer to events that are not so related to themselves. The paradox presents itself whether the (existential) event quantification is thought of as plural (380) or as singular (381), and it threatens a neo-Davidsonian analysis no matter what view of plural reference and predication accompanies it. In a blunt response, one may despair of the neo-Davidsonian analysis and exorcise events altogether as in (382) and (383) (v. Oliver & Smiley 2001, Rayo 2002, Yi 1999) and thereby forfeit, in (384) for example, adverbial modification, tense and aspect, nominalization and any other grammatical construction explained by reference to events (Parsons 1990):

(382) [The $\xi$: fires($\xi$)] expand($\xi$).
(383) [The $\xi$: fires($\xi$)] cluster($\xi$).
(384) The fires first clustered slowly along the ridgeline in several clusters and have been expanding rapidly across the leeward side while flaring out on the windward side.

Language scientists and engineers lie in wait with their reply:

“And here I go by the semanticists’ First Amendment:
The right to solve Russell’s Paradox some other time shall not be restricted.” (Landman 2000: 79)

To formalize the argument in pursuit of a way out, recall (385) and (386). Some convenient shorthand abbreviating the singular is introduced in (387)-(389).

(385) cluster[$E] \leftrightarrow \forall e (Ee \rightarrow \text{cluster}(e))$
(386) Theme[$E$,$X] \leftrightarrow \forall x (Xx \leftrightarrow \exists e (Ee \& \text{Theme}(e,x)))$
(387) cluster[$e] \leftrightarrow \text{cluster}(e)$
(388) Theme[$e$,$X] \leftrightarrow \forall x (Xx \leftrightarrow \text{Theme}(e,x))$
(389) Theme[$e$,$x] \leftrightarrow \forall y (y=x \leftrightarrow \text{Theme}(e,y))$

Recall that thematic relations as tokened in logical form, $\theta[E,X]$, are always exhaustive: according to (386), $X$ are the themes of the $Es$.  

47
Some translations of (380) and (381) end in paradox, and some do not. Suppose the subject refers specifically to the events that are not among their own themes, [the $X : \neg[\exists Y : \text{of}(X,Y)] \text{Theme}(X,Y)$):

(390) $\exists E \ [\text{the } X : \neg[\exists Y : \text{of}(X,Y)] \text{Theme}(X,Y)] \text{Theme}(E,X) \text{ expand}(E)$

Let $X$ be the events the subject refers to, and $E$, the expansions, so that, given the truth of (390), Theme$(E,X)$. If, on the one hand, of$(E,X)$, then by description, $\neg[\exists Y : \text{of}(E,Y)] \text{Theme}(E,Y)$, in particular, $\neg(\text{of}(E,X) \land \text{Theme}(E,X))$ and therefore, $\neg\text{Theme}(E,X)$, a contradiction. If, on the other hand, $\neg\text{of}(E,X)$, then by description $[\exists Y : \text{of}(E,Y)] \text{Theme}(E,Y)$, from which, by the exhaustivity of thematic relations, $\text{of}(E,X)$, contradiction again. Since nothing in this argument rests on $E$ being nonsingular, the same translation of the subject will also pitch (381) into paradox:

(391) $\exists e \ [\text{the } X : \neg[\exists Y : \text{of}(X,Y)] \text{Theme}(X,Y)] \text{Theme}(e,X) \text{ cluster}(e)$

Suppose next that the events that are not their own themes are the events each of which is not one of its own themes, [the $X : [\forall x : Xx] \neg[\exists X : Xx] \text{Theme}(x,X)$]. As translation of (381), consider:

(392) $\exists e \ [\text{the } X : [\forall x : Xx] \neg[\exists X : Xx] \text{Theme}(x,X)] \text{Theme}(e,X) \text{ cluster}(e)$

Let $X$ be the events the subject refers to, and $e$, the clustering. If, on the one hand, $Xe$, then by description $\neg[\exists X : Xe] \text{Theme}(e,X)$, implying in contradiction that $\neg Xe$. If on the other hand $\neg Xe$, then by description $[\exists X : Xe] \text{Theme}(e,X)$, from which exhaustivity implies that $Xe$, contradiction again.

In contrast, this same translation of the subject seems benign in talk about plural expansions:

(393) $\exists E \ [\text{the } X : [\forall x : Xx] \neg[\exists X : Xx] \text{Theme}(x,X)] \text{Theme}(E,X) \text{ expand}(E)$

Let $X$ be the events the subject refers to, and $E$, the expansions. If, first, $\neg\text{of}(E,X)$, then by description, for some $e$, an $E$, $[\exists X : Xe] \text{Theme}(e,X)$, which, by the exhaustivity and distributivity of thematic relations, implies $Xe$, which in turn by description implies in contradiction that $\neg[\exists X : Xe] \text{Theme}(e,X)$. If, on the other hand, $\text{of}(E,X)$, then any $e$ of $E$ meets the description, $\neg[\exists X : Xe] \text{Theme}(e,X)$, that it is not among its very own themes. Given that it is one of the themes $X$ of $E$, it is implied without apparent contradiction only that $e$ is a theme of some of the other $E$ rather than of itself.

A fully distributive interpretation of the subject’s description, referring to the events each of which is not its own theme, [the $X : [\forall x : Xx] \neg\text{Theme}(x,x)$], allows translation of both (380) and (381) to escape:

(394) $\exists e \ [\text{the } X : [\forall x : Xx] \neg\text{Theme}(x,x)] \text{Theme}(e,X) \text{ cluster}(e)$

If (394) is true, the clustering $e$ cannot be its own theme, since it is asserted that many events, all those that are not their own themes, are in fact the themes of $e$. By the same
token, since e is not its own theme, it is among the things the subject refers to, and so e is properly among the themes of e, without contradiction. Similarly, there is no contradiction in the expansions being among their own themes, as each other’s for example, while none is its own:

\[∃E [\{X: [∀x : Xx] \sim \text{Theme}[x,x]\}] \text{Theme}[E,X] \text{expand}[E]\]

The paradox latent in (380) and (381) is joined when a malicious interpreter fixes on certain interpretations of the subject or finds further disambiguating language and knows that (380) and (381) so construed or elaborated are true.

If there is a malicious interpreter, the way out from paradox for her insists on the revision to basic clause structure from §2.2.4. No sentence of her language, despite her gloss on the subject, parses the matrix as in (390)-(395). A further relation between events intrudes as in (396)-(398):

\[∃E1∃E2 [\{X : −[∃Y : \text{of}(X,Y)] \text{Theme}[X,Y]\}] \text{Theme}[E1,X] \text{Overlap}[E1,E2] \text{expand}[E2]\]

\[∃E1∃e2 [\{X : −[∃Y : \text{of}(X,Y)] \text{Theme}[X,Y]\}] \text{Theme}[E1,X] \text{Overlap}[E1,e2] \text{cluster}[e2]\]

\[∃E1∃e2 [\{X : [∀x : Xx]−[∃X : Xx] \text{Theme}[x,x]\}] \text{Theme}[E1,X] \text{Overlap}[E1,e2] \text{cluster}[e2]\]

Intending with (381) to refer to the events each of which is not one of its own themes, she means (398), which is taken to be true. Let e2 be the clustering, and X, the events the subject refers to. If Xe2, then −\text{Theme}[e2,X] and \exists E1(\text{Overlap}[E1, e2] & \text{Theme}[E1,X]), from which follows only that \exists E1(\text{Theme}[E1,X] & e2 ≠ E1), without paradox. If instead \neg Xe2, then [∃X : Xe2] \text{Theme}[e2,X] and \exists E1(\text{Overlap}[E1, e2] & \text{Theme}[E1,X]), with even less risk.

Similarly, intending with (380) to refer as in (396) to the events that are not among their own themes, let E2 be the expansions and X, the events the subject refers to. If \text{of}(E2,X), then −[∃Y : \text{of}(E2,Y)] \text{Theme}[E2,Y], in particular, −(\text{of}(E2,X) & \text{Theme}[E2,X]) and therefore, −\text{Theme}[E2,X], which comports without apparent contradiction with the requirement that for some E1, \text{Overlap}[E1, E2] & \text{Theme}[E1,X] provided that E1≠E2.

Seeking out paradox, the malicious interpreter will reach for greater havoc taking up into her description the new relations provided to the matrix clause and referring instead to the events that are not among those that even overlap their themes. As far as I can tell, translation does not come any closer to paradox:

\[∃E1∃E2[\{X : −[∃Y : \text{of}(X,Y)] \exists E1 (\text{Theme}[E1,Y] \text{Overlap}[E1,X])]\text{Theme}[E1,X] \text{Overlap}[E1,E2] \text{expand}[E2]\]

\[∃E1∃e2 [\{X : −[∃Y : \text{of}(X,Y)] \exists E1 (\text{Theme}[E1,Y] \text{Overlap}[E1,X])]\text{Theme}[E1,X] \text{Overlap}[E1,e2] \text{cluster}[e2]\]

\[∃E1∃e2 [\{X : [∀x : Xx]−[∃X : Xx] \exists E1 (\text{Theme}[E1,X] \text{Overlap}[E1,X])]\text{Theme}[E1,X] \text{Overlap}[E1,e2] \text{cluster}[e2]\]
Presuming (399) to be the logical form of a true sentence like (380), let \( E_2 \) be the expansions and \( X \), the events the subject refers to. If \( \text{of}(E_2, X) \), then \( \neg \exists Y : \text{of}(E_2, Y) \exists E_1 \) (Theme\([E_1, Y]\) Overlap\([E_1, E_2]\)) in particular, \( \neg \) (of\( E_2, X \)) \& \( \exists E_1 \) (Theme\([E_1, X]\) Overlap\([E_1, E_2]\)) and therefore \( \neg \exists E_1 \) (Theme\([E_1, X]\) Overlap\([E_1, E_2]\)), contradicting (399).

On the other hand, if \( \neg \) of\( E_2, X \), then (i) \( \exists Y : \text{of}(E_2, Y) \exists E_1 \) (Theme\([E_1, Y]\) Overlap\([E_1, E_2]\)) and yet, given (399), (ii) \( \exists E_1 \) (Theme\([E_1, X]\) Overlap\([E_1, E_2]\)). But, again, there is no apparent conflict with exhaustivity (or, distributivity) provided that the events coincident with \( E_2 \) of which \( X \) are the themes (according to (ii)) are not the same ones as those coincident with \( E_2 \) the themes of which \( E_2 \) are among (according to (i)). Recall ((371)) that events the themes of which are pizza slices may very well coincide with events the themes of which are different, pizzas whole. As again these remarks do not rely on \( E_2 \) being nonsingular, they also show (400) mutatis mutandis to be a benign translation of (381) about a single clustering.

If the subject is taken instead to refer (semi-)distributively as in (401) to the events each of which does not overlap its own themes, paradox is equally remote. Let \( X \) be the events the subject refers to, and \( e_2 \), the clustering. If \( X e_2 \), then by description \( \neg \exists X : X e_2 \) \( \exists E_1 \) (Theme\([E_1, X]\) Overlap\([E_1, e_2]\)), implying in particular that \( \neg \exists E_1 \) (Theme\([E_1, X]\) Overlap\([E_1, e_2]\)), contradicting (401). But, if \( \neg X e_2 \), then (i) \( \exists X : X e_2 \) \( \exists E_1 \) (Theme\([E_1, X]\) Overlap\([E_1, e_2]\)), and yet, given (401), (ii) \( \exists E_1 \) (Theme\([E_1, X]\) Overlap\([E_1, e_2]\)). Both conditions (i) and (ii) are met when the events with themes \( X \) and coincident with \( e_2 \) are distinct from the events coincident with \( e_2 \) of which \( e_2 \) is itself a theme.

In summary, granted the truth of (380) and (381), the malicious interpreter finds paradoxical translations just in case the neo-Davidsonian clause is unrevised as in (402); translation eludes paradox under the revised clause structure (403).

\[
\begin{align*}
(402) \ & \text{...Theme}[E, X] \ \text{expand}[E] \\
& \text{...Theme}[e, X] \ \text{cluster}[e]
\end{align*}
\]

\[
\begin{align*}
(403) \ & \text{...Theme}[E_1, X] \ \text{Overlap}[E_1, E_2] \ \text{expand}[E_2] \\
& \text{...Theme}[E_1, X] \ \text{Overlap}[E_1, e_2] \ \text{cluster}[e_2]
\end{align*}
\]

If so, the divorce between linguistics and philosophy looming earlier can be rescheduled pending further inquiry (perhaps into alleged events that are themes in events coincident with themselves). In the meantime, Russell’s paradox joins the empirical considerations of §2.2.4 in urging a particular articulation of the neo-Davidsonian clause.\textsuperscript{70}
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NOTES

1 Rejecting, for example, that (4) could be a negation entailing the negations of sentences like (11), as in:
\[\neg[\exists n: n>0]\] the nonselfidentical custards are \(n\) in number.

2 The definite description as quantifier rather than referring term is expedient; nothing hinges on it.

3 \(\Phi(v_i,v_j), \text{nonselfidentical}[\xi]\) and the like to indicate a formula of arbitrary complexity in the free variables indicated. \(\Phi(v_i,v_j), \text{off}(\xi,\zeta), Fx, Gxy\), etc. for primitive predicates and relations.

4 “If I say ‘Venus has 0 moons,’ there simply does not exist any moon or agglomeration of moons for anything to be asserted of; but, what happens is that a property is assigned to the concept ‘moon of Venus,’ namely that of including nothing under it.” (Frege 1884 §46)

5 The partitive construction is defined below.

6 Note that it is not enough that a speaker of any (of the) \(F(s)\),… be taken to implicate or presuppose that there are \(Fs\) unless taken to know that there are no \(Fs\). While it serves talk about nonselfidentical custard that its existence is thereby never implicated, presupposed or entailed in (22)-(26), in (34)b.-(38)b. domain restriction to existent custard remains necessary for the truth of these sentences, even if custard’s existence is presupposed or implicated.

7 Accepting (1) may reflect practical reasoning about one’s experience with nouns and their tendency in fact to be distributive as much as any \textit{a priori} knowledge about their grammar.

8 Likewise, if, as remarked in n. 31, it is hard to imagine how to approach the count/mass distinction (e.g., \textit{water droplets} vs. \textit{water vapor}) without privileging the singular referents that fall under these concepts, this fusing over onesies rather than twosies should not be taken to indicate that the syntax and semantics of the primitive vocabulary privileges singular reference. Nor should it be taken to privilege singular reference that whenever twosies implicate arcane, non-logical knowledge, the language reverts to the singular to distinguish \textit{a} duo from \textit{a} couple, say.

9 I hedged that it is “largely” an empirical question. A natural language with primitive plural reference that does not go beyond a designated relation \textit{‘is one of’} may be fit into monadic second order logic and enjoys
whatever conceptual advantages can be claimed for that achievement. That and $1.50 gets a ride on the subway.

10 I ignore accidents of English that some lexical nouns do not occur as both count and mass terms— *All bonfire is bonfire— and hold such a sentence to be analytically true too. See Sharvy 1978 and the extensive development in Borer 2005. If, on the contrary, the gaps are not accidental, it will suffice for the argument to follow that English and other natural languages contain at least some nouns that occur as both count and mass and that the class of such nouns be extensible.


12 If one only had in mind a contrast in distributive quantification between simple count and mass terms as in (i) and (ii), one could suppose that a mass noun occurs in (i) modified by mass-term morphology, a count noun in (ii) modified by singular morphology, and they happen to prompt homophonous number agreement on the verb.

(i) F Any patisserie is a pastry.
   F No patisserie is more than one pastry.
(ii) Any tartlet is a pastry.
     No tartlet is more than one pastry.

On this view, the logical forms in (iii) for (i) are simply ungrammatical, it having been stipulated that singular morphology does not co-occur with mass nouns:

(iii) *
   *[any \( \xi \) : sg.patisserie[\( \xi \)] is a pastry[\( \xi \)]
   *[No \( \xi \) : sg.patisserie[\( \xi \)] is a pastry[\( \xi \)]

The falsity of (i) does not, on this view, indicate that any quantity of patisserie, which may be several pastries, is patisserie, since in no sense is (i) singular quantification. Rather, (i) becomes the mass term counterpart to (iv):

(iv) F Any pastries are a pastry.
     F No pastries are more than one pastry.

The partitive construction discussed in the text is problematic for this view. By hypothesis, the same lexical items, any, of and the occur in (v) and (vi), and thus the grammar in ruling out (vii) is driven to stipulate a co-occurrence restriction between the singular morpheme and a remote phrase, the deeply embedded mass nominal:

(v) F Any of the patisserie is a pastry.
     F None of the patisserie is more than one pastry.
(vi) Any of the tartlets is a pastry.
     None of the tartlets is more than one pastry.
(vii) *
   *[any \( \xi \) : sg. of [DP the [NP patisserie]] [\( \xi \)] is a pastry[\( \xi \)]
   *[none \( \xi \) : sg. of [DP the [NP patisserie]] [\( \xi \)] is a pastry[\( \xi \)]

I would rather skip the ball than appear in such costume. It can be disguised, feature-passing [-count] up the tree of the partitive construction. Even so, if it should be further assumed that the partitive of is a relation (§1.2), this view is embarrassed by its suggestion that morphological properties of the direct object of of fixes the morphology of its subject.

On the other hand, if one is determined to forestall the conclusion that any quantity of pastry is pastry, a more abstract analysis of the partitive construction can be called upon, where it is proposed always to contain a null pronoun the content of which is copied from its antecedent, the partitive complement. That is, spoken ‘any of the patisserie’ and ‘any of the tartlets’ are underlingly any patisserie of the patisserie and any tartlet(s) of the tartlets, to which the co-occurrence restrictions for simple, non-partitive nominals may be applied. The same should then be said for spoken ‘any of that’ and ‘any of them’—that, underlingly, they are in effect any –at of the-at and any –em of the-em (v. Elbourne 2002). I believe that
such a proposal when coupled to the claim that mass-term and singular number agreement happen to be homophonous answers the argument from the falsity of (65) and (69) that any quantity of pastry is pastry. This conclusion on other grounds is however quite common in the literature (v. n. 11) and is corroborated below. Thus, even advancing the abstract analysis, I would reject the accidental homonymy and suppose that spoken ‘any of the patisserie’ is any sg.patisserie of the patisserie with singular quantification over (quantities of) patisserie.

The text argues from the distribution of bare nouns in inferences engaging count and mass terms and from the extensibility of this vocabulary that the same bare noun is a constituent of both mass terms and count terms. Such an argument does not apply to closed-class vocabulary, the, quantifiers and partitive of. One may recoil from the homophony among an alleged three thes and two ofs; but, within a given natural language, it is a cavil whether the language has five lexical items rather than two. That $\text{thes}_q=\text{thes}_p=\text{thes}_g$ and $\text{of}_p[\xi,\varsigma]=\text{of}_q[\xi,\varsigma]$ is an empirical generalization, which is defended on the basis of cross-linguistic evidence that the homophony is endemic.

Alongside the plural and mass partitives, there is a partitive of singular object in most of Gershwin’s “They Can’t Take That Away from Me”. If homophony among these three is also endemic in the world’s languages, then either syntax and semantics should provide that $\text{of}_p[\xi,\varsigma]=\text{of}_q[\xi,\varsigma]=\text{of}_q[\xi,\varsigma]$, or concede that homophony in the same place is a recurrent lightning bolt in human history. If the latter, then the homophony of the three thes loses its bite as an argument for their identity. The partitive of singular object might assimilate to the plural and mass partitive if, for example, most of you in (i) is rather like most of your ways, and more of him in (ii) is variously more of his money, more of his presence and more of his moral fiber.

(i) Most of you, they can’t take away from me—the way you wear your hat, the way you sip your tea, the way your smile just beams, the way you sing off key, the way you haunt my dreams, the way you hold your knife, the way we danced till three, the way you changed my life.

(ii) Slim put up with most of him and his ways, figuring to put away more and more of him in her bankbook. In twenty years, she got more of him around the house and more of him to crease her bed sheets and take out the garbage.

The claim would be that a partitive of singular object and the measure quantifier embedding it are incoherent in the absence of a concept fixing what is measured and how what is measured is of you in (i) or of him in (ii). One would seek cover in grammatical studies that for very different considerations of comparative syntax find that even simple names like Gershwin involve a complex DP with internal movements (v. Burge 1973, Longobardi 2005, Elbourne 2002).

As far as I can tell, there is no single lexical item of English with only the target plural partitive meaning. Sometimes among is recruited in paraphrase (e.g. McKay forthcoming); but, (iii) is ambiguous between a truth of 1853 reporting Burton’s pilgrimage in disguise to Mecca and a falsehood that he has converted to become one of the faithful.

(iii) Sir Richard Burton is among the faithful.

If it is a persistent feature of the world’s languages to confound these two senses in a single lexical item, one might hope for a univocal meaning. Perhaps among the faithful is a locative expression, among the faithful(e), and the variation is with respect to the contextually-understood locative space. At minimum, the space includes those points the plural complement refers to. The variation is from whether or not the space interpolates intervening points. The locative expression asserts location in the understood space. If that space includes only the points referred to, then to locate at such a point implies identity with its occupant—hence, the usage of among that resembles is one of. If other points fill in the space around those whose occupants are referred to, then of course there is no implication that the subject located is one of them. Contexts that do not make salient an interpolated space result in minimal contrasts such as between (iv) and (v), which are alleviated as soon as one is provided as in (vi).
(iv) The tsunami of December 26, 2004 is among the great natural disasters.  
The tsunami of December 26, 2004 is among several great natural disasters.

(v) *The tsunami of December 26, 2004 is among the other great natural disasters.  
*The tsunami of December 26, 2004 is among several other great natural disasters.

(vi) The tsunami of December 26, 2004 is among the other great natural disasters in the (recent) history of the Indian Ocean.  
The tsunami of December 26, 2004 is among several other great natural disasters in the (recent) history of the Indian Ocean.


16 ‘(∀Φ)’, ‘(∃Φ)’, quantifiers in parentheses to indicate substitutional quantification.


21 Cf. n. 20.


24 The final condition that ∀γ(∀ς(of[ς,γ] → of[ξ,γ])) is that what is referred to be the least such to meet the antecedent conditions. Let there be just one buffet blanketed in custard(s). The definite description should refer to this custard alone and not to this custard and in addition some blancmange.

25 Already for a construction as elementary as More than 365 custards blanketed the buffet, Hackl (2001, 2003ab) proves that the quantification conceals full-blown comparative clauses of degree. Presumably that syntax and semantics should be fit into The more than 365 custards blanketed the buffet, disappointing any expectation that a measure phrase will compose as simply as in (162). In the light of this result, (163) is at best provisional even if equivalent to the target meaning.


27 Necessary but not sufficient (as Kathrin Koslcki reminds me) in those contexts where the plural NP is felicitous but one still does not know how to count what is referred to:

(i) Those branches bear enough snow to topple the tree.

(ii) The cresting waves will splinter the pier.


29 Thanks to Kathrin Koslcki for pertinent discussion.

30 See Marcelo Ferreira 2005 on the wives of an astronaut for pertinent discussion.

31 Representing ‘pl[ξ]’ as extensional implies that coextensive concepts will not differ in their countability, slighting the contrast between (i) and (ii).

(i) The vortices (that fill the void) are zero in number.

(ii) *The ether (that fills the void) is zero in number.

Referring to nothing as vortices, it counts as zero. Referring to it as ether, a mass, it doesn’t. It suffices for present purposes to have dissociated the plural morpheme from what is quantified over, that some fires is [some ξ : fire[ξ] & Ψ], where ξ is not free in Ψ, leaving to a discussion of the count/mass distinction whether Ψ is as shown or more like ‘pl[fire]’, v. Koslcki 1997, 1999, 2005, in preparation and the references cited therein.

In that discussion, it will be pointed out that if there were ether, it would appear to the observer to be something to be measured, and if there were vortices, they would appear to the observer to be something to
be counted. If so, then whenever so-and-so is in fact presented to the observer, the so-and-so is expected to meet certain conditions. This expectation underlies the speaker’s judgments in thought experiments about (blue) fire(s) in Carmel. That is, ‘pl[*fire*]’ even if intentional entails conditions ‘pl*[ξ]’ for which, relative to any context of evaluation, [ιξ: fire[ξ]]pl*[ξ].

These speculations foretell of two consequences important for the problem at hand. First, as in the thought experiment, any judgment of countability takes into consideration all that be fire, and, thus in the language of thought, there occur definite descriptions [ιξ: fire[ξ]] that do not themselves contain singular, plural or mass morphology and thus deviate from those spoken. Second, despite the absent morphology and a ‘pre-individuative’ concept, singular constitution is well-defined: [ιξ: fire[ξ]]sg.of[ς,ξ] ↔ sg.fire[ς]. It is hard to imagine how to approach the count/mass distinction without recourse to singular fire as it were.

Consider, for example, a thesis that a concept is countable under given conditions of observation if anything that falls under it does not overlap in space, as when there are many fires each of which is a solitary burning bush. The logic of nonsingular partitive ‘off[ς,ξ]’ makes it immediately self-defeating to formulate the thesis that any of what is fire here does not overlap in space anything else of what is fire. Rather, that which is a thing under consideration and fire does not overlap in space anything else that is a thing under consideration and fire.

32 And subject to the same qualifications (n. 31).


34 Schein 1993: 35.


36 Linnebo 2004, McKay forthcoming, Oliver & Smiley 2001, Rayo 2002, and Yi forthcoming deplore singularist (v. §1.8.2) theories of plural reference, theories that deploy only one sort of variable, the value of which is a singular object, in part because consistency in such a theory is achieved only by restricting comprehension and thus withdrawing from quantifying over everything there is, a defect that Boolos (1984, 1985ab)) remedies basing plural reference on second-order logic (v. also Lewis 1991, Cartwright 1994, Linnebo 2003, 2004, Pietroski 2003, Williamson 2003). Linnebo 2004, McKay forthcoming, Oliver & Smiley 2001, Rayo 2002, and Yi forthcoming graduate to plural variables that refer plurally while their language retains only one sort of variable, and in this respect closely resembles the language of the partitive construction, including a constitution relation similar to of. Because one-sorted, the proposed plural language is as much at risk from Russell’s paradox as the deplored singularist accounts and so comprehension must be restricted just the same (cf. McKay forthcoming, chapter 6). Higginbotham (1998, 2000) and Linnebo (2003, 2004) also favor plural terms *sui generis* rather than second-order expressions but advance a two-sorted logical syntax.

37 Self-reference in the partitive construction itself, *any of themselves, *any not of themselves is ungrammatical, but consider attempts to reproduce Russell’s paradox in the object language periphrastically, exploiting the inferential relations between tensed sentences:

(i) They are one of the decks of cards if and only if they are a deck of cards.

(ii) The three cells are one of the multi-celled organisms if and only if they are a multi-celled organism.

(iii) They are one of the Fs if and only if they are a(n) F.

The validity of (iii) is extensible to novel F; and Russell’s paradox threatens its generalization if the second clause in (ii) is predication of F as in (iv), and the Fs is taken to be *the things that are not one of themselves:

(iv) They are one of the Fs if and only if F(they).

If it is objected that *the things that are not one of themselves is ungrammatical (v. Higginbotham 1998, 2000), its meaning finds alternative expression in the things that are not one of the things that they are, and thus grammar cannot be counted on to fend off an unwanted consequence. There remain two ways out. The first, as in the text, restricts substitutions for F to singular descriptions, noting that the subject of *are
one of the Fs’ is an essential plural. The second denies that (iv) is the translation of (iii). Rather, the copula should be taken to express a relation, and $F$ is never predicated of them:

(v) They constitute one of the Fs if and only if they constitute an $F$.

38 Nouns, adjectives and abstractions on clauses in a single variable belong to the same logical type but are not always interchangeable. A speaker’s presumption that a noun is distributive or even an adjective (allowing her to infer (238) from (237)) may reflect grammatical conditions on what the meaning of a noun or adjective can be. Perhaps, ‘$\neg$sg[ξ]’ is ungrammatical as a noun or adjective, neither being distributive nor enough like the small class of adjectives and nouns such as neighbor, relative, associate that are systematic exceptions.


42 Higginbotham (1998, 2000) appears to invoke such considerations when he says that there seems to be nothing predicational about the plural demonstrative when the speaker waves his hand at some boys, saying ‘They built a boat yesterday’. Even if the speaker thinks ‘They—the only things in that corner of the room that could have built a boat— built a boat yesterday’, he cannot be taken to have intended to communicate this thought or to be disappointed if the hearer understands instead ‘They—the only things that this schmuck could be referring to without telling me what he is referring to— built a boat yesterday’. Verbs too are acquired demonstratively, without intervention from anything ‘predicational’, as when a clarinet sounds in demonstration of a nonce verb to chalumeau or John Cleese displays the meaning of to Silly-Walk relying on the learner’s exquisite but inarticulate sense for sound and gesture to grasp the subtleties of chalumeau-ing and Silly-Walking.

43 See Platts (1979) and Larson & Segal (1995) for further discussion.

44 “…if I place a pile of playing cards in [someone’s] hands with the words: find the number of these, this does not tell him whether I wish to know the number of cards, or of complete packs of cards, or even, say, of honor cards at skat. To have given him the pile in his hands is not yet to have given him completely the object he is to investigate; I must add some further word—cards, or packs, or honors.” (Frege 1884 §22)

“While looking at one and the same external phenomenon, I can say with equal truth both “It is a copse” and “It is five trees,” or both “Here are four companies” and “Here are 500 men.” Now what changes from one judgment to the other is neither any individual object, nor the whole, the agglomeration of them, but rather my terminology. But that is itself only a sign that one concept has been substituted for another.” (ibid. §46)

45 To expose the (subdoxastic) concepts that resolve demonstrative reference,

(i) They are (all) separated from each other.

(ii) 

judge (i) against contexts that present to the subject figures such as (ii), varying the parameters familiar from experiments in gestalt perception, e.g., ratio of enclosed area to interstitial area, ratio of the area of individual enclosures to area of aggregate enclosure, ratio of aggregate enclosed area to interstitial area, ratio of aggregate area to the number aggregate, absolute number aggregated, geometric regularity of individual enclosures, similarity and scaling, heterogeneity, symmetry and axial orientation and alignment, color, shading, (partial) occlusion, animation with rigid vs. elastic motion. In a forced choice between truth and falsity, what things does the subject seize upon to judge whether they are separated or not, and does the individuation of them track the same conditions as object recognition in general?
A tempting but ultimately spurious argument looks like it confounds even more directly the distinction between plural terms and predicates. If single token of an antecedent can be counted on to bind variables of only the same type, quantifying in a quantifier’s restriction and into the partitive construction, as in (1) or (2) would show that the object of the partitive construction, allegedly a plural term (Higginbotham 1998, 2000; Linnebo 2004; McKay forthcoming; Oliver & Smiley 2001; Rayo 2002; Yi forthcoming), and the quantifier’s restriction, the positions occupied by the variable $\xi$, belong to the same semantic type:

(1) \[
[\text{the } \xi : \Phi]…[Q : \xi]…o[\zeta, \xi]…[Q : \xi]…
\]

(2) \[
[\text{the } \xi : \Phi]…[Q : \xi]…o[\zeta, \xi]…[Q : \xi]…
\]

Despite an (irregular) alternation of pronounced and unpronounced pronouns and variables, partially illustrated below and investigated extensively in the linguistics literature (v. xxx), logical forms joining the semantic type of the partitive’s second argument to that of quantificational restrictions appear prima facie to underlie (4), (5), (15) and (21), instantiating (1), and (8), (9), (12), (13) and again (21), instantiating (2).

(3) Which garments did Sid try on three of and buy four of?
(4) Which garments did Sid try on three and buy four of?
(5) Which garments did Sid try on three of and buy four?
(6) Which garments did Sid try on three and buy four?

(7) The garments that Sid tried on three of and bought four of were heavily marked down.
(8) The garments that Sid tried on three and bought four of were heavily marked down.
(9) The garments that Sid tried on three of and bought four were heavily marked down.
(10) The garments that Sid tried on three and bought four were heavily marked down.

(11) The garments such that Sid tried on three of *(them) and bought four of *(them) were heavily marked down.
(12) The garments such that Sid tried on three and bought four of *(them) were heavily marked down.
(13) The garments such that Sid tried on three of *(them) and bought four were heavily marked down.
(14) The garments such that Sid tried on three and bought four were heavily marked down.

(15) Which garments did Sid try on many of without purchasing any?
(16) Which garments did Sid try on many of without purchasing any of?
(17) Which garments did Sid try on many of without purchasing any of?

(18) ? Which garments did Sid try on many without purchasing any of?
(19) *Which garments did Sid try on many without purchasing any of?
(20) ?? Which garments did Sid try on many without purchasing any of?

(21) The students that were completing PhDs in a year when many (of them) fell in love with a few (of them) were none of them writing dissertations and all writing love poems instead.

[Cited in Schwarzschild 1996, chapter 7:
(i) They are none of them very enthusiastic. (Quirk et. al. 1985: 1399)
My sisters don’t either of them eat enough.
Wilkinson (1996) shows however that the analysis of the partitive construction implied by (1) and (2) requires emendment. Observing that the ungrammaticality of bare plurals in partitives (22) does not prevent a pronoun in that construction from being anaphorically related to a bare plural as in (23), Wilkinson points out that we cannot maintain both that the partitive constraint excluding the bare plural in (22) is semantic and that the anaphoric relation in (23) is coreference.

(22) *all of garments, *some of garments, *many of garments.
   all of the garments, some of the garments, many of the garments.
(23) Men’s garments were strewn about on the cutting-room floor. So Sid arranged all/some many of them in a neat pile.

Instead, the pronoun does not refer to what the bare plural refers to. It is rather a definite description that merely shares descriptive content with its antecedent, (24) rather than (25):

(24) [Qξ : men’s garments[ξ]] ξ were strewn about on the cutting-room floor. So Sid arranged all/some many of [the ξ : men’s garments[ξ]] in a neat pile. (where Q is the operator special to the semantics of bare nominals).
(25) * [Qξ : men’s garments[ξ]] (ξ were strewn about on the cutting-room floor. So Sid arranged all/some many of ξ in a neat pile.)
   * [Qξ : men’s garments[ξ]] ξ were strewn about on the cutting-room floor. So Sid arranged all/some many of [Q ξ : men’s garments[ξ]] in a neat pile.

In this respect, it conforms to previous and subsequent analyses (v. Elbourne 2001abc, 2002) that pronouns are always full-blooded nominals, Determiner-NP, never to be translated as simple, unstructured variables. Accordingly, the logical form for (21) and the like should be emended to (26) rather than (27):

(26) [The ξ : students…[ξ]] were [none ζ : [the ξ : ξ[ζ]] of[ζ,ξ]] writing dissertations and [all ζ : ξ[ζ]] writing love poems instead.
(27) *[The ξ : students…[ξ]] were [none ζ : of[ζ,ξ]] writing dissertations and [all ξ : ξ[ζ]] writing love poems instead.

In (26), the students quantifies in two restrictions, the restriction to all and now to the definite description that the pronoun them abbreviates. If correct for (21), it no longer shows directly that the object of the partitive construction itself belongs to the same semantic type as a quantifier’s restriction (cf. (27)). The latter conclusion derives only from the more general consideration adduced in the text. If plural definite descriptions are univocal in their semantic type, on which therefore the students… and them in (21) agree, and plural definite descriptions sometimes quantify in the restrictions to quantifiers, then the variables of plural quantification, ξ and ζ in (26), always belong to the same type, the type of predicates. Sentences (4), (5), (8), (9) and (15) with gaps rather than spoken pronouns in the partitive construction may look like better evidence joining the types of partitive object and quantifier restriction, but not if the unspoken gap contains as much structure as its spoken pronominal counterpart, as much recent research indicates (v. the copy theory of movement, see also Rayo & Yablo 2001 on the ontological commitments of non-nominal quantifiers, their discussion of Prior 1971 and their Argument from Instances and Argument from Entailments (p. 81), which would follow from a copy theory of movement.).

48 See Hossack 2000, Linnebo 2004, Rayo 2002 on the equi-interpretability of monadic second-order logic and what I have called the language of the partitive construction.
Better than (281), why not a more disquotational (1) or (2), “learn[ing] to use the higher-order languages as our home language (Williamson 2003)”, and thus in effect deriving (3) rather than (4)?

(1) \[ \Sigma \text{ satisfy } 'V_{V'} ' \leftrightarrow \\
\exists X \exists y (\forall z (\Sigma (<x,y>) \leftrightarrow z = y) \& \forall z (\Sigma (<z, V'>) \leftrightarrow Xz \& Xy) \]

(2) \[ \Sigma \text{ satisfy } 'V_{V'} ' \leftrightarrow \\
\exists X \exists y (\forall Z (\Sigma (<z, V'>) \leftrightarrow \text{sg.E.of}[Z, y]) \& \forall Z (\Sigma (<Z, V'>) \leftrightarrow \text{sg.E.of}[Z, x]) \& Xy) \]

(3) ‘∃X∃xXx’ is true if and only if Something somethings.

(4) ‘∃X∃xXx’ is true if and only if Something is one of some things.

If the theorist’s language does not itself quantify in matrix predicates, some other locution must be recruited in the metalanguage to explain the semantics of the second-order quantification that occurs in the object language, recruiting a locution that is felt to be readily available, as in (4) (v. Boolos 1984, 1985ab; Higginbotham 1998, 2000), or extending the theorist’s language as in (3). The latter would be more faithful to the language under analysis except for the suspicion that (3) is coherent only in so far as it translates into (4)—a suspicion that the theorist cannot really make herself at home in the higher-order language. Even philosophers who agree with Boolos in rejecting the tradition that joins at the hip second-order logic and Frege-speak about concepts may feel compelled to apologize for his use of the locution ‘is one of’ in (4), caught as they are between what makes sense and how they think a faithful semantics for second-order logic should read, namely, as in (3). I think however that fealty to (3) and mistrust of (4) are overrated. Surely a disquotational and homophonic semantics, wearing its own infallibility, should be treasured whenever it can be had. Yet if it seems to the theorist who doubts her understanding of (3) that her language will not support it, she is no worse off here than she is elsewhere in much of her linguistic analysis. Whenever she meets a bound morpheme, such as re- in English, and attempts a semantics for ‘re-V’, she retreats to a circumlocution such as “do V-ing again” displaying little of the syntax of that fragment of the object language under analysis. She cannot make free use of re- on the RHS. It is a bound morpheme after all. At the same time, it would be silly to conclude from circumlocution in the semantics that re- is other than a bound morpheme. It has the syntax that it has. A semantics in English for a language with a bound causative morpheme illustrates the point as well. No doubt the causative morpheme in the object language has neither the syntax nor exact meaning of cause in English, and the English theorist’s best efforts to convey the notion of direct causation that the bound morpheme expresses are not also an effort to revise the syntax of the object language which cannot be simulated in English. A speaker of the object language who attempts the semantics for her own language will also be driven to circumlocution for the semantics of her bound causative morpheme, as was the English theorist facing re-. As it turns out, the language of the partitive construction relies on a charitable view of circumlocution too. The reader may have accepted without challenge that of as it occurs in the partitive construction is a dyadic relation; but, its semantics is not disquotational and homophonic, to the extent that (5) and (6) are no better than (3):

(5) ‘sg.E.of[ξ, ζ]’ is true of <it, them> if and only if it ofs them.

(6) ‘sg.E.of[ξ, ζ]’ is true of <it, them> if and only if it is of them.

Rather, the semantics in (7) resorts to a circumlocution with a rather complex syntax including quantification (Cf. It is one of these and one of those. It is one and no more than one of them. One of them, it is), notwithstanding the self-deception of occasionally writing is-one-of:

(7) ‘sg.E.of[ξ, ζ]’ is true of <it, them> if and only if it is one of them.

Yet, no one who offers (7) commits herself to revising the syntax of object language One of them as if it should become ‘One who is one of them’ to reflect the quantificational structure of her circumlocution in the semantics for of. The syntax of One of them remains whatever grammar and inference in the object language requires of it. Likewise, should grammar or logic prompt parsing a natural language construction
as an expression of second-order logic, circumlocution in its semantics, Boolos’ use in (4) of the locution ‘is one of’, is no grounds to rescind that analysis.

The analysis in the text favors taking the natural language at its superficial word. Thus the natural language contains both second-order quantification, attested at least when quantifying in a quantifier’s restriction, and quantification into the partitive relation of. In the natural language, then, one finds a synonymy \((v. (156))\) between predication and the partitive construction:

\(V_{\text{sg.E.of} [v_j, V_i]} \leftrightarrow V_{i} \)

Deduction within the language of the partitive construction, whether deductions in the object language or deductions of the semantic theory couched in that language, relies both on a logic for \(o\) (cf. Hossack 2000, Linnebo 2004, McKay forthcoming: chapter 6, Rayo 2002, Yi forthcoming) governing the use of that lexical item in inference and on monadic second-order logic governing plural quantification elsewhere. Regimentation, translation from the language of the partitive construction into the language of second-order logic and from the language of second-order logic into the language of the partitive construction, demonstrates the deductive equivalence of second-order logic and the logic of the partitive construction \((v. (217) \text{ and } (222))\).

50 Empirical arguments (Schein 1993: chapter 1, p. 6; chapter 3§1, chapter 7; chapter 9§5; chapter 10n.3, p. 350ff.) that divide natural language quantifiers between the second-order, viz., indefinite and definite descriptions, and the (semi-)distributive or first-order, most, many, few, every, etc. are recast in the monosortal language as showing that the latter are obligatorily restricted to the singular—\([\text{most } \xi; \text{ sg.}\Phi [\xi]], [*\{\text{most } \xi; \Phi [\xi]\}—\text{in contrast to the former, } [\text{some } \xi; \text{ sg.}\Phi [\xi]], [[\text{some } \xi; \Phi [\xi]]\). Noteworthy is that the restriction to the singular holds of all distributive quantifiers despite their superficial plural morphology, most books.

51 Yi (1999b) alleges that being two is a first-order property.


53 (293)-(297) casts a wide net, as appropriate for the claim that any possible natural language quantifier can be characterized within the language of the partitive construction (or equivalently within monadic second-order logic). A further restriction is that the natural language quantifiers \(Q\) that occur as determiners are conservative (Barwise & Cooper 1981, Keenan & Stavi 1986), which Pietroski 2003 derives within a semantics formulated in monadic second-order logic for a language that is closer in its detail to natural language, where the restriction to a determiner is a NP, a predicate denoting objects, but its matrix is an open sentence denoting merely truth values (under an assignment).

54 A difference remains between spoken clauses as in (301) and unspoken (302)-(303) in that the latter for reasons both syntactic and semantic are always tenseless (prompting my use of the subjunctive in the translation), see Hackl 2001 for discussion.

55 Note that if (298) is underlyingly (i) rather than (ii) as Hackl (2003ab) argues on the basis of both comparative grammar and psycholinguistic processing, the comparison can be translated within monadic second-order logic without reference to the cardinality of the cardinals \((cf. (287)-(288))\).

\begin{align*}
(298) & \quad \text{Most cardinals are ordinals.} \\
(i) & \quad \text{More cardinals are ordinals than are not.} \\
(ii) & \quad \text{More than } \frac{1}{2} \text{ the cardinals are ordinals.}
\end{align*}

The difference in meaning between ‘\(\text{card}\{Z,X\}\)’ and ‘\(\text{card}\{n,X\}\)’ \((v. (283)-(286))\) also translates the contrast between (iv) and (v):

\begin{align*}
(iv) & \quad \text{T More cardinals than any cardinal are ordinals. } (cf. \text{ More cardinals than nine are ordinals.}) \\
(v) & \quad \text{F More cardinals than any cardinals are ordinals. } (cf. \text{ More Cardinals than any Orioles play in the World Series.})
\end{align*}

Perhaps the sense in which speakers understand (299) to be true derives from restrictions on the conditions of measurement as in (vi):

\begin{align*}
(299) & \quad \text{Most cardinals are ordinals.} \\
\end{align*}
Most natural numbers are composite.
Few natural numbers are prime.

More natural numbers are composite as one proceeds through the number line than are not.
Few natural numbers are prime as one proceeds through the number line.

Decomposition in the syntax of the object language is defended on grounds unrelated to plurals in 

More examples—

(1) The zebra mussels are choking each other in the drainpipe.
(2) The bamboo shoots smother each other.
(3) The politicians stifle each other.
(4) The motors overheated each other.

It is shown that a sentence such as (1) is expressed by no logical form that is the quantificational closure
by three ATMs, two new clients and exactly two passwords of a formula \( \Phi_{E,\xi,\zeta,\varsigma} \), \( \Phi_{e,\xi,\zeta,\varsigma} \) or \( \Phi_{\xi,\zeta,\varsigma} \)
in any combination of singular or plural variables \( \xi,\zeta \) and \( \varsigma \) with or without a singular or plural event
argument. The quantificational closures targeted cast a wide net including schemas for composing three
ATMs, two new clients and exactly two passwords into branching or \( n \)-ary quantifiers:

(1) (In exactly 24 hours,) three ATMs gave two new clients each exactly two passwords.
(2) * Three ATMs; two new clients(each); exactly two passwords \( \Phi_{E,\xi,\zeta,\varsigma} \)
* Three ATMs; two new clients(each); exactly two passwords \( \Phi_{e,\xi,\zeta,\varsigma} \)
* Three ATMs; two new clients(each); exactly two passwords \( \Phi_{\xi,\zeta,\varsigma} \)

There is thus no accurate logical form that translates give without its decomposition and separation of its
constituents among the nominal quantifiers, three ATMs, two new clients, exactly two new passwords.

The argument engages two observations about the relevant interpretation of (1): i) that it is true if
what happens in 24 hrs. is as shown in (3),

where two new clients, \( c_1 \) and \( c_2 \), are each given exactly two passwords \( p_i \), and three ATMs, \( t_1 \), \( t_2 \), and \( t_3 \)
give them their passwords; and, ii) that it is false in (4),

where two new clients, \( c_1 \) and \( c_2 \), are each given exactly two passwords \( p_i \), and three ATMs, \( t_1 \), \( t_2 \), and \( t_3 \)
give them their passwords; and, ii) that it is false in (4),
where within those 24 hours, the first client $c_1$ has been given four passwords, exceeding the exactly two that (1) reports. (Notice that the time-frame adverbial is deployed to force evaluation with respect to all of (4), the events later than (below) $p_1$ and $p_2$'s being just those of (3), which verify (1). Alternatively, one could imagine the simultaneity of what is depicted.)

The first observation, the truth of (1) in (3), suffices to put (1) beyond the reach of three ATMs, two new clients, and exactly two new passwords, when these are unary, restricted quantifiers that close off formulae under some linear assignment of scope. The second observation enters in the discussion of more exotic quantificational schemas that can be imagined for $n$-ary or branching quantifiers. Relevant for (1) would be the composition of three ATMs and two new clients (each) into a binary, restricted quantifier that included within its scope exactly two new passwords and the rest of the sentence:

(5) three ATMs gave exactly two passwords
two new clients each

It is important to note—although Schein 1993 leaves it implicit—that to render (1) true in (3), the branching quantifier must absorb each along with the two quantifiers felt to be independent of one another, three ATMs and two new clients. That is, the schema must be expansive enough to allow more than one quantifier within a branch and little thought given to how these quantifiers might separately contribute to the meaning of their branch. If pathology in the grammar of these quantifiers is no reason for quarantine, the best among the binary, restricted quantifiers interpreting (5) in fact manage (Schein 1993: chapter 4 §4, p.72ff.) both to render (1) true in (3) and false in (4). One such interpretation of (5) is (6) with the paraphrase shown:

(6) $[\exists X: \text{ATMs}(X)][\forall x: \text{of}(x,X)][\exists X': \text{of}(x,X') \& \text{of}(X',X)]$

$[\exists y: \text{new client}(y)][\text{exactly 2 z: password}(z)][\exists Z: \text{of}(z,Z) \& \text{passwords}(Z)] \text{give}(X,y,Z) \&$

$[\text{Two Y: new clients}(Y)][\text{each y: of}(y,Y)]$

$[\exists X': \text{ATMs}(X')][\text{exactly 2 z: password}(z)][\exists Z: \text{of}(z,Z) \& \text{passwords}(Z)] \text{give}(X',y,Z))$

Three ATMs each of them with perhaps some of the others give a new client, exactly two passwords; and, two new clients are each given by some ATMs exactly two passwords.

Notice that the variant in (7) replaces ‘give($X',y,Z$)’ which refers to passwords plurally with ‘give($X',y,z$)’ which distributes over them singularly:

(7) $[\exists X: \text{ATMs}(X)][\forall x: \text{of}(x,X)][\exists X': \text{of}(x,X') \& \text{of}(X',X)]$

$[\exists y: \text{new client}(y)][\text{exactly 2 z: password}(z)] \text{give}(X',y,z) \&$

$[2 Y: \text{new clients}(Y)][\forall y: \text{of}(y,Y)][\exists X': \text{ATMs}(X')][\text{exactly 2 z: password}(z)] \text{give}(X',y,z)$

Distributing over passwords, (7) fails to be true in (3): the atms $t_1$ and $t_3$ that give $c_2$ her passwords $p_3$ and $p_4$ do not together give her $p_1$ and together give her $p_3$.

The better binary quantifier (6) only fails—but fail it does—in sentences with further quantification:
Three ATMs gave two new clients each exactly two passwords each on two slips of paper.

\[ s_1 \]
\[ p_1 \langle \]
\[ s_2 \]
\[ c_1 \langle \quad \rangle t_1 \]
\[ s_3 \]
\[ p_2 \langle \]
\[ s_4 \]
\[ s_5 \]
\[ p_3 \langle \quad \rangle t_2 \]
\[ s_6 \]
\[ c_2 \langle \]
\[ s_7 \]
\[ p_4 \langle \quad \rangle t_3 \]
\[ s_8 \]

The relevant interpretation of (8), following along with (1), says that three ATMs give two clients passwords and each is given exactly two and adds that each password is given on two slips of paper (copied or in two parts). The further remark does not affect the disposition of (8) in (9), which, relates ATM, client and password exactly as (3) does.

Three ATMs gave exactly two passwords each on two slips of paper.

two new clients each

Interpretation of the binary quantification in (10) must however now be developed as in (7), each on two slips of paper now forcing distributive reference to passwords, and as remarked above the resulting interpretation is mistakenly false in (9).

Chapter 4§4 objects to the binary quantifier with a different continuation of (1):

Three ATMs gave two new clients each exactly two passwords on a single slip of paper.

As before, the relevant interpretation of (11), following along with (1), says that three ATMs give two clients passwords and each is given exactly two and adds instead that each is given her passwords on a single slip of paper. The further remark does not affect the disposition of (11) in (12), which, relating ATM, client and password exactly as (4) does, falsifies (11) (and (1)) for the same reason—the first client
Given four passwords exceeds the exactly two reported. But, the binary quantifier, if applied as in (13), will leave exactly two passwords to measure what the individual client is given on an individual slip of paper, thus mistaking (11) for true in (12), where \( c_1 \) is indeed given exactly two passwords on \( s_1 \) and exactly two passwords on \( s_2 \).

\[
(13) \quad \text{three ATMs} \quad \text{gave on a single slip of paper exactly two passwords.} \\
\text{two new clients each}
\]

The observation is correct, as far as it goes, but a second look at chapter 4§4 will show that it shows only that the intended interpretation of (11) is not represented by the combination of a binary quantifier in (13) with a linear ordering of the remaining quantifiers. Succumbing to the full pathology, I should have allowed the first branching quantifier to properly contain within its scope another branching quantifier as in (14), which may very well derive under the proposed interpretation for branching correct truth conditions for (11) at least for these contexts.

\[
(14) \quad \text{three ATMs} \quad \text{exactly two passwords} \quad \text{gave.} \\
\text{two new clients each on a single slip of paper}
\]

This quantification will however fail with the next turn of the screw, reverting to contexts where the targeted interpretation is true but the considered \( n \)-ary quantification renders it false:

\[
(15) \quad \text{Three ATMs gave two new clients each exactly two passwords on two slips of paper each in a separate transaction.}
\]

\[
(16) \quad \text{three ATMs} \quad \text{exactly two passwords} \quad \text{gave in a separate transaction.} \\
\text{two new clients each on two slips of paper each}
\]

\[
(17) \\
\begin{align*}
\text{ATMs, clients and passwords relate to each other in (17) as they have in (3). Sentence (15) goes on to say that for each client, the two passwords were on two slips of paper, also true in (17), and that each slip of paper was passed in a separate transaction, again true. Quantification according to (16) will incorrectly negotiate the interaction of three ATMs and two slips of paper each. The latter must be distributive with respect to a separate transaction, but then the atms \( t_1 \) and \( t_2 \) that give \( c_2 \) her passwords do not both participate in each transaction. It repeats the objection to binary quantification already seen in the interpretation of (8).}
\end{align*}
\]

A modest respect in which the syntax of natural language relates systematically to logical form is that logical form does not discard the words spoken, preserving the integrity of phrases spoken as best it can. If the speaker has uttered three ATMs, two new clients (each) and exactly two passwords, these are the quantifiers she means to use. If these quantifiers are linearly ordered and the semantics for each supplied
by a clause in the form of (18), it is quick work to show that no prefix from these quantifiers closes off a formula that renders (1) accurately.

\[(18) \quad \Sigma \text{ satisfy } [Q : \Phi]^\Psi \iff \ldots \]

Although I am skeptical of its applications to natural language (1993: chapter 12), the interest of \(n\)-ary quantification (Barwise 1979, Carlson 1982, Higginbotham & May 1981, Keenan 1987, Lindström 1966, Scha 1981, Sher 1990, Westerståhl 1987) is its promise to expand quantification while preserving with a hope and a prayer the integrity of the quantifiers spoken. The suggestion is to introduce semantic clauses or schemas that allow interpretation to take up several quantifiers at once:

\[(19) \quad \Sigma \text{ satisfy } [Q_1 : \Phi_1] \ldots [Q_n : \Phi_n] \Psi \iff \ldots \]

The RHS of (19) should say something about \(Q_1\)-many things that \(\Phi_1\) is true of, \(\ldots\), \(Q_n\)-many things that \(\Phi_n\) is true of and things that \(\Psi\) is true of. Unsurprisingly, establishing an expressive limit on such an open invitation is more work. What has been observed so far is that a binary quantifier applied in (5) to interpret (1) as (6) fails (8), a continuation of (1), when parsed as in (10) where the binary quantifier now includes a distributive quantifier in its scope. But, why not define under (19)’s flag a ternary quantifier, parsing (8) as (20), with the result of interpreting (8) as (21)?

\[(20) \quad \text{three ATMs two new clients each } \implies \text{ gave on two slips of paper.} \]

\[(21) \quad [\text{Three } X : \text{ATMs}[X]][\text{Two } Y : \text{new clients}[Y]][\text{each } y : \text{of}[y,Y]][\text{exactly } 2 \ z : \text{password}[z]][\text{Two } W : \text{papers}[W]][\exists X^* : \text{of}[X^*,X]][\text{give}(X^*,y,z,W) \land \forall X : \text{of}[x,X]][\exists Y^* : \text{of}[Y^*,Y]][\text{each } y : \text{of}[y,Y]][\text{exactly } 2 \ z : \text{password}[z]][\text{Two } W : \text{papers}[W]]
\]

\[\exists X^* : \text{of}[x,X^*] \land \text{of}[X^*,X][\text{give}(X^*,y,z,W)]\]

The contribution of \textit{three ATMs} is in boldface. The existential quantifier that closes off the argument position for givers must appear within the scope of any and all distributive quantifiers that intervene between \textit{three ATMs} and \textit{give}, and indeed the binary quantifier (10) fails the intended interpretation in that this existential quantifier falls outside the scope of \textit{exactly two passwords (each)}. Any attempt at logical form by quantificational closure of a relation \textit{give} will have to chase down the unbounded dependency between \textit{three ATMs} and the existential quantification of the givers across whatever distributive quantification intervenes. In the case of (15), what was observed was a mis-interpretation even allowing that one binary quantifier include another within its scope as in (16). The interpretation of that first binary quantifier still introduces some ATMs, those that give \(e_2\) passwords (17), and it stumbles over the distributive quantification in its scope, since these ATMs do not participate in each of \(e_2\)’s transactions. Clinging to \(n\)-ary quantification, (15) should rather prompt definition of a single quadernary quantifier with a binary internal structure reflecting the sentence’s combination of relative scope and scopal independence.

In characterizing a speaker’s judgment of all these sentences, (1), (8), (11), (15), the thought has been that in parsing \textit{three ATMs} and \textit{two new clients} as scopally independent she understands right off that three ATMs are givers and two new clients are receivers no matter how exactly that is done and indifferent to whatever continuations subsequently refine its description. A proponent of \(n\)-ary quantification will have mistakenly considered this thought formalized in the proposal that (15) is scanned so that first the two independent quantifiers \textit{three ATMs} and \textit{two new clients (each)} are a binary quantifier and then it is continued that the binary quantifier distributes in some fashion over the sentence’s continuation which
itself begins with two independent quantifiers, another binary quantifier and so on, any semantics for which neglects the extent of the speaker’s indifference to how exactly ATMs relate to passwords, slips of paper and transactions. Instead, this thought must be abandoned, and for every n, the semantics will contain clauses interpreting the compounding of n-quantifiers into a single n-ary quantifier. For any n, there will need to be as many clauses as there are different ways of combining scopal independence and relative scope among the n quantifiers internal to the n-ary quantifier being defined (v. (15)). This is only the threat of an infinitary semantics since I do not know that the class of n-ary quantifiers could not be inductively defined, from which all those interpreting clauses could emerge as theorems.

The argument in this section §2.2.1.2 for predicate decomposition and the separation of its constituents has been different in character from that of the preceding §2.2.1.1, where the existence of subatomic analogues to the Geach-Kaplan sentence shows that there are sentences of the natural language that cannot be translated into a logical language without the vocabulary of decomposition. Here, in contrast, the discussion presumes that (6) means what (1) is supposed to mean and even that (21) means what (8) means, although neither (6) nor (21) is in the favored logical language. The argument here is rather that there will be no satisfactory syntax and semantics for the natural language even under promiscuous assumptions about a lawful relationship between the quantifiers pronounced and the interpretation of the sentence in which they occur. Observing in (21) that there is an unbounded dependency between three ATMs and the existential quantifier closing off the relevant argument position lays a finger on the formal problem to be overcome. The argument for separation here is ephemeral, namely, that any theory that denies it is cornered into a promise to provide an infinite number of semantic clauses for infinitely-many ways of compounding arbitrary numbers of quantifiers into quantifiers that are otherwise unattested in natural language.

In his demurral to this argument, McKay (forthcoming, chapter 10) points to an equivalent of (7), but I take him to mean (6) for the reasons given above, as “the logical content” of (1), acknowledging the lack of a systematic derivation for the meaning of quantifiers that are unscoped relative to one another. Thus his discussion is mercifully uncluttered by a detour through n-ary quantification, and its significance rests on finding logical content in the juxtaposition of (1) and (7) (or, (6)). Incidental to his formulation of (7) (or, (6)), exactly two passwords dissolves into “two passwords…” and “all passwords that… are among them”, which he recognizes as a further problem for the correspondence between logical content and natural language sentence. I should mention a respect in which the argument above is robust under even more promiscuous assumptions. Both unary, restricted quantification and the schemas for n-ary quantification use the quantifiers as spoken. If the speaker has uttered exactly two passwords, this is the quantifier she means, and it has appeared on the RHS in all the above renditions. We may however countenance some deviation from her exact words and imagine that exactly two passwords is a variant of two and no more than two passwords, which in turn is just two passwords and no more than two passwords, so that (1) were (22) in disguise:

(22) Three ATMs gave two new clients each two passwords and no more than two passwords.

One might then cull a transformation that derives a logical form by distributing the coordination internal to the quantifier phrase over its entire scope:

(23) [Two passwords and no more than two passwords] Φ ⇒ Two passwords Φ and No more than two passwords Φ.

One countenances such a deviation from the quantifier spoken having in mind that it might be a regular feature of the grammar to provide it. Even so, one will not find a standard logical form good for (22), and one resorts to n-ary quantification as discussed here and in Schein (1993: chapter 4§4). In McKay’s rendition of (7) (or (6)) exactly two passwords dissolves into constituents that do not share the same content Φ, implying that the quantifier is ripped apart and its parts assigned different scope. In other words, any crooked paraphrase passes for “logical content”.
The logical form (328) simplifies and slights an important aspect of the meaning of (327), which for present purposes we can ignore. The two workbenches’ being each covered with two bedspreads is not merely part of the three hundred patches’ covering but completely coincides with it. v. Schein 1993, p. 146ff.

It is a puzzle to reconcile in (i) apparent collective reference to the same vaudevillians dancing and singing with the scope of *no more than three ballads*…

(i) No vaudevillians danced together to no more than three ballads that they sang together.

(ii) *[No x : vaudevillian(x)][∃ X: Xx & vaudevillians[X]]…together[X]…[no more than three ballads that X sang together[X]]…

Sentence (i) is not the vacuous falsehood in (ii) that no vaudevillian is among some who dance and sing together no more than three ballads. Of course, (ii) is falsified by any vaudevillian, e.g., Fanny Brice, for whom there is at least one other whom she has not appeared on stage with, Al Jolson. Rather, (i) means that no vaudevillian has no more than three ballads that he sang and danced to together with other vaudevillians, which would be true if vaudevillians were many years on the circuit with the same troupe performing the same routines. The puzzle is how to extract this meaning from a translation of (i) that conforms to its syntax. Its solution provides further argument for the separation of thematic relations in the logical syntax of the object language (Schein 1993: chapter 8).

Again, the argument assumes that the same morpheme *collaborate* is involved in judgments involving its nominalization that there is not a collaboration and in assertions involving the verb that so-and-so collaborate (Parsons 1990).

Any attempt at plural reference addresses itself to a couple of basic concerns (Schein 1993: 86): that the participants the plural refers to might do different things, and that the participants do all there is to do to bring about the reported event.

(i) The students cooked dinner.

Some students might salt water while others scrub pots, uncork wine or finish sauces and so on. Yet, a distributive, first-order thematic relation says that each did the same thing for that dinner, taking (i) to report a single event e:

(ii) ∀ x (Xx ↔ Agent(e, x))

That same thing that each did was to salt water or scrub pots or uncork wine or finish sauces… The students would not have been any less Agents if all they had done was salt water. There would indeed have been an event e*, a salting water, such that:

(iii) ∀ x (Xx ↔ Agent(e*, x))

But, it would not have been cooking dinner; it is left to ‘cook(e*)’ to exclude those events where the Agents have not done all of what is supposed to be done to put dinner on the table.

Landman (2000: 79ff.) looks to collective action to locate an objection to first-order, distributive thematic relations, to observations that only the top boy of a human pyramid need make contact with the ceiling, (v), and that a number of the school boys in the chorus may be faking without actually singing along, (vii):

(iv) John touches the ceiling.

(v) The boys touch the ceiling.

(vi) John sings ‘Rock of Ages’.

(vii) The school boys sing ‘Rock of Ages’.
The worry is that with boys nowhere near the ceiling and boys hanging out not singing, a distributive thematic relation that must be true of each boy becomes so dilute as to deprive Agenthood of meaningful agency, the restoration of which requires an essentially plural thematic relation. Remarking on (iv) and (v), there is no denying that Agenthood as understood through one’s grasp of lexical content and the context of evaluation entails something about physical contact, no matter if some of the Agent or Agents are removed from it. According to Landman, a robust meaning for Agent entails that:

(viii) The Agent brings part of self in physical contact with the Goal.

This constraint on meaning allegedly applies in the same fashion to (iv) and (v) if ‘the Agent’ and ‘self’ refers to John in (iv) and to the plurality of the boys in (v). Some part of the boys needs to be in contact with the ceiling, and some part of John does too. (viii) tells you neither which individual boys nor which parts of John touch the ceiling, but at least it captures the same meaning for singular (iv) and plural (v), in a way that presumably escapes accounts with only first-order distributive thematic relations.

It is true that not all of John touches the ceiling and not all the boys touch it, and that not all of John sings and not all the boys sing. It is also true that not just any part of John touching the ceiling counts towards his achievement. I, for example, don’t touch the sink losing hair over it. If the thought is that John touches the ceiling just in case some part of him does, then the relevant notion of part is theory-laden with questions of personal identity, intentional action and so on. Also, it cannot be that the boys touch the ceiling just in case one or more of them does, or that they sing just in case one does. That would make all boys everywhere, referring to them as the boys, ceiling-touchers and choristers, not just those in the human pyramid or those on stage. The relevant notion of part is not the one constituting plural reference if the thought is to be that the boys touch the ceiling just in case some part of them does. The relevant notion of part is here too theory-laden and interest-dependent. And, even on the same semantic field, the relevant notions of part for singular reference may diverge from that appropriate for plural reference:

(ix) The USPS lead bicycle (Lance Armstrong) crossed the finish line to claim the prize.
(x) The USPS bicycles crossed the finish line to claim the prize.
(xi) The USPS bicycle team crossed the finish line to claim the prize.

The truth of (x) and (xi) does not imply that most of the USPS bicycles or most of the USPS team crossed the finish line to claim the prize; but presumably, (ix) does imply that most of the lead bicycle did. All of these observations can be culled from the literature on mereology (Burge 1972, Simons 1987, Moltmann 1997, Koslicki 1997, 1999, 2005 and references cited therein). Much as they show the claim that (viii) is the same for (iv) and (v) to rely on equivocation, Landman’s interest in a robust notion of agency common to all attributions of Agenthood, singular or plural, remains. In fact, recognition that any notion of parthood would be hopelessly concept- and interest-dependent might recommend an essentially plural thematic relation, at least in that (xii) is a clearly unsuccessful reduction to the singular.

(xii) \[ \text{Agent}(e, X) \leftrightarrow [Q_X: \text{part-of}(x, X)] \text{Agent}(e, x) \]

So, without having understood the emphasis on (viii), let’s see if Landman’s worry and the facts of these examples can be addressed, and let us start with his assumption of thematic relations that are essentially plural. The thematic relation ‘Agent(e, X)’ exercises a certain oracular authority. Our analysis might stumble over the eccentric and variable conditions that determine collective Agency, conditions that qualify diverse individuals, among them some hangers on and fakers, as the Agents of an event; but, by hypothesis, the thematic relation is decisive in recognizing that the boys in the pyramid rather than they along with some other boys nearby are the Agents of that event. They and these other boys, without doing anything different, might at the same time be the Agents of a holding up the circus tent, all of which the thematic relations discerns:

(xiii) \[ \text{Agent}(e_1, B_{1-n}) \land \neg \text{Agent}(e_1, B_{1-n+1}) \]
\[ \text{Agent}(e_2, B_{1-n+1}) \land \text{Agent}(e_2, B_{1-n}) \]
With Agenthood entailing bona fide agency, it should result in a speaker’s understanding that something makes actual physical contact with the ceiling. I take it from Landman’s discussion that that thing is—more often than not—not the same as the collective Agents themselves. With a view towards other sentences, abbreviate talk about physical contact or vocalization as a relation between that thing, the bona fide agent, and the event:

\[ \text{Agent} \left[ e, X \right] \rightarrow \exists x \text{agent} \left( e, x \right) \]

\(\text{(xiv)}\) recasts some of what Landman intends with (viii). If the boys or John are the possibly collective Agents of an event \( e \)— however that plural relation is construed—something in that event touches the ceiling for real (Note other things could touch the ceiling too in this event, ‘agent \( e, x \)’ is not a function on events). Something further relates the bona fide agent to the Agents. It suffices that:

\[ \text{Agent} \left[ e, X \right] \rightarrow \exists x \left( Xx \& \text{agent} \left( e, x \right) \right) \]

If John is alone among the \( X \), he himself touches the ceiling or sings the whole hymn. That is, some Agents are fake agents only if some are real. As formulated, (xv) leaves in place an essentially plural thematic relation. But, argument is entirely lacking that meaning is better served here by an essential plural. Its authority to say of a particular event as in (xiii) that its Agents are them (agents and fakers meeting some other intuited standard, but not any others) is no better than a first-order authority where, in effect, (xvii) takes over for (xvi):

\[ \text{Agent} \left[ e, B_{1-n} \right] \& \neg \text{Agent} \left[ e, B_{1-n+1} \right] \]

\[ \text{(xvi)} \]

\[ \text{Agent} \left( e, b_{1} \right) \& \ldots \& \text{Agent} \left( e, b_{n} \right) \& \neg \text{Agent} \left( e, b_{n+1} \right) \]

\[ \text{(xvii)} \]

That is, to say the thematic relations tokened in sentences are the first-order, distributive relations assumed in the text:

\[ \text{Agent} \left[ E, X \right] \leftrightarrow_{\text{def}} \forall x \left( Xx \leftrightarrow \exists e \left( Ee \& \text{agent} \left( e, x \right) \right) \right) \]

\(\text{(316)}\)

Speakers who grasp these relations grasp much else \textit{a priori}, e.g., that an agent of physical action is itself a physical object, all of which is cashed out in further theory. Agenthood in (316) entails robust agency, serving both singular and plural subjects, in that speakers also understand that:

\[ \text{Agent} \left( e, x \right) \rightarrow \exists x \text{agent} \left( e, x \right) \]

\[ \text{(xviii)} \]

\[ \text{agent} \left( e, x \right) \rightarrow \text{Agent} \left( e, x \right) \]

\[ \text{(xix)} \]

As far as I can tell, (316), (xviii) and (xix), in which all primitive relations are first-order distributive, answer Landman’s worry.

There is however a latent concern here, which does not trouble Landman. The problem of polysemy challenges an assumption that thematic roles such as Agent are absolute in meaning and invariant in context and asks to what extent this be true of such apparently heterogeneous usage of the preposition \textit{to} and the thematic role Agent as reflected in the following:

\textit{Brutus tossed the ball to Caesar.}
\textit{Brutus explained the proposal to Caesar.}
\textit{Brutus buried Caesar.}
\textit{Brutus exhumed the discussion.}

For further discussion, see Schein 2002 and references therein.

\[ \text{See Schein (in preparation) for further discussion and a proposal.} \]

\[ \text{It is faith in this generalization that prompts everybody else to say that ‘} b_{1} \text{and...and } b_{n} \text{’ is an expression with the same reference as ‘the } n \text{ Bs’ and to invent an operator and yielding this result.} \]
If, as in Schein in preparation, chapter 3, nominal descriptive content becomes adverbial descriptive content, (365) and (367) do relevantly differ in their logical syntax and semantics:

(365) The sportscasters at the press conference, sportscasting at the press conference, peppered the Managing General Partner of the Texas Rangers with question he couldn’t answer.

(367) The sportscasters from ABC, CBS and NBC, sportscasting from ABC, CBS and NBC, peppered the Managing General Partner of the Texas Rangers with question he couldn’t answer.

The acceptability conditions may in turn reflect a semantic difference between saying that a single event of sportscasting at the press conference amounted to peppering the MGP with questions and saying that three events from ABC, CBS and NBC did so.

The formula suppresses the device of cross-reference; but, I have in mind descriptive anaphora: There was some hooting, there was some hollering, and (all) that drowned out the lecture. (Schein in preparation).

Overest(x,y)
Overest(x,y) ↔ Overest(y,x)
\( x \leq y \leftrightarrow \forall z (\text{Overest}(z,x) \rightarrow \text{Overest}(z,y)) \)
\( x=y \leftrightarrow (x \leq y \& y \leq x) \)
\( \text{Overest}(x,y) \leftrightarrow \exists z \forall u (\text{Overest}(u,z) \leftrightarrow (\text{Overest}(u,x) \& \text{Overest}(u,y))) \) (meet)
\( \exists z \forall u (\text{Overest}(u,z) \leftrightarrow (\text{Overest}(u,x) \lor \text{Overest}(u,y))) \) (join)

Thanks to Paul Pietroski and Philippe Schlenker for critical discussion, March 2002.

The monosortal language of the partitive construction could be used just as well at the cost of further eyestrain.

Schein in preparation, for further discussion.