How Much is at Stake for the Pragmatic Encroacher

Jeffrey Sanford Russell

September 4, 2013

Many people are saying nowadays that what you know partly depends on what practical decisions you face (e.g. Stanley 2005; Fantl and McGrath 2009). This “pragmatic encroachment” thesis usually involves two different ideas. One idea is that knowledge plays a distinctive role in practical reasoning: you can act on what you know. The other idea is that knowledge is harder to achieve when more is at stake. But it isn’t obvious that these two ideas fit together the way that is generally assumed. Charity Anderson and John Hawthorne (ms) have recently argued that when “high stakes” is made precise in some natural ways, situations where more is at stake are not necessarily the ones which make stricter demands on practical decisions—so the two ideas come apart. Of course, their discussion is not conclusive: there are other reasonable ways of trying to sharpen what stakes are, and Anderson and Hawthorne invite others to investigate “to what extent the choice of other sharpenings makes a structural difference”.

I doubt their invitation will be ignored. The discovery of unspoiled conceptual territory—close to home, theoretically important, and thus far unanalysed—is too enticing to epistemologists. The usual method for these explorations is to elicit intuitions about particular cases, and then try to fit an account to those intuitions. The danger of this method is that it may lead to an account of stakes that doesn’t do the theoretical work that originally made the notion interesting. So I’ll work in the opposite direction. First I’ll outline the job that “high stakes” is supposed to do according to pragmatic encroachers. Then with a bit of decision theory I’ll work out what “high stakes” would have to mean in order for it to do this job. What we end up with may or may not be an intuitively plausible account of our ordinary meaning of “high stakes” (if the ordinary use is definite enough at all) but even if it isn’t, I hope this account will focus the investigation in a productive direction. Since it follows directly from a certain version of pragmatic encroachment together with some standard decision theory, anyone who hopes to advance a rival account of stakes should not just present intuitive cases in support of it, but also explain what alternative theoretical work there is for it to do.
Here’s what I’m taking the pragmatic encroacher to say. First, the idea that you can act on what you know. Whatever you know is certain enough to form a basis for rational action—to be taken as given for practical purposes. In other words, if you know \( X \), then you are **practically certain** of \( X \), in the following sense: for any action \( A \), if it is rational for you to do \( A \) conditional on \( X \), then it is unconditionally rational for you to do \( A \). So (following Anderson and Hawthorne) I take pragmatic encroachment to involve this thesis.\(^1\)

**Practical Adequacy:** If you know \( X \) then you are practically certain of \( X \).

The second idea is that this practical condition on knowing \( X \) goes along with how much is at stake in \( X \) being true. There are two pieces to this. The first is that practical adequacy imposes a certain **threshold** on the “strength of epistemic position” or “epistemic probability” that you need in order to know \( X \) (e.g. Fantl and McGrath 2009, 29). I assume that what we’re talking about here is the kind of probability that figures in decision theory, often called your **credence** in \( X \). The second piece is that the practical probability-threshold is higher when more is at stake. Here is a way of stating the pragmatic encroachers’ second idea.

**Stakes Set Threshold:** For a practical situation \( s \) and a proposition \( X \), there is a threshold \( p(X, s) \) such that to be practically certain of \( X \) in \( s \) requires having credence \( C(X) \geq p(X, s) \). More is at stake in \( X \) in \( s \) than in \( Y \) in \( t \) iff the threshold \( p(X, s) \) is higher than the threshold \( p(Y, t) \).

This thesis is the main thing I’m taking the pragmatic encroacher to say about stakes.

I’ll also assume a bit of standard decision theory (in the style of Jeffrey 1983). For each proposition \( A \), there is a certain quantity \( V(A) \), your **expected value** of \( A \). In a given practical situation, there are certain **available actions**, the alternative things you are in a position to willingly do. For simplicity we identify each action with a corresponding **proposition** (such as “I bring an umbrella”). These are the assumptions I’ll use about expected values:

**Decision Theory:** It is rational to do \( A \) iff for each available \( B \), \( V(A) \geq V(B) \).

It is rational to do \( A \) conditional on \( X \) iff for each available \( B \), \( V(A \land X) \geq V(B \land X) \).

Expected value is a weighted sum of cases:

\[
V(A) = V(A \land X) \cdot C(X \mid A) + V(A \land \neg X) \cdot C(\neg X \mid A)
\]

(That is, the value of doing \( A \) is the weighted average of the value of doing \( A \) when \( X \) turns out to be true and the value of doing \( A \) when \( X \) turns out to be false, with each case weighted by how probable it is if you do \( A \).

---

Finally, let’s make some simplifying assumptions about the kind of decision situation that you face. (I’ll relax some of these later.) Suppose you face a forced choice between two actions, A and B. Suppose also that the proposition X is simple with respect to your decision: whether X is true is independent of which action you choose. (We aren’t concerned here with the more complicated issues involved in knowing things that you are in a position to bring about or prevent. If your decision is about whether to bring an umbrella, then “It will rain today” is simple, but “My umbrella will get lost today” is not.)

This means:

\[ C(X \mid A) = C(X \mid B) = C(X) \]

\[ C(\neg X \mid A) = C(\neg X \mid B) = C(\neg X) \]

Finally, suppose that conditional on X, A is strictly better than B:

\[ V(A \land X) > V(B \land X) \]

These assumptions straightforwardly imply a particular account of how much is at stake. In this situation acting on X amounts to doing A rather than B. So you are practically certain of X if and only if it is unconditionally rational for you to do A—that is, iff

\[ V(A) \geq V(B) \]

Since expected value is a weighted sum of cases, we can rewrite this condition:

\[ V(A \land X) \cdot C(X \mid A) + V(A \land \neg X) \cdot C(\neg X \mid A) \geq V(B \land X) \cdot C(X \mid B) + V(B \land \neg X) \cdot C(\neg X \mid B) \]

Because X is simple, this can be simplified and rearranged:

\[ \left( V(A \land X) - V(B \land X) \right) \cdot C(X) \geq \left( V(B \land \neg X) - V(A \land \neg X) \right) \cdot C(\neg X) \]

So, since the left side is positive, it follows that being practically certain of X is equivalent to the following condition:

\[ \frac{C(X)}{C(\neg X)} \geq \frac{V(B \land \neg X) - V(A \land \neg X)}{V(A \land X) - V(B \land X)} \]

The number on the left side of this inequality is your odds of X. So in the simple case of just two available actions, you are practically certain of X iff your odds of X are higher than the number on the right side. Let’s call that number the stakes ratio, \( S(X, s) \) (where s labels the simple decision scenario just described).

Your probability of X is high enough for practical certainty just in case your odds of X are at least \( S(X, s) \). Since your probability of X gets higher as your odds get higher, this means that the probability-threshold for practical certainty is higher when \( S(X, s) \) is higher. And according to Stakes Set Threshold there is more at stake just when the probability-threshold is higher. So we can conclude:

\[ ^2 \text{I'm also assuming that the kinds of trickiness that distinguish evidential and causal decision theories don't arise in the kinds of cases under consideration. If they did, we might need to work with "interventionist" credences and expected values instead (Cartwright 1979, inter alia).} \]

\[ ^3 \text{Since } C(\neg X) = 1 - C(X), \text{ in fact the probability threshold } p(X, s) = S(X, s)/(1 + S(X, s)). \]
Stakes: More is at stake in \( X \) in \( s \) than in \( Y \) in \( t \) iff \( S(X, s) > S(Y, t) \)

Pragmatic encroachment and decision theory together imply this account of how much is at stake.

Let's look at what the stakes ratio \( S(X) \) is like. (I'll drop the mention of the situation when it's clear in context.) It is a ratio between two quantities: the **cost of being wrong about** \( X \), \( V(B \land \neg X) - V(A \land \neg X) \), and the **benefit of being right about** \( X \), \( V(A \land X) - V(B \land X) \). Recall that \( A \) is the best action conditional on \( X \)—so if you rationally act on \( X \) then \( A \) is what you do. So \( V(A \land \neg X) \) is the expected value of acting on \( X \) when \( X \) turns out to be false. The cost of being wrong about \( X \) is how much worse acting on the mistaken assumption of \( X \) is than the alternative action would be. As this cost gets bigger, more is at stake in \( X \) being true. The benefit of being right about \( X \) is how much better it is to act on \( X \) than its alternative, supposing \( X \) turns out to be true. As this benefit increases, the stakes ratio goes down: since there is more in favour of acting on \( X \), it takes less confidence to make it rational to do so.

Here's an example: Keith DeRose's classic "bank case" (1992). First version. It's Friday afternoon, you need to deposit your paycheck, and there are long lines at the bank. The bank might be open Saturday, and if it is, the lines will be much shorter. If you wait till Saturday and it turns out to be closed, that would be kind of a nuisance.

<table>
<thead>
<tr>
<th>Bank open Saturday</th>
<th>Bank closed Saturday</th>
</tr>
</thead>
<tbody>
<tr>
<td>Go Saturday</td>
<td>Short wait</td>
</tr>
<tr>
<td></td>
<td>Nuisance</td>
</tr>
<tr>
<td>Go Friday</td>
<td>Long wait</td>
</tr>
<tr>
<td></td>
<td>Long wait</td>
</tr>
</tbody>
</table>

In this case the stakes ratio is:

\[
S(\text{Bank open Saturday}) = \frac{\text{Long wait} - \text{Nuisance}}{\text{Short wait} - \text{Long wait}}
\]

Second version: same as before, except if you don't make the deposit on Friday or Saturday then an important check will bounce, which would be really terrible.

<table>
<thead>
<tr>
<th>Bank open Saturday</th>
<th>Bank closed Saturday</th>
</tr>
</thead>
<tbody>
<tr>
<td>Go Saturday</td>
<td>Short wait</td>
</tr>
<tr>
<td></td>
<td>Terrible things</td>
</tr>
<tr>
<td>Go Friday</td>
<td>Long wait</td>
</tr>
<tr>
<td></td>
<td>Long wait</td>
</tr>
</tbody>
</table>

\[
S(\text{Bank open Saturday}) = \frac{\text{Long wait} - \text{Terrible things}}{\text{Short wait} - \text{Long wait}}
\]

In this version, there is a much bigger cost of being wrong about whether the
bank is open Saturday: there is a much bigger gap in value between a long wait and the terrible things that may ensue than there is between the long wait and the minor nuisance. So there is more at stake in the bank being open in the second situation. (This is what you might hope, since “Low Stakes” and “High Stakes” are standard labels for these two cases).

Note that if which action is best doesn’t turn on $X$ at all—the case where $V(B \land \neg X)$ isn’t any better than $V(A \land \neg X)$—then the stakes ratio isn’t even positive. In that case practical adequacy doesn’t impose any constraint on your probabilities at all.

(The more general case where more than two actions are available only requires a slight modification. In that case, practical certainty requires that $V(A)$ be at least as good as each alternative. So by the same reasoning, in the more general case

$$S(X) = \max_{B \neq A} \frac{V(B \land \neg X) - V(A \land \neg X)}{V(A \land X) - V(B \land X)}$$

This calculation is still based on the assumption that $X$ is simple.)

The stakes ratio has a peculiar feature: it isn’t sensitive to the absolute magnitudes of the costs and benefits involved, just their proportions. For example suppose I have to choose between two boxes, exactly one of which contains £1. And suppose you also have to choose between two boxes, exactly one of which contains £1000.

<table>
<thead>
<tr>
<th>Money in Box A</th>
<th>Money in Box B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Box A</td>
<td>£1</td>
</tr>
<tr>
<td>Box B</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Money in Box A</th>
<th>Money in Box B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Box A</td>
<td>£1000</td>
</tr>
<tr>
<td>Box B</td>
<td>0</td>
</tr>
</tbody>
</table>

In each of our situations, the stakes ratio for the money being in Box A is 1 : 1, since the cost of wrongly acting on the assumption that the money is in Box A is just the same as the benefit of rightly acting on that assumption. So we get the conclusion that there is exactly as much at stake in the money being in Box A in the £1 situation as there is in the £1000 situation. In each case, the probability that makes it rational to choose one box over the other is just $\frac{1}{2}$. So what it takes to make it rational to act on the money being in Box A is equally undemanding for each of us (assuming that choosing a box is the only relevant action). But as a judgment about how much is at

---

\(^4\)Thanks to Charity Anderson for helpful discussion of this case.
stake this is at least a bit counter-intuitive. It seems more natural to say that there is more at stake in the money being in Box A when there is more money involved.

But remember, we didn’t get this far by trying to respect intuitions about cases. Rather, we derived consequences from certain pragmatic encroachment theses together with decision theory. So we face a choice. Give up standard decision theory. Give up the idea that how much is at stake is what sets the threshold for a practical condition on knowledge, either by giving up pragmatic encroachment altogether or by finding some other way of characterising the view. Or accept an account of how much is at stake which is at odds with some of our intuitive responses to cases like these.⁵

References

Anderson, Charity, and John Hawthorne. ms. “Knowledge, Practical Adequacy, and Stakes.”


⁵Thanks to Charity Anderson, Julien Dutant, Daniel Eaton, John Hawthorne, Tim Pickavance, and the other participants at the workshop on Religious Epistemology, Contextualism, and Pragmatic Encroachment at Oxford in March 2013. This project was made possible through the support of a grant from the John Templeton Foundation. The opinions expressed in this publication are those of the author and do not necessarily reflect the views of the John Templeton Foundation.