Frege's *FA*, Week 6

Frege's definitions of number

1. $\#F = 0$ iff $\forall x \neg Fx$
   $\#F = Sn$ iff $\exists x(Fx \wedge \#(\lambda y.x \neq y \wedge Fy) = n)$

2. $\#F = \#G$ iff $F \sim G$

3. $\#F = \text{ext}(\lambda X.X \sim F)$

Basic Law V: $\text{ext}\, F = \text{ext}\, G$ iff $\forall x(Fx \leftrightarrow Gx)$

This is inconsistent! Let $R = \lambda x.\exists F(x = \text{ext}\, F \wedge \neg Fx)$.

Frege's response

Set theory

Extensionality: For all sets A and B, if $\forall x(x \in A \leftrightarrow x \in B)$ then $A = B$

Separation: For any set A, there is some set B such that $\forall x(x \in B \leftrightarrow x \in A$ and $\varphi(x))$

Replacement: For any set A, if for each $x \in A$ there is exactly one y such that $\varphi(x, y)$, then there is a set B such that $\forall y(y \in B \leftrightarrow \exists x(x \in A$ and $\varphi(x, y))$

Type theory

extensional/intensional

Neo-Fregeanism

second-order logic + HP