Frege’s FA, Week 6

Frege’s definitions of number

1. \( \#F = 0 \) iff \( \forall x \neg Fx \)
   \( \#F = Sn \) iff \( \exists x (Fx \land \#(\lambda y. x \neq y \land Fy) = n) \)
2. \( \#F = \#G \) iff \( F \sim G \)
3. \( \#F = \text{ext}(\lambda x. X \sim F) \)

**Basic Law V**: \( \text{ext} F = \text{ext} G \) iff \( \forall x (Fx \leftrightarrow Gx) \)

This is inconsistent! Let \( R = \lambda x. \exists F (x = \text{ext} F \land \neg Fx) \).

Frege’s response

Set theory

**Extensionality**: For all sets \( A \) and \( B \), if \( \forall x (x \in A \leftrightarrow x \in B) \) then \( A = B \)

**Separation**: For any set \( A \), there is some set \( B \) such that \( \forall x (x \in B \leftrightarrow x \in A \land \varphi(x)) \)

**Replacement**: For any set \( A \), if for each \( x \in A \) there is exactly one \( y \) such that \( \varphi(x, y) \),
then there is a set \( B \) such that \( \forall y (y \in B \leftrightarrow \exists x (x \in A \text{ and } \varphi(x, y)) \)

Type theory

extensional/intensional

Neo-Fregeanism

second-order logic + HP