3. (20 points) Find the mass of the solid bounded above by the hemisphere \( x^2 + y^2 + z^2 = 8, \ z > 0 \) and below by the cone \( z^2 = x^2 + y^2, \ z > 0 \) if the mass density function is \( \sigma(x, y, z) = z. \)

\[
x^2 + y^2 + z^2 = 8
\]

radius \( = \sqrt{8} = 2\sqrt{2} \)

Intersection of cone and hemisphere:

\[
z^2 = x^2 + y^2
\]

\[
(x^2 + y^2) + z^2 = 8
\]

\[
z^2 + 2z = 8
\]

\[
z = 2
\]

\[
\iiint_S \sigma \, dv
\]

\[
= \iiint_S z \, dx \, dy \, dz + \iiint_S z \, dx \, dy \, dz
\]

\[
= \int_0^2 \int_{x^2+y^2=2^2} \left( \int_{x^2+y^2=2^2} z \, dx \, dy \right) \, dz + \int_2^{2\sqrt{2}} \int_{x^2+y^2=8-z^2} \left( \int_{x^2+y^2=8-z^2} z \, dx \, dy \right) \, dz
\]

\[
= \int_0^2 \int_{x^2+y^2=2^2} z \left( \pi z^2 \right) \, dz + \int_2^{2\sqrt{2}} \int_{x^2+y^2=8-z^2} z \left( \pi (8-z^2) \right) \, dz
\]

\[
= \pi \int_0^{2\sqrt{2}} z^3 \, dz + \pi \int_2^{2\sqrt{2}} 8z - z^3 \, dz
\]

\[
= \pi \left[ \frac{1}{4} z^4 \right]_0 + \pi \left[ \frac{8}{2} z^2 - \frac{1}{4} z^4 \right]_2
\]

\[
= \pi \left[ 4 - 0 \right] + \pi \left[ (32 - 16) - (16 - 4) \right]
\]

\[
= 4\pi + 4\pi = 8\pi
\]