Homework 4

#7.8

*Omitted variable bias:* If you leave out the years of experience \( (X_3) \) from the model, the coefficient of education \( (X_2) \) will be biased in general, the nature of the bias depending on the correlation between \( X_2 \) and \( X_3 \). The standard error, the residual sum of squares, and \( R^2 \) will all be affected as a result of this omission.

#7.19

a. Model (5) seems to be the best as it includes all the economically relevant variables, including the composite real price of chicken substitutes, which should help alleviate the multicollinearity problem that may exist in model (4) between the price of beef and price of pork. Model (1) contains no substitute good information, and models (2) and (3) have limited substitute good information.

b. The coefficient of \( \ln X_{2t} \) represents income elasticity; the coefficient of \( \ln X_{3t} \) represents own-price elasticity.

c. Model (2) considers only pork as a substitute good, while model (4) considers both pork and beef.

d. There may be a problem of multicollinearity among the price of beef, pork, and chicken.

e. yes, this might alleviate the problem of multicollinearity.

f. They should be substitute goods because they compete with chicken as a food consumption product.

g.

Dependent Variable: \( Y \)
Method: Least Squares
Date: 03/21/05 Time: 15:44
Sample: 1960 1982
Included observations: 23

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>2.029865</td>
<td>0.118682</td>
<td>17.10338</td>
<td>0.0000</td>
</tr>
<tr>
<td>X2</td>
<td>0.481286</td>
<td>0.068188</td>
<td>7.058251</td>
<td>0.0000</td>
</tr>
<tr>
<td>X3</td>
<td>-0.350628</td>
<td>0.079394</td>
<td>-4.416310</td>
<td>0.0003</td>
</tr>
<tr>
<td>X6</td>
<td>-0.061035</td>
<td>0.129960</td>
<td>-0.469645</td>
<td>0.6440</td>
</tr>
</tbody>
</table>
modified $R^2 = (1 - \frac{K}{n})R^2 = (1 - \frac{4}{23}) \times 0.98 = 0.81$.

Coefficient estimates verify the positive income elasticity and negative own-price elasticity. The coefficient of $X_6$ is not significant.

h.

Dependent Variable: Y
Method: Least Squares
Date: 03/21/05 Time: 16:01
Sample: 1960 1982
Included observations: 23

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
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<td>0.137882</td>
<td>15.41533</td>
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<td>X2</td>
<td>0.405924</td>
<td>0.044791</td>
<td>9.062535</td>
<td>0.000</td>
</tr>
<tr>
<td>X3</td>
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<td>0.083332</td>
<td>-5.265956</td>
<td>0.000</td>
</tr>
<tr>
<td>X4</td>
<td>0.106656</td>
<td>0.087838</td>
<td>1.214228</td>
<td>0.239</td>
</tr>
</tbody>
</table>

Estimation results show that Model (2) underestimate the income elasticity (0.41 compared to 0.48; downward bias) and overestimate the own-price elasticity (in absolute term; 0.43 compared to 0.35).
The rate of change of $Y$ with respect to $X_2$ and $X_3$ is:

$$\frac{\partial Y}{\partial X_2} = \beta_2 + \beta_4 X_3, \quad \frac{\partial Y}{\partial X_3} = \beta_3 + \beta_4 X_2.$$ 

As you can see, the marginal effect of income (on consumption expenditure) depends on the level of wealth ($X_3$), and the marginal effect of wealth depends on the level of income ($X_2$). That is, the interaction term reflects the fact that the variable income and wealth interact in this way. Since MPC is independent of the wealth when $\beta_4 = 0$, we can test this null hypothesis($H_0 : \beta_4 = 0$) using individual $t$-test.

#8.19 $n = 23, K = 3$. $t^*(20) = 2.086$

For income elasticity,

$H_0 : \beta_2 = 1 \quad H_1 : \beta_2 \neq 1 \quad t = \frac{0.4515-1}{0.0247} = -22.2065$.

As $|t| > 2.086$, we reject the null. The income elasticity is significantly different from 1.

For price elasticity,

$H_0 : \beta_3 = -1 \quad H_1 : \beta_3 \neq -1 \quad t = \frac{-0.3772-(-1)}{0.0635} = 9.808$.

As $|t| > 2.086$, we reject the null. The price elasticity is significantly different from $-1$.

#8.32 Estimation results are as follows (Model I and Model II in order)

<table>
<thead>
<tr>
<th>Variable</th>
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<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>22.16269</td>
<td>7.089479</td>
<td>3.126139</td>
<td>0.0056</td>
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<tr>
<td>ADEXP</td>
<td>0.363174</td>
<td>0.097120</td>
<td>3.739425</td>
<td>0.0014</td>
</tr>
</tbody>
</table>

R-squared | 0.423951 | Mean dependent var | 40.46667 |
Adjusted R-squared | 0.393633 | S.D. dependent var | 30.18061 |
S.E. of regression  | 23.50152 | Akaike info criterion | 9.242400 |
Sum squared resid    | 10494.11  | Schwarz criterion    | 9.341878  |
Log likelihood       | -95.04520 | F-statistic          | 13.98330  |
Durbin-Watson stat   | 2.371839  | Prob(F-statistic)    | 0.001389  |

Dependent Variable: IMPRESSION
Method: Least Squares
a. Model I includes only the advertisement expenditure as the independent variable, while Model II includes the squared value of advertisement, too. In the result, the marginal effect of the advertisement expenditure changes in Model II. First, the marginal effect of advertisement expenditure in Model I will be:

$$\frac{dY}{dX} = 0.3631;$$

whereas in Model II:

$$\frac{dY}{dX} = 1.0847 - 0.008X.$$  

This suggests that retained impression increases at a decreasing rate as advertising expenditure increases.

b. It depends on our premise, but Model II would be better if \(X^2\) is a relevant variable.

c. \(H_0 : \beta_3 = 0 \quad H_1 : \beta_3 \neq 0.\)

We can conduct either \(F\)–test using two models, or \(t\)–test using Model II only.

\[
F = \frac{(0.5299 - 0.4241)/1}{(1 - 0.529)/18} = 4.013 < 4.41 = F^*(1, 18). \text{ Therefore, we cannot reject the null. We choose Model I.}
\]

\[
t = \frac{0.00399}{0.00168} = 2.011 < 2.101 = t^*(18). \text{ Therefore, we cannot reject the null. We choose Model I.}
\]

Note that the conclusions from \(F\)–test and \(t\)–test for the same null hypothesis coincide.

d. In Model II, diminishing returns to advertising occur when \(\frac{dY}{dX} = 1.0847 - 0.008X < 0\), i.e. \(X > \frac{1.0847}{0.008} = 135.38\). That is, after \(X = 135.38\), it does not pay to advertise.