Market Mechanisms for Buying Random Wind

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Market Mechanisms for Buying Random Wind

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Abstract—The intermittent nature of wind power leads to the question how wind power generators can participate in a deregulated electricity market. In the proposed paradigm, wind power generators bid probability distributions of generation, instead of bidding supply functions as conventional power generators do. We focus on designing incentive compatible auction mechanisms to accommodate various objectives of the aggregator. Specifically, we propose a variant of the VCG mechanism. We also propose another class of mechanisms which have a desired commitment-with-penalty payment structure. We then consider a networked setting in the context of the economic dispatch problem.

Index Terms—Game theory, mechanism design, renewable energy integration, smart grid, stochastic resource auction.

I. INTRODUCTION

WITH the increasing penetration of wind power, challenges arise in integrating such random generation into the current electricity grid and market. While wind power generators are commonly treated as negative loads and receive feed-in tariffs, this scheme would be no longer applicable when the penetration level is high, with substantial reserve margin needed.

Alternatively, wind power generators can be required to participate in the competitive electricity pool through a two-settlement market system. In the ex ante day-ahead (DA) forward market, the wind power generator commits to a fixed amount of generation; in the ex post real-time (RT) spot market, it pays a penalty for the shortfall. However, it is still unclear what are the appropriate bidding and pricing mechanisms. We observe that it may not be efficient for wind power generators to bid supply functions as conventional power generators do. Rather, the value of aggregation can be possibly exploited to benefit the social welfare.

This motivates our proposed paradigm referred to as the stochastic resource auction. The underlying assumption is that the current two-settlement market system does not change, but aggregators are allowed to enter the market. The aggregators procure wind power through auctions according to certain objectives, assuming any risk due to the uncertainty of wind power generation. The key feature of a stochastic resource auction is that while the realized generation of each wind power generator is random, the probability distribution can be learned beforehand as its private information.

Regarding the literature, the problem of optimizing the contracted amount can be studied as a decision-making problem [1]. In [2], they introduce risky power contracts in addition to firm power contracts to enable flexible and efficient wind power aggregation. They focus on how optimal offerings and equilibria depend on exogenous price signals, and on deriving concept and expressions for critical prices from the perspective of wind power generators. To address the risk of not meeting operating constraints such as power balance due to the uncertainty of wind power generation, a new paradigm for power system operation called risk-limiting dispatch (RLD) has been proposed [3]. The RLD employs real-time information about supply and demand, taking into account the stochastic nature of wind power generation, and determines a risk-constrained stochastic optimal dispatch.

However, the major issue of the aforementioned work is that such implementations require accurate stochastic models for wind power generation, which are possibly private information of wind power generators. Without proper market mechanisms, they may not have incentives to reveal their private information truthfully. Our work is aim to design incentive compatible auction mechanisms to accommodate various objectives of the aggregator. Once the truthful distributions of generation are elicited, the aggregator is then able to make the optimal plan against risk.

Auction and market design for electricity markets is a well-studied problem [4], [5], [6], [7], [8]. However, almost all of economic and auction theory deals with classical goods. The auction design problem we introduce is for stochastic goods, which has received only scant attention, if at all.

We note that following the same motivation, one can approach the same problem from a contract design point of view that typically considers a principal and a single agent [9], [10]. In [11], they consider an aggregator procuring both conventional and renewable power from a single producer that has multi-dimensional private information, and show that the optimal mechanism is a menu of contracts. As another context, [12] proposes a dynamic contract design problem with an application to indirect load control. The key characteristic of their setting is that the principal has no capability to monitor the agent’s control or the state of the engineered system, whereas the agent has perfect observations.

The paper is organized as follows. In Section II, we make an argument why it is more efficient for wind power generators to bid probability distribution of generation rather than supply functions, and introduce the stochastic resource auction paradigm. In Section III, we propose two basic mechanisms. One is a variant of the VCG mechanism. The other one has a desired commitment-with-penalty payment structure. We generalize the results in Section IV. We then consider a networked setting in the context of the economic dispatch problem in Section V. In Section VI, we present a case study to complement the analysis in the previous sections. Section VII concludes the paper.
II. PROBLEM STATEMENT

A. Model

Consider $N$ wind power generators as strategic players (or agents), indexed by $i = 1, \ldots, N$. Agent $i$’s wind power generation is a random variable $W_i \in [0, \bar{w}]$, where $\bar{w}$ denotes the maximum capacity among the agents. The probability distribution of $W_i$ can be parameterized by $\theta_i \in \Theta_i$, where $\theta_i$ and $\Theta_i$ are referred to as agent $i$’s type and type space, respectively.

Denote by $w_i$ the realization of $W_i$. While $w_i$ cannot be known a priori, agent $i$ learns $\theta_i$ (and hence the probability distribution of $W_i$) beforehand as its private information. Let $\Theta = \times_i \Theta_i$ and $\Theta_{-i} = \times_{j \neq i} \Theta_j$. Let $\theta = (\theta_1, \ldots, \theta_N)$ and $\theta_{-i} = (\theta_1, \ldots, \theta_{i-1}, \theta_{i+1}, \ldots, \theta_N)$. While $\theta_i$ is agent $i$’s private information, $\theta$ is drawn from a commonly known prior distribution $\phi(\cdot)$. Since the wind farms can be geographically close, $\theta_1, \ldots, \theta_N$ are possibly correlated.

B. Bidding Supply Functions

Despite the uncertainty of generation, the wind power generators can still bid supply functions as conventional power generators do. This is also a common approach in practice.

Assume that the price in the RT market can be precisely predicted, denoted by a constant $\pi_{RT} > 0$. When agent $i$ commits to supply $x_i$ amount of power in the RT market, the shortfall is $(x_i - w_i)_+$, where we define $x_+ = \max\{x, 0\}$.

The cost function of agent $i$ is the expected payment made by agent $i$ to make up for the shortfall:

$$c_i(x_i) = \mathbb{E}[\pi_{RT}(x_i - W_i)_+]$$

Such a cost function has some appealing properties. Apparently, $c_i(\cdot)$ is increasing. It is also convex, since it is the expectation of a convex function. Therefore, the cost function of a wind power generator has the same structure as that of a conventional power generator. This suggests that wind power generators can bid supply functions in the DA market as conventional power generators do. In such a market architecture, the agents bear the risk, since it is their own responsibility to buy power in the RT market to make up for the shortfall, if there is any.

For comparison purposes, we consider a simple problem. Assume that the demand can be represented by a valuation function $v(\cdot)$ that is increasing and concave. For example, a linear inverse demand function corresponds to a quadratic valuation function. The problem is to decide the optimal demand $y$ and supply $x_i$ of each agent to maximize the social welfare. Formally, it is a convex optimization problem:

$$\begin{align*}
\text{maximize} & \quad v(y) - \sum_i c_i(x_i) \quad (1a) \\
\text{subject to} & \quad y = \sum_i x_i, \quad (1b) \\
& \quad x_1, \ldots, x_N \geq 0. \quad (1c)
\end{align*}$$

C. Bidding Probability Distributions

When the agents bid supply functions, it is their own responsibility to fulfill the committed amount. One implication is that the excess wind power $(w_i - x_i)_+$ for each agent $i$ is disposed. Alternatively, they can bid probability distributions, and the problem for the aggregator is to decide the optimal demand $y$. Formally, it is a convex optimization problem:

$$\begin{align*}
\text{maximize} & \quad v(y) - \mathbb{E}[\pi_{RT}(y - \sum_i W_i)_+] \quad (2a) \\
\text{subject to} & \quad y \geq 0. \quad (2b)
\end{align*}$$

In such a market architecture, the aggregator bears the risk of buying power in the RT market to make up for the shortfall, if there is any. The following result shows that this market architecture is more efficient in terms of social welfare.

**Proposition 1:** The optimal value of problem (2) is greater than or equal to that of problem (1).

**Proof:** For any optimal solution $(x_1^*, \ldots, x_N^*, y^*)$ to problem (1), we have

$$\begin{align*}
v(y^*) - \mathbb{E}[\pi_{RT}(y^* - \sum_i W_i)_+] & = v(y^*) - \mathbb{E}[\pi_{RT}(\sum_i x_i^* - \sum_i W_i)_+] \\
& \geq v(y^*) - \mathbb{E}[\pi_{RT}(\sum_i (x_i^* - W_i)_+)] \\
& = v(y^*) - \sum_i c_i(x_i^*),
\end{align*}$$

where the inequality follows from the fact that $(x + y)_+ \leq x_+ + y_+$ for any $x$ and $y$. Therefore,

$$\max_{y \geq 0} \{v(y) - \mathbb{E}[\pi_{RT}(y - \sum_i W_i)_+]\} \geq v(y^*) - \sum_i c_i(x_i^*),$$

where the left-hand side is the optimal value of problem (2), and the right-hand side is that of problem (1).

D. Stochastic Resource Auction

The message conveyed by the preceding result is that bidding probability distributions can potentially utilize the value of aggregation so as to enhance the social welfare. This motivates the proposed paradigm referred to as the stochastic resource auction, as shown in Fig. 1.
The timeline of the stochastic resource auction is the following. In the DA market, nature draws $\theta$ according to the joint distribution $\phi(\cdot)$. Each agent $i$ learns its own type $\theta_i$ and updates its belief in the distribution of the others’ types, to the conditional distribution $\phi_i(\cdot|\theta_i)$. Then each agent $i$ submits a bid $\hat{\theta}_i$ as its reported type, which could be different from the true type $\theta_i$, to the aggregator. Based on the bid profile $\hat{\theta}$, $M$ out of the $N$ agents are selected as the wind power providers, the set of which is denoted by $I$. The unselected agents get zero payoffs and leave the auction. Each selected agent $i$ makes a payment $t^\text{DA}_i(\hat{\theta})$ to the aggregator. In the RT market, upon the realization of $X_i$, each selected agent $i$ gets paid an amount of $t^\text{RT}_i(\hat{\theta}, w_i)$ from the aggregator. Therefore, the payoff function of agent $i \in I$ is given by
\[ u_i(\hat{\theta}, w_i) = t^\text{RT}_i(\hat{\theta}, w_i) - t^\text{DA}_i(\hat{\theta}). \]

Note that we allow $t^\text{DA}_i$ or $t^\text{RT}_i$ to be negative, indicating a payment in the reverse direction. Also, we will denote by $\hat{W}_i$ the random variable parameterized by $\hat{\theta}_i$.

In a stochastic resource auction, the selection and payment schemes define a direct revelation mechanism $\Gamma = \{I, t^\text{DA}_i, t^\text{RT}_i\}$. We can focus on direct and truthful mechanisms without loss of generality by the revelation principle. While the mechanism is theoretically static, the paradigm is aligned with the existing two-settlement market system.

Since the agents are rational and selfish, they may not have incentives to reveal their probability distributions truthfully. The focus of this paper is to design incentive compatible auction mechanisms to accommodate various objectives of the aggregator. Once the truthful distributions of generation are elicited, the aggregator is then able to make the optimal plan against risk. Since we will focus on dominant strategy implementation, correlation among the agents’ types is not critical. Indeed, we did not assume that they are independent. The reader can consult [13] for the standard game-theoretic definitions used throughout the paper.

We will first study the scenarios where there are only one or multiple agents selected as the wind power providers in an auction. We then consider the networked setting in the context of the economic dispatch problem, where transmission constraints are taken into account. In such a scenario, all of the agents are present as wind power providers.

### III. Basic Mechanisms

An important feature in our context is that the agents have no intrinsic valuation for the wind power generation they produce. This suggests that we may derive a richer class of incentive compatible mechanisms beyond the standard mechanism design framework. We are interested in mechanisms with proper practical interpretations.

We first consider the basic scenario where only one agent is selected as the provider, i.e., $I = \{i\}$, for technical or regulatory reasons. The objective is to identify the agent who yields the highest expected generation. That is, the aggregator’s problem is
\[ \text{maximize } \mathbb{E}[W_i]. \]

Under certain requirements, we derive two mechanisms. The first one is a variant of the well-known Vickrey-Clarke-Grove (VCG) mechanism [13]. The second one has a desired commitment-with-penalty payment structure [1].

#### A. The Stochastic VCG Mechanism

The first mechanism is derived subject to the following requirement:
\[ t^\text{RT}_i(\hat{\theta}, w_i) = \pi w_i, \quad (4) \]
where $\pi > 0$ is a constant, denoting the price the aggregator pays the selected agent for its realized generation. It does not need to be the price in the DA market, nor that in the RT market.

In the proposed mechanism, the selection scheme is to select the agent who claims to yield the highest expected generation:
\[ i \in \arg \max_j \mathbb{E}[\hat{W}_j]. \quad (5) \]

This is natural since agent $i$ is indeed the desired provider once the incentive compatibility is achieved. Also, note that any tie-breaking rule applies.

It remains to design $t^\text{DA}_i(\hat{\theta})$ to satisfy the incentive compatibility. We have
\[ \mathbb{E}[u_i(\hat{\theta}, W_i)] = \pi \mathbb{E}[W_i] - t^\text{DA}_i(\hat{\theta}). \]

Given $\hat{\theta}_{-i}$, agent $i$ should be indifferent to all the $\hat{\theta}_i$’s that make it be selected. Thus, $t^\text{DA}_i(\hat{\theta})$ is independent of $\hat{\theta}_i$. On the other hand, if a truthful report $\theta_i$ does not make it be selected given $\hat{\theta}_{-i}$, then a misreport $\hat{\theta}_i$ that makes it be selected should not give it a positive payoff. That is,
\[ t^\text{DA}_i(\hat{\theta}) \geq \pi \mathbb{E}[W_i]. \]

Therefore, one possible choice is
\[ t^\text{DA}_i(\hat{\theta}) = \pi \max_{j \neq i} \mathbb{E}[\hat{W}_j], \quad (6) \]
and the payoff function of agent $i$ is given by
\[ u_i(\hat{\theta}, w_i) = \pi w_i - \pi \max_{j \neq i} \mathbb{E}[\hat{W}_j]. \]

We call the proposed mechanism the stochastic VCG (SVCG) mechanism, since it is an analogue of the standard VCG mechanism. The expected RT payment $\pi \mathbb{E}[W_i]$ can be viewed as the counterpart of the valuation in the standard VCG mechanism, and the DA payment $\pi \max_{j \neq i} \mathbb{E}[\hat{W}_j]$ as the counterpart of the usual payment (or externality). Unlike the standard setting, the valuation here is not intrinsic; it is also in the form of a payment. This opens the possibility of other kinds of incentive compatible mechanisms.

**Theorem 1:** The SVCG mechanism specified by (4)–(6) is dominant strategy incentive compatible.

We will give a formal proof for the general scenario in Section IV. The SVCG mechanism can be interpreted in the following way. In the DA market, the selected agent makes a contractual payment $\pi \max_{j \neq i} \mathbb{E}[\hat{W}_j]$ to the aggregator, which depends on the reported expected generation of the second highest bidder. In the RT market, the aggregator makes a
payment \( \pi \mathbb{E}[W_i] \) to the selected agent for the actual generation at a price \( \pi \). \textit{Dominant strategy incentive compatibility} means that each agent will truthfully report its probability distribution, independent of what the other agents do. Consequently, truth-telling for all agents is a so-called \textit{dominant strategy equilibrium}.

\section{The Stochastic Shortfall Penalty Mechanism}

Recall that \( W_i \in [0, \bar{w}] \) for all \( i \). Ideally, if \( W_i = \bar{w} \) with probability 1, then agent \( i \) is the desired provider, given the objective (3). This suggests that \( \bar{w} \) can serve as a reference point in a commitment-with-penalty payment structure. Specifically, we require

\begin{equation}
\hat{t}_i^{\text{DA}}(\hat{\theta}) = -\pi \bar{w},
\end{equation}

and

\begin{equation}
\hat{t}_i^{\text{RT}}(\hat{\theta}, w_i) = -\lambda(\hat{\theta})(\bar{w} - w_i),
\end{equation}

where \( \lambda(\cdot) \) is a function of \( \hat{\theta} \). Then the payoff function of agent \( i \) is given by

\[ u_i(\hat{\theta}, w_i) = \pi \bar{w} - \lambda(\hat{\theta})(\bar{w} - w_i). \]

The interpretation is the following. In the DA market, the aggregator makes a payment to the selected agent for the full capacity generation \( \bar{w} \) at a price \( \pi \). In the RT market, the selected agent is penalized for the shortfall \( \bar{w} - w_i \) at a penalty price \( \lambda(\hat{\theta}) \).

Clearly, the selection scheme should be the same as before:

\[ i \in \arg \max_j \mathbb{E}[\hat{W}_j]. \]

\section{Generalizations}

\subsection{General Objective Functions}

In the basic scenario, the objective is to contract with the agent who yields the highest expected generation. Now we generalize the objective. The aggregator wants to contract with agent \( i \) who yields the highest \( \mathbb{E}[g(W_i)] \):

\[ \text{maximize } \mathbb{E}[g(W_i)], \]

where \( g : [0, \bar{w}] \rightarrow \mathbb{R} \) is referred to as the objective function. We assume that \( g(\cdot) \) is continuous, which satisfies most practical purposes, so that it attains a maximum and a minimum. We can consider \(-g(\cdot)\) in (11) when we have a minimization problem. Without loss of generality, we assume that \( g \geq 0 \), since we can always add a sufficiently large constant to \( g(\cdot) \).

We give some examples of the objective function.

\textbf{Example 1:} When \( g(w) = w \), we recover the basic scenario.

\textbf{Example 2:} When \( g(w) = \min\{w, D\} \), the interpretation is that there is an upper bound \( D \in [0, \bar{w}] \) on the demand in the RT market, beyond which the aggregator does not care about how much more would be generated.

We now generalize the SVCG and SSP mechanisms. In both mechanisms, the selection scheme is given by

\[ i \in \arg \max_j \mathbb{E}[g(\hat{W}_j)]. \]

\textbf{1) Generalized SVCG Mechanism:} Define

\[ \hat{t}_i^{\text{DA}}(\hat{\theta}) = \max_{j \neq i} \mathbb{E}[g(W_j)], \]

and

\[ \hat{t}_i^{\text{RT}}(\hat{\theta}, w_i) = g(w_i). \]

\textbf{Theorem 3:} The generalized SVCG mechanism specified by (12)–(14) is dominant strategy incentive compatible.

\textbf{Proof:} First, we note that conditioned on that agent \( i \) is selected, his expected payoff is independent of \( \hat{\theta}_i \). On the other hand, if a truthful report \( \theta_i \) does not make it be selected given \( \hat{\theta}_i \), then a misreport \( \hat{\theta}_i \) that makes it be selected should not give it a positive payoff. That is,

\[ \lambda(\hat{\theta}) \geq \frac{\pi \bar{w}}{\bar{w} - \mathbb{E}[W_i]}. \]

Therefore, one possible choice is

\[ \lambda(\hat{\theta}) = \frac{\pi \bar{w}}{\bar{w} - \max_{j \neq i} \mathbb{E}[W_j]}. \]

Note that \( \lambda(\hat{\theta}) \geq \pi \), which is necessary for the mechanism to be incentive compatible.

\textbf{Theorem 2:} The SSP mechanism specified by (7)–(10) is dominant strategy incentive compatible.

Similarly as in the SVCG mechanism, each agent has no incentive to lie, whatever the others report. We will give a formal proof for the general scenario in Section IV. For now, it is interesting to note the duality between the SVCG and the SSP mechanisms: In the DA market, money flows from the selected agent to the aggregator in the SVCG mechanism (which depends on the second highest bid), while it flows from the aggregator to the selected agent in the SSP mechanism (which is a constant); In the RT market, money flows from the aggregator to the selected agent in the SVCG mechanism (which depends on the realization), while it flows from the selected agent to the aggregator in the SSP mechanism (which depends on both the second highest bid and the realization).
Suppose that agent \( i \) is not selected by bidding \( \hat{\theta}_i \), which means
\[
E[g(W_i)] \leq \max_{j \neq i} E[g(\hat{W}_j)],
\]
so that his current payoff is zero. Consider that he changes his bid to \( \bar{\theta}_i \). If he is not selected, he still gets a zero payoff. If he is selected, his expected payoff given by (15) is nonnegative. In either case, his expected payoff does not increase. Thus, he has no incentive to deviate.

We have shown that truth-telling is a dominant strategy for each agent. Therefore, the generalized SVCG mechanism is dominant strategy incentive compatible.

2) Generalized SSP Mechanism: Let
\[
g^* = \max_{0 \leq w \leq \bar{w}} g(w).
\]

Define
\[
t_i^{DA}(\hat{\theta}) = -g^*,
\]
and
\[
t_i^{RT}(\hat{\theta}, w_i) = -\lambda(\hat{\theta})(g^* - g(w_i)),
\]
where
\[
\lambda(\hat{\theta}) = \frac{g^*}{g^* - \max_{j \neq i} E[g(W_j)]}.
\]

**Theorem 4:** The generalized SSP mechanism specified by (12) and (16)-(18) is dominant strategy incentive compatible.

**Proof:** First, we note that conditioned on that agent \( i \) is selected, his expected payoff is independent of \( \hat{\theta}_i \):
\[
E[u_i(\hat{\theta}, W_i)] = E[t_i^{RT}(\hat{\theta}, W_i)] - t_i^{DA}(\hat{\theta})
\]
\[
= g^* - \lambda(\hat{\theta})(g^* - E[g(W_i)])
\]
\[
= \frac{g^* [E[g(W_i)] - \max_{j \neq i} E[g(W_j)]]}{g^* - \max_{j \neq i} E[g(W_j)]},
\]
\[
(19)
\]
Now fix an agent \( i \) and any \( \hat{\theta}_i \). We show that bidding \( \hat{\theta}_i \) is the best response of agent \( i \).

Suppose that agent \( i \) is selected by bidding \( \hat{\theta}_i \), which means
\[
E[g(W_i)] \geq E[g(W_j)], \quad \forall j \neq i,
\]
so that his current expected payoff given by (19) is nonnegative. Consider that he changes his bid to \( \bar{\theta}_i \). If he is still selected, his expected payoff remains the same. If he is not selected, he gets a zero payoff. In either case, his expected payoff does not increase. Thus, he has no incentive to deviate.

Suppose that agent \( i \) is not selected by bidding \( \hat{\theta}_i \), which means
\[
E[g(W_i)] \leq \max_{j \neq i} E[g(\hat{W}_j)],
\]
so that his current payoff is zero. Consider that he changes his bid to \( \bar{\theta}_i \). If he is not selected, he still gets a zero payoff. If he is selected, his expected payoff given by (19) is nonnegative. In either case, his expected payoff does not increase. Thus, he has no incentive to deviate.

We have shown that truth-telling is a dominant strategy for each agent. Therefore, the generalized SVCG mechanism is dominant strategy incentive compatible.

The preceding theorems tell that in a very general setting, each agent will truthfully reveal its private information, whether the others do so or not. We illustrate the application of the results in a networked setting where line capacity constraints are taken into account.

**Example 3:** Suppose that the \( N \) wind power generators are located at different buses in a power network, while the aggregator has demand of power at a certain bus. Due to line capacity constraints, there is an upper bound \( P_i \) of each agent \( i \)’s supply to that bus. The \( P_i \)'s can be easily determined once the dispatch for the conventional power generators is given.

In this case, we let \( g(w_i) = \max\{w_i, P_i\} \) and consider the following problem:
\[
\maximize_i E[g_i(W_i)],
\]
(20)
which is a further generalization of problem (11). One can derive the mechanisms following the same ideas as before.

As another case, suppose that the aggregator has demand of power at various buses. Due to line capacity constraints, the value of power at different buses can vary to the aggregator. Let \( \pi_i^{DA} \) be the locational marginal price in the DA market of the bus at which agent \( i \) is located. Then we can let \( g_i(w_i) = \pi_i^{DA}w_i \) and again consider problem (20).

B. Selecting Multiple Providers

We can make further generalizations by considering the case in which multiple agents are selected based on some criteria. Specifically, the aggregator wants to contract with \( M \) (where \( M < N \)) agents according to a continuous objective function \( g : [0, \bar{w}]^M \to \mathbb{R}_+^+ \):
\[
\maximize_{i_1, \ldots, i_M} E[g(W_{i_1}, \ldots, W_{i_M})].
\]
We give some examples of the objective function.

**Example 4:** When \( g(w_{i_1}, w_{i_2}) = (w_{i_1} - w_{i_2})^2 \), the interpretation is that the aggregator wants to contract with two agents who have the closest distributions in a certain sense.

**Example 5:** When \( g(w_{i_1}, \ldots, w_{i_M}) = w_{i_1} + \cdots + w_{i_M} \), the interpretation is that the aggregator wants to contract with \( M \) agents who yield the highest expected generation.

We propose the generalized SVCG and SSP mechanisms and omit the proofs due to space constraints.

**Theorem 5:** Let the selection scheme be
\[
(i_1, \ldots, i_M) \in \arg \max_{j_1, \ldots, j_M} E[g(\hat{W}_{j_1}, \ldots, \hat{W}_{j_M})].
\]
For all \( k = 1, \ldots, M \), let the DA payment be
\[
t_k^{DA}(\hat{\theta}) = \max_{j_1 \neq k, \ldots, j_M \neq i_k} E[g(\hat{W}_{j_1}, \ldots, \hat{W}_{j_M})],
\]
and the RT payment be
\[
t_k^{RT}(\hat{\theta}, w_{i_k}) = E[g(\hat{W}_{i_1}, \ldots, \hat{W}_{i_M})|\hat{W}_{i_k} = w_{i_k}].
\]
Then the generalized SVCG mechanism is dominant strategy incentive compatible.

**Theorem 6:** Let the selection scheme be
\[
(i_1, \ldots, i_M) \in \arg \max_{j_1, \ldots, j_M} E[g(\hat{W}_{j_1}, \ldots, \hat{W}_{j_M})].
\]
For all \( k = 1, \ldots, M \), let
\[
g^*_k(\theta) = \max_{0 \leq w \leq \bar{w}} \mathbb{E}[g(\hat{W}_{1k}, \ldots, \hat{W}_{iM})|\hat{W}_{ik} = w].
\]
Let the DA payment be
\[
i^\text{DA}_{ik}(\hat{\theta}) = -g^*_k(\hat{\theta}),
\]
and the RT payment be
\[
i^\text{RT}_{ik}(\hat{\theta}, w_{ik}) = -\lambda_{ik}(\hat{\theta})g^*_k(\hat{\theta}) - \mathbb{E}[g(\hat{W}_{1k}, \ldots, \hat{W}_{iM})|\hat{W}_{ik} = w_{ik}],
\]
where
\[
\lambda_{ik}(\hat{\theta}) = \frac{g^*_k(\hat{\theta})}{g^*_k(\hat{\theta}) - \max_{j \neq i, \ldots, j \neq M} \mathbb{E}[g(\hat{W}_{j1}, \ldots, \hat{W}_{jM})]}.\]
Then the generalized SSP mechanism is dominant strategy incentive compatible.

The preceding results show that each agent has no incentive to lie even if the aggregator wants to select multiple agents. For certain objective functions, the mechanisms have an explicit form. Consider Example 5. We rank the agents in order of the reported expected generation:
\[
\mathbb{E}[\hat{W}_{1k}] \geq \cdots \geq \mathbb{E}[\hat{W}_{iN}].
\]
In both mechanisms, agents \( i_1, \ldots, i_M \) are selected. In the generalized SVCG mechanism, for all \( k = 1, \ldots, M \), we have
\[
i^\text{DA}_{ik}(\hat{\theta}) = \mathbb{E}[\hat{W}_{iM+i}], \quad i^\text{RT}_{ik}(\hat{\theta}, w_{ik}) = w_{ik}.
\]
In the generalized SSP mechanism, for all \( k = 1, \ldots, M \), we have
\[
i^\text{DA}_{ik}(\hat{\theta}) = -\bar{w}, \quad i^\text{RT}_{ik}(\hat{\theta}, w_{ik}) = -\lambda(\bar{w} - w_{ik}),
\]
where
\[
\lambda(\bar{w}) = \frac{\bar{w}}{\bar{w} - \mathbb{E}[\hat{W}_{iM+i}]}.
\]

V. ECONOMIC DISPATCH WITH WIND POWER INTEGRATION

In Example 3, we consider a networked stochastic resource auction from an aggregator’s point of view. In this section, we study a similar problem from an independent system operator’s (ISO’s) perspective, and demonstrate how wind power generators can participate in the wholesale electricity market directly in the context of the economic dispatch problem, where all of the agents are present as wind power providers.

A. Model

We consider a connected power network which consists of \( N \) buses indexed by \( i = 1, \ldots, N \). For simplicity, we assume that each bus \( i \) has exactly one wind power generator, referred to as agent \( i \), whose model is the same as before. In particular, agent \( i \)’s wind power generation is a random variable \( W_i \in [0, \bar{w}] \) parameterized by \( \theta_i \), whose realization is denoted by \( w_i \). Since we focus on the strategic behavior of the wind power generators, we assume that the supply from conventional power at each bus \( i \) is competitive, represented by a cost function \( c_i(\cdot) \) which is increasing, convex and differentiable. The demand at each bus \( i \) is also competitive, represented by a valuation function \( v_i(\cdot) \) which is increasing, concave and differentiable.

For analytical and computational simplicity, we adopt a DC power flow model as a common practice [14]. It is derived via a sequence of approximations:

1) Branch resistances are negligible;
2) Voltage phase angle differences are small;
3) Voltage magnitudes are constant at 1.0 per unit.

Let \( B_{ij} \) be the magnitude of the susceptance (which is the imaginary part of admittance) of the branch connecting bus \( i \) and \( j \), with \( B_{ij} = B_{ji} \geq 0 \). Let \( \phi_i \) be the voltage phase angle at bus \( i \). Moreover, let \( f^\text{max}_{ij} \) be the flow limit of the branch connecting bus \( i \) and \( j \), with \( f^\text{max}_{ij} = f^\text{max}_{ji} \geq 0 \). Then the power flow from bus \( i \) to \( j \) is given by
\[
f_{ij} = B_{ij}(\phi_i - \phi_j) \leq f^\text{max}_{ij}.
\]
Let \( x = (x_1, \ldots, x_N) \) be the conventional power generation profile, \( y = (y_1, \ldots, y_N) \) be the demand profile, \( z = (z_1, \ldots, z_N) \) be the wind power generation profile, \( \phi = (\phi_1, \ldots, \phi_N) \) be the voltage phase angle profile, \( W = (W_1, \ldots, W_N) \) be the random vector of wind power generation, and \( w = (w_1, \ldots, w_N) \) be the realized wind power generation profile. Note that \( x \) and \( y \) should be determined in the DA market, while \( z \) and \( \phi \) are subject to adjustments in the RT market due to the uncertainty of \( w \).

In the DA market, the ISO determines an optimal dispatch \((x, y, z, \phi)\) that maximizes the expected social welfare:

\[
\begin{align*}
\text{maximize} & \quad \sum_i (v_i(y_i) - c_i(x_i)) - \mathbb{E}[Q(x, y, W)] \quad (21a) \\
\text{subject to} & \quad x_i + z_i - y_i = \sum_j B_{ij}(\phi_i - \phi_j), \quad \forall i, \\
& \quad B_{ij}(\phi_i - \phi_j) \leq f^\text{max}_{ij}, \quad \forall (i,j), \\
& \quad x_i, z_i, y_i \geq 0, \quad \forall i,
\end{align*}
\]

where \( Q(x, y, w) \) is the recourse cost in the RT market due to the uncertainty of \( w \). (21b) is the bus power balance equation, (21c) is the branch power flow constraint, and (21d) is the feasibility constraint.

The recourse cost \( Q(x, y, w) \) is the minimum cost to balance the power throughout the network in the RT market. Let \( \pi^+_i \) and \( \pi^-_i \) be the price of procuring and disposing power, respectively, at each bus \( i \). Let \( z^+_i \) and \( z^-_i \) be the amount of power procured and disposed, respectively, at each bus \( i \). Let \( z^+ = (z^+_1, \ldots, z^+_N) \) and \( z^- = (z^-_1, \ldots, z^-_N) \). In the RT market, the ISO determines an optimal dispatch \((z^+, z^-, \phi)\) that minimizes the cost for balancing power:

\[
\begin{align*}
\text{minimize} & \quad \sum_i (p^+_i z^+_i + p^-_i z^-_i) \quad (22a) \\
\text{subject to} & \quad x_i + w_i + z^+_i - y_i - z^-_i = \sum_j B_{ij}(\phi_i - \phi_j), \quad \forall i, \\
& \quad B_{ij}(\phi_i - \phi_j) \leq f^\text{max}_{ij}, \quad \forall (i,j), \\
& \quad z^+_i, z^-_i \geq 0, \quad \forall i.
\end{align*}
\]

The optimal value of problem (22) gives the recourse cost \( Q(x, y, w) \).
Thus, we have formulated a two-stage stochastic programming problem (21)–(22). We first show the convex structure of the problem.

**Proposition 2:** The first-stage problem (21) is a convex optimization problem.

**Proof:** We only need to show that \( Q(x, y, w) \) is convex in \((x, y)\), since expectation preserves convexity.

Clearly, the linear programming problem (22) has an optimal solution, so that strong duality holds. Consider its dual problem. Then \( Q(x, y, w) \) takes the following form:

\[
Q(x, y, w) = \max_{(\mu, v) \in \mathcal{D}} \left( \mu^T (x + w) - y \right) + \sum_{i,j} \nu_{ij} f_{ij}^{\max},
\]

where \( \mu \) : \((\mu_1, \ldots, \mu_N) \) and \( \nu_{ij} \)'s are dual variables whose domain is \( \mathcal{D} \). Since pointwise maximum preserves convexity, it follows that \( Q(x, y, w) \) is convex in \((x, y)\).

To solve the stochastic programming problem (21)–(22), one can apply the sample average approximation method, which is not the focus of this paper.

**B. Mechanism**

To solve the exact system problem (21)–(22), the ISO needs to know the true distribution of \( W \). We propose an incentive compatible mechanism following the same idea of the SVCG mechanism.

Let \((x^*, y^*, z^*, \phi^*)\) be a solution to the following optimization problem:

\[
\begin{align*}
\text{maximize} & \quad \sum_i (v_i(y_i) - c_i(x_i)) - \mathbb{E}[Q(x, y, \hat{W})] \\
\text{subject to} & \quad x_i + z_i - y_i = \sum_j B_{ij}(\phi_i - \phi_j), \quad \forall i, \\
& \quad B_{ij}(\phi_i - \phi_j) \leq f_{ij}^{\max}, \quad \forall (i,j), \\
& \quad x_i, y_i, z_i \geq 0, \quad \forall i.
\end{align*}
\]

Let \((x^{-k}, y^{-k}, z^{-k}, \phi^{-k})\) be a solution to the following optimization problem:

\[
\begin{align*}
\text{maximize} & \quad \sum_i (v_i(y_i) - c_i(x_i)) - \mathbb{E}[Q(x, y, \hat{W})] | \hat{W}_i = 0] \\
\text{subject to} & \quad x_i + z_i - y_i = \sum_j B_{ij}(\phi_i - \phi_j), \quad \forall i, \\
& \quad B_{ij}(\phi_i - \phi_j) \leq f_{ij}^{\max}, \quad \forall (i,j), \\
& \quad x_i, y_i, z_i \geq 0, \quad \forall i.
\end{align*}
\]

In the DA market, each agent \(k\) makes a payment \(t_{k}^{DA}(\hat{\theta})\):

\[
t_{k}^{DA}(\hat{\theta}) = \sum_i (v_i(y^{-k}_i) - c_i(x^{-k}_i)) - \mathbb{E}[Q(x^{-k}, y^{-k}, \hat{W}) | \hat{W}_i = 0]. \tag{23}
\]

In the RT market, each agent \(k\) gets paid \(t_{k}^{RT}(\hat{\theta}, w_k)\):

\[
t_{k}^{RT}(\hat{\theta}, w_k) = \sum_i (v_i(y^*_i) - c_i(x^*_i)) - \mathbb{E}[Q(x^*, y^*, \hat{W}) | \hat{W}_k = w_k]. \tag{24}
\]

When \(\hat{\theta}_k = \theta_k\), the expected payoff of agent \(k\) is given by

\[
\mathbb{E}[u_k(\hat{\theta}, w_k)] = \mathbb{E}[t_{k}^{RT}(\hat{\theta}, w_k)] - t_{k}^{DA}(\hat{\theta})
\]

\[
= \sum_i (v_i(y^*_i) - c_i(x^*_i)) - \mathbb{E}[Q(x^*, y^*, \hat{W})] - \sum_i (v_i(y^{-k}_i) - c_i(x^{-k}_i)) \]

\[
+ \mathbb{E}[Q(x^{-k}, y^{-k}, \hat{W}) | \hat{W}_k = 0] \geq 0.
\]

**Theorem 7:** The mechanism specified by (23)–(24) is dominant strategy incentive compatible.

This result shows that each wind power generator will truthfully reveal its distribution in the DA market so that the ISO can have the right information to compute the economic dispatch. We omit the proof due to space constraints.

### VI. CASE STUDIES

In this section, we present a case study based on the IEEE 57-bus system [15]. Our focus is to show that the stochastic resource auction paradigm can reduce the system cost.

There are 57 buses, 7 conventional power generators and 42 loads in this power network. There is an inelastic demand at each bus, so that the objective is to minimize the total expected cost. Note that the IEEE test cases do not have branch flow limits. Inclusion of branch flow limits is left for future work.

For the cost functions for conventional power, we use the data from MATPOWER [16], which is listed in Table I.

<table>
<thead>
<tr>
<th>Generator</th>
<th>Bus</th>
<th>Cost ($/hr) (x: MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0.0770x^2 + 20x</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>0.0100x^2 + 40x</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>0.2500x^2 + 20x</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>0.1000x^2 + 40x</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>0.0222x^2 + 20x</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>0.0100x^2 + 40x</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>0.0323x^2 + 20x</td>
</tr>
</tbody>
</table>

We compute the optimal dispatch by solving a convex quadratic programming problem, and the payment made to each wind power producer under locational marginal pricing.

Consider the architecture where the wind power generators bid supply functions. The cost function of each agent is

\[
c_i(x_i) = \mathbb{E}[p(x_i - W_i)] = \pi RT x_i^2 / 200.
\]

We can compute the optimal dispatch by solving a convex quadratic programming problem, and the payment made to each wind power producer under locational marginal pricing.

Consider the architecture where the wind power producers bid probability distributions. Since the distribution of the aggregate wind power generation is sophisticated, we apply the Monte Carlo method to obtain the optimal dispatch, i.e., to determine the generation from conventional power generators. We use the mechanism specified by (23)–(24) to compute the payment made to each wind power producer.

Fig. 2 shows the expected total cost versus penalty price under two architectures. We can see that the stochastic resource auction architecture (in which wind power producers bid probability distributions) can achieve a lower system cost than the traditional architecture (in which wind power producers bid supply functions as conventional power generators do). Moreover, as the penalty price increases, the total expected cost also increases in both architectures.
probability distributions instead of bidding supply functions as in the traditional approach. We derive an incentive compatible mechanism that has a natural form corresponding to the two-settlement market system. In the end, we present a case study to complement the analysis in the previous sections.

In the future, we will consider a repeated version of stochastic resource auctions with spot market prices that vary according to a Markov process. We would also consider a double-sided market architecture wherein there are buyers (the utility companies) as well as both types of sellers (conventional as well as renewable energy generators). It is an open question whether it is even possible to design such a market with desirable equilibrium properties.

The presented work, along with the proposed future work, will potentially provide economic solutions for integrating renewable energy generators into smart-grid networks.

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