Spectrum Sharing Through Contracts

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Abstract—Development of dynamic spectrum access and allocation techniques recently [6] have made feasible the vision of cognitive radio systems. However, a fundamental question arises: Why would licensed primary users of a spectrum band allow secondary users to share the band and degrade performance for them? This incentive issue has been sought to be addressed by designing incentive-compatible auction mechanisms [4]. This, however, does not solve the problem. It is not clear who acts as the auctioneer. If the primary himself does, why would the secondary trust the primary to not manipulate the auction. We propose that a more appropriate mechanism to solve this incentive problem is a contractual mechanism. In this paper, we consider a simple setting: A single primary transmitter-receiver pair and a single secondary transmitter-receiver pair with a Gaussian interference channel between them. We consider the setting of complete information when channel attenuation coefficients and noise levels at the receivers are common knowledge. We consider when receivers cooperate to do successive-interference cancellation. Unlike the results of [7] for unlicensed bands, we show that it is possible to achieve socially optimal rate allocations with contracts in licensed bands.

Index Terms—Cognitive radio systems, Spectrum sharing, Game Theory, Principal-Agent models, Contract design.

I. INTRODUCTION

The scarcity of spectrum is becoming an impediment to the growth of more capable wireless networks. Several measures are sought to address this problem: Freeing up unused spectrum, sharing of spectrum through new paradigms such as cognitive radio sensing, as well as sophisticated information theoretic schemes and network coding methods. Nearly all such methods presume perfect user cooperation. This, however, is an unjustified assumption. And with non-cooperative, selfish users who act strategically, network (sum-rate) capacity can be arbitrarily bad as shown for the single-hop Gaussian interference channel in unlicensed bands [7].

For licensed bands, one of the key challenges is when users have cognitive radio capability, why would primary users give up their ownership rights over their spectrum and share it with secondary users at the cost of performance degradation to themselves? FCC mandates are not going to solve the problem as primary users can always transmit junk to keep channels busy and deter secondary users [17], [1].

This incentive issue has been sought to be addressed by guaranteeing the primary user a payment in lieu of sharing his spectrum and suffering some performance degradation. This has been sought to be implemented in various ways: dynamic competitive pricing [19], [12], [16], and spectrum auctions [10], [9], [4]. While competitive pricing is usually not incentive-compatible and not robust to manipulation by strategic users, carefully designed auctions can potentially be strategy-proof and yield socially optimal outcomes. In many scenarios, we can even operate them as double-sided auctions or markets when there are both buyers and sellers. Unfortunately, for auctions to be practical, they must be operated by a neutral, disinterested party as an auctioneer. Otherwise, the auctioneer can manipulate the auctions to his advantage. This situation is unlikely to arise in most cognitive radio systems. The spectrum sharing and allocation must happen as a direct result of interaction between a primary user and one or more secondary users. Thus, a principal-agent model is more appropriate for such a scenario [15], [2]. One user (possibly the primary) acts as a principal, and offers several contracts to the agent(s) (possibly the secondary user(s)). The agent(s) then picks one of the possible contracts or may reject all of them. Our hope is to specify the class of contracts that a primary user can offer such that both the primary and the secondary users are able to maximize their individual utilities while still achieving a social welfare objective.

The principal-agent model has been used in solving some problems in communication networks such as network formation [11] and wireless multihop routing [8]. The principal-agent model and the contractual mechanism approach to spectrum sharing is new. We focus on a two-user setting and assume that their radios are sophisticated enough for them to employ successive-interference cancellation (SIC) techniques [5]. One of the receivers acts as a dominant user and decodes his signal the last after decoding signals of all the other users [13]. Typically, this would be the primary user but there can be scenarios where the secondary user radio is more sophisticated and acts as a dominant user. Again, either the dominant or the non-dominant user could act as the principal and offer contracts for the other user to choose from. In each case, we specify the class of contracts that are incentive-compatible. Furthermore, we are able to show that these are also optimal in the sense of maximizing the sum-rate. We first propose contracts that do not allow time or frequency division multiplexing (TDMA, FDMA), and then allow for this possibility.

The paper is organized as follows. In Section II, we describe
the physical layer channel model and some related work. In Section III, we introduce the contract design problem in a cognitive radio setting. In Section IV and V, we consider optimal spectrum sharing contracts without and then with time-sharing. We end with a discussion of future work.

II. MODEL AND RELATED WORK

Suppose there are \( M \) transmitter-receiver pairs that share the Gaussian Interference Channel (GIC) of bandwidth \( W \). We can model it as the following,

\[
y_{i}[n] = \sum_{j=1}^{M} h_{j,i} x_{j}[n] + z_{i}[n]; i = 1, \ldots, M
\]

where \( x_{i} \) is the signal from the transmitter \( i \), \( y_{i} \) is the signal received at the receiver \( i \), \( h_{j,i} \) is the channel attenuation coefficient from transmitter \( j \) to receiver \( i \), and the noise process \( z_{i} \) are i.i.d. over time with distribution \( \mathcal{N}(0, N_0) \). Each user could treat the signal from other users as interference. This is well justified as the practical limitations such as decoder complexity and uncertainty in the estimation of channel coefficients may preclude the use of interference cancellation algorithms. We also assume that the users use random Gaussian codebooks for transmission. Then, the maximum rate that the system can achieve is given by

\[
R_i = W \log \left( 1 + \frac{c_{i,i} P_i}{N_0 W + \sum_{j \neq i} c_{j,i} P_j} \right), \quad \forall i
\]

where \( P_i \) is the transmitted power of the user \( i \), and \( c_{i,j} = |h_{i,j}|^2 \). Also, due to power constraints, \( P_i \) must satisfy \( P_i \leq \bar{P}_i \) for all \( i \).

The spectrum sharing problem is to determine a set of power allocation \( P = (P_1, \ldots, P_M) \) that maximizes a given global utility function (such as the achievable sum-rate \( \sum_i R_i \)) while satisfying the power constraints. However, users are selfish and may not cooperate with each wanting to maximize its’ own rate. Thus, they pick their power allocations \( P \) each wanting to maximize their own rate and leading to a spectrum sharing game between them. To predict the outcome of such a game, we look at its’ Nash equilibrium \( P^* \) such that given the power allocations of all the other users \( P_{-i} \), user \( i \)’s rate is maximized at \( P_{i}^* \), i.e.,

\[
R_i(P_{i}^*, P_{-i}) \geq R_i(P_i, P_{-i}), \quad \forall P_i \leq \bar{P}_i.
\]

In [7], a richer strategy space was considered such that a user could chose any power spectral density \( p_i(f) \) as long as it satisfied its total power constraint, i.e.,

\[
\int_{W} p_i(f) df \leq \bar{P}_i.
\]

Furthermore, it was shown that in a flat fading GIC a Nash equilibrium (NE) exists, all NE are pure strategy equilibria, and under certain conditions, full-spread power allocation (i.e., \( p_i(f) = \bar{P}_i/W \) for all \( i \)) is a NE. Moreover they have shown that under some conditions, full-spread is the unique NE. However, in most cases, the set of rates that result from the full-spread NE is not Pareto efficient. So there may be a significant performance loss if the \( M \) users operate at this point due to lack of cooperation. In fact, in many cases this inefficient outcome is the only possible outcome of the game. For the more general parallel GIC, existence of Nash equilibrium was proved in [20].

The above discussion assumed that users did not use cooperative schemes from multi-user information theory. A particular scheme of relevance is Successive Interference Cancellation (SIC) which works as follows. Suppose user 1 decodes his own signal by treating interference from all other users as noise, then he can achieve a rate

\[
R_1 = \log \left( 1 + \frac{c_{1,1} P_1}{N_0 W + \sum_{j > 1} c_{j,1} P_j} \right).
\]

Now, user 2 can do the same and decode user 1’s signal first as above. Then, he can subtract this signal from the received signal, and decode his own signal by treating all other users as noise. Thus, he can achieve a rate

\[
R_2 = \log \left( 1 + \frac{c_{2,2} P_2}{N_0 W + \sum_{j > 2} c_{j,2} P_j} \right).
\]

Note that this is greater than what he could have received if he had treated user 1’s signal as noise as well. Proceeding in this way, user \( M \) then achieves a rate

\[
R_M = \log \left( 1 + \frac{c_{M,M} P_M}{N_0 W} \right).
\]

We will call user \( M \) to be the first-dominant user. If there are only two users, we will call such a user as a dominant user, and the other as a non-dominant user. If we proceed according to a different permutation on the \( M \) users, we will achieve a different rate vector \( R' \). By time-sharing between the various achievable rate vectors, we get an achievable rate region \( \mathcal{R}_{SIC} \) that is strictly larger than that obtained with the naive non-cooperative scheme \( \mathcal{R}_{naive} \). Thus, all users could potential gain if we could find a way for them to cooperate. This would be successful if their incentives are aligned for cooperation.

III. CONTRACT DESIGNS FOR SPECTRUM SHARING

As we discussed above, spectrum sharing with naive coding, i.e., treating interference from other users as noise can lead to inefficient outcomes as Nash equilibria. Thus, a question arises whether it is possible to alleviate this inefficiency by introducing an incentive alignment mechanism. Our focus in this paper is a licensed band setting with cognitive radios where there is a primary user who owns the spectrum band and a secondary user who wants to share the spectrum with the primary user. Spectrum sharing is desirable for two reasons. First, the primary user may not be using the channel all the time. When it is not being used, it can be used by the secondary user. Second, even when the primary user is using the channel, it can still share the spectrum with the secondary user with little or no performance degradation to itself. For example, when there are only two users sharing a Gaussian interference channel, if they both agree to cooperate in doing successive interference cancellation, the dominant user suffers...
no performance degradation at all. His achievable rate is the same as if the other user were not present.

This, however, does not imply that if the primary user acts as the dominant user, it does not suffer any externality cost due to the presence of the other user. For one, the achievable rate region $R_{SIC}$ is possible only asymptotically in the codeword length. So, with any practical SIC codes, there is going to be some performance loss. Second, SIC requires cooperation between the users in codebook design. Thus, coding and decoding at the primary transmitter and receiver is more complex than before again imposing complexity externality. Even if these issues are ignored, the primary user still may have an incentive to not share the spectrum unless he is compensated for it in some way. Furthermore, in some scenarios complexity considerations can entail that the primary acts as a non-dominant user while the secondary acts as a dominant user. Thus, it is amply clear that there is a need to introduce incentive alignment schemes that will induce the primary user to share spectrum with the secondary user, and moreover cooperate in doing so by using advanced communication schemes such as SIC.

As we argued before, auction mechanisms [9][10] are not the right framework for this problem since there is no independent, impartial entity that can coordinate the auction and act as an auctioneer or a broker. Here, the primary user is an interested party with incentives to manipulate the auction. We, thus, consider this as a principal-agent model [15][2] where the principal offers one or more contracts to the agent. The agent then selects one or rejects all.

\[ \max_{P_p,P_a,\lambda} R_p(P_p,P_a) + \lambda(P_p,P_a) \]  \hspace{1cm} \text{(3)}

subject to:

\[ [\text{IR}]: \quad R_a(P_a,P_p) - \lambda(P_p,P_a) \geq \bar{U}_a \]

\[ [\text{IC}]: \quad R_a(P_a',P_p) - \lambda(P_p',P_a') \geq R_a(P_a,P_p) - \lambda(P_p,P_a), \forall P_a' \leq P_a. \]

The rate functions $R_p, R_a$ above are given by $R_1(P_1, P_2) = W \log(1 + \frac{P_1 h_1}{N_0 + c_1 P_2})$. Note that the power budget constraints $P_p$ and $P_a$ on $P_p$ and $P_a$ are implicit. The optimization problem $\text{CD-OPT}$ above is a non-convex, variational problem, solving which, in general, is difficult. The existence of a solution $\lambda^*, P_p^*, P_a^*$ to it can be established by using the extreme value theorem. In subsequent discussion, we will actually establish it by construction.

We assume a complete information setting wherein rate functions of all players are known to all players. When agents have asymmetric information about each each other, it can lead to non-optimal solutions to $\text{CD-OPT}$. Such adverse selection models are also of relevance for the spectrum sharing problem and will be considered in future work. There is also an issue of moral hazards due to hidden actions.

Once a contract has been agreed upon, if the players cannot directly observe each other’s power usage, there is the chance of each player violating his agreement by transmitting at a different rate than agreed upon. This moral hazard problem due to hidden actions can also lead to sub-optimality of operation. Thus, the designed contracts must be robust to the moral hazard problem.

In subsequent discussion, we assume a complete information setting. The primary and the secondary user, if they agree to a contract, use successive interference cancellation (SIC). Either the primary or the secondary user can act as a dominant user. Furthermore, either of the dominant or the non-dominant user can be the principal, i.e., the one who proposes the contract. To ensure the security, the non-dominant user can employ a data encryption scheme on its source coded bits before passing it to the channel coder. Thus the dominant user won’t be able to decrypt the actual data of the non-dominant user though it can get the channel bits as a part of SIC decoding.

\[ U_p \] \hspace{1cm} \text{and} \hspace{1cm} \bar{U}_a \] denote the reservation utilities for the principal and the agent that they can derive if the contract is not accepted. The, the agent will accept a contract only if he can find a feasible operating point at which his payoff is at least as large as his reservation utility $\bar{U}_a$. This is called an individual rationality (IR) constraint, i.e., it has to be rational for the agent to participate. Furthermore, among the IR and IC operating points, he will pick one that maximizes his payoff. This is called incentive compatibility (IC). Thus, in designing the contracts, the principal should take both the IR and IC constraints into account. The principal’s problem is then given by the following optimization problem.
We denote the transmitting powers of the dominant and non-dominant users as $P_d$ and $P_{nd}$ respectively with power constraints $P_d \leq \bar{P}_d$ and $P_{nd} \leq \bar{P}_{nd}$. For the ease of illustration and without loss of generality, we will assume $W = 1$ and $N_0 = 1$. We will also interpret the powers to be received powers rather than transmit powers. The power budgets can also be scaled accordingly. This simplifies the exposition without affecting the results. Due to asymmetry in the channel coefficient, the received power in different receiver can potentially be different. However, since the rates are determined based on the signal power at the dominant user’s receiver, $P_d$ and $P_{nd}$ denote the received signal power of dominant user and the non-dominant user at the dominant user’s receiver.

Now, the SINR seen by the dominant user’s receiver for the signal from the non-dominant user is $\Gamma_{nd} = \left( \frac{P_{nd}}{1 + P_d} \right)$, it can decode any data stream from the non-dominant user when the rate is less than or equal to $\log(1 + \Gamma_{nd})$. Thus, the maximum rate at which the non-dominant user can transmit is

$$R_{nd}(P_{nd}, P_d) = \log \left( 1 + \frac{P_{nd}}{1 + P_d} \right).$$

In the less interference scenario, i.e., when channel cross gains $c_{i,j}$ is less than the channel direct gains $c_{i,i}$, the SINR seen by the non-dominant user’s receiver towards its own signal will always be greater than $\Gamma_{nd}$ and it is perfectly possible for the non-dominant user to transmit at the rate $R_{nd}(P_{nd}, P_d)$ with arbitrary small error rate. Now, the dominant user’s receiver decodes this data and subtracts it from the aggregate signal. Since the interference is removed, the dominant user can transmit at a maximum rate of

$$R_d(P_d, P_{nd}) = \log(1 + P_d).$$

In Figure 1, if user 1 acts as dominant user and both transmit at maximum power, the operating point will be at the point $A$. If user 2 acts as a dominant user and both transmit at maximum power, the operating point is $B$. It is quite apparent that a user derives a higher rate if he is a dominant user than if he is non-dominant user.

The utilities of the two users will be assumed to be equal to their rates, i.e.,

$$u_d(P_d, P_{nd}) = R_d(P_d, P_{nd}),$$

$$u_{nd}(P_{nd}, P_d) = R_{nd}(P_{nd}, P_d).$$

We define a social welfare function as $S(P_d, P_{nd}) = u_d(P_d, P_{nd}) + u_{nd}(P_{nd}, P_d)$. We say that rate allocation $(R_d^{**}, R_{nd}^{**})$ is socially optimal if it is achieved by a power allocation $(P_d^{**}, P_{nd}^{**})$ that maximizes the social welfare function subject to the power constraints. We will say that a spectrum contract is optimal if it achieves a socially optimal rate allocation.

It is easy to conclude the following.

**Proposition 1:** In a Gaussian interference channel with a primary and a secondary user, both of whom cooperatively do successive-interference cancellation without time-sharing (i.e., one always acts as the dominant user), the socially optimal power allocation is $(P_d, P_{nd})$. The socially optimal rate allocation is given by $(R_d(P_d, P_{nd}), R_{nd}(P_{nd}, P_d))$. The optimal value of social welfare is given by $\log(1 + P_d + P_{nd})$.

**Proof:** The socially optimal power allocation can be found as the solution for the following optimization problem.

$$\max_{P_d \leq \bar{P}_d, P_{nd} \leq \bar{P}_{nd}} S(P_d, P_{nd})$$

which is equal to

$$\max_{P_d \leq \bar{P}_d, P_{nd} \leq \bar{P}_{nd}} u_d(P_d, P_{nd}) + u_{nd}(P_{nd}, P_d)$$

Substituting the value of $u_d$ and $u_{nd}$ will reduce the above problem to

$$\max_{P_d \leq \bar{P}_d, P_{nd} \leq \bar{P}_{nd}} \log(1 + P_d + P_{nd})$$

The solution for the above problem is clearly $(P_d^{**}, P_{nd}^{**}) = (\bar{P}_d, \bar{P}_{nd})$. Then the corresponding socially optimal rate allocation will be $(R_d^{**}, R_{nd}^{**}) = (R_d(\bar{P}_d, \bar{P}_{nd}), R_{nd}(\bar{P}_{nd}, \bar{P}_d))$. Now, the optimal value of the social welfare is

$$S(P_d^{**}, P_{nd}^{**}) = u_d(P_d^{**}, P_{nd}^{**}) + u_{nd}(P_{nd}^{**}, P_d^{**}) = \log(1 + \bar{P}_d + \bar{P}_{nd}).$$

When both users can act as a dominant user, there is also the issue of who will be the dominant user. This possibility can also be allowed as part of a contract definition. Furthermore, both users can time-share on who acts as a dominant user. This is considered in Section V.

In order to maximize individual rates, each user may want to act as dominant user. So, without any proper incentive, they may not participate in this spectrum sharing game and act as non-dominant user. So, the problem of the principal’s is to design a contract which will give enough incentive for the agent to participate in the spectrum sharing game and at the same time maximizes the principal’s net utility.
IV. OPTIMAL CONTRACTS WITHOUT TIME-SHARING

In this section, we focus on the cases when either the primary or the secondary acts as a dominant user. However, they cannot both act as dominant users by time-sharing. We will also consider cases when either the dominant user is the principal or the agent. Thus, four cases arise, each potentially leading to a different solution of the contract design optimization problem.

We denote the transmitting power of the primary and secondary users as $P_p$ and $P_s$, respectively with power constraints $P_p \leq P_p^*$ and $P_s \leq P_s^*$. The primary user has a reservation utility $U_p = \log (1 + P_p)$, the utility it can receive without sharing the spectrum with the secondary. The reservation utility of the secondary user $U_s$ shall be assumed to be zero for simplicity but this can easily be relaxed. Also we denote reservation utility of the agent as $\hat{U}_a$. Clearly $\hat{U}_a = U_p$ when the primary acts as agent and $\hat{U}_a = U_s$ when the secondary acts as agent.

The principal will offer a suite of contracts to the agent by specifying $\lambda(P_p, P_s)$, the payment the agent will have to make if it chooses an operating point $(P_p, P_s)$. The agent chooses the operating point $(P_p, P_s)$ and makes the corresponding payment to the principal. Thus, the agreed upon payment and operating point $(\lambda(P_p, P_s), P_p, P_s)$ will be called a contractual agreement.

The principal designs the contract $\lambda$ by solving the CD-OPT problem which will maximize his payoff at some operating point $(P_p^*, P_s^*)$ while ensuring it is Individual-Rational and Incentive-Compatible for the agent to accept the contract and choose the desired operating point.

We now look at two cases. The other two cases are symmetric to these two.

Case (i): Primary user is the Principal and the Dominant user: In this case, the primary user’s optimization problem is the following.

$$\max_{P_p, P_s, \lambda} \quad u_{nd}(P_s, P_p) + \lambda(P_p, P_s) \quad \text{s.t.} \quad \text{[IR]}: \quad u_{nd}(P_s, P_p) \geq \lambda(P_p, P_s) \geq 0 \quad \text{[IC]}: \quad u_{nd}(P_s', P_p) - \lambda(P_p, P_s') \geq 0, \quad \forall P_s' \leq P_s.$$  

The [IR] constraint implies that the maximum payment $\lambda$ that the primary can get is $u_{nd}(P_s, P_p)$. Thus, the principal can offer a contract

$$\lambda^*(P_p, P_s) = u_{nd}(P_s, P_p) \quad \text{(5)}$$

which satisfies the [IR] constraint with equality. The [IC] constraint is now trivially satisfied. With this, the principal’s optimization problem reduces to

$$\max_{P_p \leq P_p^*} \quad \log (1 + P_p + P_s) \quad \text{(6)}$$

The solution for the above is $P_p^* = \bar{P}_p, P_s^* = \bar{P}_s$. So, the operating point is point $A$ in Figure (2). The rates (and utilities) of the users are given by

$$(R_p^*, R_s^*) = (u_{nd}(\bar{P}_p, \bar{P}_s), u_{nd}(\bar{P}_s, \bar{P}_p)).$$

The primary user is able to extract the entire sum utility, $U_p^* = \log (1 + \bar{P}_p + \bar{P}_s)$ while the secondary user gets $U_s^* = 0$.

Case (ii): Secondary user is the Principal, Primary is the Dominant user: Now, the principal’s (secondary user’s) optimization problem is the following.

$$\max_{P_p, P_s, \lambda} \quad u_{nd}(P_s, P_p) - \lambda(P_s, P_p) \quad \text{s.t.} \quad \text{[IR]}: \quad u_{nd}(P_s, P_p) + \lambda(P_s, P_p) \geq \hat{U}_p \quad \text{[IC]}: \quad u_{nd}(P_s, P_p) + \lambda(P_s, P_p) \geq 0, \quad \forall P_p' \leq P_p.$$  

The [IR] constraint implies that the minimum payment $\lambda$ that the primary need to get is $\hat{U}_p - u_{nd}(P_s, P_p)$. Thus, the principal (secondary) can offer a contract

$$\lambda^*_s(P_p, P_s) = \hat{U}_p - u_{nd}(P_s, P_p) = \log (1 + P_p + P_s) \quad \text{(8)}$$

which satisfies both [IR] and [IC] constraints and minimizes his payment as well. Substituting $\lambda^*_s(P_p, P_s)$ in the optimization problem of the principal reduces it to

$$\max_{P_p \leq P_p^*, P_s \leq P_s} \quad \log (1 + P_p + P_s) \quad \text{(9)}$$

The solution for the above is, as before, $P_p^* = \bar{P}_p, P_s^* = \bar{P}_s$ and the operating point is also at $A$ in Figure (2). So there is no change in the rates of users compared to the previous case.

The rates of the users are given by

$$(R_p^*, R_s^*) = (u_{nd}(\bar{P}_p, \bar{P}_s), u_{nd}(\bar{P}_s, \bar{P}_p)).$$

However, since the secondary is acting as principal there is change in the payment made. This results in a different net payoffs obtained by the users. The net payoff can be calculated as

$$(U_p^*, U_s^*) = \left( \log (1 + \bar{P}_p), \log (1 + \bar{P}_s) \right).$$

Thus, as in the case above, the principal is able to extract maximum achievable social welfare. Thus, the two contracts lead to socially optimal outcomes, i.e., those that maximize the sum of the utilities $S = U_p + U_s$. It is quite apparent from the above that the operating point depends on who is the dominant user while the distribution of the surplus depends on who has the bargaining power (i.e., who is the principal).

In both cases above, we have considered the primary user as the dominant user. A very similar analysis yields the optimal contracts in the remaining two cases, when the secondary user is dominant. In that case the operating point will shift to the point $B$ in Figure (2). Since the individual rates depend on the operating point, the rates in these two cases can be calculated as

$$(R_p^*, R_s^*) = (u_{nd}(\bar{P}_p, \bar{P}_s), u_{nd}(\bar{P}_s, \bar{P}_p)).$$

The net payoffs for the users depend on who is acting as principal.

We formalize the above discussion.

Theorem 1: Consider a Gaussian interference channel with a primary and a secondary user when both cooperate in
successive interference cancellation and have power budgets $\hat{P}_p$ and $\hat{P}_a$. If the dominant user acts as the principal, he offers contracts $\lambda_d(P_1, P_2) = u_{nd}(P_2, P_1) − \bar{U}_a$ to the other. If the non-dominant user acts as the principal, he offers contracts $\lambda_{nd}(P_1, P_2) = \bar{U}_a − u_d(P_2, P_1)$. Then, we can establish the following:

(i) These contracts are incentive-compatible and satisfy individual rationality. (ii) If the primary acts as the dominant user, the operating point is $(R^{*p}_p, R^{*p}_a) = (R_d(\hat{P}_p, \hat{P}_a), R_{nd}(\hat{P}_p, \hat{P}_a))$ (point A in Figure (2)). (iii) If the primary acts as the principal, the payoffs are $(\bar{U}^*_p, \bar{U}^*_a) = (\log (1 + \hat{P}_p + \hat{P}_a), 0)$. (iv) In all cases, the social welfare $S(P_p, P_a) = R_p + R_a$ is maximized and equals $\log (1 + \hat{P}_p + \hat{P}_a)$.

Statements similar to (ii) and (iii) hold for the secondary user as well.

V. Optimal Contracts with Time-Sharing

In the previous section, we considered the setting when only one user, the primary or the secondary is a dominant user, and then the operating point is either A or B in Figure (2) independent of who is the principal. However, that does not address the question how do they agree on who will be the dominant user. In the following discussion, we are addressing this issue by allowing time sharing between the users. We restrict our attention to the symmetric channel case where $c_{i,i} = c_{i,j}$. In this case, we only need to consider the received power at either of the receivers since the other being the same due to symmetry.

We thus allow for time-sharing where the principal acts as dominant user $\alpha$ fraction of the time, and as a non-dominant user $\bar{\alpha} = 1 - \alpha$ fraction of the time with $\alpha \in [0, 1]$. Thus, if the principal and the agent use powers $(P_p, P_a)$ respectively, with power constraints $P_p \leq \hat{P}_p, P_a \leq \hat{P}_a$, then with a given $\alpha$, the rate allocation of the principal is $R_p(P_p, P_a) = \alpha R_d(P_p, P_a) + \bar{\alpha} R_{nd}(P_p, P_a)$ and that of the agent is $R_a(P_a, P_p) = \bar{\alpha} R_d(P_a, P_p) + \alpha R_{nd}(P_a, P_p)$.

We define a social welfare function as $S(P_p, P_a, \alpha) = R_p(P_p, P_a) + R_a(P_a, P_p)$. We say that a rate allocation $(R^{*p*}_p, R^{*a*}_a)$ is socially optimal with time-sharing if it is achieved by a power allocation $(P^{***}_p, P^{***}_a)$ and a time-sharing parameter $\alpha$ that maximizes the social welfare function subject to the power constraints and allowing for time-sharing. It is now easy to conclude the following.

Proposition 2: In a Gaussian interference channel with a primary and a secondary user, one acting as principal and the other as agent, both of whom cooperatively do successive-interference cancellation with time-sharing (i.e., principal acts as dominant user $\alpha$ fraction of the time, and as a non-dominant user $\bar{\alpha}$ fraction of the time), the socially optimal power allocation is $(P^{***}_p, P^{***}_a)$. The socially optimal rate allocation is given by $(R_p(P^{***}_p, P^{***}_a), R_a(P^{***}_a, P^{***}_p))$. The optimal value of social welfare is given by

$$\log (1 + \hat{P}_a + \hat{P}_p).$$

Proof: The socially optimal power allocation can be found as the solution for the following optimization problem.

$$\max_{P_p, P_a, \alpha} \max_{\bar{\alpha}} S(P_p, P_a)$$

which is equal to

$$\max_{P_p, P_a, \alpha} \alpha R_d(P_p, P_a) + \bar{\alpha} R_{nd}(P_p, P_a) + \bar{\alpha} R_d(P_a, P_p) + \alpha R_{nd}(P_a, P_p).$$

Substituting the value of $R_d$ and $R_{nd}$ will reduce the above problem to

$$\max_{P_p, P_a, \alpha} \log (1 + P_p + P_a).$$

The solution for the above problem is clearly $(P^{***}_p, P^{***}_a) = (\hat{P}_p, \hat{P}_a)$, which means that both users transmit at their maximum power. Interestingly, the solution doesn’t depend on the time sharing parameter $\alpha$. This implies that all points on the pareto-optimal boundary (i.e., the line segment AB in Figure (2)) are socially optimal operating points. Thus, the corresponding socially optimal rate allocation will be $(R^{***}_p, R^{***}_a) = (R_d(\hat{P}_p, \hat{P}_a), R_a(\hat{P}_a, \hat{P}_p))$. Now, the optimal value of the social welfare is $S(R^{***}_d, R^{***}_nd) = R^{***}_p + R^{***}_a = \log (1 + \hat{P}_p + \hat{P}_a)$. 

We define a time-sharing spectrum contract to be the tuple $(\lambda(P_p, P_a, \alpha), P_p, P_a, \alpha)$ such that the operating point is $(P_p, P_a)$ with the principal acting as the dominant user $\alpha$ fraction of the time, and the agent acting as the dominant user $\bar{\alpha}$ fraction of the time, and the payment is $\lambda(P_p, P_a, \alpha)$. The principal offers such a contract to the agent, and the agent chooses an operating point $(P_p, P_a, \alpha)$. Thus, the agreed upon payment and operating point $(\lambda(P_p, P_a), P_p, P_a, \alpha)$ will be called a time-sharing contractual agreement. We will say that a time-sharing spectrum contract is optimal if it achieves a socially optimal rate allocation with time-sharing.

The principal wants to design a contract payment function $\lambda(P_p, P_a, \alpha)$ that maximizes his payoff $R_p + \lambda$ once the agent has accepted and picked the operating point $((P_p, P_a), \alpha)$. As before, we have individual rationality (IR) and incentive-compatibility (IC) constraints for the agent. Thus, the principal picks an optimal contract payment function $\lambda^*(P_p, P_a, \alpha)$ that maximizes his payoff while satisfying IR and IC constraints for the agent. We thus, have the following optimization problem to find the optimal contract.

As before, we now have two basic cases.

Case (i): Primary user is the Principle: In this case, we have the following optimization problem to find the optimal contract.

\[ \max_{P_p, P_a, \alpha} \alpha u_d(P_p, P_a) + \bar{\alpha} u_{nd}(P_p, P_a) + \lambda(P_p, P_a, \alpha) \quad (10) \]
subject to:

\[ \textbf{[IR]}: \alpha u_{nd}(P_s, P_p) + \tilde{\alpha} u_d(P_s, P_p) - \lambda(P_p, P_s, \alpha) \geq 0 \]

\[ \textbf{[IC]}: \alpha u_{nd}(P_s, P_p) + \tilde{\alpha} u_d(P_s, P_p) - \lambda(P_p, P_s, \alpha) \geq \\
\alpha u_{nd}(P'_s, P_p) + \tilde{\alpha} u_d(P'_s, P_p) - \lambda(P_p, P'_s, \alpha), \forall P'_s \leq P_p. \]

The \textbf{[IR]} constraint implies that the maximum payment \( \lambda \) that the principal can get is \( \alpha u_{nd}(P_s, P_p) + \tilde{\alpha} u_d(P_s, P_p) \). Thus, the principal can offer a contract

\[ \lambda^*_p(P_p, P_s, \alpha) = \alpha u_{nd}(P_s, P_p) + \tilde{\alpha} u_d(P_s, P_p) \]

\[ = \log \left( \frac{(1 + P_p + P_s)\alpha}{(1 + P_s)^\alpha} \right). \]

which satisfies the \textbf{[IR]} constraint with equality. The \textbf{[IC]} constraint is now trivially satisfied. With this, the principle’s optimization problem reduces to

\[ \max_{P_p, P_s, \alpha} \alpha u_d(P_p, P_s) + \tilde{\alpha} u_{nd}(P_p, P_s) \]

\[ + \alpha u_{nd}(P_p, P_s) + \tilde{\alpha} u_d(P_p, P_s) \]

which further reduces to

\[ \max_{P_p, P_s, \alpha} \log (1 + P_p + P_s). \]

The solution for this is clearly \((P^*_p, P^*_s) = (P^*_p, P^*_s)\) which says that the operating point is on the Pareto-efficient frontier. More interestingly, the solution doesn’t depend on the time sharing parameter \( \alpha \). This implies that no matter what parameter \( \alpha \) the secondary user picks, the principal will still achieve his maximum payoff over all feasible operating points \((P_p, P_s, \alpha)\).

The equilibrium rates of the primary and secondary are given by

\[ R^*_p = \log \left( \frac{(1 + P^*_p + P^*_s)\alpha(1 + P^*_p)\alpha}{(1 + P^*_p)^\alpha} \right) \]

and

\[ R^*_s = \log \left( \frac{(1 + P^*_p + P^*_s)\alpha(1 + P^*_s)\alpha}{(1 + P^*_s)^\alpha} \right). \]

The net payoffs at equilibrium are

\[ (U^*_p, U^*_s) = \left( \log (1 + P^*_p + P^*_s), 0 \right), \]

that is, the principle is able to extract the entire surplus from the agent.

Case (ii): Secondary user is the Principal: In this case we can set up the optimization problem in the same way as in Case (i), with two differences. Here, the payment is made by the principal and the agent (primary) has a reservation utility \( \log(1 + P^*_p) \).

\[ \textbf{CD-OPT-TS2:} \]

\[ \max_{P_p, P_s, \alpha, \lambda} \alpha u_d(P_s, P_p) + \tilde{\alpha} u_{nd}(P_s, P_p) - \lambda(P_p, P_s, \alpha) \]

subject to:

\[ \textbf{[IR]}: \alpha u_{nd}(P_s, P_p) + \tilde{\alpha} u_d(P_s, P_p) + \lambda(P_p, P_s, \alpha) \geq \hat{U}_p \]

\[ \textbf{[IC]}: \alpha u_{nd}(P_p, P_s) + \tilde{\alpha} u_d(P_p, P_s) + \lambda(P_p, P_s, \alpha) \geq \\
\alpha u_{nd}(P_p', P_s) + \tilde{\alpha} u_d(P_p', P_s) + \lambda(P_s, P_p', \alpha), \forall P'_p \leq P_p. \]

The \textbf{[IR]} constraint implies that the minimum payment \( \lambda \) that the primary need to get is \( \hat{U}_p - \alpha u_{nd}(P_p, P_s) - \tilde{\alpha} u_d(P_p, P_s) \) Thus, the principle (secondary) can offer a contract

\[ \lambda^*_s(P_p, P_s, \alpha) = \hat{U}_p - \alpha u_{nd}(P_p, P_s) - \tilde{\alpha} u_d(P_p, P_s) \]

\[ = \log \left( \frac{(1 + P_p + P_s)\alpha(1 + P^*_p)\alpha}{(1 + P^*_p + P_s)^\alpha(1 + P^*_p)^\alpha} \right). \]

which satisfies both \textbf{[IR]} and \textbf{[IC]} constraints and minimizes his payment as well. Substituting \( \lambda^*_s(P_p, P_s, \alpha) \) in the optimization problem of the principal reduces it to

\[ \max_{P_p, P_s, \alpha} \log (1 + P_p + P_s). \]

The solution is, clearly, \((P^*_p, P^*_s) = (P^*_p, P^*_s)\). As in the previous case, the solution is Pareto-efficient frontier and the it doesn’t depend on the time sharing parameter \( \alpha \). The equilibrium rates of the principal and agent are given by

\[ R^*_p = \log \left( \frac{(1 + P^*_p + P^*_s)\alpha(1 + P^*_p)\alpha}{(1 + P^*_p)^\alpha} \right) \]

and

\[ R^*_s = \log \left( \frac{(1 + P^*_p + P^*_s)\alpha(1 + P^*_s)\alpha}{(1 + P^*_s)^\alpha} \right). \]

The net payoffs at equilibrium are

\[ (U^*_p, U^*_s) = \left( \log (1 + P^*_p), \log \left( 1 + \frac{P^*_s}{1 + P^*_p} \right) \right), \]

The values \( \alpha = 0 \) and \( \alpha = 1 \) will give the rates and utilities in the points \( A \) and \( B \).

We now formalize the above discussion.

**Theorem 2:** Consider a Gaussian interference channel with a primary and a secondary, with a power budgets \( \hat{P}_p \) and \( \hat{P}_s \), one user acting as principal and the other as agent, both of whom cooperatively do successive-interference cancellation with time-sharing. If the primary user acts as the principal, he offers contracts \( \lambda^*_p(P_p, P_s, \alpha) = \alpha u_{nd}(P_p, P_p) + \tilde{\alpha} u_d(P_p, P_p) \) to the secondary. If the secondary user acts as the principal, he offers contracts \( \lambda^*_s(P_p, P_s, \alpha) = \hat{U}_p - \alpha u_{nd}(P_p, P_p) - \tilde{\alpha} u_d(P_p, P_p) \). Then, we can establish the following:

(i) These contracts are incentive-compatible and satisfy individual rationality. (ii) The operating point can be anywhere on the pareto-optimal boundary, depending on the value of the time sharing parameter \( \alpha \). (iii) If the primary acts as the principal, the payoffs are \((U^*_p, U^*_s) = (\log (1 + P^*_p + P^*_s), 0)\). If the secondary acts as principal, the payoffs are \((U^*_p, U^*_s) = \left( \log (1 + P^*_p), \log \left( 1 + \frac{P^*_s}{1 + P^*_p} \right) \right)\). (iv) In all cases, the social welfare \( S(P_p, P_s) = R_p + R_s \) is maximized and equals \( \log (1 + P^*_p + P^*_s) \).

A. Stability of the contract mechanisms

Once the principal and agent have agreed on a time-sharing contractual agreement to transmit at powers \((P_p, P_s) = (P^*_p, P^*_s)\) and a time sharing parameter \( \alpha \), the agent makes the agreed upon payment to the principal. The question now is whether the optimal time-sharing spectrum contracts have
a moral hazard with hidden action, i.e., can either of them strictly improve their payoff by violating the agreement when their actions cannot be observed by the other?

We note that at the optimal time-sharing contractual agreements, both users transmit at full power available to them. By reducing their transmission powers, they can only decrease the rate that they achieve without affecting the other users. Thus, deviation in transmission is not a concern. However, the players can still renege on the agreed upon $\alpha$ if they can improve their payoff. Thus, if a player reneges on an agreed time-sharing $\alpha$, it means it would start to act as a dominant (non-dominant) user when the other player believes it is acting as a non-dominant (dominant) user thus potentially causing successive-interference cancellation to fail. We now argue that this is not possible, and the optimal time-sharing spectrum contracts are stable against such deviations.

**Proposition 3:** An optimal time-sharing contractual agreement $(\lambda^*, \bar{P}_p, \bar{P}_a, \alpha)$ is a Nash equilibrium.

**Proof:** Without loss of generality assume that the principal and agent have agreed to act as dominant and non-dominant user at some specific time. Thus, the agent should transmit at rate at most $R_{nd}(\bar{P}_a, \bar{P}_d)$ and the principal transmits at rate $R_p = R_d(\bar{P}_d, \bar{P}_{nd})$. Now suppose that the agent transmits at some rate $R_a > R_{nd}$. However, the SINR seen by the agent’s receiver is $\frac{R_a}{1 + R_a \bar{P}_{nd}}$. Since $\alpha > R_{nd} = \log(1 + \frac{\bar{P}_{nd}}{1 + \bar{P}_p})$, to decode the agent’s transmitted signal, it must first decode and cancel the principal’s signal. It can decode the principal’s signal only if $R_p \leq \log(1 + \frac{\bar{P}_p}{1 + \bar{P}_p})$ but this is not true. Hence, the agent’s receiver fails to decode its own transmitted signal. Now, if the agent is transmitting at rate $R_a = R_{nd}(\bar{P}_{nd}, \bar{P}_d)$, then the principal’s rate will only reduce it if acts as a non-dominant user and does not decode the agent’s signal first. Thus, it is clear that neither the principal nor the agent can strictly improve their payoffs by deviating from their agreed roles in successive-interference cancellation. Hence, the optimal time-sharing contractual agreement is a Nash equilibrium.

VI. DISCUSSION AND FURTHER WORK

This paper presents a new approach to incentivized spectrum sharing for licensed bands in cognitive radio systems. Building on multi-user information theory [14], cooperative communication schemes are fairly well-developed [13]. It is well-known that significant gains in network capacity and performance can be achieved by their deployment. One of the impediments is that such schemes require cooperation between various users each of whom is an independent and selfish user. There is little justification in assuming that users will expend their resources and particularly battery power to aid communications of other users. Thus, studies of strategic interaction between users in communication systems have largely obtained negative results [7]. Others who have tried to improve the incentives so that the system operates more efficiently have devised mechanisms (e.g., auction mechanisms) that are not practical for wireless systems. In comparison, we have proposed an incentive mechanism approach that not only enables deployment of sophisticated cooperative communication schemes (such as SIC) but also is natural and easy to implement. Furthermore, it leads to very robust results. In particular, the contract mechanism always yields social welfare or sum-rate maximizing rate allocations. Moreover, we are able to show that the spectrum contractual agreements are a Nash equilibria and thus the mechanism is stable against unilateral deviation.

The results we have presented in this paper lead to a number of questions. For example, we have assumed that the utility functions of the users are equal to their rates. Would the results still hold if the the utilities are some concave increasing function if the rates? Can we extend to the setting when there are multiple secondary users and one primary user? Can we obtain stable time-sharing contracts in the non-symmetric channel case? We have also ignored the issue of fairness.

The setting we have considered in this papers is that of complete information, i.e., channel gains and power budget are common knowledge. However, the asymmetric information scenario when channel gains of other users are not known, and furthermore, the actions (i.e., the transmit powers used) by the other players cannot be observed, is a lot more realistic and interesting. This can be viewed as a double sided moral-hazard adverse selection problem that to our knowledge has not been solved even in the game theory literature [15], [2].

As part of further work, we hope to address the above issues and develop a similar framework for the asymmetric information scenario.

REFERENCES


