

Financial Markets and Wages*

Claudio Michelacci
CEMFI and CEPR

Vincenzo Quadrini
University of Southern California, CEPR and NBER

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Abstract

We study a labor market equilibrium model in which firms sign optimal long-term contracts with workers. Firms that are financially constrained offer an increasing wage profile: They pay lower wages today in exchange of higher future wages once they become unconstrained. Because constrained firms grow faster, the model predicts a positive correlation between the growth of wages and the growth of the firm. Under some conditions, the model also generates a positive relation between firm size and wages. Using matched employer-employee data from Finland and the National Longitudinal Survey of Youth for the US, we show that the key dynamic properties of the model are supported by the data.

JEL classification: G31, J31, E24

Key words: Investment financing, long-term contracts, wages

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1 Introduction

This paper studies how financial markets frictions affect the wage policy of the firm and, through this mechanism, how the dynamics of wages is related to the dynamics of the firm.

We develop a model in which firms sign optimal long-term (implicit) contracts with workers as in Harris and Holmstrom (1982), Wright (1988) and Burdett and Coles (2003). Due to limited enforceability, external investors are willing to finance the firm only in exchange of collateralizable capital. If the funds supplied by external investors are limited—that is, the firm is financially constrained—the optimal wage contracts are characterized by increasing wage profiles until the firm becomes unconstrained. By paying lower wages today, the firm generates higher cash flows in the current period, *implicitly* borrowing from workers. This allows the firm to grow faster. Therefore, a key prediction of the model is that the start-up wage of a new hired worker is negatively correlated to the future growth rate of the firm. Another prediction is that wages grow faster in fast growing firms. As we will see below, these predictions of the model find strong empirical support in the data.

In addition to generating clear predictions about the relation between the dynamics of the firm and the dynamics of its wages, the model also generates a positive cross-sectional relation between firm size and wages, which is a well-known empirical finding. In this respect our paper complements previous theoretical studies such as Burdett and Mortensen (1998) and Zabojnik and Bernhardt (2001), that provide explanations for this empirical finding but do not rely on financial markets frictions.

An important theoretical question is why firms are able to borrow (implicitly) from workers beyond what they can borrow from external investors. There are two features in the model that explain this. First, if a worker quits, the firm loses part of the accumulated capital. This could derive from recruiting costs, training expenses and/or worker's productivity enhanced through learning. The firm's loss of valuable capital endows the worker with a punishment tool which is not available to external investors. Second, a worker provides effort in the working place only if he or she believes that the effort will be rewarded by the firm. But when the firm reneges its wage promises, the worker will also expect the firm to renege future promises and it becomes optimal to quit. The threat of quitting guarantees that the firm does not renege the long-term wage contract. This mechanism provides the

worker with an additional (implicit) form of collateral that is not available to external investors.

To evaluate the key properties of the model, we use matched employer-employee data from Finland, available for the period 1988-2002, and the 1979-2002 National Longitudinal Survey of Youth (NLSY) for the United States. Using these two data sets, we show that the within-job growth of individual wages is positively correlated to the growth rate of the firm even after controlling for other dynamic features of the firm. Furthermore, the start-up wage of new hired workers is negatively correlated to the future growth rates of the firm. These are two of the dynamic predictions of our model that are induced by financial markets frictions.

The plan of the paper is as follows. In the next section we review the main empirical contributions that are relevant for our paper. Section 3 describes the basic theoretical framework and characterizes the firm's dynamics. Section 4 extends the model to allow for firms' and workers' turnover, derives the labor market equilibrium and studies its properties quantitatively. Section 5 describes how the long-term contract can be sustained as a sub-game perfect equilibrium of the strategic interaction between the firm and the worker. Section 6 conducts the empirical analysis and Section 7 concludes.

2 Existing empirical findings

There are two main goals in this paper. The first goal is to characterize the dynamic properties of firms and wages induced by financial markets frictions. The second goal is to show how these dynamic properties reproduce a set of empirical regularities shown by previous studies, with special attention to the firm size-wage relation. In this section we will describe these regularities. Then, in Section 6, we will provide new empirical evidence about the relation between the dynamics of wages and the dynamics of firms that is consistent with the predictions of our model.

1. *Larger firms pay higher wages.* The positive relation between firm size and wages is robust to the introduction of several controls for worker's and firm's characteristics. See Brown and Medoff (1989) and Oi and Idson (1999) for a review. It does not arise just because larger firms employ more skilled workers. Abowd and Kramarz (2000) report that, both in France and in the US, variation in firms' characteristics explains about 70 per cent of the firm

size-wage differential.¹

2. *The link between firm age and wages is not clear-cut.* Doms, Dunne, and Troske (1997), Troske (1999), and Brown and Medoff (2003) find that the effect of firm's age on wages is positive without controlling for worker's characteristics but it becomes negative (albeit not significant) after controlling for worker's experience and firm's size.

3. *Fast growing firms pay lower wages.* Bronars and Famulari (2001) and Hanka (1998) report that firm's growth (in terms of employment and sales) has a negative effect on wages in a regression that controls for several workers' and firms' characteristics, including firm size.

4. *Firms that are in financial distress have lower employment and pay lower wages.* Nickell and Wadhvani (1991) document the negative relation between debt and employment. Other studies provide some evidence that indicators of financial pressure are associated with lower wages. See Nickell and Nicolitsas (1999), Hanka (1998), Blanchflower, Oswald, and Garrett (1990).

The first and second findings relate the level of wages to the size of the firm and its age. The third finding relates the level of wages to the dynamics of the firm. The fourth suggests that financial factors could be important for the wage policy of the firm. In the following sections we will show how financial factors affect the wage policy of the firm in a way that is consistent with the above empirical findings.

3 The basic model

We start describing a simple version of the model to illustrate the key dynamics of firms and wages. The analysis of this simple model will facilitate the understanding of the general model studied in Section 4.

Consider a risk-neutral infinitely lived entrepreneur with initial wealth a_0 and with lifetime utility $E_0 \sum_{t=0}^{\infty} \beta^t c_t$, where β is the intertemporal discount factor and c_t is consumption.

The entrepreneur has the managerial skills to run an investment project that generates revenues $y = A \cdot N$. The variable N denotes the number of hired workers and A is a constant. The project is subject to the capacity

¹This study revises the previous estimate reported in Abowd, Kramarz, and Margolis (1999) that was based on an approximation of the estimation problem.

constraint $N \leq \bar{N}$. In the general model studied in Section 4, the capacity constraint \bar{N} is allowed to differ across entrepreneurs or firms.

The employment of each worker requires two types of fixed investment: fungible investment, κ_f , and worker-specific investment, κ_w . The first investment, κ_f , has an external value and can be resold at no cost. The second type, κ_w , represents the cost incurred by the firm for recruiting and training a new worker. This is lost if the worker quits or is fired. We will denote by $\kappa = \kappa_f + \kappa_w$ the sum of the two components. The total capital accumulated at the end of time t by a firm created at time zero is $\kappa \sum_{\tau=0}^t n_\tau$, where n_τ is the number of workers hired at time τ (who start producing at time $\tau + 1$). The output produced by the firm at $t + 1$ is $y_{t+1} = A \sum_{\tau=0}^t n_\tau$.

Workers are infinitely lived with lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t [U(c_t) + \ell_t], \quad U(c_t) = \frac{c_t^{1-\sigma} - 1}{1 - \sigma},$$

where β is the discount factor, σ is the coefficient of relative risk aversion, c_t is consumption, and $\ell_t \in \{0, \bar{\ell}\}$ denotes the utility of leisure which is forgone when the worker provides working effort. The assumption that there is some forgone utility is relevant only for the analysis of renegotiation studied in Section 5. In equilibrium workers provide effort and in the analysis that precedes Section 5 we impose $\ell_t = 0$.

Workers do not save and can not borrow by pledging their future labor income. Therefore, consumption is simply equal to their wages.²

Funds are provided by *investors* who are risk-neutral and discount future payments at rate r . The individual supply is infinitesimal, but the total number of investors is large enough to guarantee that the aggregate supply of funds is perfectly elastic at rate r . We assume that $\beta \leq 1/(1+r)$ so that internal financing does not dominate external financing.

The investment to employ a worker, κ , is what creates the financial need. Using the renegotiation idea of Hart and Moore (1994) and Kiyotaki and Moore (1997), the entrepreneur can borrow only the amount that can be collateralized, that is, κ_f . Since the collateral must also guarantee the interests on the loan, the firm can borrow at most $\bar{\kappa}_f = \kappa_f/(1+r)$, per each worker.

²The assumption that workers do not save is without loss of generality. Because we assume that the return from savings is smaller than $1/\beta - 1$ and, as we will see, wages do not decrease over time, the worker would not save even if he or she were allowed to. For the general model of Section 4, it is further required that β is sufficiently small.

The borrowing limit, then, can be written as $b_t \leq \bar{\kappa}_f \sum_{\tau=0}^t n_\tau$, where b_t denotes the debt contracted at time t . We will show in Section 5 that under certain assumptions this is the only feasible contract with investors.

When a worker is hired, the firm signs a long-term contract that specifies the whole sequence of wages. The assumption of long-term wage contracts is supported by the empirical studies of Beaudry and DiNardo (1991) and McDonald and Worswick (1999). By assuming that the labor market is *competitive*, the initial lifetime utility provided to the worker is equal to the utility earned by re-entering the market. This utility, denoted by q_{res} , is for the moment exogenous. We will make it endogenous in Section 4.

3.1 The firm's problem

We start analyzing the optimization problem assuming that firms and workers commit to the long-term contracts. In Section 5 we will describe the conditions under which the parties (firms and workers) never renege on their promises and the contract can be supported as a sub-game perfect equilibrium of the repeated game played by the firm with each individual worker.

Let $\{w_{t,t+j}\}_{j=1}^\infty$ be the sequence of wages that the firm promises to the workers hired at time t . Here $w_{t,t+j}$ denotes the wage paid at time $t+j$ to workers hired at time t . Then the total wage payments at time $t+1$ are $\sum_{\tau=0}^t n_\tau w_{\tau,t+1}$. Let a_t denote the *net worth* at the end of period t —that is, after production and after the payment of wages and interests. The sum of the firm's net worth, a_t , and debt financing, b_t , equals the sum of firm's capital, $\kappa \sum_{\tau=0}^t n_\tau$, and dividend payments, d_t . Thus, $d_t = a_t + b_t - \kappa \sum_{\tau=0}^t n_\tau$.

Given the initial assets a_0 , the firm maximizes the discounted value of the entrepreneur's consumption, which always equals dividends since the entrepreneur is at least as impatient as the market, $\beta \leq 1/(1+r)$. Thus, at time zero, the firm chooses the whole sequence of debt, employment and wages, $\left\{b_t, n_t, \{w_{t,t+j}\}_{j=1}^\infty\right\}_{t=0}^\infty$, to solve the problem:

$$V(a_0) = \max \sum_{t=0}^{\infty} \beta^t \left(a_t + b_t - \kappa \sum_{\tau=0}^t n_\tau \right) \quad (1)$$

subject to

$$a_t + b_t - \kappa \sum_{\tau=0}^t n_\tau \geq 0, \quad (2)$$

$$b_t \leq \bar{\kappa}_f \sum_{\tau=0}^t n_\tau, \quad (3)$$

$$\sum_{j=1}^{\infty} \beta^j U(w_{t,t+j}) \geq q_{res}, \quad (4)$$

$$a_{t+1} = (\kappa + A) \sum_{\tau=0}^t n_\tau - \sum_{\tau=0}^t n_\tau w_{\tau,t+1} - (1+r)b_t, \quad (5)$$

$$\sum_{\tau=0}^t n_\tau \leq \bar{N} \quad (6)$$

which all have to hold for all $t \geq 0$. Constraint (2) imposes the non-negativity of dividends. This results from the limited liability of the entrepreneur together with the non-negativity of consumption. Constraint (3) imposes the borrowing limit and (4) is the worker's participation constraint. This imposes that the sequence of wages offered to each cohort of new recruits cannot be smaller than the reservation utility q_{res} . This constraint should be imposed not only when the worker is hired, but also in all future periods. However, as we will show below, wages never decrease. Therefore, if the participation constraint is satisfied when the worker is hired, it will also be satisfied at any future date. Constraint (5) defines the law of motion for the end-of-period net worth and the last constraint imposes the capacity constraint.

Let γ_t and $\lambda_t n_t$ be the lagrange multipliers for constraints (2) and (4), respectively. Appendix A shows that the first order condition with respect to $w_{\tau,t}$ is:

$$\lambda_\tau U_c(w_{\tau,t}) = 1 + \gamma_t, \quad (7)$$

where U_c denotes the marginal utility of consumption. The variable λ_τ is the marginal cost to the firm of providing one unit of utility to a worker hired at time τ . Thus, $\lambda_\tau U_c(w_{\tau,t})$ represents the marginal cost of reducing wages. The term $1 + \gamma_t$ is the value of one additional unit of internal funds. Therefore, equation (7) says that the optimal wage policy of the firm is such that the marginal cost of reducing wages is equal to the marginal value of internal funds. In other words, the firm 'borrows' from a worker until the cost of borrowing is equal to the marginal value of internal funds.

The multiplier γ_t captures the tightness of financial constraints and depends on the firm's net worth a_t . As the firm retains earnings, its assets increase over time and the variable γ_t converges to zero. Then, equation (7) implies that:

Property 1 *The wage received by each worker grows over time until the firm becomes unconstrained, that is, $\gamma_t = 0$.*

Equation (7) also implies that the ratio of marginal utilities between workers of different cohorts remains constant over time. If we consider (7) for two cohorts, indexed by τ_1 and τ_2 , and divide side by side we obtain

$$\frac{U_c(w_{\tau_1,t})}{U_c(w_{\tau_2,t})} = \frac{\lambda_{\tau_2}}{\lambda_{\tau_1}}.$$

Since the right-hand-side does not change over time, this condition implies:

Property 2 *The ratios of marginal utilities for different cohorts of workers remain constant over time.*

In the next section we take advantage of this property to rewrite the problem recursively with a limited number of state variables. The recursive formulation will be convenient in the next section when we study the general model with entry and exit.

3.2 Recursive formulation of the firm's problem

Let $q_{\tau,t} = \sum_{j=1}^{\infty} \beta^j U(w_{\tau,t+j})$ be the lifetime utility promised at the end of time t to a worker hired at time τ , with $\tau \leq t$. Notice that $q_{\tau,t}$ follows the recursive form

$$q_{\tau,t} = \beta [U(w_{\tau,t+1}) + q_{\tau,t+1}] \quad (8)$$

with $q_{\tau,\tau} = q_{res}$.

With the utility function $U(c) = (c^{1-\sigma} - 1) / (1 - \sigma)$, Property 2 implies that the ratios of wages paid to workers of different cohorts remain constant over time.³ This property also implies that the ratios of lifetime utilities promised to different cohorts of workers remain constant over time. Thus, if we consider the last and the first cohort of workers, we have that, at any given point in time, their relative lifetime utilities and wages are linked by

$$\frac{q_{t,t}}{q_{0,t}} = \left(\frac{w_{t,t+1}}{w_{0,t+1}} \right)^{1-\sigma} = \frac{q_{res}}{q_{0,t}},$$

³This implies that cohort of workers who earn more on entry maintain their advantage over time. The existence of these cohort effects in the wage policy of the firm is documented by Baker, Gibbs, and Holmstrom (1994).

where the last equality uses the fact that $q_{t,t} = q_{res}$. Inverting the second equality provides an expression for the wage ratio between the cohort hired at time t and the cohort hired at time zero, which reads as

$$\frac{w_{t,t+1}}{w_{0,t+1}} = \left(\frac{q_{res}}{q_{0,t}} \right)^{\frac{1}{1-\sigma}} = \psi(q_{0,t}).$$

From now on we omit the zero subscript to identify the first cohort of workers. Therefore, w_t and q_t denote the time- t wage and promised utility of the first cohort of workers (the oldest one). The total wages paid by the firm at time t can be written as $H_t w_t$, where

$$H_t = \sum_{\tau=0}^{t-1} \psi(q_\tau) n_\tau,$$

which evolves recursively as

$$H_{t+1} = H_t + \psi(q_t) n_t. \quad (9)$$

Once we know H_t and the utility promised to the first cohort of workers, q_t , the determination of the whole wage structure paid by the firm at time $t+1$ only requires the determination of the wage for the first cohort of workers, that is w_{t+1} . This allows us to write the firm's problem recursively with a limited number of state variables as follows:

$$V(a, q, N, H) = \max_{b, w', q', N' \leq \bar{N}} \left\{ d + \beta V(a', q', N', H') \right\} \quad (10)$$

subject to

$$d = a + b - \kappa N' \geq 0, \quad (11)$$

$$b \leq \bar{\kappa}_f N', \quad (12)$$

$$q = \beta [U(w') + q'], \quad (13)$$

$$a' = (\kappa + A)N' - H'w' - (1+r)b, \quad (14)$$

$$H' = H + \psi(q)(N' - N). \quad (15)$$

The variable N denotes the current employment of the firm and the prime denotes the next period value. Thus $N' - N$ is the change in employment, that

is, the number of workers hired in the current period (who start producing in the next period). Constraints (11) and (12) impose the non-negativity of dividends and the borrowing limit, respectively. Equation (13) is the promise-keeping constraint for the first cohort of workers. Finally, equations (14) and (15) are the laws of motion for the states a and H , respectively.

Let γ and $\lambda H'$ denote the lagrange multipliers associated with constraints (11) and (13), respectively. Appendix B shows that the first order conditions for the above problem imply that

$$\lambda U_{w'} = 1 + \gamma', \quad (16)$$

$$\lambda = \lambda'. \quad (17)$$

The first condition is analogous to (7) while the second says that the lagrange multiplier for the worker's participation constraint is constant over time.

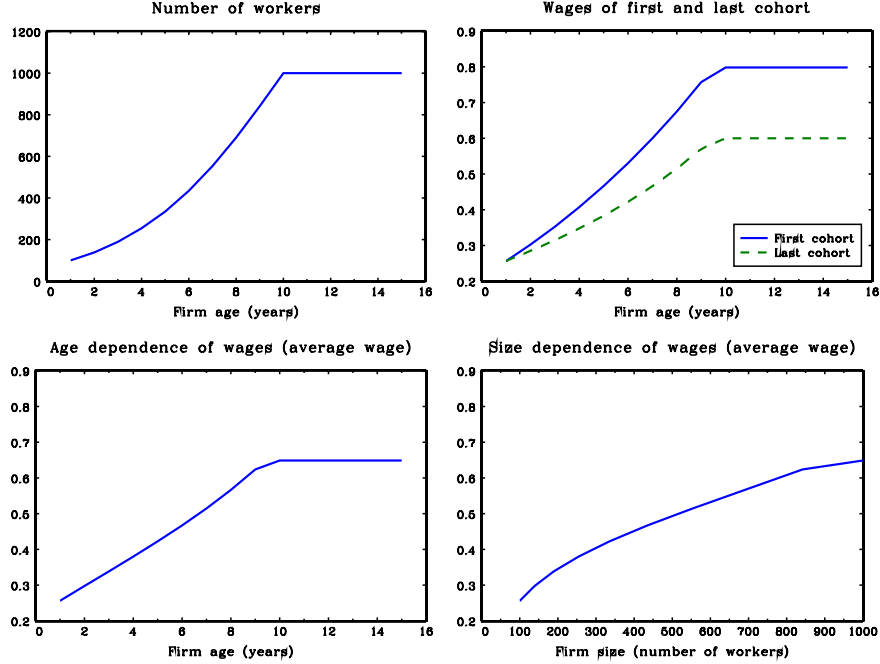
These two conditions characterize the wage dynamics of the firm. As observed in the previous section, the lagrange multiplier γ decreases over time until it becomes zero. From equation (16) we can see that the wage paid to the first cohort of workers increases over time until $\gamma' = 0$. Because the wages paid to all other cohorts of workers are proportional to the wage paid to the first cohort, we also have that the average wages increase over time until $\gamma' = 0$. Wages differ across workers of different cohorts. In fact, because all workers start with $q = q_{res}$, after which the promised utility grows over time, older workers receive higher wages than younger workers. Once the firm becomes unconstrained, that is, $\gamma = 0$, the firm would like to increase employment beyond \bar{N} , but the capacity constraint binds.

3.3 A numerical example

Figure 1 shows some of the properties of the model with a numerical example. The parameter values are as follows: $r = 0.03$, $\beta = 0.934$, $\sigma = 1$, $q_{res} = U(0.6)/(1 - \beta)$, $\bar{N} = 1,000$, $A = 1$, $\kappa_f = 1$ and $\kappa_w = 1.8$. We will then consider several values of a_0 . The numerical example considered here is provided only for illustrative purposes. A formal quantitative exercise will be conducted in Section 4.3, after the specification of the general model.

The first panel of Figure 1 plots the employment dynamics. The firm starts with an initial employment of 100 workers and then gradually grows until it reaches the optimal size $\bar{N} = 1,000$. The transition takes place in 11 periods. The second panel plots the wage profile of the first cohort of workers

Figure 1: Employment dynamics and wage patterns over age and size.



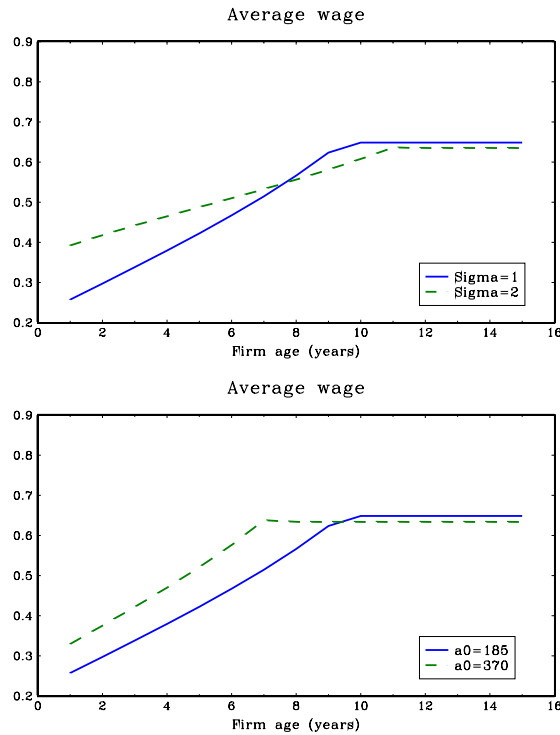
(those hired at time 0) and the initial wage paid to newly hired workers. The wage profile of the first cohort (continuous line) is increasing until the firm reaches the unconstrained status. The dashed line shows the start-up wage earned by workers hired in different periods. As the firm gets closer to the optimal scale, it offers higher initial wages, and therefore, the wage profile of newer workers is less steep overall. Therefore, growing firms pay wages that are lower initially and grow overtime. This property will be tested in the empirical analysis conducted Section 6.

The third panel plots the average wage paid by the firm as a function of its age and the fourth panel the average wage as a function of its size (measured by the number of employees). The average wage increases with the size and age of the firm. This is a direct consequence of the fact that, when the firm is young and constrained, it operates at a suboptimal scale and offers an increasing profile of wages.

The concavity of the utility function, σ , and the initial assets, a_0 , play an important role in shaping the dynamics of wages. The first panel of Figure

2 shows that the size dependence of wages is stronger for lower degrees of concavity. Clearly, with a smaller σ the worker is more willing to accept a non-flat consumption profile and it becomes cheaper for the firm to borrow from workers. In the extreme case in which $\sigma = 0$ (linear utility), the financing premium required by the workers is zero. In this case the firm would pay zero wages until it reaches the optimal scale.

Figure 2: Wage dynamics for different utility curvatures and initial assets.



The initial assets of the firm also play an important role. For given \bar{N} , smaller values of a_0 imply tighter constraints, that is, the firm starts with a smaller scale. This also implies that the firm has a greater incentive to rely on its wage policy to finance its growth. As a result, it pays smaller initial wages as shown in the second panel of Figure 2. The dependence of the wage dynamics on a_0 will be important for generating a firm size-wage relation in the general model studied in the next section.

4 General model and simulated regressions

In the simple model studied so far, the profile of wages is fully captured by the age of the firm. Therefore, once we control for age, firm size becomes irrelevant. However, in a cross section of firms, size could have an independent effect because there are other sources of heterogeneity. In particular, firms could differ in the capacities \bar{N} and in the initial assets a_0 . To capture this additional source of heterogeneity, we need to extend the model and specify the whole industry structure, including entrance and exit.

We extend the model in three main directions. We allow for (i) firm heterogeneity in technology \bar{N} and initial wealth a_0 ; (ii) firm entry and exit; and (iii) turnover of workers. The first extension allows us to generate a size distribution of firms similar to the data. The second guarantees that at each point in time there is a fraction of firms that are financially constrained. The third is introduced for robustness.

4.1 Model description

At each point in time, workers die with probability $1 - \eta$ and firms become unproductive with probability $1 - p$. The workers of exiting firms re-enter the market as unskilled workers, that is, they require a new investment κ_w . The exit of firms and the subsequent entrance of new firms (entrepreneurs) guarantee that there are constrained firms at any point in time.

Firms are heterogeneous in the project capacity \bar{N} , which is constant over time. New firms draw \bar{N} from the distribution $\Gamma(\bar{N})$. Their initial wealth is related to the project capacity as follows:

$$a_0 = \alpha \cdot \bar{N}^\rho.$$

This is a simple way to formalize the idea that the initial wealth of the entrepreneur is correlated with the project capacity. For instance, entrepreneurs with more promising projects may be able to raise more funds initially by pooling a larger number of founders. Alternatively, we could think that the probability of drawing large capacity projects increases with the ability of the entrepreneur, which in turn could be correlated with the initial wealth.

The parameters α and ρ determine the degree of financial tightness for new firms, as a function of the project capacity. Given the linearity of the production function and the borrowing limit, the financial tightness of a new

firm can be defined as:

$$FTI \equiv \frac{(\kappa - \bar{\kappa}_f) \cdot \bar{N}}{a_0} = \frac{(\kappa - \bar{\kappa}_f) \cdot \bar{N}^{1-\rho}}{\alpha},$$

where FTI stands for ‘Financial Tightness Index’. The numerator is the total capital that must be financed internally when the firm operates at the optimal scale \bar{N} . The denominator is the initial net worth. When this ratio is greater than 1 the firm is financially constrained. Lower values of α increases the financial tightness for all new firms while the parameter ρ differentiates the tightness across different types of firms. When $\rho = 1$, the tightness is independent of the firm’s capacity. When $\rho < 1$, firms with larger capacities face tighter constraints initially.

We also allow for job-to-job mobility by making the following assumptions. In each period a firm matches with m workers employed in other firms, who can transfer the worker-specific capital and do not require a new investment κ_w . As in Burdett and Vishwanath (1988), the matching technology is balanced in the sense that the number of matched workers is proportional to its size, that is, $m = \chi N$, where χ is constant. The worker-specific capital can be transferred only to firms with the same characteristics which, in this environment, are given by the project capacity \bar{N} and its age.⁴

After matching with the worker, the firm makes a *take-it or leave-it* offer. Offers are private information and there is a cost to make an offer verifiable to the employer. This assumption implies that the worker is unable to let the current and new employers compete over his or her skills. Anticipating this, the contacting firm offers a contract that gives to the worker the same utility q received in the incumbent firm. See Hashimoto (1981) and Anderlini and Felli (2001) for a formal characterization of these types of equilibria.⁵

⁴Alternatively, we can assume that the worker-specific capital can be transferred with some probability to any firm. However, this probability is higher for firms with the same characteristics. Assuming that the contact ability of each firm is limited, firms will concentrate their search among firms with the same characteristics.

⁵To make the offer verifiable to the employer, the worker needs to exercise some effort. The current employer would match the external offer if the worker demands to renegotiate the contract. However, because the renegotiation of the contract requires effort from the worker, the utility from renegotiating is smaller than the utility from accepting the external offer. This generates an hold-up problem and the worker never tries to renegotiate. Anticipating this, the poaching firm offers an expected utility only slightly higher than the utility that the worker earns by staying with the current employer.

To keep the model tractable we treat each firm as if it employs a continuum of workers. This implies that the death probability $1 - \eta$ is also the fraction of workers who die in an individual firm and χ is the fraction of workers contacted by other firms.

4.2 Optimization problem for the general model

Let N be the number of workers employed by a surviving firm. Of those, $(1 - \eta)N$ will be lost because of death. Moreover, χN workers will be lost because contacted by other firms. On the other hand, the firm matches with χN workers with transferable skills. Therefore, the only net employment loss is $(1 - \eta)N$. Notice that, by treating the firm as if it employs a continuum of workers, the utilities offered to the workers contacted by the firm are equal to the utilities of the workers who switch to another employer.

Limiting the analysis to steady state equilibria in which the price for unskilled workers q_{res} is constant, the promise-keeping constraint for a hired worker can be written as follows:

$$q_{\tau,t} = \beta \left[U(w_{\tau,t+1}) + p \cdot \eta \cdot q_{\tau,t+1} + (1 - p) \cdot \eta \cdot q_{res} \right].$$

We are assuming that the viability of the project and the separation of the worker is observed after paying the current wage (but before the new investment). Consequently, the current wage is not renegotiated.

For the analysis that follows it will be convenient to rescale the promised utility $q_{\tau,t}$ by the constant term $\beta(1 - p)\eta q_{res}/(1 - \beta p\eta)$, that is,

$$z_{\tau,t} = q_{\tau,t} - \frac{\beta(1 - p)\eta q_{res}}{1 - \beta p\eta}.$$

Using this transformation, the promise-keeping constraint can be written as:

$$z_{\tau,t} = \beta \left[U(w_{\tau,t+1}) + p\eta z_{\tau,t+1} \right]. \quad (18)$$

Since the ratios of marginal utilities between different cohorts of workers is constant over time (*i.e.* Property 2 remains valid), the wage ratio between a new worker hired without skills and the first cohort of workers, also hired without skills, satisfies:

$$\frac{w_{t,t+1}}{w_{0,t+1}} = \left(\frac{z_{t,t}}{z_{0,t}} \right)^{\frac{1}{1-\sigma}} = \psi(z_t)$$

which identifies the constant relative wage earned by the workers hired unskilled at time t . Notice that we maintain the convention of omitting the zero subscript to identify the first cohort of workers.

As in the previous section, we use the variable H to summarize the compensation structure of the firm, which now evolves as follows:

$$H' = \eta H + \psi(z)(N' - \eta N), \quad (19)$$

where $N' - \eta N$ is the number of unskilled workers hired in the current period.

The law of motion for the next period value of the firm's asset is:

$$a' = (k + A)N' - H'w' - (1 + r)b. \quad (20)$$

The recursive representation is similar to that of section 3.2, once we use z as a state variable in place of q . The problem solved by a surviving firm with capacity \bar{N} can be written recursively as follows:

$$V(a, z, N, H) = \max_{\substack{b, w', z', \\ N' \leq \bar{N}}} \left\{ d + \beta \left[p \cdot V(a', z', N', H') + (1 - p) \cdot L' \right] \right\} \quad (21)$$

subject to

$$d = a + b - \kappa N' \geq 0 \quad (22)$$

$$b \leq \bar{\kappa}_f N' \quad (23)$$

$$z = \beta \left[U(w') + p\eta z' \right] \quad (24)$$

$$a' = (\kappa + A)N' - H'w' - (1 + r)b \quad (25)$$

$$H' = \eta H + \psi(z)(N' - \eta N) \quad (26)$$

$$L' = a' - \kappa_w N' \quad (27)$$

The firm survives with probability p . In this case the continuation value is $V(a', z', N', H')$. If the firm exits, the liquidation value is $L' = a' - \kappa_w N'$ because the worker-specific capital, κ_w , becomes useless outside the firm. Notice that the firm finds out about its viability only after paying the wages but before hiring new workers.

The derivation of the first order conditions are in Appendix C. We are now able to define a steady state labor market equilibrium.

Definition 1 *A steady state labor market equilibrium is defined by: (i) Policy rules $b(a, z, N, H)$, $w(a, z, N, H)$, $z(a, z, N, H)$, $N(a, z, N, H)$ and value functions $V(a, z, N, H)$ for each firm type \bar{N} ; (ii) Aggregate demand and supply of unskilled workers; (iii) A price for unskilled workers q_{res} ; (iv) A distribution (measure) of firms $M_{\bar{N}}(a, z, N, H)$; (v) A transition function for the distribution of firms. Such that: (a) For each firm type \bar{N} , the policy rules solve the firm's problem (21) and $V(a, z, N, H)$ is the associated value function; (b) The market for unskilled workers clears at the equilibrium price q_{res} ; (c) The transition function is consistent with the firm policies, the probability distribution of initial capacities, $\Gamma(\bar{N})$, and the initial distribution of wealth $a_0 = \alpha \bar{N}^\rho$; (d) The next period distribution generated by the transition function is equal to the current distribution.*

4.3 Quantitative analysis

In this section we study the properties of the model by estimating wage regressions on model-generated data, similar to those estimated in the empirical literature. We show that i) that the model generates a positive firm size-wage relation; ii) the relation holds also after controlling for the age of the firm; and iii) that fast growing firms pay on average lower wages. We first describe the parametrization of the model and then we report the regression results. Appendix D describes how we solve for the equilibrium.

Parametrization: The interest rate on secured debt is set to $r = 0.03$ and the intertemporal discount factor to $\beta = 0.934$. This implies a discount rate for entrepreneurs equal to $1/\beta - 1 \approx 0.07$, which is close to the post-war stock market return in the US. The risk-aversion parameter is set to $\sigma = 1$ (log-utility). The per-worker investment is chosen to have a capital-output ratio of 2.8. With the normalization $A = 1$, this requires $\kappa = 2.8$. The non-sunk fraction of capital, κ_f , determines the leverage of the firm. We set it to 0.35 which is in line with the average leverage of Compustat companies.

The survival probability of workers is set to $\eta = 0.9778$. This corresponds to a working life duration of about 45 years, which is consistent with the calibration of life-cycle models such as Auerbach and Kotlikoff (1987) and Rios-Rull (1996). The probability of firms' survival p and the matching probability χ determine the flow of workers who re-enter the labor market as unskilled workers and the flow of skilled workers who switch employer without re-entering the market. We interpret the first group of workers as

experiencing a transition from employment to unemployment, the second as a job-to-job transition. We set $1 - p$ to 0.05 and χ to 0.15. The chosen values of p and χ , together with the value of η used in the calibration, imply that about 80 percent of workers have more than one year of tenure with their employer. This is approximately the number reported for the U.S. economy by Farber (1999). The magnitude of χ relative to η and p comes from the fact that, in the US, job-to-job transitions are more than twice the transitions from employment to unemployment. See Fallick and Fleischman (2001).

The employment capacity \bar{N} can take eight values. These values and the corresponding probabilities $\Gamma(\bar{N})$ are determined jointly with the parameters of the function $a_0 = \alpha \cdot \bar{N}^\rho$. We use a simulated method of moments to pin down these parameters. More specifically, we minimize the sum of square errors between specific moments generated by the model and those observed in the data. The moments are the size distribution of new and incumbent firms as reported in Table 1, plus a capital income share of 40 percent.⁶

Table 2 reports the estimated distribution of new projects and their initial financial tightness. The estimated parameters imply that firms with larger projects face higher initial tightness. This is a consequence of the fact that the distribution of new firms shown in Table 1 is much more concentrated toward small firms than the distribution of incumbent firms. The values of the two parameters that determine how the initial wealth of the firm is related to its capacity are $\alpha = 1.892$ and $\rho = 0.708$.

Simulated regression: Using the steady state distribution of firms, we estimate the following regression:

$$\ln(\text{Wage}_{i,j}) = \bar{\alpha} + \alpha_T \cdot \text{WorkerTenure}_{i,j} + \alpha_{T^2} \cdot \text{WorkerTenure}_{i,j}^2 + \alpha_A \cdot \text{FirmAge}_j + \alpha_S \cdot \ln(\text{FirmSize}_j) + \alpha_G \cdot \text{FirmGrowth}_j$$

The index i identifies the worker and j the firm where the worker is employed. This specification is similar to the one used in the empirical literature although we include a smaller set of controls consistent with the

⁶The size distribution reported in Table 1 gives us 20 independent moments. With the addition of the capital income share we have 21 moments to match but only 17 parameters: eight values of \bar{N} , seven probabilities $\Gamma(\bar{N})$, plus α and ρ . Once we have the values of these parameters we also have the labor supply. The implied value is $L = 25.7$.

Table 1: Size distribution of firms in the U.S. economy, 2001.

<i>Firm size (Employees)</i>	<i>Firms</i>	<i>Employees</i>	$\frac{\textit{Employees}}{\textit{Firms}}$
New firms			
<i>1-19</i>	95.37%	53.28%	3.3
<i>20-499</i>	4.58%	37.66%	48.0
<i>500+</i>	0.05%	9.06%	1,022.7
<i>Total</i>	100.00%	100.00%	5.8
All firms			
<i>1-19</i>	87.46%	17.90%	4.7
<i>20-49</i>	7.94%	10.27%	30.0
<i>50-99</i>	2.53%	7.43%	68.4
<i>100-499</i>	1.72%	14.26%	192.4
<i>500-999</i>	0.17%	5.13%	689.0
<i>1,000-1,499</i>	0.06%	3.02%	1,217.4
<i>1,500-2,499</i>	0.05%	3.84%	1,915.8
<i>2,500+</i>	0.07%	38.13%	12,074.1
<i>Total</i>	100.00%	100.00%	23.2

Source: Small Business Administration, Office of Advocacy, data from U.S. Census Bureau, Statistics of U.S. Businesses. <http://www.sba.gov/advo/stats/data.html>.

structure of our model. The goal of these regressions is to investigate whether the data generated by the model produces estimation results that are similar to those obtained in the empirical literature. The results are reported in Table 3 with standard errors in parenthesis.

The first column reports the coefficient estimates when all variables are included in the regression. All the estimates are statistically significant. Of special interest are the coefficients of firm's size and growth. The estimates for these two parameters are consistent with the findings of the empirical literature. In particular, while the size of the firm has a positive impact on wages, the effect of firm's grow is negative. We discuss in details each of the coefficient estimates.

The firm size effect: To understand the effect of firm size on wages, it is important to take into account that firms with different capacity \bar{N} face different financial constraints initially. This is determined by the parameter ρ estimated to be 0.708. This implies that the financial tightness of new firms increases with \bar{N} (see Table 2). As a result, we would have that firms with a high value of \bar{N} pay a steeper wage profile. This also implies that these firms pay higher wages than any other firm once they become unconstrained.

Table 2: Distribution of new projects and financial tightness.

\bar{N}	$\Gamma(\bar{N})$	FTI
7.3	0.83653	1.74
31.2	0.10052	2.66
67.6	0.03816	3.34
277.2	0.02020	5.04
716.7	0.00265	6.65
1,444.0	0.00067	8.16
2,196.8	0.00075	9.22
18,929.3	0.00051	17.28

Because these are also the largest firms, this mechanism generates a positive correlation between firm size and wages. If ρ was equal to 1—implying that all new firms face the same financial tightness—then the differences in wages would be fully captured by the age of the firm.⁷

The effect of firm size is important and comparable to those found in the empirical literature. Brown and Medoff (1989) survey the empirical studies and report estimates of the log-size coefficient that ranges from 0.01 to 0.03. Our estimated coefficient of 0.0104 implies that the average wage paid by the largest size class of firms (those with more than 2,500 employees) is about 8 percent higher than the average wage paid by the first size class of firms (those with less than 20 employees).

The firm growth effect: The second important result is the negative effect of firm growth on wages. The intuition for this result arises naturally from the discussion above: firms that grow are those with binding financing constraints. Because of the constraints, these firms pay lower wages today in exchange of higher future wages when they operate at the optimal scale. Quantitatively, the estimates of this coefficient is not very different from those found in the empirical literature. Bronars and Famulari (2001) report a coefficient of firm growth that ranges from -0.4 to -0.35.

Tenure and firm age: The other two variables included in the regression is the worker’s tenure and the age of the firm. The positive effect of the

⁷Indeed, if we constrain ρ to be one and we control for firm age, the estimated coefficient for the size of the firm becomes insignificant after controlling for the age of the firm. On the other hand, the sign and significance of the coefficient for size is not affected by α . The parameter α is important for the coefficient of firm’s growth.

Table 3: Wage equation estimation from model-generated data.

<i>Description</i>	(1)	(2)	(3)	(4)	(5)	(6)
Constant	-0.5452 (0.0003)	-0.5692 (0.0003)	-0.5188 (0.0003)	-0.5421 (0.0002)	-0.6280 (0.0003)	-0.6236 (0.0003)
Worker tenure	0.0072 (0.0001)	0.0074 (0.0001)	- -	- -	0.0150 (0.0001)	0.0154 (0.0001)
Worker tenure ² /1,000	-0.1998 (0.0028)	-0.2401 (0.0028)	- -	- -	-0.4756 (0.0031)	-0.4753 (0.0031)
Firm age	-0.0011 (0.0000)	- -	-0.0011 (0.0000)	- -	-0.0003 (0.0000)	- -
Firm log-size	0.0104 (0.0000)	0.0092 (0.0000)	0.0105 (0.0000)	0.0093 (0.0000)	0.0085 (0.0000)	0.0089 (0.0000)
Firm growth	-0.6762 (0.0011)	-0.5795 (0.0010)	-0.7233 (0.0011)	-0.6284 (0.0010)	- -	- -
<i>R-square</i>	0.285	0.265	0.271	0.253	0.103	0.101
<i>Observations</i>	1,369,505	1,369,505	1,369,505	1,369,505	1,369,505	1,369,505

Notes: OLS estimates. Standard errors in parenthesis.

worker's tenure derives from the fact that the wages paid by constrained firms increase over time, and therefore, with the tenure of workers. The return to tenure is smaller than the one estimated by Topel (1991), but comparable to the estimates of Altonji and Shakotko (1987). The estimated coefficient for firm's age is negative. However, the sign and magnitude of this coefficient depends on the variables we include in the regression. For instance, if we exclude worker's tenure, the coefficient of firm's age decreases significantly and it becomes positive if we also exclude firm size from the regression. In brief, the unconditional correlation between wage and firm's age is positive while it becomes negative after controlling for some workers and firms characteristics. The fact that the relation between firm's age and wages depends on the variables included in the regression is consistent with the empirical findings that the effect of age is not clear cut (see Section 2).

Sensitivity analysis: Table 4 reports the estimates for alternative values of the coefficient of risk aversion σ . When $\sigma = 0.5$ (low concavity), the firm size-wage effect increases more than 20 percent. In this case, the wages of

firms with more than 2,500 employees are about 10 percent higher than the wages paid by firms in the size class 1-19. This derives from the fact that the cost of offering an increasing wage profile is smaller when the intertemporal elasticity of substitution is high. Consequently, firms offer a steeper wage profile and the effects of firm’s size and growth are stronger. The opposite is true when $\sigma = 2$. In the limit case in which $\sigma = \infty$, all firms would pay a constant wage and the model would not generate any wage differential.

Table 4: Sensitivity analysis

<i>Description</i>	<i>(1)</i> $\sigma = 0.5$	<i>(2)</i> $\sigma = 1.0$	<i>(3)</i> $\sigma = 2.0$
Constant	-0.5401 (0.0005)	-0.5452 (0.0003)	-0.5376 (0.0002)
Worker tenure	0.0102 (0.0001)	0.0072 (0.0001)	0.0051 (0.0000)
Worker tenure ² /1,000	-0.2897 (0.0046)	-0.1998 (0.0028)	-0.1388 (0.0017)
Firm age	-0.0020 (0.0000)	-0.0011 (0.0000)	-0.0006 (0.0000)
Firm log-size	0.0133 (0.0001)	0.0104 (0.0000)	0.0061 (0.0000)
Firm growth	-1.2797 (0.0016)	-0.6762 (0.0011)	-0.3418 (0.0008)
<i>R-square</i>	0.393	0.265	0.219
<i>Observations</i>	1,356,111	1,369,505	1,365,043

Notes: OLS estimates. Standard error in parenthesis.

5 Contracts implementation

In the previous sections we have assumed that firms commit to the long-term wage contracts. Commitment could be problematic because the promised utilities increase over time until the firm becomes unconstrained. More specifically, a new worker starts with $q_t = q_{res}$ and, as long as the firm survives, he or she receives $q_{t+j} \geq q_{res}$, for all $j > 0$. Because new workers can be hired with initial utility q_{res} , the firm may have an incentive to renege promises that exceed q_{res} . The goal of this section is to discuss the conditions that

prevent the firm from renegotiating the long-term contract. We then discuss why collateralized debt is the only form of external financing for the firm.

Before continuing, it will be convenient to summarize the timing of the model. First, workers decide whether to provide effort—which has a cost $\bar{\ell}$ in forgone utility—and whether to quit the firm. Then production takes place and the firm observes whether the worker has provided effort. At this point the firm could renege its wage promises. Afterwards, the firm decides whether to renegotiate the debt. Renegotiation entitles the investors to seize the firm’s assets. After the payment of the wages and the repayment of the debt, the survival of the firm is observed.

5.1 Worker-firm relationship

If both the worker and the entrepreneur cooperate (the worker by exerting effort and the entrepreneur by paying the promised wage), output is produced and the worker earns the promised wage. The only Nash Equilibrium of each period sub-game is the one in which the firm reneges its promises and pays zero wages. Anticipating that, the worker withdraws effort and quits. In the repeated game, however, cooperation can be sustained through trigger strategies, provided that replacing the worker is sufficiently costly for the firm. Suppose that the worker and the firm follow these strategies (which for simplicity are specified independently of the investors’ past history):

- **Worker:** The worker provides effort as long as the firm pays the contracted wages. If one of the two parties has reneged sometimes in the past (either the worker has shirked or the firm has paid a wage different from the one contracted), the worker withdraws effort and quits.
- **Firm:** The firm pays the contracted wages as long as the worker provides effort. If one of the two parties has reneged sometimes in the past (either the worker has shirked or the firm has paid a wage different from the one contracted), it sets the wage to zero.

The equilibrium associated with these strategies is sub-game perfect. To see this, let’s consider first the worker. Providing low effort would trigger a wage cut which forces the worker to quit the firm and be left with the reservation value q_{res} starting from the next period. But the utility from doing so, $U(0) + \bar{\ell} + \eta q_{res}$, is not bigger than the utility obtained from providing effort, that is, $U(w_t) + p\eta q_t + (1 - p)\eta q_{res}$. Thus, along the equilibrium path,

the worker never shirks and quits. If the firm has sometimes paid a different wage from the one contracted, quitting is optimal since the firm would pay a zero wage both today and in the future.

Consider now the firm. When the firm expects the worker to quit tomorrow, setting the wage to zero today is always the firm's best response. Thus, given each worker's strategy, paying zero wages is optimal when the worker has sometimes shirked. Along the equilibrium path, the firm never finds optimal to deviate from the promised long-term contract because, if the firm reneges its wage promises, the worker quits and the firm loses the sunk investment κ_w . Therefore, the assumptions that part of the investment is worker-specific, is key to prevent the firm from renegotiating the contract.

Of course, there is a limit to this. If the worker's utility becomes very big, the gains from reducing the wage obligations (by reneging the long-term contract and hiring a new worker) become higher than the loss of sunk investment. This happens if $\bar{\kappa}_f/\kappa$ is close to 1 and the initial assets of the firm, a_0 , are small. In this paper we have implicitly assumed that $\bar{\kappa}_f/\kappa$ is sufficiently small and a_0 sufficiently large so that this never arises in equilibrium.

To show that the non-renegotiation condition is satisfied in the numerical exercises conducted in the paper, Table 5 reports the maximum gains that can be obtained by replacing an existing worker (and paying lower wages afterwards). The maximum gain can be achieved by firms with the largest capacity \bar{N} once they become unconstrained.⁸ These firms are paying the highest wages to the first cohort of workers. Denote the wage paid to this cohort by w_{max} . A firm could replace these workers with new workers receiving a constant wage w_{res} . This is the wage that gives the reservation utility $q_{res} = \beta U(w_{res})/(1 - \beta\eta)$. By doing so, the firm would save $w_{max} - w_{res}$ in wage payments in each period, with expected discounted value given by

$$RG(\mathbf{P}) \equiv \frac{\beta(w_{max} - w_{res})}{1 - \beta p \eta (1 - \chi)},$$

where RG stands for Renegotiation Gains and \mathbf{P} are the model's parameters. Notice that the term $\beta p \eta (1 - \chi)$ becomes the discount factor with which the firm discounts future gains: the firm survives with probability p and

⁸It can be shown that the maximal promised utility for which the firm does not renegotiate is decreasing in the age of the firm. This together with the fact that the promised utility of workers increases with tenure (till the firm becomes unconstrained), proves that the incentive to renegotiate is the highest when the firm is unconstrained.

the worker with probability η . Moreover, conditional on firm’s and worker’s survival, the worker does not switch to a new employer with probability $1 - \chi$.

Table 5 reports the renegotiation gains for different curvatures of the utility function. As expected from the theoretical analysis, the renegotiation gains increase as we decrease σ . This is because with a lower σ it is cheaper to borrow from workers and the profile of wages is steeper. The renegotiation gains are compared to the loss of worker-specific capital κ_w , which in the parameterized model takes the value of 1.820. For the baseline parametrization with $\sigma = 1$, the non-renegotiation condition is satisfied. However, for smaller values of σ this is no longer the case.

Table 5: Renegotiation gains for different curvatures of the utility function.

	$\sigma = 0.5$	$\sigma = 1.0$	$\sigma = 2.0$
$RG(\mathbf{P})$	4.080	1.597	0.334

The value of κ_w required to support the long-term contract is relatively big in the calibrated model. We should emphasize, however, that this cost proxies for all possible losses that the firm incurs when the worker quits. Recruiting and training costs is only part of this. The loss of productivity acquired through learning is an important and possibly large loss for the firm.

5.2 Investors-firm relationship

Suppose that when the entrepreneur renegotiates (defaults on) the debt contract, investors have the right to liquidate the assets of the firm but cannot exclude the entrepreneur from participating in financial markets. In other words, the entrepreneur can get new financing from other investors. Furthermore, when the firm refinances investment, it can retain the hired workers. This implies that the investment in recruitment and training is not lost.

Under the above conditions, collateralized debt is the only type of financing that the firm can get from investors. To see this, suppose that the firm could borrow above the value of the collateral. After receiving the loan, the entrepreneur would renegotiate down the part of the debt in excess of the collateral and obtain a new (identical) financial contract from other investors. Anticipating this, only secured loans will be offered.

6 Empirical analysis

We have seen that our model is consistent with several empirical findings. In particular, the fact that large firms pay higher wages (Brown and Medoff (1989)) and that, on average, fast growing firms pay lower wages (Hanka (1998) and Bronars and Famulari (2001)). In the model the wage policy and the dynamics of the firm are both affected by its financial conditions, that evolve over time. This interaction generates two specific predictions about the relation between the dynamics of wages and the dynamics of firms which, to our knowledge, have not been tested yet and that lies at the core of the mechanism that generates a firm size-wage relation. We summarize these predictions below.

Firm growth and within-job wage growth: The first prediction of the model is based on the first order condition (7) but derived from the general model. This is equation (47) in the appendix. Assuming a CES utility function and taking logs, this condition can be written as:

$$\ln \lambda_\tau - \sigma \ln w_{\tau,t} = \ln(1 + p\gamma_t), \quad \forall t > \tau.$$

After evaluating this condition at two points in time and then taking differences side by side, we obtain:

$$\ln w_{\tau,t+1} - \ln w_{\tau,t} = \frac{1}{\sigma} \ln \left(\frac{1 + p\gamma_t}{1 + p\gamma_{t+1}} \right).$$

The left-hand-side is the within-job growth of wages while the right-hand-side is increasing in the growth of the firm. To see this remember that the variable γ_t is the shadow value of internal funds and identifies the tightness of the financial constraints for the firm. As the firm retains earnings, its financial tightness decreases and the firm grows until γ_t converges to zero. A large decline in γ_t is obtained when the firm experiences a large size expansion and gets closer to the optimal scale. This implies that the right-hand-side term is bigger when the firm experiences faster growth. Thus, one key prediction of the model is that within-job wage growth is higher when the firm experiences faster growth. This gives rise to the following testing relation:

Test 1 *Individual wages grow faster in fast growing firms.*

To test this relation, we estimate the following equation:

$$\ln W_{ijt} - \ln W_{ijt-1} = \beta_x \cdot X_{ijt} + \alpha_{G1} \cdot G_{ijt} \quad (28)$$

where W_{ijt} is the real wage income earned by worker i in job j at time t ; X_{ijt} is a set of controls for the worker's and firm's characteristics as described below; G_{ijt} is the yearly growth rate of the firm j . We are interested in the sign of the coefficient α_{G1} , which we expect to be positive.⁹

Firm growth and start-up wages: In the model, firms with tight financial constraints pay lower start-up wages in the promise of higher future wages. To see this, consider the first order condition for a new worker hired without skills (and requiring the investment κ_w). This is the first order condition considered above when $\tau = t - 1$. The condition reads:

$$\ln \lambda_{t-1} - \sigma \ln w_{t-1,t} = \ln(1 + p\gamma_t).$$

As observed above, the financial tightness of the firm is captured by the variable γ_t , which declines over time. As this variable decreases, the left-hand-side term must also decrease. This requires a higher wage. The variable λ_{t-1} also declines. However, it can be proved that the decline in λ_{t-1} is not sufficient to compensate the decline in γ_t . This can also be seen in the numerical example presented in Figure 2 for the simplified model. Because firms with tighter constraints (higher γ_t) will grow faster in the future, we have that start-up wages are negatively correlated with the future growth rate of the firm.

In the general model, some of the new workers are hired with skills. Therefore, their start-up wage depends on the promised utility achieved before switching to the new employer. However, independently of the promised utility achieved with the previous employer, the new wages will grow for a longer period of time if the worker is hired by a firm with a higher γ_t . In fact, a higher γ_t implies that the firm will take more time to reach \bar{N} . For

⁹Equation (28) is different from the first difference of a standard wage equation, since the first difference is now taken only for workers within the same job. Indeed equation (28) is borrowed directly from Topel (1991), (see his equation (4)), who stresses that the use of within-job first differences eliminates any worker or job specific fixed effect that could in principle bias the estimates. Notice that we are not interested in separately identifying whether firm's growth increases the return to tenure or labor market experience, since, in the extended version of the model, both contribute to wage growth.

given lifetime utility, this implies that the start-up wage of the newly hired worker is lower. Thus, we have the following testing relation:

Test 2 *Start-up wages are lower in firms with higher future growth.*

To test this relation, we estimate the following equation:

$$\ln W_{ijt} = \beta_x \cdot X_{ijt} + \alpha_{G2} \cdot G_{ijt+1}, \quad (29)$$

where W_{ijt} is the real wage income earned by the new hired worker i in firm j at time t ; X_{ijt} is a set of controls for the worker's characteristics as described below; G_{ijt+1} is the yearly growth rate of the employer j . We are interested in the sign of the coefficient α_{G2} , which we expect to be negative.

6.1 Data sources

We use the Finnish Longitudinal Employer-Employee Data (FLEED) released by Statistics Finland. It contains information on *all* Finnish firms and *all* individuals in the age group 16-70 living in Finland between 1988 and 2002. The longitudinal nature of the data set is crucial for our analysis because we can observe the same worker and firm at any different date. Appendix E describes in details how we select the final sample and defines the main variables of interest.

To investigate whether similar results hold for the United States we also use the National Longitudinal Survey of Youth, started in 1979 (NLSY79). This is a nationally representative sample of 12,686 young men and women who were 14-22 years old when they were first interviewed in 1979. Appendix E describes how we select our final sample and the main variables of interest.

We have chosen to work with these two data set because of its longitudinal structure. This allows us to compute the growth rate in the size of the firm and of individual wages. However, there are some drawbacks with the NLSY79 data. One problem is that there is information only on the size of the establishment, not the size of the firm. Because we also have information on whether the employer has more than one establishment, we can conduct the analysis only on single establishment firms. Another problem is that the size of the establishment is self-reported by the worker. Thus the reported numbers are likely to be plagued by substantial measurement errors. A third problem is that we can calculate the growth rate of the firm only for

those workers who remain in the same job for two consecutive years. These problems do not arise in the Finnish data because we have data at the firm level which is directly reported by the firm, not the employees. However, despite these problems and despite the fact that the two data sets are for different countries, we obtain consistent results.

6.2 Empirical results

Table 6 reports some descriptive statistics for the variables used in the estimation of equations (28) and (29). Panel A is for the FLEED data and Panel B is for the NLSY79 sample.

We first investigate whether the set of relevant stylized facts documented in the literature for the United States also hold in Finland. We find that: i) the magnitude of the firm’s size effect is of the same order of magnitude as those found using US data; ii) fast growing firms pay on average lower wages; and iii) smaller firms grow faster. For economy of space we do not report the full details of these findings because they have been investigated in previous studies and are consistent with them.¹⁰

Firm growth and within-job wage growth: The top section of Table 8 reports the OLS estimations of equation (28) using the FLEED data.¹¹ The basic estimation is reported in columns 1. In column 2 we also add firm’s size. All regressions include the set of standard controls described at the bottom of the table plus the productivity growth of the firm, where productivity is measured as value added per worker.

The inclusion of productivity growth is important in two respects. First, we do not have information about hours worked. Therefore, we want to rule out the possibility that workers in fast growing firms experience faster wage growth simply because they work longer hours. If this is the case, then the growth in wages should be captured by the growth in the productivity of the

¹⁰By running a standard OLS wage regression that includes all variables reported in Table 6, plus the square of tenure and age, a full set of year dummies, six industry dummies, and nine educational dummies, we find that the coefficient for the current (logged) firm size is about 0.03, which is close to the values typically found in US data. When we also add the current growth rate of the firm and its two lagged values, we find that the sum of the three coefficients is about -0.01 and it is statistically significant.

¹¹In the regressions below, a Hausman specification test does not reject the null hypothesis that the fixed effects are uncorrelated with the independent variables at a five per cent level of significance. So OLS yields consistent estimates.

Table 6: Sample Statistics

A) FLEED Data		
<i>Variable</i>	<i>Mean</i>	<i>Std. Dev.</i>
Monthly wage income	1.54	3.78
Male	0.63	0.48
Age	36.86	10.38
Tenure	6.54	8.09
Firm's size	1656.7	3969.9
Firm's Growth	0.095	0.455
Productivity growth	0.0126	0.420
<i>No. of observations</i>	7,266,473	
B) NLSY79 Data		
<i>Variable</i>	<i>Mean</i>	<i>Std. Dev.</i>
Hourly wage	12.58	15.68
Male	.45	.49
Black	.06	.24
White	.88	.32
Experience	13.64	4.95
Tenure	3.67	2.70
Firm size	52.16	260.91
Firm growth	.013	.18
<i>No. of observations</i>	1,999	

Panel A (FLEED): Tenure and age in years. Monthly wage income is in local currency divided by the CPI index. The original measure of yearly income is divided by the number of months worked in the year. Firm's growth rate is the yearly growth rate of the firm. Productivity Growth is the yearly growth rate of the ratio between value added and total employment.

Panel B (NLSY79): Tenure and labor market experience are in years. Weekly tenure is converted into years dividing by 52. Hourly wages are in dollars. White refers to individuals that are neither black nor hispanics. Firm's growth rate is the yearly growth rate of the establishment for single establishment employers.

firm, given that productivity is measured per worker. The second reason we want to control for productivity growth is because firms may become more profitable as they grow. If there is some form of rent sharing, this would imply that fast growing firms pay fast growing wages. By controlling for productivity growth, we (at least partially) rule out this channel.

The coefficient estimates of firm's growth, α_{G1} , have the expected sign and they are statistically significant at the conventional levels. This remains true after controlling for firm productivity growth and the size of the firm. Therefore, growing firms offer steeper wage profiles as predicted by our model.

Productivity growth is positive and statistically significant. This is a feature of the data that cannot be captured by our model because workers' productivity remains constant in the model. An extension in which the firm technology A evolves stochastically may be able to capture this feature of the data. In our set-up, however, the characterization of the optimal contract with firm level productivity shocks is not trivial because we can no longer apply the aggregation results used to write the firm's problem recursively.

The coefficient on firm's size is not statistically different from zero. This shows that the impact of firm growth on within-job wage growth is more important than the impact of firm size. It also shows that the wage policy of the firm changes as the firm evolves over the life cycle, which is one of the key implications of the model.¹²

The second section of Table 7 reports the estimation results using the NLSY79 data. The basic estimation is reported in column 1. Column 2 adds the current size of the firm to the set of regressors. The full set of regressors is described at the bottom of the table. The coefficient estimates for the growth rate of the firm, α_{G1} , have the expected sign and are statistically significant at the conventional levels. Firm size is not statistically significant as in the Finnish data. This is in line with Hu (2003) who find that the return to tenure is not clearly associated with firm size.

Firm growth and start-up wages: To estimate equation (29) we need to observe both the initial wage of the worker in the job and the future dynamics of the firm. This information is readily available in the Finnish Data because we have information about the firm for the whole sample period even if the worker quits. In the NLSY79, instead, this information is available only for

¹²We find similar results when we restrict the sample to male workers, female workers or firms with less than 500 employees.

Table 7: Firm growth and within-job wage growth (Test 1).

A) FLEED Data		
<i>Description</i>	<i>(1)</i>	<i>(2)</i>
Firm growth	0.052** (0.0007)	0.052** (0.001)
Productivity Growth	0.007** (0.0004)	0.007** (0.0004)
Firm Size		-0.00005 (0.00007)
<i>R-square</i>	0.012	0.014
<i>No. of Observations</i>	4,576,731	4,576,731
B) NLSY79 Data		
<i>Description</i>	<i>(1)</i>	<i>(2)</i>
Firm growth	0.087* (0.041)	0.083* (0.041)
Firm size		0.004 (0.004)
<i>R-square</i>	0.016	0.017
<i>No. of Observations</i>	1,794	1,794

Panel A: Finland data The dependant variable is the log monthly real wage income. All regressions include age in level and squared; tenure in level and squared; fifteen year dummies; six industry dummies; a male dummy; and nine education dummies.

Panel B: US data The dependant variable is the log hourly wage of workers with at least 35 working hours per week. All regressions include experience in level and squared; tenure in level and squared; twelve year dummies; a male dummy; four region dummies; a dummy for working in a metropolitan area; and twelve industry dummies.

* Significant at 5 percent level; ** Significant at 1 percent level.

workers who remain with the same employer for at least two consecutive years. Because selecting only these workers would reduce the sample size considerably, we will conduct this test only with FLEED data.

Table 8 reports the OLS estimation of equation (29), where initial wages are the wage incomes earned by workers with only one year of tenure. The basic estimation is reported in columns 1. In column 2 we also control for the dynamics of firm productivity in the years before and after the hiring, to rule out alternative mechanisms whereby future firm's growth could lead to lower initial wages. All regressions include the set of standard controls described at the bottom of the table.

Table 8: Firm Growth and start-up wages (Test 2).

FLEED Data		
<i>Description</i>	(1)	(2)
Firm growth between t and t+1	-0.016** (0.001)	-0.005** (0.001)
Firm growth between t+1 and t+2	-0.002* (0.001)	-0.006** (0.001)
Initial firm size	0.030** (0.0002)	0.024** (0.0003)
<i>R-square</i>	0.28	0.29
<i>No. of Observations</i>	1,704,117	1,704,117

Finland data The dependant variable is the log monthly real wage income of a worker in their first year of tenure. All regressions include age in level and squared; fifteen year dummies; six industry dummies; a male dummy; nine education dummies; and the (logged) firm productivity. In Column two we also control for the growth of firm productivity in the two years before and after hiring.

* Significant at 5 percent level; ** Significant at 1 percent level.

The coefficient estimates of firm growth, α_{G2} , have the expected sign and they are statistically significant at the conventional levels. We have also included the initial size of the firm which has a positive impact. This is also consistent with our model because firms that pay lower initial wages are those that are financial constrained and operate at a suboptimal scale (and therefore, are small). Therefore, small and fast growing firms pay lower

initial wages to their workers. Given the results reported in Table 7, these workers will then experience faster wage growth in subsequent periods.

7 Conclusion

This paper studies how financial constraints affect the compensation structure of workers. Firms that are financially constrained offer an increasing profile of wages to alleviate their financial restrictions. This allows firms to generate higher cash flows which can be used to finance firm growth. We test the key predictions of the model using a longitudinal matched employer-employees data from Finland and the National Longitudinal Survey of Youth for the United States. The estimation results support our theory. In particular, we find that the growth of wages is positively correlated with the growth of the firm and that start-up wages are negatively correlated with the future growth rate of the firm.

As a result of the mechanism described above, the model can also generate a positive relation between firm size and wages, which is a well-known empirical finding. There are several theoretical contributions that try to explain why large firms pay higher wages. For example, large firms may employ workers with higher skills or human capital as in Zabojnik and Bernhardt (2001) or in Kremer and Maskin (1996). Others have suggested a theory based on efficiency wages a la Shapiro and Stiglitz (1984) where large firms pay higher wages because detecting shirking is more difficult. Wage bargaining is another possibility if workers of larger firms have higher bargaining power. These theories capture only part of the relation between firm size and wages. In fact, even after controlling for variables that proxy for these explanations, firm size is still an important determinant of wages.¹³ Burdett and Mortensen (1998) and the extension with optimal wage contracts of Burdett and Coles (2003) propose a theory based on wage posting and search frictions whose full implications have not been tested yet. Our paper provides an additional (and complementary) explanation in which financial markets frictions play a central role.¹⁴

¹³This is clearly stated in Troske (1999) who concludes: “*After testing several possible explanations we are still left with the question: why do large firms pay higher wages?*”.

¹⁴These frictions generate a firm size-wage relation through a dynamic mechanism that is different from Burdett and Mortensen (1998). In particular, our model captures the fact that wages are negatively correlated with the growth of the firm. In Burdett and Mortensen, instead, firms that grow faster should be the ones that pay higher wages.

The centerpiece of our theory is the result that financially constrained firms offer increasing wage profiles, implicitly borrowing from workers. This rises the question of why firms are able to borrow from workers beyond what they can borrow from the financial markets. In our model this is possible because workers can use a punishment mechanism that is not available to external investors. An external investor can punish the firm only by confiscating the physical assets. A worker, instead, can punish the firm by quitting because of the loss of the job-specific investment for the firm. This gives the worker a credible punishment tool in the event of repudiation that is not available to investors.

Indeed, there is both direct and indirect evidence that firms borrow from their employees. In some cases, the borrowing is explicit.¹⁵ In others, the loan is implicit in the compensation structure of employees, as in our model. For example, the widespread use of stock options and/or stock grants to ordinary workers, such as middle-run managers, secretaries and clerks—whose effort, when individually considered, is likely to have a negligible effect on the overall value of the firm—can hardly be justified as a way to provide incentives. This view is also expressed in Hall and Murphy (2003). Most likely, stock options are used to delay the cash compensation of employees and retain more funds in the firm. In accordance with this interpretation, Blasi, Kruse, and Bernstein (2003) find that stock options were especially rewarding for workers hired before their companies went public—*i.e.*, companies that are more likely to be financially constrained. Also consistent with this interpretation is the finding of Core and Guay (2001) for which the use of stock options is more common in firms that are financially constrained.

It should be point out, however, that this is only a conjecture since the firm dynamics generated by this model has not been fully explored.

¹⁵An example is Energy Services Group International, an energy-services engineering and construction company in Williamsburg, VA. The company got a major new contract from an electric utility in Florida but it could not persuade banks to lend any more money. Only employees came forward with investments that ranged from \$200 to \$74,000 in exchange of promissory notes. See Inc. Magazine, January 1992, <http://www.inc.com/magazine/19920101/3886.html>.

A Characterization of the firm's problem

Let γ_t , μ_t , $\lambda_t n_t$ and θ_t denote the lagrange multipliers associated with constraints (2), (3), (4) and (5) respectively. Then the Lagrangian can be written as:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left\{ \begin{aligned} & \left(a_t + b_t - \kappa \sum_{\tau=0}^t n_{\tau} \right) + \\ & \gamma_t \left[a_t + b_t - \kappa \sum_{\tau=0}^t n_{\tau} \right] + \\ & \mu_t \left[\bar{\kappa}_f \sum_{\tau=0}^t n_{\tau} - b_t \right] + \\ & \lambda_t n_t \left[\sum_{j=1}^{\infty} \beta^j U(w_{t,t+j}) - q_{res} \right] + \\ & \theta_t \left[\sum_{\tau=0}^t (\kappa + A - w_{\tau,t+1}) n_{\tau} - (1+r)b_t - a_{t+1} \right] \end{aligned} \right\}.$$

The first order conditions with respect to $w_{\tau,t}$ and a_t , for $t \geq 1$, are

$$\beta \lambda_{\tau} U_c(w_{\tau,t}) = \theta_{t-1}, \quad \forall \tau \leq t \quad (30)$$

and

$$\theta_{t-1} = \beta(1 + \gamma_t), \quad (31)$$

respectively. Using (31) to substitute for θ_{t-1} in (30) yields (7) in the text.

B First order conditions for the recursive problem

The Lagrangian can be written as:

$$\begin{aligned} \mathcal{L} &= a + b - \kappa N' + \beta V(a', q', N', H') \\ &+ \gamma (a + b - \kappa N') \\ &+ \mu (\bar{\kappa}_f N' - b_t) \\ &+ \lambda H' [\beta (U(w') + q') - q] \end{aligned}$$

where γ , μ and $\lambda H'$ are lagrange multipliers. The problem is also subject to the laws of motion for the next period value of a and H , that is, constraints (14) and (15), respectively.

The first order conditions are:

$$b : \quad 1 + \gamma - \mu = \beta(1 + r)V_{a'} \quad (32)$$

$$w' : \quad V_{a'} = \lambda U_{c'} \quad (33)$$

$$q' : \quad V_{q'} + \lambda H' = 0 \quad (34)$$

$$N' : \quad \beta \left[(\kappa + A - \psi(q)w')V_{a'} + V_{N'} + \psi(q)V_{H'} \right] \geq (1 + \gamma)\kappa - \mu\bar{\kappa}_f \quad (35)$$

where the last condition is satisfied with equality if $N' < \bar{N}$. The envelope conditions are:

$$V_a = 1 + \gamma \quad (36)$$

$$V_q = -\beta\psi_q(N' - N) \left[w'V_{a'} - V_{H'} \right] - \lambda H' \quad (37)$$

$$V_N = (1 + \gamma)\kappa + \beta\psi(q) \left[w'V_{a'} - V_{H'} \right] \quad (38)$$

$$V_H = -\beta \left[w'V_{a'} - V_{H'} \right] \quad (39)$$

Equation (16) in the text comes from using (36) to substitute for V_a in (33). We now show that the above conditions also imply that $\lambda = \lambda'$.

By substituting (36) in (39) we get:

$$-V_H = \beta \left[(1 + \gamma')w' - V_{H'} \right]. \quad (40)$$

From (33) we have that $(1 + \gamma')w' = \lambda(w')^{1-\sigma} = \lambda(1 - \sigma)U(w')$, which substituted in (40) yields

$$-V_H = \beta \left[(1 - \sigma)\lambda U(w') - V_{H'} \right]. \quad (41)$$

Now consider the promise-keeping constraint $q = \beta[U(w') + q']$. Multiplying the left and right-hand side by $(1 - \sigma)\lambda$ we get:

$$(1 - \sigma)\lambda q = \beta \left[(1 - \sigma)\lambda U(w') + (1 - \sigma)\lambda q' \right]. \quad (42)$$

Equations (41) and (42) imply:

$$-V_H = (1 - \sigma)\lambda q \quad (43)$$

$$-V_{H'} = (1 - \sigma)\lambda q' \quad (44)$$

Updating the first term we also have that:

$$-V_{H'} = (1 - \sigma)\lambda'q' \quad (45)$$

Condition (44) and (45) then imply that $\lambda = \lambda'$.

C First order conditions for the general model

Let γ , μ and $\lambda H'$ be the lagrange multipliers associated with the constraints (22), (23), and (24), respectively. Following the same steps as in Appendix B we obtain the first order conditions:

$$b: \quad 1 + \gamma - \mu = \beta(1 + r)(1 + p\gamma') \quad (46)$$

$$w': \quad 1 + p\gamma' = \lambda U_{c'} \quad (47)$$

$$z': \quad V_{z'} + \eta\lambda H' = 0 \quad (48)$$

$$N': \quad \beta \left[(1 + p\gamma') (\kappa + A - \psi(z)w' - (1 - \eta)\kappa_w) - (1 - p)k_w + \right. \\ \left. p(V_{N'} + \psi(z)V_{H'}) \right] \geq (1 + \gamma)\kappa - \mu\bar{\kappa}_f \quad (49)$$

where the last equation is satisfied with equality if $N' < \bar{N}$. Notice that (46), (47) and (49) make use of the envelope condition $V_a = 1 + \gamma$. The remaining envelope conditions are:

$$V_z = \beta\psi_z(N' - \eta N) [pV_{H'} - (1 + p\gamma')w'] - \lambda H' \quad (50)$$

$$V_N = -\psi(z)V_H \quad (51)$$

$$V_H = -\eta\beta [(1 + p\gamma')w' - pV_{H'}] \quad (52)$$

D Computation of the equilibrium

Solving for the firm's problem: For given \bar{N} and q_{res} , the firm problem is solved backward starting from the state in which the firm becomes unconstrained.

Let's assume that the firm takes T periods to become unconstrained. Therefore, we know that $N_{T+1} = \bar{N}$ and $\gamma_T = \gamma_{T+1} = 0$.

We start by guessing the value of $w_{T+1} = w_{\bar{N}}$ and H_{T+1} . Using the first order condition $1 = \lambda U_c(w_{T+1})$, we determine the lagrange multiplier λ . Using the promise-keeping constraint $z_T = \beta[U(w_{T+1}) + p\eta z_{T+1}]$, and imposing $z_T = z_{T+1}$, we determine the (transformed) promised utility at time $T + 1$. Using conditions (51) and (52) with the terminal condition $V_{H,T} = V_{H,T+1}$, we determine the partial derivative of the value function with respect to H . Finally, we determine b_T using the borrowing limit $b_T = \bar{k}_f N_{T+1}$ and μ_T using the first order condition $\mu_T = 1 + \gamma_T - \beta(1 + r)(1 + p\gamma_{T+1})$. At this point we have all the terminal conditions to solve the problem backward. These are the variables $(N_{T+1}, H_{T+1}, w_{T+1}, b_T, z_T, V_{N,T}, V_{H,T}, \mu_T, \gamma_T)$. The solution at each point $t = T, T - 1, \dots, 0$ is determined as follows:

1. The value of $V_{N,t}$ is determined by condition (51), that is,

$$V_{N_t} = -\psi(z_t)V_{H,t}$$

2. The wage w_t is determined using the first order condition:

$$1 + p\gamma_t = \lambda U_c(w_t)$$

3. We now determine the a_t , N_t , H_t and b_{t-1} using the budget constraint with $d_t = 0$, the laws of motion for a_t and H_{t+1} , and the borrowing limit:

$$\begin{aligned} a_t &= \kappa N_{t+1} - b_t \\ a_t &= (\kappa + A)N_t - H_t w_t - (1 + r)b_{t-1} \\ H_{t+1} &= \eta H_t + \psi(z_t)(N_{t+1} - \eta N_t) \\ b_{t-1} &= \bar{k}_f N_t \end{aligned}$$

4. The values of z_{t-1} and $V_{H,t-1}$ are determined using the promise-keeping constraint and condition (52), that is:

$$\begin{aligned} z_{t-1} &= \beta[U(w_t) + p\eta z_t] \\ V_{H,t-1} &= -\eta\beta[(1 + p\gamma_t)w_t - pV_{H,t}] \end{aligned}$$

5. The values of μ_{t-1} and γ_{t-1} are then determined using the first order conditions for debt and employment, that is:

$$1 + \gamma_{t-1} - \mu_{t-1} = \beta(1 + r)(1 + p\gamma_t)$$

$$\beta \left[(1 + p\gamma_t) (\kappa + A - \psi(z_{t-1})w_t - (1 - \eta)\kappa_w) - (1 - p)k_w + p(V_{N,t} + \psi(z_{t-1})V_{H,t}) \right] \geq (1 + \gamma_{t-1})\kappa - \mu_{t-1}\bar{\kappa}_f$$

After solving for all $t = T, T-1, \dots, 0$, we check whether $z_0 = z_{res}$ and $H_1 = N_1$. The condition $H_1 = N_1$ implies that $N_0 = H_0 = 0$. If the two conditions are not satisfied, we change the guesses for w_{T+1} and H_{T+1} until convergence.

In the solution of the model we also solve for the initial assets a_0 . If a_0 is bigger than the initial assets, we increase T . This takes advantage of the fact that smaller are the initial assets of the entrepreneur and longer is the transition to the unconstrained status.

Labor market equilibrium: To compute the labor market equilibrium we start by guessing the equilibrium value of z_{res} . Given this value we solve the firm's problem for all \bar{N} as described above. After finding the invariant distribution of firms, we compute the aggregate demand of labor and check the clearing condition in the labor market. We update z_{res} until the labor market clears.

E Data appendix

E.1 Finnish Longitudinal Employer-Employee Data

We select all employees who are in the age group 16-65 and working full time. We eliminate those for which data on personal characteristics and/or wages are missing. The final sample includes 7,266,473 observations, corresponding to more than one million individuals. Following is a description of the variables we use in the analysis.

Gender Dummies. This corresponds to variable SP in the survey.

Age. This corresponds to variable IKA in the survey

Firm's Size. This corresponds to the variable TPHENK, that reports the Personnel of the firm. It comes from the financial statement of the firm.

Industry Dummies. This is the variable TAY in the survey. There are six industry grouping: 1) Manufacturing; 2) Construction; 3) Trade, Hotels and Restaurants; 4) Transportation, Storage and Telecommunications; 5) Business Services and the Financial Sector; 6) Otherwise.

Firm's Productivity. This is the ratio of the firm's value added (variable JAL in the survey) and the Firm's size. Value added is the sum of corrected operating profits, wages and salaries, and other personnel expenses.

Employer Tenure. Difference between the current year and the year when the employment relationship started (variable ALKU3 in the survey).

New vs. Continuing Jobs. FLEED explicitly report whether the worker has changed employer.

Firm's Growth Rates. Log change in firm's size.

Monthly Wage Income. FLEED report the total yearly wage income of the individual. In case the worker is employed for the whole year the monthly wage income is the total yearly wage income divided by 12. In case the worker experiences unemployment, monthly income is determined by dividing yearly income by the total number of months in employment.

Education Dummies. There are nine education grouping: 1) Pre-primary education; 2) Primary Education; 3) Lower Secondary education; 4) Upper Secondary Education; 5) Post-Secondary non-tertiary education; 6) Lowest level tertiary education; 7) Lower-degree level tertiary education; 8) Higher-degree level tertiary education; 9) Doctorate or equivalent level tertiary.

E.2 National Longitudinal Survey of Youth

We focus on a sample of 6,111 individuals designed to be representative of the non-institutionalized civilian segment of the U.S. young population. We consider only the 13 more recent waves, from 1986 to 2002, because the size of the establishment is not always reported in the previous waves. NLSY79 contains information only on the size of the establishment, not the size of the firm. We also know, however, whether the employer has more than one establishment. To make sure that the size of the establishment is a good measure of the size of the firm, we select only workers employed in single establishment firms. The sample is restricted to full time workers (working a minimum of 35 hours per week) with reliable data on wages and with positive labor market experience. We also restrict the sample to observations for which the annual growth rate of the firm is smaller than plus or minus 50 percent. This eliminates outliers that are likely the results of measurement errors. This leads to our final sample of 1,991 observations for 771 individuals. Following is the description of the main variables.

Regional Dummies. There are four regional dummies constructed from the variable "Region of current residence".

Schooling. This is the "Highest grade completed as of May 1 survey year".

Experience. Age of worker at interview date, minus years of schooling, minus six.

Working Hours. Until 1993 the number of working hours per week is obtained from the variable “Hours per week usually worked at current/most recent job”. Starting in 1994, job 1 always coincides with the CPS job and information about working hours is obtained from the variable “Hours per week worked at job 1”.

Metropolitan Area. This is obtained from the question “Is Respondent current residence Urban/Rural?”.

Establishment’s size. Until 1993, this is equal to “Number of employees at location of current job”. Starting in 1994 we use “Number of employees at location of job 1”. We set to missing value observations with a reported value of either 99995 or 99996.

Multiple Establishments. Until 1993, information about whether the firm has multiple establishments is obtained from the question “Does employer at current job have greater-than-one location?”. Starting in 1994, we use the question “Does employer at job 1 have greater-than-one location?”

Industry Dummies. Until 1993 the industry dummies were constructed by using the variable “Type of business or industry of most recent job (Census 3 digit)”. Starting in 1994 we used the variable “Type of business or industry job 1 (Census 3 digit)”. From these variables we constructed twelve industry dummies.

Hourly wage. Until 1993 the hourly wage in dollar is obtained from the variable “Hourly rate of pay current job”. Starting in 1994 we used the variable “Hourly rate of pay of job 1”. To eliminate obvious data entry errors we drop observations whose hourly wage is greater than \$500 or less than half the minimum wage.

Employer Tenure. This is obtained from the five variables “Total Tenure in weeks with employer job 1 (2, 3, 4, 5)”. We then identify whether job 1, 2, 3, 4 or 5 corresponds to the CPS job by using the questions “Internal Check: Is job 1 (2, 3, 4, 5) the same as current job”. After 1993 the CPS job corresponds to job 1.

New vs. Continuing Jobs. To identify whether the current CPS job is a *new* or a *continuing* job, we follow the procedure detailed in Appendix 9 of the user’s guide to NLSY79.

Firm’s Growth Rates. Log change in establishment’s size.

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