Risky Investments with Limited Commitment*

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Abstract
Over the last three decades there has been a dramatic increase in the size of the financial sector and in the compensation of financial executives. This increase has been associated with greater risk-taking and the use of more complex financial instruments. Parallel to this trend, the organizational structure of the financial sector has changed with the traditional partnership replaced by public companies. The organizational change has increased the competition for managerial talent and weakened the commitment between investors and managers. We show how the increased competition and the weaker commitment can raise the managerial incentives to undertake risky investment. In aggregate, this results in higher risk-taking, a larger and more productive financial sector, greater income inequality, and lower market valuation of financial institutions.

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1 Introduction

The past several decades have been characterized by dramatic changes in the size and structure of financial firms in the United States and elsewhere. What was once an industry dominated by partnerships has evolved into a much more concentrated sector dominated by large public firms. In this paper we argue that this evolution has altered the structure of contractual arrangements between investors and managers in ways that weakened commitment and increased the managers’ incentives to undertake risky investments. At the aggregate level, the change resulted in a larger and more productive financial sector, higher compensation of financial executive and greater income inequality.

The increase in size and importance of the financial sector in the US economy is documented in Phillipon (2008) and Philippon and Resheff (2009). Figure 1 shows that the GDP share of the financial industry doubled in size between 1970 and 2011. The share of employment has also increased but by less than the contribution to GDP. This is especially noticeable starting in the mid 1980s when the share of employment stopped growing while the share of value added continued to expand. Accordingly, we observe a significant increase in value added per worker compared to the whole economy.

![Size of Finance and Insurance](image)

Figure 1: Share of Value Added and Employment. Source: Bureau of Economic Analysis.

The increase in size was associated with a sharp increase in compensation. Clementi and Cooley (2009) show that between 1993 and 2006 the average compensation levels of CEOs in the financial sector increased from parity with other sectors of the economy to nearly double. At the same time compensation of managers became more unequal in the financial sector. Figure 2 shows that the income concentration in the financial sector (measured by the income share of the top 5%) has increased significantly compared to the rest of the economy. Other concentration indices provide a similar picture.

Although value added per worker in the financial sector has increased compared to
the other sectors of the economy, the valuation of financial companies has not increased as much as nonfinancial companies. Figure 3 plots the average ratio of market to book value of equity for publicly listed financial and nonfinancial firms. Starting in the early 1980’s, the market valuation of financial firms displays a flat trend while the valuation of nonfinancial firms has continued to grow. This may be a reflection of compensation practices in a sector where managers retain a larger share of the surplus.

The changes described above took place during a period in which the organizational structure of the financial sector was also changing, with traditional partnerships replaced by public companies. Until 1970 the New York Stock Exchange prohibited member firms from being public companies. When the organizational restriction on financial companies was relaxed, there was a movement to go public and partnerships began to disappear. Merrill Lynch went public in 1971, followed by Bear Stearns in 1985, Morgan Stanley in 1985, Lehman Brothers in 1994 and Goldman Sachs in 1999. Other venerable investment banks were taken public and either absorbed by commercial banks or converted to bank holding companies. Today, there are very few partnerships remaining and they are small. The same evolution occurred in Britain where the closed ownership Merchant Banks virtually disappeared.

What were the implications of the organizational change from partnerships to public companies? In this study we emphasize two implications. The first was to increase the competition among financial firms to hire managerial talent. The second was to alter the structure of contractual arrangements between investors and managers in ways that weakened commitments.\(^1\)

\(^1\)The transition from partnerships to public companies also affected the liability of partners and managers. In this paper we do not study this particular aspect of the organizational change.
Let’s first consider the increased competition or demand for managerial talent. By becoming public companies, financial firms had greater access to capital (through the sales of shares) which facilitated their growth. But capital is only one of the production factors. Human capital is also important. Therefore, as more financial capital was coming in, more managerial capital was needed and this increased the competition (demand) for managers.  

Consider now the weakened commitment. Many have argued that a partnership was a preferred form of organization for investment firms because managers and investors were the same people and it was the partners own assets that were at risk. Public companies, on the other hand, are organizations with significant separation between ownership (shareholders) and investment control (managers), and it is well understood that they are characterized by significant agency problems. Partnerships have historically evolved from, what we call, ‘the traditional partnership’, which few partners invested their own capital, to ‘the modern partnership’ where, with a relatively large number of younger partners, some partners effectively acted as ‘delegated managers’, making investment decisions on firm’s portfolios far beyond their capital share in the partnership.

Agency problems were also present in ‘the modern partnership’. However, in a world

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2Roy Smith, a former partner at Goldman Sachs described the evolution of the relationship between compensation and firm structure as follows: “In time there was an erosion of the simple principles of the partnership days. Compensation for top managers followed the trend into excess set by other public companies. Competition for talent made recruitment and retention more difficult and thus tilted negotiating power further in favor of stars. You had to pay everyone well because you never knew what next year would bring, and because there was always someone trying to poach your best trained people, whom you didn’t want to lose even if they were not superstars. Consequently, bonuses in general became more automatic and less tied to superior performance. Compensation became the industry’s largest expense, accounting for about 50% of net revenues”, Wall Street Journal February 7, 2009.
where the firm is fully committed to honor its contracts, agency problems are solved with the optimal design of contracts and if ‘delegated managers’ are tempted to leave the partnership, they are retained with appropriate compensation, as long as it is efficient to do so (i.e. the contract accounts for limited enforcement constraints). Similarly, the contract is designed to prevent the manager from distorting investments in her favour (i.e. the contract accounts for incentive compatibility constraints). The modern partnership used the cost of liquidating partners’ shares, covenants, and other contractual schemes to make it costly for the ‘delegated managers’ to quit. In the transition from the ‘the modern partnership’ to the ‘the financial public company’ these costs were hardly enough to retain managers lured by competing firms and any attempt to retain them resulted in a higher share of the surplus going to them. This effect, together with the separation between shareholders and managers, resulted in weaken the commitment of investors.

In this paper we ask whether the implications of the organizational changes from partnerships to public companies—specifically, the higher competition for managers and the weaker enforcement of contracts—contributed to generate (i) a larger financial sector with a higher value added per worker; (ii) greater income inequality; (iii) lower stock market valuation of financial institutions.

We address this question by developing a model in which investors compete for and hire managers to run investment projects, with each investor-manager pair representing a financial firm. Two features of the model are especially important. The first feature is that production depends on the human capital of the manager which can be enhanced, within the firm, with costly (and risky) investment. The second important feature is that human capital can be transferred outside the firm by managers. This generates a conflict of interest between investors and managers: while the interest of investors is for the value of human capital inside the firm, managers also care about the outside value. As a result, the investment desired by investors may be different than the investment desired by managers. Then, if investors cannot control the firm policies either directly or indirectly through a credible compensation scheme, managers may deviate from the optimal policies.

We first characterize the optimal contract with one-sided limited commitment. In this environment the investor commits to the contract but there is limited commitment of managers. We interpret this case as capturing the economic environment that prevailed in the period preceding the change in organizational structure from ‘modern partnerships’ to public companies, in which the firm, acting as representative of all partners—including those working as ‘delegated managers’—fulfils all the promises made in the contract.

3This is largely consistent with the literature on incomplete contracts. According to Grossman and Hart (1986) and Hart and Moore (1990), more efficient organizational forms are those where the agents who control the investment surplus own a larger share of the assets.

4The New York Stock Exchange regulatory change mentioned above was an important factor because it allowed financial partnerships to become public companies. However, this does not tell us why they chose to do so. In several cases firms were simply acquired by public companies but in others it was an important strategic decision. Charles Ellis (2008) in his history of Goldman Sachs—the last major firm to go public—suggests that one major motivation for going public was to increase the capital available for their proprietary trading operations through an IPO. The goal of this paper is not to understand why financial firms chose to go public. Rather, we want to understand the consequences of the choice.
but partners have discretionary control of investments and may leave the partnership. However, to simplify the analysis, we abstract from non-pecuniary costs and rewards, typically used in ‘modern partnerships’ to deter ‘delegated managers’ from quitting.

After analyzing the environment with one-sided limited commitment, we study the optimal contract with double-sided limited commitment, which we interpret as representative of the most recent period dominated by public companies. In this environment, in addition to the already mentioned effect of ‘the competition for managers weakening the commitment from investors’ the firm operates on behalf of shareholders who are distinct from managers. This separation implies that, if the firm could bring some ex-post gains to the shareholders by reneging on promises made to managers, it may choose to do so. This motivates our choice of modelling public companies as investor-manager relationships with double-sided limited commitment.

A key result of the paper is that the impact of more competition for managers on risk-taking depends on the contractual environment—i.e. one-sided vs. double-sided limited commitment. Risk taking is not exogenous in the model but depends on both competition and commitment. When investors commit, promises of higher future compensation are credible and they can be structured to deter managers from choosing riskier investments. As a result, higher competition for managerial talent is costly to the partnership (the residual claimants) and the optimal contract with one-sided commitment reacts to an increase in competition by reducing risk taking. However, when investors do not commit to long-term contracts, promises of future payments are not credible and managers cannot be discouraged from choosing less efficient investments. In this case a manager chooses the investment that maximizes her outside value, ignoring the cost that this imposes on the firm. As competition for managerial talent increases, so does the incentive to raise the outside value. Therefore, in the environment with double-sided limited commitment, risk taking rises with competition.

To make the outside value of managers endogenous and to study the implications of the organizational change for the whole economy, we embed the micro structure in a general equilibrium model with two sectors: financial and nonfinancial. In the general model we formalize the change from partnership to public companies as lowering the cost of capital and, as a result, increasing the competition for managers, while the weakened commitment, due to the increasing competition for managers and the transition from partners to shareholders, is captured by the shift from a regime with one-sided limited commitment to a regime with double-sided limited commitment. We then show that these changes can generate (i) greater risk-taking in the financial sector; (ii) a larger financial sector with higher value added per worker; (iii) greater income inequality between sectors and within the financial sector; and (iv) lower valuation of financial firms.

Since the transformation from a partnership to a public company implies weaker commitment and, therefore, lower efficiency in the contractual relation, it is natural to ask why financial firms have chosen to become public companies. One of the reasons is that public corporations have access to a broader set of investors. To the extent that the benefits from cheaper funds dominate the losses of lower contractual efficiency, the partners of financial firms (ex-post) prefer to become public companies. As partnerships become public companies, the demand for managerial talent increases and this is what
creates, in the general equilibrium, the increased competition or demand for managerial talents, which, in turn, weakens the commitment of the investors.

The organization of the paper is as follows. After relating the paper to the existing literature, Section 2 describes the theoretical model. Section 3 characterizes the optimal contract under different assumptions about commitment. Since the model is linear in human capital, which grows over time, Section 4 reformulates the optimal contract with normalized variables. Section 5 embeds the micro structure in a general equilibrium and Section 6 studies the consequences of the organizational change with the general model. Section 7 concludes.

1.1 Relation to the literature

The basic framework often used to study executive compensation is adapted from the principle-agent model of dynamic moral hazard with private information by Spear and Srivastava (1987). Examples include Wang (1997), Quadrini (2004), Clementi and Hopenhayn (2006), Fishman and DeMarzo (2007). Albuquerque and Hopenhayn (2004) is also in this class of models even though the agency frictions are based on limited enforcement instead of information asymmetry.

An assumption typically made in this class of models is that the outside option of the agent is exogenous. As argued above, however, an important consequence of the demise of the partnership form is that financial managers are no longer constrained by the limited liquidity of the portion of their wealth that is tied to the firm and it is easier for them to seek outside employment. Since the value of seeking outside employment depends on market conditions for managers, it becomes important to derive these conditions endogenously. A second assumption typically made in principal-agent models is that investors fully commit to the contract. However, the clearer separation between investors and managers that followed the transformation of financial partnerships to public companies and the associated competition for managerial talent, could have also reduced the commitment of investors.

In this paper we relax both assumptions: we make the outside option of managers endogenous and we allow for the limited commitment of investors. As we will see, the relaxation of both assumptions are key for the central results of the paper.  

The empirical facts described in the introduction have also motivated other studies. The models used in these studies can capture some of the empirical facts but we are not aware of models that capture all of them. We are also unaware of any study that connects changes in the organizational structure of the financial sector with the increased competition for managerial talent. Cheng, Hong and Scheinkman (2012) and, in a general equilibrium Edmans and Gabaix (2011), explain how in a Principal-Agent relationship with a fixed sharing rule, an exogenous increase in risk can result in higher compensation

\[5\]Cooley, Marimon and Quadrini (2004) endogenized the outside value of entrepreneurs but kept the assumption that investors commit to the long-term contract. Marimon and Quadrini (2011) relaxed both assumptions and showed that differences in “barriers to competition”, result in income differences across countries. In these two papers, however, uncertainty did not play a role while it is central to the analysis of the current paper. Furthermore, the current model features two sectors (financial and nonfinancial) and shows that changes in one sector can have important effects, in general equilibrium, on the other.
for risk-averse financial managers in order to satisfy their participation and incentive constraints. Bolton, Santos and Scheinkman (2012) argue that it is “cream skimming” in the more opaque financial transactions—those taking place in over-the-counter or bespoke markets—that have encouraged excessive compensation of financial managers and the large share of GDP of the financial industry. In our paper, instead, we propose a model where the increase in risk is generated endogenously as a consequence of greater competition and weaker commitment which we see as direct consequences of the change in organizational structure from partnerships to public companies.

2 The model

We start with the description of the financial sector and the contracting relationships that are at the core of the model. After the characterization of the financial sector, we will embed it in a general equilibrium in Section 5.

Managers are skilled workers with the ability to produce and develop innovative projects. Managers are mobile and when they choose to leave the firm, at least part of the know-how created with innovative projects can be transferred by them to other firms. The employment relationship is regulated by a contract between the firm and the manager.

A central difference between a partnership and a public company is the ownership structure. In a partnership the firm is owned by the partners. Consequently, the objective of the firm is to maximize the partners’ welfare. Because the firm operates on behalf of the partners, there is no risk that the firm reneges on promises made to individual partners, even if in ‘modern partnerships’ some partners act as ‘delegated managers’ (i.e. their investment decisions mostly regard partnership’s capital, beyond their initial share of the partnership’s rents). Therefore, to characterize the contractual relationship in a partnership, we assume that the firm commits to the long-term contract but managers do not commit (one-sided limited commitment).

In a public company, instead, the owners of the firm, the shareholders, are distinct from partners and managers. Since the firm operates on behalf of shareholders, not partners and managers, reneging on previous promises to managers becomes a possibility in a public company. Of course, if it were possible to write formal contracts in which future promises are legally binding, the firm would not be able to renege on these promises. However, making all promises legally binding may be ex-ante very costly, and possibly not even feasible. Based on this premise, we characterize the contractual relationship in a public company under the assumption of double-sided limited commitment.

Preferences and technology. There are two types of agents, investors and managers. Investors are risk-neutral with expected lifetime utility

$$V_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left( \Pi_t - C_t \right),$$
where $\Pi_t$ denotes the cash flows generated by the firm and $C_t$ is the compensation of managers. Thus, $\Pi_t - C_t$ represents the payment received by investors.

Managers are risk averse with expected lifetime utility,

$$Q_0 = E_t \sum_{t=0}^{\infty} \beta^t \left[ u(C_t) - e(\lambda_t) \right],$$

where $C_t$ is the manager’s compensation (consumption) and $\lambda_t$ is the effort needed to innovate as described below. The period utility satisfies $u' > 0$, $u'' < 0$ and $e' > 0$, $e'' > 0$, $e(0) = 0$, $e'(0) = 0$. Managers are characterized by human capital $h_t$. The output generated by the firm is equal to $h_t$.

Human capital can be increased by investing $I_t = \kappa(\lambda_t)h_t$. The investment generates a new implementable project $i_{t+1}$ according to the technology,

$$i_{t+1} = \lambda_t h_t \varepsilon_{t+1}.$$

The variable $\lambda_t$ determines the scale of the investment. A larger scale is associated with a higher cost for the firm and requires more effort from the manager. The variable $\varepsilon_{t+1} \in \{\varepsilon_L, \varepsilon_H\}$ is stochastic and i.i.d., with $\mathbb{E}(\varepsilon) > 0$. The function $\kappa(\cdot)$ is strictly increasing, strictly convex and satisfies $\kappa(0) = 0$.

We think of $i_{t+1}$ as a new project that enhances the human capital of the manager only if the project is implemented in a firm—either the current or new firm. If the new project $i_{t+1}$ is implemented, the human capital of the manager becomes $h_{t+1} = h_t + i_{t+1}$. If the new project $i_{t+1}$ is not implemented—for instance, if the manager leaves the financial sector and finds occupation outside the financial industry—her human capital remains $h_t$. Therefore, if a new project is implemented after the development stage, it becomes embedded human capital. Otherwise it fully depreciates. The importance of this assumption will become clear later.

To use a compact notation, we denote by $\pi(\lambda_t) = 1 - \kappa(\lambda_t)$ the output, net of the investment expenditures, generated by the firm per each unit of human capital. Furthermore, we denote by $g(\lambda_t, \varepsilon_{t+1}) = 1 + \lambda_t \varepsilon_{t+1}$ the gross growth rate of human capital, provided the manager remains employed in a financial firm. Then, the firm’s cash flow and the evolution of human capital can be written as

$$\Pi_t = \pi(\lambda_t)h_t, \quad (1)$$

$$h_{t+1} = g(\lambda_t, \varepsilon_{t+1})h_t. \quad (2)$$

Information structure and timing. All variables are public information with the exception of $\lambda_t$, the innovation scale chosen by the manager. Although this variable is not

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6The assumption that the pre-existing human capital does not depreciate when the manager leaves the financial industry is not essential for the qualitative properties of the model. It is only made to maintain the linear homogeneity in $h_t$. The alternative assumption that the whole human capital depreciates when the manager leaves the financial sector would lead to similar properties. However, we would lose the linear homogeneity property in $h_t$. As we will see, this property allows us to work with a representative firm even if there is heterogeneity in $h_t$. 

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observable, the investor can infer the value of $\lambda_t$ by observing the investment $I_t = \kappa(\lambda_t)h_t$, since $h_t$ is public information. However, we assume that the investor observes $I_t$ (and indirectly $\lambda_t$) at the end of the period, after the payment of the manager’s compensation $C_t$. This assumption implies that $C_t$ cannot be conditional on the current choice of $\lambda_t$. It can only be contingent on past innovations. An alternative assumption could be that, given the choice of $\lambda_t$, the investment cost is incurred at $t+1$. In this way the firm has to wait next period before inferring the value of $\lambda_t$. This alternative assumption does not change the properties of the model but it requires a more cumbersome notation.

**Agency issues for managers.** Managers have an option to quit and search for an offer in a new firm. If a manager chooses to quit, she will receive an offer with probability $\rho \in [0,1]$. This probability captures the degree of competition for managers, that is, the ease with which a manager finds occupation in the financial sector after quitting the current employer. Higher values of $\rho$ denote more competition for managers. Since we are assuming that an implementable project of size $i_{t+1}$ fully depreciates if not implemented in a firm, the human capital of a manager who chooses to quit at the beginning of $t+1$ will be $h_t + i_{t+1}$ only if she receives an offer. Otherwise, her human capital remains $h_t$.

Denote by $Q_{t+1}(h_t)$ the manager’s outside value at the beginning of period $t+1$ without an external offer and by $\overline{Q}_{t+1}(h_{t+1})$ the outside value with an offer. The expected outside value at $t+1$ of a manager with human capital $h_t$ is equal to

$$D(h_t, h_{t+1}, \rho) = (1 - \rho) \cdot Q_{t+1}(h_t) + \rho \cdot \overline{Q}_{t+1}(h_{t+1}).$$

(3)

For the moment we take the probability $\rho$ and the outside value functions $Q_{t+1}(h_t)$ and $\overline{Q}_{t+1}(h_{t+1})$ as exogenous. At this stage we only assume that $Q_{t+1}(h_t)$ and $\overline{Q}_{t+1}(h_{t+1})$ are strictly increasing and differentiable. However, when we extend the model to a general equilibrium in Section 5, these terms will be derived endogenously. This is an important element that will be central for some of the key results of this paper.

In addition to having the ability to quit, the manager has full control over $\lambda_t$. Full control is allowed by the assumption that this variable is not directly observable by investors. Investors can only infer the value of $\lambda_t$ at the end of the period, after the payment of $C_t$. This implies that, in absence of proper incentives, the value $\lambda_t$ chosen by the manager may not be efficient. In particular, since the manager does not take into account the investment cost $I_t$ incurred by the firm, she may be tempted to increase $\lambda_t$ in order to raise her outside value. Therefore, there are two types of enforcement problems on the side of managers: ability to quit and discretion in the choice of $\lambda_t$.

**Agency issues for investors: partnership vs. public companies.** Agency issues could also emerge for firms since they could renge on past promises made to managers. The limited commitment of firms, however, depends on their ownership structure. In a partnership the firm operates on behalf of partners. Therefore, a firm cannot gain from reneging on promises made to the partners (or partners-managers) it represents. Based on this observation, we characterize the contractual relationship in a partnership
assuming one-sided limited commitment: the firm commits to the contract but individual partners-managers do not commit.

In a public company, instead, the firm operates on behalf of shareholders who are distinct from managers. Because of this separation, the temptation of firms to renege past promises made to managers becomes possible in public companies. To formalize this possibility we assume that in a public company there is double-sided limited commitment: managers could quit and investors could renege past promises.

3 Optimal contract

In both cases of partnerships and public companies we make the simplifying assumption that there is constant return to scale in the operation of the firm. Consequently, we can limit the analysis to the optimal contract between a risk-neutral investor and a risk-averse manager. Following is a formal definition of a contract.

Definition 1 A contract between an investor and a manager with initial human capital \( h_0 \) consists of sequences of investments \( \{\lambda(H^t,\Lambda^{t-1})\}_{t=0}^{\infty} \), payments to the manager \( \{C(H^t,\Lambda^{t-1})\}_{t=0}^{\infty} \) and to the investor \( \{\pi(\lambda(H^t,\Lambda^{t-1}))h_t - C(H^t,\Lambda^{t-1})\}_{t=0}^{\infty} \), conditional on the history of human capital \( H^t = (h_0,\ldots,h_t) \) and investment \( \Lambda^{t-1} \equiv (\lambda_0,\ldots,\lambda_{t-1}) \).

Notice that the manager’s compensation \( C_t \) is not conditional on \( \lambda_t \) but only on past values. This is because \( \lambda_t \) becomes public information only after the payment of \( C_t \).

3.1 One-sided limited commitment: The case of a partnership

The optimal contract is characterized by solving a planner’s problem that maximizes the weighted sum of utilities for the investor and the manager but subject to a set of constraints. These constraints guarantee that the allocation chosen by the planner is enforceable in the sense that both parties choose to participate and the manager has no incentive to take actions other than those prescribed by the contract. We first characterize the key constraints and then we specify the optimization problem.

The allocation chosen by the planner must be such that the value of the contract for the manager is not smaller than the value of quitting at the beginning of every period. This gives rise to the enforcement constraint,

\[
E_t \sum_{n=0}^{\infty} \beta^n \left[ u(C_{t+n}) - e(\lambda_{t+n}) \right] \geq D(h_{t-1},h_t,\rho),
\]

which must be satisfied for all \( t \geq 1 \). Notice that the contract starts at time zero but the constraint must be satisfied starting at \( t \geq 1 \).

A second constraint takes into account that the manager has full control in the choice of investment and could deviate from the \( \lambda_t \) recommended by the planner (incentive-compatibility). Denote by \( \hat{\lambda}_t \) the innovation chosen by the manager when she deviates from the recommended innovation. By deviating, the manager anticipates leaving the
firm at the beginning of the next period. Therefore, $\hat{\lambda}_t$ maximizes the anticipated value of quitting, that is,

$$\hat{\lambda}_t = \arg\max_{\lambda \in [0,1]} \left\{ u(C_t) - e(\lambda) + \beta E_t D(h_t, h_{t+1}, \rho) \right\},$$ \hspace{1cm} (5)$$

where $h_{t+1} = g(\lambda, \varepsilon_{t+1}) h_t$.

It is important to point out that, the assumption that the manager quits at the beginning of next period after deviating is made to simplify the presentation but it is without loss of generality. In fact, the manager could still continue employment with the current firm after deviating. However, the continuation value received at $t + 1$ (after deviating) would be $D(h_t, h_{t+1}, \rho)$. This is because it is ex-ante optimal for the planner to impose the maximum punishment in case of deviation. Given the manager’s option to quit, the maximum punishment is the value of quitting.

The optimal deviation $\hat{\lambda}_t$ can be characterized with the first order condition,

$$e_1(\hat{\lambda}_t) = \beta E_t D_2(h_t, h_{t+1}, \rho) g_1(\hat{\lambda}_t, \varepsilon_{t+1}) h_t.$$ \hspace{1cm} (6)$$

We have used numerical subscripts to denote the derivative of a function with respect to a particular argument. The assumed properties of the function $e(.)$ guarantee an interior solution, that is, $\hat{\lambda}_t \in (0,1)$. From now on, we will denote with the hat sign the innovation that maximizes the expected outside value net of dis-utility.

An important feature of the optimal deviation $\hat{\lambda}_t$ is that it is not affected by current compensation $C_t$ because of the assumption that $\lambda_t$ becomes public information only after paying the manager’s compensation. The manager can still be punished at $t + 1$ by cutting $C_{t+1}$. However, at that point, the ability to quit sets a lower bound to the feasible punishment. If $C_t$ could be conditioned on $\lambda_t$, investors could punish managers’ deviation by reducing $C_t$. With a utility function that satisfies $u(0) = -\infty$, the planner would have unlimited power to punish.

Given the optimal deviation, $\hat{\lambda}_t$, the incentive-compatibility constraint at time $t$ can be written as,

$$u(C_t) - e(\hat{\lambda}_t) + \beta E_t \sum_{n=0}^{\infty} \beta^n \left( u(C_{t+n+1}) - e(\lambda_{t+n+1}) \right) \geq$$

$$u(C_t) - e(\hat{\lambda}_t) + \beta E_t D(h_t, g(\hat{\lambda}_t, \varepsilon_{t+1}) h_t, \rho).$$ \hspace{1cm} (7)$$

The left-hand-side is the manager’s value if she chooses the innovation recommended by the planner, $\lambda_t$. The right-hand-side is the value achieved by deviating from the recommended policy and choosing $\hat{\lambda}_t$ as determined in (5).

We now have all the ingredients to write down the optimization problem solved by the planner in a regime with one-sided limited commitment (partnership).

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7Specifically, $D_2(.,.,.)$ denotes the derivative of the outside value with respect to the second argument and $e_1(.)$ is the derivative with respect to the first and only argument.
Let \( \tilde{\mu}_0 \) be the planner’s weight assigned to the manager and normalize to 1 the weight assigned to the investor. The planner solves the problem

\[
\max_{\{C_t, \lambda_t\}_{t=0}^\infty} E_0 \left\{ \sum_{t=0}^\infty \beta^t \left( \pi(\lambda_t) h_t - C_t \right) + \tilde{\mu}_0 \sum_{t=0}^\infty \beta^t \left( u(C_t) - e(\lambda_t) \right) \right\}
\]

(8)

s.t. (2), (4), (7).

The optimization problem is also subject to initial participation constraints for both the investor and the partner. These constraints, which for simplicity we have omitted, only restrict the admissible values for the weight \( \tilde{\mu}_0 \).

Following Marcet and Marimon (2011), the problem can be written recursively as

\[
\tilde{W}(h, \tilde{\mu}) = \min_{\tilde{\chi}, \tilde{\gamma}(\varepsilon')} \max_{C, \lambda} \left\{ \pi(\lambda) h - C + \tilde{\mu} \left( u(C) - e(\lambda) \right) - \tilde{\chi} \left( e(\lambda) - e(\tilde{\lambda}) \right) + \beta E \left[ \tilde{W}(h', \tilde{\mu}') - \left( \tilde{\chi} + \tilde{\gamma}(\varepsilon') \right) D(h, h', \rho) \right] \right\}
\]

(9)

s.t. \( h' = g(\lambda, \varepsilon') h \), \( \tilde{\mu}' = \tilde{\mu} + \tilde{\chi} + \tilde{\gamma}(\varepsilon') \),

where \( \tilde{\gamma}(\varepsilon') \) is the Lagrange multiplier for the enforcement constraint (4), \( \tilde{\chi} \) is the multiplier for the incentive-compatibility constraint (7), and prime denotes next period.

The variable \( \tilde{\mu} \) in the recursive problem captures the value of the contract for the manager: Higher values of \( \tilde{\mu} \) are associated with higher values of the contract for the manager. This variable evolves over time according to \( \tilde{\mu}' = \tilde{\mu} + \tilde{\chi} + \tilde{\gamma}(\varepsilon') \). Therefore, the value for the manager increases whenever the incentive-compatibility constraint or the enforcement constraint are binding. This is necessary to prevent the manager from deviating and quitting.

**Optimal partnership policies.** Differentiating problem (9) with respect to the manager’s consumption \( C \) we obtain,

\[
C_t = u_{\tilde{\mu}_t}^{-1} \left( \frac{1}{\tilde{\mu}_t} \right),
\]

which characterizes the *compensation policy* as a function of the state variable \( \tilde{\mu}_t \).

Since \( \tilde{\mu}_{t+1} = \tilde{\mu}_t + \tilde{\chi}_t + \tilde{\gamma}(\varepsilon_{t+1}) \), the manager’s consumption increases whenever the enforcement and/or incentive-compatibility constraints are binding. Since \( \lambda_t \) is always positive, \( h_t \) grows in expectation and with it the outside value for the manager \( D(h_t, h_{t+1}, \rho) \). This implies that the enforcement constraint becomes binding at some point in the future, raising the value of \( \tilde{\mu} \). From equation (10) we can then see that the growth in \( \tilde{\mu} \) is inherited by consumption. Therefore, the optimal partnership contract does not provide full consumption insurance to the manager.
The **investment policy** is characterized by the first-order condition with respect to $\lambda$, which can be written as

$$
\left( \frac{\bar{\mu}_t + \bar{\chi}_t}{h_t} \right) e_1(\lambda_t) - \pi_1(\lambda_t) = \beta E_t \left[ W_1(h_{t+1}, \bar{\mu}_{t+1}) - \left( \bar{\chi}_t + \tilde{\gamma}_t(\varepsilon_{t+1}) \right) D_2(h_t, h_{t+1}, \rho) \right] g_1(h_t, \varepsilon_{t+1}). \quad (11)
$$

The left-hand side is the marginal cost of innovation per unit of human capital. This term is increasing in $\lambda_t$, $\bar{\mu}_t$ and $\bar{\chi}_t$, and decreasing in $h_t$. The right-hand-side is the expected marginal benefit from innovating, net of participation costs. If incentive constraints are not binding $\lambda_t$ is expected to increase with $h_t$. However, binding incentive-compatibility and enforcement constraints imply positive values of $\bar{\chi}_t$ and $\tilde{\gamma}_t(\varepsilon_{t+1})$, which tend to reduce the right-hand-side of equation (11) and – at least, partially – offset the effect of increasing $h_t$. Therefore, binding constraints reduce innovation $\lambda_t$, with respect to the case where incentive and enforcement constraints are not binding.

Intuitively, to retain the manager, the value of staying must increase or the value of quitting must decline. The value of staying can be increased by promising higher compensation and/or by valuing higher the cost of effort; increasing $\lambda_t$ directly achieves the former and indirectly, through the change in $\bar{\mu}_{t+1}$, the latter. The value of quitting can be reduced by choosing a lower $\lambda_t$. Both channels lead to a decrease in $\lambda_t$ when the constraints are binding.

We are especially interested in understanding how higher competition (captured by a higher value of $\rho$) affects the optimal investment policy. In an economy with higher $\rho$ managers have better outside opportunities, implying that the initial $\mu_0$ is higher (given $h_0$). Furthermore, as it can be seen from equation (11), $\rho$ only plays a direct effect on investment when incentive constraints are binding (i.e. when $(\bar{\chi}_t + \tilde{\gamma}_t(\varepsilon_{t+1})) > 0$). We have the following result, with the formal proof provided in Appendix A.

**Proposition 1**

If $W_{1,2}(h_{t+1}, \bar{\mu}_{t+1}) \leq 0$, more competition for managers (higher $\rho$) results in lower innovation $\lambda_t$ when the enforcement or incentive-compatibility constraints are binding.

As we discuss in Appendix A, the condition $W_{1,2}(h_{t+1}, \bar{\mu}_{t+1}) \leq 0$ is fairly general. In particular, it is satisfied when the manager’s utility from consumption takes the logarithmic form as we will see more explicitly in Section 4.

### 3.2 Double-sided limited commitment: The case of a public company

With double-sided limited commitment, which we think as representative of the contractual environment with public companies, managers are free to leave the firm and investors (shareholders) can renege promises made to managers. In case of reneging, the parties re-bargain the contract. To simplify the analysis we assume that the investor has all bargaining power. Under this assumption, by reneging the investor is able to force the
manager to the outside option. Therefore, whenever the value for the manager exceeds her outside value, the investor reneges. In the the planner’s problem this is captured by imposing that the enforcement constraint is always satisfied with equality.\(^8\)

The limited commitment of the investor alters the optimization problem (9) in several dimensions. First, since the manager anticipates that the investor reneges whenever her outside value is lower than the continuation utility, the manager always chooses the innovation that maximizes her outside value. Therefore, with double-sided limited commitment we have that \(\lambda_t = \hat{\lambda}_t\). This also implies that the incentive-compatibility constraint is no longer relevant and \(\tilde{\chi}\) can be set to zero.

The second modification is that the variable \(\tilde{\mu}_{t+1}\), the weight assigned by the planner to the manager in the next period, is no longer dependent on \(\tilde{\mu}_t\). The dependence of \(\tilde{\mu}_{t+1}\) from \(\tilde{\mu}_t\) (through the law of motion \(\tilde{\mu}_{t+1} = \tilde{\mu}_t + \tilde{\chi}_t + \tilde{\gamma}_t(\varepsilon_{t+1})\)) captures the investor’s commitment (through the commitment of the planner) to fulfill promises made to the manager in the next period. In the environment with one-sided limited commitment, even if the enforcement constraint is not binding tomorrow, the new weight assigned to the manager will not be reduced. Without investor’s commitment, however, promises made in the past and captured by the variable \(\tilde{\mu}_t\), are no longer relevant. Therefore, \(\tilde{\mu}_{t+1}\) is exclusively determined by the multiplier associated with the enforcement constraint in the next period, that is, \(\tilde{\mu}_{t+1} = \tilde{\gamma}_t(\varepsilon_{t+1})\). The contractual problem can be written as

\[
W(h, \tilde{\mu}) = \min_{\tilde{\gamma}(\varepsilon')} \max_C \left\{ \pi(\hat{\lambda}) h - C + \tilde{\mu} \left( u(C) - e(\hat{\lambda}) \right) + \beta E \left[ W(h', \tilde{\mu}') - \tilde{\gamma}(\varepsilon') D(h, h', \rho) \right] \right\}
\]

\[s.t. \quad h' = g(\lambda, \varepsilon') h, \quad \tilde{\mu}' = \tilde{\gamma}(\varepsilon').\]

The contract prescribes a consumption plan determined by (10) with \(\tilde{\mu}' = \tilde{\gamma}(\varepsilon')\) and the innovation \(\hat{\lambda}\) solves the first order condition (6). Since \(D_{2,3} > 0\), an increase in competition captured by \(\rho\) increases the right-hand-side of (6), that is, it increases the marginal benefit of innovation for the manager and leads to more innovation. This is stated formally in the next proposition and proved in Appendix A.

**Proposition 2** With double-sided limited commitment a higher \(\rho\) results in higher innovation \(\lambda = \hat{\lambda}\).

\(^8\)It is important to point out that our definition of double-sided limited commitment is different from the definition often used in the literature. Typically, in an environment with double-sided limited commitment, parties renege whenever the value of their contract falls below the outside value. In our definition, instead, investors renege even if the value of the contract is above their outside value. Also notice that we do not allow for equilibria with more complex mechanisms such as trigger strategies. Our equilibrium with double-sided limited commitment is equivalent to an environment in which the parties re-bargain every period and investors have all bargaining power.
Propositions 1 and 2 show that the effect of more competition for managers on risk-taking depends crucially on whether investors commit to the contract. Higher competition increases risk-taking only when there is limited commitment of both investors and managers. To the extent that the organizational change from partnerships to public companies increased mobility (formalized in a higher $\rho$) and weakened commitment (especially for investors), we should observe higher risk-taking.

4 Normalization with log-utility

Since human capital grows on average over time, so does the value of the contract for the manager and the investor. It is then convenient to normalize the growing variables so that we can work with a stationary formulation of the contracting problem. This is especially convenient when the utility of managers and the outside values take the logarithmic form.

Assumption 1 The utility function and the outside values of managers take the forms

\[ u(C) - e(\lambda) = \ln(C_t) - e(\lambda_t), \]

\[ Q_{t+1}(h_t) = q + B\ln(h_t), \]

\[ \overline{Q}_{t+1}(h_{t+1}) = \bar{q} + B\ln(h_{t+1}), \]

where $q$, $\bar{q}$ and $B \equiv \frac{1}{1-\beta}$ are constant.

Although the functional forms for the outside values $Q_{t+1}(h_t)$ and $\overline{Q}_{t+1}(h_{t+1})$ may seem arbitrary at this stage, we will see that in the extension to the general equilibrium they take exactly these forms.

We start by normalizing the value of the contract for the investor. This can be expressed recursively as $V_t = \pi(\lambda_t)h_t - C_t + \beta E_t V_{t+1}$ and can be rewritten as

\[ v_t = \pi(\lambda_t) - c_t + \beta E_t g(\lambda_t, e_{t+1})v_{t+1}, \]

where $v_t = V_t/h_t$ and $c_t = C_t/h_t$.

The value of the contract for a manager is $Q_t = \ln(C_t) - e(\lambda_t) + \beta E_t Q_{t+1}$. Defining $q_t = Q_t - B\ln(h_t)$, we can rewrite it in normalized form as

\[ q_t = \ln(c_t) - e(\lambda_t) + \beta E_t \left[ B\ln \left( g(\lambda_t, e_{t+1}) \right) + q_{t+1} \right]. \]

Next we consider the enforcement constraint (4) that can be written recursively as $Q_{t+1}(h_{t+1}) \geq (1-\rho)Q_{t+1}(h_t) + \rho \cdot \overline{Q}_{t+1}(h_{t+1})$. Using $q_{t+1} = Q_{t+1}(h_{t+1}) - B\ln(h_{t+1})$ and the functional forms specified in Assumption 1, the enforcement constraint can be rewritten as

\[ q_{t+1} \geq (1-\rho)q + 1 - \rho \cdot B\ln \left( g(\lambda_t, e_{t+1}) \right). \]
The right-hand-side depends on \( \lambda_t \) (provided that \( \rho < 1 \)). Thus, investment affects the outside value of the manager and, when the enforcement constraint is binding, it affects compensation. This property is a direct consequence of the assumption that the outside value of the manager without an external offer depends on \( h_t \), while the outside value with an external offer depends on \( h_{t+1} \). If both values were dependent on the embedded human capital \( h_{t+1} \), the last term in (15) would disappear. The outside value would still depend on \( \rho \) but it would not affect the choice of \( \lambda_t \).

The incentive-compatibility (7) can be written recursively as

\[
-e(\lambda_t) + \beta E_t Q_{t+1} \left( g(\lambda_t, \varepsilon_{t+1}) h_t \right) \geq \frac{\rho \ln g(\hat{\lambda}_t, \varepsilon_{t+1})}{1 + \lambda_t \varepsilon_{t+1}}
\]

where \( \lambda_t \) is the investment recommended by the contract and \( \hat{\lambda}_t \) is the investment chosen by the manager under the assumption that she will quit at the beginning of next period. The incentive-compatibility constraint can be rewritten in normalized form as

\[
-e(\hat{\lambda}_t) + \beta E_t \left[ q_{t+1} + B \ln \left( g(\hat{\lambda}_t, \varepsilon_{t+1}) \right) \right] \geq \frac{\rho \ln g(\hat{\lambda}_t, \varepsilon_{t+1})}{1 + \hat{\lambda}_t \varepsilon_{t+1}}
\]

We can now provide a more explicit characterization of the manager’s optimal deviation \( \hat{\lambda}_t \). Using \( g(\lambda, \varepsilon) = 1 + \lambda \varepsilon \), condition (6) can be written as

\[
\epsilon_1(\hat{\lambda}_t) = \rho B E_t \left( \frac{\varepsilon_{t+1}}{1 + \hat{\lambda}_t \varepsilon_{t+1}} \right).
\]

We can now see more explicitly that \( \hat{\lambda} \) increases with \( \rho \), as stated more generally in Proposition 2. Therefore, when the manager faces better outside options, the strategic incentive to innovate increases.

**One-sided limited commitment: The case of a partnership.** The original contractual problem (8) with one-sided limited commitment can be reformulated in normalized form using the ‘promised utility’ approach: This maximizes the normalized investor’s value subject to the normalized promise-keeping, limited enforcement and incentive-compatibility constraints, that is,

\[
v(q) = \max_{\lambda, c, q(\varepsilon')} \left\{ \pi(\lambda) - c + \beta E g(\lambda, \varepsilon') v(q(\varepsilon')) \right\} \quad \text{subject to (14), (15), (16)}.
\]

Although the objective of the firm in a partnership is to maximize the value of the contract for the partners, subject to the participation of the investor, here we work with
the dual problem which maximizes the value for the investor subject to the promise-keeping constraint for the manager; both problems give the same results.

The solution to problem (18) provides the investment policy \( \lambda = \varphi^\lambda(q) \), the compensation policy \( c = \varphi^c(q) \), and the continuation utilities \( q(\varepsilon^u(q, \varepsilon')) \). Because of the normalization, these policies are independent of \( h \). However, once we know the innovation policy \( \lambda \) and the initial human capital \( h \), we can reconstruct the whole sequence of human capital using the law of motion \( h' = g(\lambda, \varepsilon') \). Then, with the sequence of \( h \) we can compute \( C = ch, \; Q = q + B \ln(h) \) and \( V = vh \) from the normalized variables.

Since there is a one-to-one mapping from the normalized policies to the original (non-normalized) variables, we can characterize the optimal contract by focusing on the normalized variables which satisfy the first order conditions

\[
\begin{align*}
c &= \mu, \\
(\mu + \chi)q_1(\lambda) - \pi_1(\lambda) &= \beta \mathbb{E}\left[ v(\varphi(\varepsilon')) + \frac{B(\mu + \chi + (1 - \rho)\gamma(\varepsilon'))}{g(\lambda, \varepsilon')} \right] g_1(\lambda, \varepsilon'), \\
\mu(\varepsilon') &= \frac{\mu + \chi + \gamma(\varepsilon')}{g(\lambda, \varepsilon')}.
\end{align*}
\]

The variables \( \mu, \gamma(\varepsilon') \) and \( \chi \) are the Lagrange multipliers, respectively, for constraints (14), (15) and (16). These multipliers are related to the multipliers used in Section 3 by \( \mu = \tilde{\mu}/h, \; \gamma(\varepsilon')/h \) and \( \tilde{\chi}/h \). The detailed derivation of the first order conditions is provided in Appendix B.

**Double-sided limited commitment: The case of a public company.** With double-sided limited commitment, investors renegotiate promises that exceed the outside value of managers. Thus, the value of the contract for the manager is always equal to the outside value, that is, the enforcement constraint is always binding. Anticipating renegotiation, the best strategy for the manager is to choose the innovation that maximizes the outside value, that is, \( \hat{\lambda} \) as determined by condition (17). Problem (12) can then be reformulated in normalized form as,

\[
v(q) = \max_{c, q(\varepsilon)} \left\{ \pi(\hat{\lambda}) - c + \beta E g(\hat{\lambda}, \varepsilon) v\left( q(\varepsilon) \right) \right\}
\]

subject to

\[
q = \ln(c) - e(\hat{\lambda}) + \beta E \left[ B \ln \left( g(\hat{\lambda}, \varepsilon) \right) + q(\varepsilon) \right]
\]

\[
q(\varepsilon) = (1 - \rho)q + \rho \bar{q} - (1 - \rho)B \ln \left( g(\hat{\lambda}, \varepsilon) \right), \quad \text{for all } \varepsilon.
\]
Problem (22) is a special case of problem (18) where we have replaced the incentive-compatibility constraint (16) with \( \lambda = \hat{\lambda} \), and we have imposed that the enforcement constraint (15) is always satisfied with equality. Notice that the decision variables \( c \) and \( q(\varepsilon) \) are fully determined by the promise-keeping and enforcement constraint. Therefore, the problem can be solved without performing any optimization, besides solving for \( \hat{\lambda} \).

4.1 Contract properties

In this subsection we present the properties of the optimal contract numerically. The specific parameter values will be described in Section 6.1 when we conduct a quantitative analysis with the general model. To facilitate the intuition for the dynamics shown here, however, we anticipate that in the calibration the two values of the shocks have been normalized to \( \varepsilon_L = 0 \) and \( \varepsilon_H = 1 \). Therefore, in case of a negative shock, human capital remains constant. The computational procedure used to solve for the optimal contract is described in Appendix D.

The dynamics of promised utilities. The top panels of Figure 4 plot the values of next period normalized continuation utilities, \( q(\varepsilon) = \varphi^q(q, \varepsilon) \), as functions of current normalized utility, \( q \), for the environments with one-sided and double-sided limited commitment. We have also plotted the 45 degree line. If the continuation utility is below (above) the 45 degree line, the next period \( q \) is smaller (bigger) than the current \( q \). The vertical line indicates the initial normalized values of the contract for the manager, \( \bar{q} \). The determination of the initial value will be specified later when we embed the model in a general equilibrium.

We discuss first the case with one-sided limited commitment. The contract starts with an initial \( \bar{q} \) indicated by the vertical line. Then, if the investment does not succeed (\( \varepsilon' = \varepsilon_L = 0 \)), the next period value of \( q \) remains the same. If the investment succeeds (\( \varepsilon' = \varepsilon_H = 1 \)), the next period \( q \) declines until it reaches a lower bound. At that point the value of \( q \) remains constant. The motion of the multipliers \( \mu \), given by (21). When constraints are not binding and investment the is successful \( \mu(\varepsilon') < \mu \) and by the Envelope theorem, \( \mu = -v'(q) \), then concavity of \( v \) implies that \( q(\varepsilon') < q \). It is important to remember, however, that these are normalized utilities. Therefore, the fact that \( q \) declines does not necessarily mean that the actual (non-normalized) utility \( Q = q + B \ln(h) \) declines. Whether \( Q \) decreases or increases after a successful investment when constraints are not binding, depends on whether the manager’s marginal cost of investment outweighs its marginal benefit or not, since the manager is given a lower share of the surplus (lower \( \mu \)) in the optimal contract.\(^9\)

9For more details, see Appendix B.
surplus shares for the manager. The partnership provides relatively high consumption to the manager from the beginning, knowing that will get high rents from her in the future; therefore, the contract also has a counterpart for the investor: his main rewards are in the future. With limited enforcement constraints eventually, the normalized utility reaches a lower bound which is indicated by the intersection of the dashed line $q(\varepsilon_H)$ with the 45 degree line. After that the continuation utilities fluctuate in an interval delimited by the intersections of the two dashed lines with the 45 degree lines. The investor anticipates this fact and, therefore, his curtailed future rents translate in lower constant consumption when constraints are not binding and in non-smooth (higher) consumption when they bind.

The optimal policy in the environment with double-sided limited commitment is shown in the second panel of Figure 4. In this environment the investor does not commit to the contract and reneges promises that exceed the outside value of the manager. As a result, the manager always receives the outside value. The only exception is in the first period when she receives the lifetime utility indicated by the vertical line. After the initial period, $q$ jumps immediately to the outside option and fluctuates between two values. The fact that the initial $q$ (indicated by the vertical line) is bigger than future values
implies that in the first period the manager receives a higher payment (consumption) relative to her human capital.

The term *double-sided limited commitment* is normally used in the literature to refer to the case of *double-sided limited enforcement*, meaning that both contractual sides have alternative default options and the contract cannot violate the default options of both agents. We have been explicit about the *limited enforcement* on the manager’s side but mute on the investor’s side. However, in our economy the natural outside option for the investor is to hire another manager and start a new contract, but if the investor can commit to the new contract there is no gain for him in hiring a new contract, since as we have seen, in our environment, *one-sided limited commitment* contracts have the property that the investor gets most of his rents in the future; that is, once a contract has started the firm prefers the existing manager to a new manager with the same $h$. Of course, one could consider intermediate cases between the two analysed here. For example, that the contract only accounts for the manager’s *limited enforcement constraint* when she shows that has received an outside offer (i.e. after having done some costly on-the-job search), or that the investor can only commit with certain probability, however there is no much additional conceptual or quantitative gain for considering them here\textsuperscript{10}. In particular, regarding the move to a *double-sided limited commitment*, it should be understood that for the investor the temptation to renege from his promises is not when the manager threatens to quit, since then he would also incur the cost of posting a new vacancy, but while the manager’s enforcement constraint is not binding and, therefore, has no credible does not have a credible threat to a curtail of promised consumption, except that then the manager will realise that the investor does not fulfil the contract and, therefore, this intermediate situation, where the time for the enforcement constraints to bind is reduced, cannot be a contractual equilibrium; in other words, once investors are not trusted by managers and investors understand we are in an environment with *two-sided limited commitment*\textsuperscript{11}.

**Investment.** The bottom panels of Figure 4 plot the investment policy $\lambda$. In the environment with one-sided limited commitment, the enforcement and incentive-compatibility constraints are not binding for high values of $q$. As a result, $\lambda$ is only determined by the investment cost, part of which is given by the effort dis-utility. For lower values of $q$, however, the enforcement constraint is either binding or close to be binding. Consequently, a higher value of $\lambda$ increases the outside value for the manager and must be associated with a higher promised utility. Since this is costly for the investor, the optimal $\lambda$ is smaller for low values of $q$.

In the environment with double-sided limited commitment $\lambda$ is independent of $q$ since

\textsuperscript{10}We have studied intermediate cases in previous versions of this work and details can be provided upon request

\textsuperscript{11}We do not consider the possibility that a partnership, or public company, may gain from having a reputation for keeping their promises, since in our competitive economy expected profits are zero and reputation requires rents to be sustained. The detailed analysis of such models is beyond the scope of this paper; see Marimon, Nicolini and Teles (2012) for a model of ‘competition and reputation for commitment’.
the manager always chooses $\lambda = \hat{\lambda}$, which is determined by condition (17). This is the value that maximizes her outside value net of the utility cost of effort. But in doing so, the manager does not take into account that investment is also costly for the firm.

For the particular parametrization considered here, the investment chosen with double-sided limited commitment is greater than in the environment with one-sided limited commitment. However, this property is not general for two reasons. First, because, as already mentioned, even with full commitment—and, therefore, with one-sided limited commitment when constraints are not binding—whether the optimal contract prescribes higher or lower risk (i.e. $\lambda$), when the manager share of the surplus decreases after a successful investment, depends on whether manager’s marginal cost of investment outweighs its marginal benefit or not. Second, because there are two contrasting mechanisms that affect innovation in the two—one-sided vs. double-sided—environments. With double-sided limited commitment, the manager chooses the investment that maximizes the outside value. Because the manager does not take into account the cost incurred by the firm, $\kappa(\lambda) h$, this leads to the choice of a higher $\lambda$ compared to the environment with one-sided limited commitment. This mechanism, however, is alleviated by the fact that innovations increase the outside value of managers with probability $\rho < 1$. Instead, when $\lambda$ is chosen to maximize the surplus of the existing contract—which is the case in the one-sided limited commitment—the innovation adds value with probability 1. This leads to a lower $\lambda$ when there is double-sided limited commitment. Therefore, to have that higher investment in the environment with double-sided limited commitment, we need that the probability of finding another occupation (the probability $\rho$) is sufficiently large.

5 General model

We now embed the financial sector in a general equilibrium. This allows us to endogenize the parameter $\rho$ and the outside values $Q_{t+1}(h_t)$ and $\overline{Q}_{t+1}(h_{t+1})$.

There are two sectors: financial and nonfinancial. Given the service nature of the financial sector, we assume that its output is used as an intermediate input in the production of final goods by nonfinancial firms. Since we are now representing the whole economy, we think of the agents in the contractual relationships more generally as workers rather than exclusively as managers.

There are two types of agents as in the baseline model—a unit mass of investors and a unit mass of risk-averse workers, with the same preferences as described earlier. We focus on the case of log-utility for workers as specified in Section 4. However, we now assume that workers die with probability $\omega$. The discount factor should then be considered the product of the actual intertemporal discount factor, $\hat{\beta}$, and the survival probability $1 - \omega$, that is, $\beta = \hat{\beta}(1 - \omega)$.

In every period a mass $\omega$ of newborn workers with initial human capital $h_0$ enter the economy so that the population size remains constant. The motivation for choosing this demographic structure is to keep the distribution of $h_t$ among living workers stationary. The finite lives of workers together with the same initial human capital $h_0$ guarantee that the cross-sectional distribution of human capital converges to an invariant distribution.
Of the newborn workers a fraction $\psi$ have the skills to work in the financial sector and they could choose to do so at any point in their lifetime if they have the opportunity as we will describe below.

Innovations, as described earlier, take place only in the financial sector. In the real economy, of course, the financial sector is not the only sector that innovates. Therefore, in the model, we should interpret the innovations that take place in the financial sector as ‘differential’ in comparison to the rest of economy. An alternative interpretation is that the financial sector encompasses all the ‘innovative segments’ of the economy with financial and nonfinancial, where similar organizational changes have taken place. In this paper we prefer to focus on the financial sector because this is the sector where the organizational and economic changes described in the introduction took place in a larger scale.

The technology and contractual structure of the financial sector is as described in the previous sections. The only difference is that the output produced by the financial sector is in the form of intermediate services purchased by nonfinancial firms. The cash flows generated by a financial firm is as specified before, that is,

$$\Pi_t = P_t \left( 1 - \kappa(\lambda_t) \right) h_t,$$

where $P_t$ is the equilibrium price for financial services. For simplicity we assume that the investment cost is measured in units of financial services. Although the price is determined in the general equilibrium, an individual firm takes it as given. Therefore, the characterization of an optimal contract is analogous to the previous sections.

We now place some structure on the hiring process. We introduce matching frictions in the labor market: Workers in search of an occupation find employment if they are matched with vacancies funded by investors. Since workers are heterogeneous in human capital, we assume directed search. Vacancies specify the level of human capital $h$ and the initial value of the contract for the worker $Q_t(h)$. We have added a time subscript to the function because, outside the steady state, this function could depend on the aggregate states. The cost of posting a vacancy is $\tau h$ where $\tau$ is a constant.

Denote by $N_t(h, \overline{Q}_t)$ the number of vacancies posted for workers with human capital $h$ and offering lifetime utility $\overline{Q}_t$. Furthermore, denote by $S_t(h, \overline{Q}_t)$ the number of workers with human capital $h$ in search of an occupation in the financial sector with posted value $\overline{Q}_t$. The number of matches is determined by the function $m_t(h, \overline{Q}_t) = N_t(h, \overline{Q}_t)^\eta S_t(h, \overline{Q}_t)^{1-\eta}$. From this matching function we derive the probabilities that a vacancy is filled, $\phi_t(h, \overline{Q}_t) = m_t(h, \overline{Q}_t)/N_t(h, \overline{Q}_t)$, and the probability that a searching worker finds occupation, $\rho_t(h, \overline{Q}_t) = m_t(h, \overline{Q}_t)/S_t(h, \overline{Q}_t)$. Free entry in the financial sector implies that, for any level of human capital, the following condition will be satisfied in equilibrium $\phi_t(h, \overline{Q}_t) V_t(h, \overline{Q}_t) = \tau h$. This is the typical free-entry condition where the value of the match for the investor, $V_t(h, \overline{Q}_t)$, multiplied by the probability of matching with a worker, $\phi_t(h, \overline{Q}_t)$, is equal to the cost of posting a vacancy, $\tau h$.

We now take advantage of the properties derived earlier in the case of log-utility for workers. We have seen in Section 4 that the value of the contract for the investor is linear in $h$, that is, $V_t(h, \overline{Q}_t) = v_t(\overline{q}_t)h$, and the normalized value for a newly hired worker can
be written as $\bar{q}_t = Q_t - B \ln(h_t)$. To determine $\bar{q}_t$ we only need to define a menu of posted contracts for all possible levels of human capital $h$. Focusing on a symmetric equilibrium in which the probability of filling a vacancy is independent of $h$, the free-entry condition in the financial sector can be rewritten in normalized form as

$$\phi_t(\bar{q}_t) v_t(\bar{q}_t) = \tau. \quad (23)$$

Appendix C discusses the equilibrium conditions in the labor market in more detail and shows that matched workers receive a fraction $1-\eta$ of the surplus. This is a standard property in models with directed search.

The next step is to characterize the outside value of workers employed in the financial sector, which is equal to

$$D_t(h_{t-1}, \lambda_{t-1}, \varepsilon_t) = (1 - \rho_t)Q_t(h_{t-1}) + \rho_t \bar{Q}_t(h_t). \quad (24)$$

We have already defined the matching probability $\rho_t$ and the value of the match for the worker $Q_t(h_t) = \bar{q}_t + B \ln(h_t)$. What is left to define is the value of not matching, $Q_t(h_{t-1})$. In this case the worker finds occupation in the nonfinancial sector. Therefore, to derive this value we need to describe the nonfinancial sector.

There are not matching frictions in the nonfinancial sector and the labor market is competitive. A worker with human capital $h_t$ produces final goods $F(x_t)h_t$, where $x_t$ is the input of financial services per unit of human capital. The function $F(\cdot)$ is strictly increasing and concave. The cash flows generated by a nonfinancial firm is then

$$\Pi_{NF}^t = (F(x_t) - x_t P_t) h_t,$$

where the superscript $NF$ clarifies that the variable is for the nonfinancial sector.

The input of financial services $x_t$ is determined by the first order condition $\partial F(x_t)/\partial x_t = P_t$. Using this condition we can rewrite the cash flows more compactly as $\Pi_{NF}^t = A(P_t) h_t$. Competition for human capital in the nonfinancial sector then implies that workers earn $A(P_t)$ per unit of human capital. This shows that the wage rate in the nonfinancial sector depends on the price of financial services. If this price falls, the income of a worker employed in the nonfinancial sector rises.

Since there are not innovations in the nonfinancial sector, $\lambda_t = 0$. Therefore, the current utility of workers reduces to $\ln(A(P_t) h_t)$. For calibration purposes we assume that a worker with financial skills has a different productivity in the nonfinancial sector than a worker without skills. In particular, the effective human capital in the nonfinancial sector of a worker with financial skills is $\zeta h_t$. The lifetime utility of a worker currently employed in the nonfinancial sector with human capital $h$ and with the skills to find an occupation in the financial sector is

$$Q_t(h) = \ln \left( A(P_t) \zeta h \right) + \beta \left[ (1 - \rho_{t+1}) \cdot Q_{t+1}(h) + \rho_{t+1} \cdot \bar{Q}_{t+1}(h) \right]. \quad (25)$$

12 The proper calibration of the parameter $\zeta$ insures that the value of searching in the financial sector for workers with financial skills is greater than the value of working in the nonfinancial sector.
The worker consumes the wage income \( \Pi_{t}^{NF} = A(P_t)\zeta h_t \) in the current period. In the next period she will find a job in the financial sector with probability \( \rho_{t+1} \). In this case the lifetime utility is \( \overline{Q}_{t+1}(h) \). This is the value of a new contract in the financial sector. With probability \( 1 - \rho_{t+1} \) she will remain employed in the nonfinancial sector and the lifetime utility is \( \overline{Q}_{t+1}(h) \). This provides the last piece needed to complete the definition of the outside value of a worker employed in the financial sector as specified in (24).

5.1 Partnerships vs. public companies

In the partial equilibrium analysis we have emphasized the importance of the separation between ownership and management to differentiate a partnership from a public company. We have then focused on the implication of this separation for the enforcement of contracts. Since a partnership operates on behalf of the partners, the firm is unlikely to renege on promises made to the partners it represents. In a public company, instead, the firm operates on behalf of investors (shareholders), who are distinct from managers. This introduces the possibility that the firm reneges on managers.

In the partial equilibrium analysis we have emphasized the importance of the separation between ownership and management to differentiate a partnership from a public company. We have then focused on the implication of this separation for the enforcement of contracts. In particular, we have modelled, as a transition from one-sided to two-sided limited commitment, the transition from the modern partnership, where the firm is committed to its contracts—in particular, with partners acting as managers—to the public company, where the firm operates on behalf of investors (shareholders), who are distinct from managers, introducing the possibility that the firm reneges on managers.

However, the weaker contractual structure of the public company is counterbalanced by a higher ability to attract external investors which do not need to become partners, just shareholders. We model this ‘investment wedge’ as a cost of partnership’s capital \( \xi h \), where \( h \) is the initial human capital of the associated manager. For expositional convenience we assume that this cost is paid upfront in the first period of the match. But this is without loss of generality because, for example, it can be reinterpreted as the discounted value of a minimum return that (a marginal) investor-partners must receive to ‘invest and commit’. In sum, investors are willing to fund the posting of a vacancy only if the following condition is satisfied

\[
\tau h \leq \phi_t(h, \overline{Q}_t) \left[ V_t(h, \overline{Q}_t) - \xi h \right].
\]

The term on the left-hand-side is the cost of a vacancy while the term on the right-hand-side is the current value of the net payments received by investors: the cash flows paid by the firm, \( V_t(h, \overline{Q}_t) \), minus the wedge cost, \( \xi h \). The net payments will be received only if the vacancy is filled, which happens with probability \( \phi_t(h, \overline{Q}_t) \).

Since the supply of funds by external investors is perfectly elastic, the above condition will be satisfied with equality in equilibrium. Therefore, in normalized form, the free-entry condition for jobs created by partnerships can be written as

\[
\phi_t(\hat{q}_t) \left[ v_t(\hat{q}_t) - \xi \right] = \tau.
\]
Effectively, the lower ability to attract external funds by partnerships reduces the value of posting a vacancy and in equilibrium we expect that fewer vacancies will be posted. In this way the equilibrium with partnerships could be characterized by a lower matching probability $\rho$ (lower competition for workers). Therefore, in the general equilibrium, when there is a partnership’s ‘investment wedge’, the change in $\rho$ is an outcome of the organizational change. Alternatively, the same effect, of increasing the competition for managers, can take place in our model if hiring costs $\tau$ decrease. For example, anticipating the weaker commitment, investors are willing to hire less sophisticated or trustworthy managers (even if formally $h$ is the same). We keep the former interpretation in what follows.

5.2 General equilibrium

We focus on steady state equilibria, which can be defined as:

**Definition 2 (Steady state)** Given the contractual regime in the financial sector (one-sided or double-sided), a steady state general equilibrium is defined by (i) Contractual policies in the financial sector $\lambda = \phi^\lambda(q)$, $c = \phi^c(q)$, $q(\varepsilon) = \phi^q(q, \varepsilon)$; (ii) Normalized utilities for newly hired workers in the financial sector, $\bar{q}$, and initial normalized value for investors, $\bar{v}$; (iii) Demand and price of financial services $X$ and $P$; (iv) Posted vacancies, $N(h)$, searching workers, $S(h)$, filling probabilities, $\phi(h)$, and finding probabilities, $\rho(h)$; (v) Wage rate in the nonfinancial sector $A(P)$; (vi) Distributions of workers in the financial sector, $M^F(h, q)$, and in the nonfinancial sector, $M^{NF}(h, q)$; (vii) Law of motion for the distribution of workers, $(M^F_{t+1}, M^{NF}_{t+1}) = \Phi(M^F_t, M^{NF}_t)$. Such that (i) The policy rules $\phi^\lambda(q)$, $\phi^c(q)$, $\phi^q(q, \varepsilon)$ solve the optimal contract in the financial sector; (ii) The normalized utility $\bar{q}$ and investor value $\bar{v}$ solve the free-entry condition (23) and investors receive a share $\eta$ of the surplus; (iii) the market for financial services clears (demand $X$ is equal to the production of financial services) and $P$ is the equilibrium price; (iv) Filling and finding probabilities satisfy $\phi(h) = m(N(h), S(h))/N(h)$ and $\rho(h) = m(N(h), S(h))/S(h)$; (v) The demand of financial services satisfies the first order condition $\partial F(x)/\partial x = P$ and workers employed in the nonfinancial sector earn the wage rate $A(P_t)$; (vi) The law of motion $\Phi(M^F, M^{NF})$ is consistent with individual policies; (vii) The distributions of workers in the financial and nonfinancial sectors are constant, that is, $(M^F, M^{NF}) = \Phi(M^F, M^{NF})$.

For the later analysis, it will be convenient to state the following properties:

**Lemma 3** In a steady state the fraction of workers employed in the nonfinancial sector is $1 - \psi \rho / [1 - (1 - \omega)(1 - \rho)]$. They all have the same human capital $h_0$ (although the effective human capital of those with financial skills is $\zeta h_0$).

**Proof.** Since the probability of finding a job in the financial sector is $\rho < 1$, in equilibrium the value of being employed in the financial sector must be bigger than the value of being employed in the nonfinancial sector. This implies that, once a worker finds occupation
in the financial sector, she will stay in that sector until retirement through death. As a result, only newborn workers (whose human capital is $h_0$) will enter the nonfinancial sector. Since there is no innovation in the nonfinancial sector, these workers will continue to have the initial human capital $h_0$. The steady state fraction of workers is found through the job flow equations. Denote by $\Gamma_s^t$ and $\Gamma_u^t$ the mass of workers employed in the nonfinancial sector with and without, respectively, the skills to work in the financial sector. The flow equations for these two groups of workers are

\[
\begin{align*}
\Gamma_{s}^{t+1} &= (1 - \omega)\Gamma_{s}^{t} + \psi \omega (1 - \rho) \\
\Gamma_{u}^{t+1} &= (1 - \omega)\Gamma_{u}^{t} + (1 - \psi)\omega
\end{align*}
\]

In a steady state $\Gamma_{s}^{t+1} = \Gamma_{s}^{t} = \Gamma_{s}^{\infty}$ and $\Gamma_{u}^{t+1} = \Gamma_{u}^{t} = \Gamma_{u}^{\infty}$. Imposing these conditions, we can derive to total number of workers employed in the nonfinancial sector $\Gamma_{s}^{\infty} + \Gamma_{u}^{\infty} = 1 - \psi \rho / [1 - (1 - \omega)(1 - \rho)]$.

Lemma 4 An increase in $\rho$ results in a higher steady-state contract value $q$ offered to the manager; i.e. $\overline{q}'(\rho) > 0$.

The lemma, proved in Appendix C, states that more competition for workers in either sectors (captured by the matching probability), redistributes rents in their favor. We will use the lemma later to establish some of the main results of the paper.

5.3 Inequality

Since the income of workers depends on human capital, we can use $h$ as a proxy for the distribution of income in the financial sector (in the nonfinancial sector all workers have the same human capital as stated in Lemma 3). As a specific index of inequality we use the coefficient of variation in human capital. In the regime with double-sided limited commitment this index can be computed analytically.

Consider the environment with double-sided limited commitment and denote by $\hat{\lambda}$ the innovation effort that maximizes the outside value of a worker in the financial sector as determined by condition (17). With double-sided limited commitment the optimal effort is the same for all workers employed in the financial sector. Therefore, the gross growth rate of human capital for an individual worker is $g(\hat{\lambda}, \varepsilon')$. Appendix E shows that the average human capital and the coefficient of variation for the cross sectional distribution of human capital are equal to

\[
\begin{align*}
\text{Ave}(h) &= h_0 \left[ \frac{\omega}{1 - (1 - \omega)Eg(\hat{\lambda}, \varepsilon)} \right], \\
\text{Std}(h) \over \text{Ave}(h) &= \sqrt{\frac{[1 - (1 - \omega)Eg(\hat{\lambda}, \varepsilon)]^2}{\omega[1 - (1 - \omega)Eg(\hat{\lambda}, \varepsilon)^2]}} - 1.
\end{align*}
\]
Therefore, the average human capital and the inequality index in the financial sector are simple functions of the investment $\lambda$ and they satisfy the following properties:

**Lemma 5** With double-sided limited commitment, the average human capital and the inequality index are strictly increasing in $\lambda$.

That average human capital increases with investment is obvious. The dependence of inequality on $\lambda$ can be explained as follows. If $\lambda = 0$, the human capital of all workers will be equal to $h_0$ and the inequality index is zero. As $\lambda$ becomes positive, inequality increases for two reasons. First, since the growth rate $g(\lambda, \varepsilon)$ is stochastic, human capital will differ within the same age cohort of workers. Second, since each age cohort experiences growth on average, the average human capital differs between age cohorts. More importantly, the cross sectional dispersion in human capital induced by these two mechanisms (the numerator of the inequality index) dominates the increase in average human capital (the denominator of the inequality index). Appendix E derives analytical expressions separately for the within and between cohort inequality.

Similar properties should also hold, approximately, for the environment with one-sided limited commitment. However, with one-sided limited commitment it is not possible to derive analytical expressions for the inequality index since the equilibrium $\lambda$ differs across workers (see Figure 4). But intuitively, the inequality index should increase with the average value of $\lambda$.

### 6 The impact of organizational changes

As emphasized in the introduction, until 1970 the New York Stock Exchange prohibited member firms from being public companies. This was a major obstacle for partnerships in the financial sector to become public companies. However, when the restriction was lifted, there was a movement to go public and partnerships began to disappear.

In our model, the transformation of a partnership to a public company eliminates the partnership’s ‘investment wedge’, which is a cost. If this cost is bigger than the efficiency losses associated with the weaker commitment, the public company may be preferred to the modern partnership. This could explain why financial companies decided to change their organizational structure when the legal restrictions were lifted. We now proceed with a quantitative analysis of such a change.

#### 6.1 Quantitative analysis

We calibrate the model annually using data for the 2000s. Since in the 2000s the partnership form of organization was no longer dominant in the financial sector, we calibrate the model under the assumption that in the 2000s this sector is characterized by the environment with double-sided limited commitment.\(^\text{13}\)

\(^{13}\) The reason we calibrate the model to the end period is because with double-sided limited commitment we can compute some of the calibration moments analytically without iteration. If we calibrate the model
The investment cost and the dis-utility cost are assumed to be quadratic, that is, $\kappa(\lambda) = \lambda^2$ and $e(\lambda) = \alpha \lambda^2$. The production function in the nonfinancial sector takes the form $F(x) = 1 + zx^\nu$. Both parameters $z$ and $\nu$ are important for determining the share of income generated by the financial sector but it is difficult to pin down them independently. Therefore, we assign $\nu = 0.9$ recognizing that this value is not based on any particular calibration moment.

The innovation shock $\varepsilon$ takes two values. The lower value is set to $\varepsilon_L = 0$ while the second value is normalized to $\varepsilon_H = 1$. The parameter of the matching function $\eta$ is also pre-set without a particular calibration target. We set this parameter to 0.5, which is customary in the labor search and matching literature. The last parameter we pre-set is the parameter $\zeta$ which determines the effective human capital of workers with financial skills employed in the financial sector. The central role of this parameter is to pin down the productivity differential of skilled workers employed the financial and nonfinancial sectors. We set $\zeta = 0.5$. This implies that, if a worker employed in the financial sector switches to the nonfinancial sector, her human capital is half as productive as in the financial sector.

Given the above specifications and after normalizing the initial human capital $h_0$ to 1, we have 7 parameters to calibrate (see the top section of Table 1). To calibrate these 7 parameters we use the 7 moments listed in the bottom section of Table 1.

Table 1: Parameters and calibration moments.

<table>
<thead>
<tr>
<th>Parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$ Discount factor</td>
<td>0.962</td>
</tr>
<tr>
<td>$\alpha$ Utility parameter for dis-utility innovation effort</td>
<td>0.309</td>
</tr>
<tr>
<td>$\omega$ Death probability</td>
<td>0.025</td>
</tr>
<tr>
<td>$\psi$ Fraction newborn workers searching for jobs in finance</td>
<td>0.061</td>
</tr>
<tr>
<td>$p$ Probability of successful innovation</td>
<td>0.034</td>
</tr>
<tr>
<td>$z$ Productivity of financial services in the nonfinancial</td>
<td>0.707</td>
</tr>
<tr>
<td>$A$ Matching efficiency</td>
<td>0.500</td>
</tr>
<tr>
<td>$\tau$ Cost of posting a vacancy in the financial sector</td>
<td>0.342</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Calibration moments</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest rate</td>
<td>0.04</td>
</tr>
<tr>
<td>Life expectancy of workers</td>
<td>40.00</td>
</tr>
<tr>
<td>Investment share</td>
<td>0.10</td>
</tr>
<tr>
<td>Employment share in finance</td>
<td>0.06</td>
</tr>
<tr>
<td>Value added share in finance</td>
<td>0.08</td>
</tr>
<tr>
<td>Inequality index (coeff. variation) in financial sector</td>
<td>4.00</td>
</tr>
<tr>
<td>Probability of matching in the financial sector</td>
<td>0.50</td>
</tr>
</tbody>
</table>

An interest rate of 4% is standard in the calibration of macroeconomic models. A lifetime of 40 years corresponds to an approximate duration of working life. Employment to the start period with one-sided limited commitment, we need to solve the model using numerical approximation techniques and the calibration requires an iteration over the values of the parameters: guess the parameter values, solve the model and check whether it replicates the calibration targets.
and value added shares for the financial sector are the approximate numbers for FIRE (finance, insurance and real estate) in the 2000s as shown in Figure 1. The inequality index comes from the 2010 Survey of Consumer Finance for occupations in the financial sector. The job finding rate for workers with experience in finance is the only target that is not based on a particular empirical observation. Appendix F provides a detailed description of how the 7 moments are mapped into the 7 parameters.

Results. Since the model has been calibrated using the 2000s data with the monitoring cost set to $\xi = 0$, for the pre-1980s period we have to assign a value to this parameter. Ideally, we would like to use a calibration target. However, given the difficulties of identifying such a target, we simply show what happens if in the pre-1980s period the additional cost to create a vacancy in the financial sector (due to the additional cost to attract external investors) is $\phi \xi = 0.5 \tau$. Therefore, the total cost of a vacancy is 50% higher than the cost with public companies. Notice that, since the probability $\phi$ is endogenous in the model, to find the value of $\xi$ that in equilibrium implies a 50% higher cost of a vacancy requires and iterative procedure.

Figure 5 plots the steady state policy $\lambda = \phi^\lambda(q)$ for financial firms in the environments with one-sided and double-sided limited commitment, and for two values of the vacancy cost, $\tau$ and $\tau + \phi \xi$. In the environment with one-sided limited commitment, more competition (lower cost) reduces slightly the investment $\lambda$. This is because, as shown in Table 2, the probability of receiving offers increases with more competition. Since this raises the outside value of managers, a larger part of the return must be shared with managers, making the investment less attractive for investors, consistent with Proposition 1.

In contrast, when neither managers nor investors can commit, more competition increases innovation, as Proposition 2 predicts. Also in this environment the probability of external offers increases, which raises the external value of managers and makes investment less attractive for investors. In order to implement the optimal $\lambda$, investors would need to promise adequate future compensation. The problem is that future promises
are not credible with double-sided limited commitment and the only way managers can increase their value is by raising the outside value. This is achieved by choosing higher $\lambda$. With a lower $\tau$, the probability of an external offer $\rho$ increases. Since the worker benefits from higher innovation only if she receives an external offer, the higher probability $\rho$ raises the manager’s incentive to innovate.

So far we have shown that the organizational change that took place in the financial sector induced more innovation and, therefore, higher risk-taking. We now show that this also generated other changes that are consistent with the observations highlighted in the introduction.

Table 2 shows that the shift to an environment with double-sided limited commitment and lower vacancy cost is associated with some increase in the share of employment in the financial sector and a larger increase in the share of output. Another important prediction of the model is that the shift is associated with a reduction in the (average) value for investors, relative to human capital. Since we do not have physical capital in the model, we use $h$ as a proxy for the book value of assets. In doing so we have in mind an extended environment in which physical capital is complementary to human capital. Table 2 also shows that the initial investor’s value is lower. This follows directly from Lemma 4 and the free entry condition $\phi(q) \cdot v(q) = \tau$ after the reduction in the vacancy cost $\tau$.

Table 2 also shows why the investor’s commitment to a long-term contract can be weakened by competition. As expected, an increase in competition for managers results in a redistribution in favour of workers, independently of the level of commitment. However,
at any level of competition, a move from one-sided to double-sided limited commitment increases the normalized \textit{ex-post} value of the investor \( Ev(q) \) and, even more, the non-normalized \textit{ex-post} value because of higher growth. Therefore, the investor may be tempted to recover his \textit{ex-post} losses due to increased competition by reneging on his commitments. Such a move to a double-sided limited commitment economy may reduce the investor’s initial value (as Table 2 shows), but increases his expected value \textit{ex-post}.

Finally, we emphasize that, even if there are no structural changes in the nonfinancial sector, per-capita income increases also in this sector. This is because the increase in the supply of financial services reduces the price of these services. As the price declines, more services are used in the production of final goods, which increases the income of nonfinancial workers. However, the relative income of nonfinancial workers increases significantly less than the income of workers employed in the financial sector. This is shown by the fact that the value added share in the financial sector increases more than the employment share and financial workers extract a larger share of the surplus from investors. Therefore, the model generates more income inequality between sectors. Furthermore, the organizational change also increases income inequality in the financial sector as shown by the inequality index captured by the coefficient of variation. This follows directly from the increase in the innovation effort \( \lambda \) that generates more cross-sectional dispersion in the distribution of human capital in the financial sector.

### 7 Conclusion

The financial crisis of 2007-2009 focused attention on the growth in size and importance of the financial sector over the past few decades, as well as the increase in risk taking in the financial sector. Much attention has also been placed on the extremely high compensation of financial professionals. Why did these trends emerge over this period of time? In this paper we argue that changes in the organizational structure of financial firms have increased competition for managerial skills and weakened the enforcement of contractual relationships between workers/managers and investors. These changes could have also played an important role in another widely documented trend occurred during the same period—the increase in income inequality.

The fact that inequality has increased over time, especially in anglo-saxon countries, is well documented (e.g. Saez and Piketty (2003)). The increase in inequality has been particularly steep for managerial occupations in financial industries (e.g. Bell and Van Reenen (2010)). In this paper we propose one possible explanation for this change. We emphasize the increase in competition for human talent and the weakened commitment that followed the organizational changes in the financial sector. In an industry where the enforcement of contractual relations is limited, the increase in competition raises manager’s incentives to undertake risky investments. Although risky innovations may have a positive effect on aggregate production, the equilibrium outcome may not be efficient and it generates greater income inequality. The higher competition for managerial talent seems consistent with the evidence that managerial turnover, although not explicitly modelled in the paper, has also increased during the last thirty years.
We have shown these effects through a dynamic general equilibrium model with long-term contracts, subject to different levels of commitment and enforcement. The model features two sectors—financial and nonfinancial. The focus on the financial sector is motivated by three considerations. First, the organizational changes described in the introduction have been more evident in the financial sector. Second, we also believe that some of the features of this sector—that our model helps to explain—are less present in other sectors (for example the relatively low book value). Third, *managerial talent* is an extremely important factor of production in the financial sector and it is particularly *inalienable* (capital and unskilled labor play a more relevant role in other innovative sectors and patents on financial instrument are *rare avis* and difficult to enforce).\(^\text{14}\)

However, the modeling of the financial sector is general and can be used to study similar organizational changes in other sectors. The general predictions of the model is that, when organizations are subject to external competition—with different effects on members of the organization—competition is likely to distort internal decisions and result in redistribution of *ex-post* rents. With enough commitment (in our model, one-sided limited commitment), the organization can internalize these distortions. Nevertheless, an organizational structure with weaker commitment may be preferred since it may also allow for new ventures or capital, even if it is characterized by lower operational inefficiency.

The high-tech industry may also share some of tensions emphasized here. If internal incentive problems are resolved by granting patent rights to researchers, such structure may deter venture capitalists from investing, while the ability to trade the firm’s patents can attract them and, as a result, researchers may need to have a constant flow of new ideas (resulting in patents) to prevent their salaries from falling, in relation to the firms’ revenues. While such effect seems to go against the researchers, now there is more capital in the high-tech industry and, therefore, more competition for researchers and, possibly, more inequality among them. Whether higher competition, innovation, risk, and inequality is preferable for the society as a whole, it depends on the details of the model.

It can be argued that modern financial organizations have many credible instruments (bonuses, etc.) to overcome the investor’s commitment problem and, therefore, that our model with two-sided limited commitment is a poor description of innovative financial firms. We have explicitly chosen to work with a simplified model in order to sharpen the key mechanism that emerges in the presence of limited commitment. Sophisticated compensation packages for CEOs and workers in general are just partial forms of limited commitment compared to the internal compensation schemes that dominated in the previous organizational form, that is, the traditional partnership.

\(^\text{14}\)Although these differences with other innovative sectors may be a question of degree “But perhaps the most significant change has been to human capital. Recent changes in the nature of organizations, the extent and requirements of markets, and the availability of financing have made specialized human capital much more important, and also much more mobile. But human capital is inalienable, and power over it has to be obtained through mechanisms other than ownership”. Rajan and Zingales (2000).
Appendices

A Proof of Propositions 1 and 2

Proposition 1

In order to prove Proposition 1, first notice that the contractual Problem (9) takes the following form when it is normalised by $h$; that is, $\mu = \tilde{\mu}/h$, $\tilde{\gamma}(\varepsilon')/h$ and $\tilde{\chi}/h$ and $\tilde{W}(h, \tilde{\mu}) = v(\mu)h + \mu Q(h, \mu)h$:

$$\min_{\chi, \gamma(\varepsilon')} \max_{c, \lambda} \left\{ \begin{array}{c} \beta \pi(\lambda) - c + \mu \left( u(ch) - e(\lambda) \right) - \chi \left( e(\lambda) - e(\tilde{\lambda}) \right) \\
+ \beta E \left[ v(\mu')g(\lambda, \varepsilon') + (\mu + \chi + \gamma(\varepsilon')) Q(h', \mu') \right] \\
- \chi D \left( h, g(\tilde{\lambda}, \varepsilon')h, \mu \right) - \gamma(\varepsilon')D(h, h', \rho) \end{array} \right\}$$

s.t. $h' = g(\lambda, \varepsilon')h$, $\mu' = (\mu + \chi + \gamma(\varepsilon'))/g(\lambda, \varepsilon')$, and the corresponding first-order condition with respect to $\lambda$ is given by (11):

$$(\mu + \chi) e_1(\lambda) - \beta \pi_\lambda(\lambda) \geq \beta E \left[ v(\mu') + (\mu + \chi + \gamma(\varepsilon')) Q(h', \mu') \right] - \gamma(\varepsilon') D(h, h', \rho) \varepsilon'.$$

An increase in $\rho$, before $\lambda$ is chosen, has a direct effect on the enforcement constraint when $\gamma_3(\varepsilon') > 0$ and it is given by $D_{2,3}(h, h', \rho)$. By the definition of $D$, (3), $D_{2,3} > 0$ and, therefore, this direct effect of the enforcement constraint makes investment more costly. Furthermore, an increase in $\rho$, by making the incentive and enforcement constraints tighter, increases the value of the respective multipliers – possibly, from zero to a positive value – since $D_3 > 0$, which in turn increases $\mu'$. In turn, the effects of increasing $\mu'$ are given by $v'(\mu') < 0$ and $Q_{h,\mu'}$. Therefore if, as we assume, $Q_{h,\mu} \leq 0$, the effect of an increase in $\rho$ is, unambiguously, a lower optimal $\lambda^*$.

Comment to Proposition 1. The assumption $Q_{h,\mu} \leq 0$ may not hold and the result of Proposition 1 remain the same, since the effect on $Q_{h,\mu}$ is likely to be dominated by the other unambiguous effects. Nevertheless, the assumption is fairly general: it only says that the increase in the manager’s value due to an increase in $h$ is not complemented by an additional increase when $\mu$ also raises. In particular, if the manager has CRRA preferences for consumption, of the form

$$u(C) = \frac{C^{1-\sigma}}{1-\sigma}$$
the optimal consumption policy, (10), takes the form: 
\[(ch)^{-\sigma} = (h\mu)^{-1}\]; that is, 
\[u(h\mu) = \frac{(h\mu)^{\frac{1-\sigma}{\sigma}}}{1 - \sigma},\]
and, therefore, 
\[u_{h,\mu}(h\mu) = \frac{1 - \sigma}{\sigma^2} (h\mu)^{\frac{1-2\sigma}{\sigma}}.\]

In sum, \(u_{h,\mu}(h\mu) \leq 0\) if and only if \(\sigma \geq 1\); i.e. if and only if the intertemporal elasticity of substitution is less or equal one. Otherwise, if \(1/\sigma > 1\) the optimal contract will tend to lower current consumption in exchange for compensating the manager in the future with \(Q_{h,\mu}(h\mu) > 0\). Notice that, given the separability between consumption and effort \(Q\) inherits its differentiability properties from \(u\) (we abstract from some technicalities in making this claim). We analyse in detail the particular case of \(\sigma = 1\); i.e. \(Q_{h,\mu}(h\mu) = 0\).

**Proposition 2**

The proof follows directly from the first-order condition for the optimal deviation \(\hat{\lambda}\), (6):
\[e_1(\hat{\lambda}_t) = \beta E_t D_2(h_t, h_{t+1}, \rho) \frac{1}{\sigma^2} (h\mu)^{\frac{1-2\sigma}{\sigma}}.\]

Taking derivatives, with respect to \(\rho\), we obtain:
\[e_{1,1}(\hat{\lambda}_t)\hat{\lambda}_1(\rho) = \beta E_t D_{2,3}(h_t, h_{t+1}, \rho) \frac{1}{\sigma^2} (h\mu)^{\frac{1-2\sigma}{\sigma}}.\]

By our convexity assumption \(e_{1,1} > 0\) and, furthermore, \(D_{2,3} > 0\). It follows that \(\hat{\lambda}_1 > 0\).

**B  The first-order conditions of Problem (18)**

Let \(\mu, \gamma(\varepsilon)\) and \(\chi\) be the Lagrange multipliers associated with the promise-keeping, enforcement and incentive-compatibility constraints. The lagrangian can be written as
\[v(q) = \pi(\lambda) - c + \beta \sum_{\varepsilon'} g(\lambda, \varepsilon') v(q(\varepsilon'))p(\varepsilon') + \mu \left\{ \ln(c) - e(\lambda) + \beta \sum_{\varepsilon'} \left[ \mathcal{B} \ln \left( g(\lambda, \varepsilon') \right) + q(\varepsilon') \right] p(\varepsilon') - q \right\} + \beta \sum_{\varepsilon'} \left[ q(\varepsilon') + (1 - \rho)\mathcal{B} \ln \left( g(\lambda, \varepsilon') \right) - \hat{d} \right] \gamma(\varepsilon') p(\varepsilon') + \chi \left\{ -e(\lambda) + \beta \sum_{\varepsilon'} \left[ \mathcal{B} \ln \left( g(\lambda, \varepsilon') \right) + q(\varepsilon') \right] p(\varepsilon') - d(\hat{\lambda}) \right\}.\]
The terms $d$ and $d(\hat{\lambda})$ collect variables and functions that are not affected by the contract policies $c$, $\lambda$ and $q(\varepsilon)$. The first order conditions with respect to these three variables are, respectively,

$$-1 + \frac{\mu}{c} = 0$$

$$\pi_1(\lambda) - (\mu + \chi)e_1(\lambda) + \beta \sum_{\varepsilon'} \left[ v(q(\varepsilon')) + \frac{B(\mu + \chi + (1 - \rho)\gamma(\varepsilon'))}{g(\lambda, \varepsilon')} \right] g_1(\lambda, \varepsilon') p(\varepsilon') = 0$$

$$g(\lambda, \varepsilon') v_1(q(\varepsilon')) + \mu + \chi + \gamma(\varepsilon') = 0$$

Substituting the envelope condition $v_1(q) = -\mu$ we obtain equations (19)-(21).

**A remark on investment with full commitment.** As it has been discussed, the characterisation of the investment policy in the one-sided limited commitment problem when constraints are not binding is given by the investment policy under full commitment. As an illustration of such policy, consider the case that: $\varepsilon^H = 1$, $\varepsilon^L = 0$, $p = \Pr\{\varepsilon' = \varepsilon^H\}$ and $\chi = \gamma(\varepsilon') = 0$. Then, the FOC with respect to $\lambda$ reduces to

$$\mu \left[ e'(\lambda) - \frac{\beta}{1 - \beta} \frac{p}{1 + \lambda} \right] + k'(\lambda) = \beta p v(q(\varepsilon^H)).$$

Notice that the term within brackets on the right-hand side is the difference between the marginal cost and the marginal benefit to the manager, while $\mu$ is the manager’s share of the surplus. Since $\bar{\mu} = \mu h = ch = C$ is constant with full commitment, $\lambda^*$ can only be constant if $\left[ e'(\lambda^*) - \frac{\beta}{1 - \beta} \frac{p}{1 + \lambda^*} \right] = 0$ (For example, for the parameters of our computation there is no stationary solution since, for $\lambda \in [0, 1]$, $\left[ e'(\lambda) - \frac{\beta}{1 - \beta} \frac{p}{1 + \lambda} \right] < 0$). Furthermore, taking the derivative of the above equation, with respect to $\mu$, and using the envelope condition (i.e. $v'(q(\varepsilon^H)) = -\mu(\varepsilon^H) = -\mu/(1 + \lambda)$), we obtain

$$\left[ e'(\lambda) - \frac{\beta}{1 - \beta} \frac{p}{1 + \lambda} \right] + \beta p \frac{\mu}{1 + \lambda} v''(q(\varepsilon^H)) = -\left( \mu \left[ e''(\lambda) + \frac{\beta}{1 - \beta} \frac{p}{(1 + \lambda)^2} \right] + k''(\lambda) \right) \lambda'(\mu).$$

It follows that if the first term on the left-hand side is negative, for low values of $\mu$, $\lambda'(\mu) > 0$ (in fact, for the parameters of our computation, $\lambda'(\mu) > 0$ for all $\mu \leq 1$). In other words, when the manager’s marginal benefit from increasing $\lambda$ exceeds its marginal cost, lowering its share in the partnership contract results in lower risk. With a different cost structure the reverse may be true\textsuperscript{15}.

\textsuperscript{15}Previous versions of this work provided examples with increasing risk through full commitment investment paths.
C The posted contract

As it is well known, with directed search there is an indeterminacy of rational expectations equilibria based on agents coordinating on arbitrary beliefs. Following the literature on directed search, we restrict beliefs by assuming that searching managers believe that small variations in matching value are compensated by small variations in matching probabilities so that the expected application value remains constant (see, for example, Shi 2006). More specifically, if \( \overline{Q}_t(h) \) is the value of the equilibrium contract, then for any \( Q_t(h) \) in a neighbourhood of \( \overline{Q}_t(h) \), the following condition is satisfied,

\[
\rho_t\left(h, \overline{Q}_t(h) \right) \cdot \left[ \overline{Q}_t(h) - Q_t(h) \right] = \rho_t\left(h, \overline{Q}_t^*(h) \right) \cdot \left[ \overline{Q}_t^*(h) - Q_t(h) \right],
\]

(28)

where we have made explicit that the probability of a match depends on the value received by the manager.

This condition says that workers are indifferent in applying for a job among employers who offer different contracts since lower values are associated with higher probabilities of matching. In a competitive equilibrium with directed search, investors take \( \overline{Q}_t(h) \) as given and choose the contract by solving the problem

\[
\max_{Q_t(h)} \left\{ \phi_t\left(h, \overline{Q}_t(h) \right) \cdot V_t\left(h, \overline{Q}_t(h) \right) \right\},
\]

subject to (28).

The term \( \phi_t(h, \overline{Q}_t(h)) \) is the probability that the vacancy is filled and \( V_t(h, Q_t) \) is the value for the match for the investor.

We have shown in the analysis of the optimal contract after matching that the investor’s value is a function of the value promised to the manager. The equilibrium solution also provides the initial value of the contract for the investor \( V_t(h, \overline{Q}_t(h)) \).

For any \( h \), if \( \overline{Q}_t(h) \) is also the value of an equilibrium contract, the investor must be indifferent: \( \phi_t(h, \overline{Q}_t(h)) \cdot V_t(h, \overline{Q}_t(h)) = \phi_t(h, \overline{Q}_t^*(h)) \cdot V_t(h, \overline{Q}_t^*(h)) \). Therefore, we will only consider symmetric equilibria where investors offer the same contract \( \overline{Q}_t(h) \) for the same \( h \). Furthermore, competition in posting vacancies implies that, for any level of human capital \( h \), the following free entry condition must be satisfied in equilibrium,

\[
\phi_t\left(h, \overline{Q}_t(h) \right) \cdot V_t\left(h, \overline{Q}_t(h) \right) = \tau h.
\]

(30)

We can now take advantage of the linear property of the model to normalize the above equations. We have shown that the value of a contract for the investor is linear in \( h \), that is, \( V_t(h, Q_t(h)) = v_t(q_t) h_t \). Therefore, the free entry condition can be rewritten in normalized form as

\[
\phi_t(\overline{q}_t) \cdot v_t(\overline{q}_t) = \tau.
\]

(31)

This takes also into account that we focus on a symmetric equilibrium in which the probability of filling a vacancy is independent of \( h \) (which justifies the omission of \( h \) as an explicit argument in the probability \( \phi_t \)).
In normalized variables, the investor’s problem (29) can be rewritten as

\[ \bar{q}_t = \arg \max_q \left\{ \phi_t(q) \cdot v_t(q) \right\} \]

subject to

\[ \rho_t(q - \bar{q}_t) = \rho_t(\bar{q}_t^*) (\bar{q}_t^* - \bar{q}_t), \]

where the maximization over \( q \) is restricted in a neighborhood of \( \bar{q}_t^* \).

We can solve for the normalized initial utility \( \bar{q}_t \) by deriving the first order condition

\[ 1 - \eta = \frac{-v'_t(\bar{q}_t)(\bar{q}_t - \bar{q}_t)}{v_t(\bar{q}_t) - v'_t(\bar{q}_t)(\bar{q}_t - \bar{q}_t)}. \] (32)

The right-hand side is the share of the surplus created by the match (in utility terms) going to the worker.

We now turn to Lemma 4, which is a special case of a more general result we prove here. Let \( v_e(\bar{q}) \) denote the elasticity of the investor’s value function; i.e. \( v_e(\bar{q}) \equiv -\frac{v'(\bar{q})}{v(\bar{q})} \).

Our log-linear specification implies that \( v'_e(\bar{q}) > 0 \).

Lemma 4A \( v'_e(\bar{q}) > 0 \) implies \( \bar{q}'(\rho) > 0 \).

The optimality condition (32) can be written as

\[ \frac{1 - \eta}{\eta} = v_e(\bar{q}) \frac{\bar{q} - \bar{q}}{\bar{q}}. \] (33)

In a stationary equilibrium, using (25) we obtain:

\[ \bar{q} - \bar{q} = \bar{q} - \left\{ a + \beta \left[ (1 - \rho) \bar{q} + \rho \bar{q} \right] \right\} \]
\[ = (1 - \beta) \bar{q} + \beta (1 - \rho) (\bar{q} - \bar{q}) \]
\[ = (1 - \beta (1 - \rho))^{-1} (1 - \beta) \bar{q}, \]

where \( a \equiv \ln \left( A(P_t) \right) \) is taken as given and, therefore, it can be added to \( \bar{q} \), without changing the derivative of \( \bar{q} \) with respect to \( \rho \). We simplify notation, by simply not distinguishing between \( \bar{q} + a \) and \( \bar{q} \) in what follows:

\[ v_e(\bar{q}) = \frac{1 - \eta}{\eta} \frac{\bar{q}}{\bar{q} - \bar{q}} = \frac{1 - \eta}{\eta} \frac{(1 - \beta (1 - \rho))}{(1 - \beta)}. \]

Taking derivatives with respect to \( \rho \),

\[ v'_e(\bar{q})\bar{q}'(\rho) = \frac{1 - \eta}{\eta} \frac{\beta}{1 - \beta} > 0; \]

it follows that \( \bar{q}'(\rho) > 0 \) if \( v'_e(\bar{q}) > 0 \).
D Numerical solution

We describe here the numerical procedure used to solve Problem (18) for exogenous outside values \( q \) and \( \tilde{q} \) and for exogenous finding probability \( \rho \). The numerical solution is based on an iterative procedure that starts with the guess of two functions

\[
\mu = \psi(q) \\
v = \Psi(q).
\]

The first function returns the multiplier \( \mu \) (derivative of the normalized investor’s value) as a function of the promised utility. The second returns the normalized investor value \( v \) also as a function of the promised utility. These two functions are approximated with piece-wise linear functions on an equally spaced grid for \( q \). Therefore, the first step is to construct a grid for the state variable \( q \).

Given the guessed for the approximated functions \( \psi(q) \) and \( \Psi(q) \), for each grid point of \( q \) we solve the following system of equations,

\[
\begin{align*}
c = \mu & \quad \text{(34)} \\
(\mu + \chi)e_1(\lambda) - \pi_1(\lambda) = \beta E \left[ v(q(\varepsilon')) + \frac{B(\mu + (1 - \rho)\gamma(\varepsilon'))}{g(\lambda, \varepsilon')} \right] g_1(\lambda, \varepsilon') & \quad \text{(35)} \\
g(\lambda, \varepsilon')\psi(q(\varepsilon')) = \mu + \chi + \gamma(\varepsilon') & \quad \text{(36)} \\
v = \pi(\lambda) - c + \beta E g(\lambda, \varepsilon')\Psi(q(\varepsilon')) & \quad \text{(37)} \\
q = \ln(c) - e(\lambda) + \beta E \left( B \ln \left( g(\lambda, \varepsilon') \right) + q(\varepsilon') \right) & \quad \text{(38)} \\
\gamma(\varepsilon') \left[ q(\varepsilon') - (1 - \rho)q - \rho q + (1 - \rho)B \ln \left( g(\lambda, \varepsilon') \right) \right] = 0 & \quad \text{(39)} \\
\chi \left\{ -e(\lambda) + \beta E \left[ q(\varepsilon') + B \ln \left( g(\lambda, \varepsilon') \right) \right] + e(\tilde{\lambda}) \\
-\beta E \left[ (1 - \rho)q + \rho q + \rho B \ln \left( g(\tilde{\lambda}, \varepsilon') \right) \right] \right\} = 0 & \quad \text{(40)}
\end{align*}
\]

The first three equations are the first order conditions with respect to \( c, \lambda, q(\varepsilon') \), respectively. The remaining equations are, respectively, the contract value for the investor, the promise-keeping constraint, the Kuhn-Tucker condition for the enforcement constraints and the Kuhn-Tucker condition for the incentive-compatibility constraint.

Notice that conditions (36) and (39) must be satisfied for all values of \( \varepsilon' \), which can take two values. Therefore, we have a system of 9 equations in 9 unknowns: \( c, \lambda, v, \mu, \)
\( \chi, q(\varepsilon_L), q(\varepsilon_H), \gamma(\varepsilon_L), \gamma(\varepsilon_H) \). Once we have solved for the unknowns at each grid point for \( q \), we can update the functions \( \psi(q) \) and \( \Psi(q) \) using the solutions for \( v \) and \( \mu \) and iterate on these two (approximated) functions until convergence.

### E Derivation of the inequality index

With double-sided limited commitment in both sectors, the inequality index takes the same form. Therefore, we derive the general formula without specifying the sector.

In each period there are different cohorts of active workers who have been employed for \( k \) periods. Because workers die with probability \( \omega \), the fraction of active workers in the \( k \) cohort (composed of workers employed for \( k \) periods) is equal to \( \omega(1 - \omega)^k \). Denote by \( h_k \) the human capital of a worker who have been employed for \( k \) periods. Since human capital grows at the gross rate \( g(\lambda, \varepsilon) \), we have that \( h_k = h_0 \Pi_{t=1}^k g(\lambda, \varepsilon) \). Of course, this differs across workers of the same cohort because the growth rate is stochastic. The average human capital is then computed as

\[
\bar{h} = \omega \sum_{k=0}^{\infty} (1 - \omega)^k E_k h_k,
\]

where \( E_k \) averages the human capital of all agents in the \( k \)-cohort. Because growth rates are serially independent, we have that \( E_k h_k = h_0 E g(\lambda, \varepsilon) \). Substituting in the above expression and solving we get

\[
\bar{h} = \frac{h_0 \omega}{1 - (1 - \omega) E g(\lambda, \varepsilon)}.
\]

We now turn to the variance which is calculated as

\[
\text{Var}(h) = \omega \sum_{k=0}^{\infty} (1 - \omega)^j E_k (h_k - \bar{h})^2.
\]

This can be rewritten as

\[
\text{Var}(h) = \omega \sum_{j=0}^{\infty} (1 - \omega)^j \left( E_k h_k^2 - \bar{h}^2 \right).
\]

Using the serial independence of the growth rates, we have that \( E_k h_k^2 = h_0^2 [E g(\lambda, \varepsilon)^2]^k \). Substituting and solving we get

\[
\text{Var}(h) = \frac{h_0^2 \omega}{1 - (1 - \omega) E g(\lambda, \varepsilon)^2} - \bar{h}^2
\]

To compute the inequality index we simply divide the standard deviation (square root of (42)) by the mean (defined in (41)). This returns the inequality index (26).
We can separate the *within* and *between* components of the inequality index starting by rewriting the formula for the variance of $h$ as,

$$\text{Var}(h) = \omega \sum_{k=0}^{\infty} (1 - \omega)^k \left[ (E_k h_k^2 - \bar{h}_k^2) - (\bar{h}_k^2 - \bar{h}^2) \right],$$

where $\bar{h}_k = E_k h_k = h_0 E g(\hat{\lambda}, \varepsilon)^k$ is the average human capital for the $k$ cohort. Substituting the expression for $h_k$ and $\bar{h}_k$ and solving we get

$$\text{Var}(h) = \left( \frac{h^2 \omega}{1 - (1 - \omega) E g(\hat{\lambda}, \varepsilon)^2} - \frac{h^2 \omega}{1 - (1 - \omega)(E g(\hat{\lambda}, \varepsilon))^2} \right) + \left( \frac{h^2 \omega}{1 - (1 - \omega)(E g(\hat{\lambda}, \varepsilon))^2} - \bar{h}^2 \right).$$

Dividing by $\bar{h}^2$ using the expression for $\bar{h}$ derived in (41), we are able to write the square of the inequality index as

$$\text{Inequality index}^2 = \left( \frac{[1 - (1 - \omega) E g(\hat{\lambda}, \varepsilon)]^2}{\omega [1 - (1 - \omega) E g(\hat{\lambda}, \varepsilon)]^2} - \frac{[1 - (1 - \omega) E g(\hat{\lambda}, \varepsilon)]^2}{\omega [1 - (1 - \omega)(E g(\hat{\lambda}, \varepsilon))^2]} \right) + \left( \frac{[1 - (1 - \omega) E g(\hat{\lambda}, \varepsilon)]^2}{\omega [1 - (1 - \omega)(E g(\hat{\lambda}, \varepsilon))^2]} - 1 \right).$$  \hspace{1cm} (43)

The first term is the *within* cohorts inequality while the second term is the *between* cohorts inequality. Both terms are strictly increasing in $\hat{\lambda}$.

\section*{F Calibration}

We calibrate the 7 parameters listed in the first part of Table 1 using the 7 moments listed at the bottom of the same table.

- The discount factor is pinned down by the interest rate target, that is, $\hat{\beta} = 1/1.04$.
- The death probability is pinned down by the life expectancy, that is, $\omega = 1/40$. Given the calibration of $\hat{\beta}$, the effective discount factor is $\beta = (1 - \omega)\hat{\beta} = 0.9375$.
- The share of employment in the financial sector (see Lemma 3) is

$$s_N = \frac{\psi \rho}{1 - (1 - \omega)(1 - \rho)}.$$

Given the calibrated value of $\omega$ and the calibration targets $\rho$ and $s_N$, this equation pins down $\psi$. 

40
To pin down the probability \( p \) we use the inequality index in the financial sector as measured by the coefficient of variation which is equal to
\[
CV = \sqrt{\frac{[1 - (1 - \omega)Eg(\lambda, \varepsilon)]^2}{\omega[1 - (1 - \omega)Eg(\lambda, \varepsilon)^2]}} - 1.
\]
In order to solve for \( p \) we need the steady state value of \( \lambda \) which can be determined using the investment ratio in the financial sector \( r_I = \lambda^2 \). Given the calibration target \( r_I \) this allows us to determine \( \lambda \). We can then use this value of \( \lambda \) in the above equation and solve for \( p \) given the calibration target for the coefficient of variation.

The dis-utility parameter \( \alpha \) is determined by the first order condition for effort
\[
2\alpha\lambda = \rho\betaBE \left( \frac{\varepsilon'}{1 + \lambda\varepsilon'} \right).
\]
Given \( p \), \( \lambda \) and \( \rho \) we can solve this equation for \( \alpha \).

Next we pin down the parameter \( z \). To do so, we have to consider five equations evaluated at the steady state. The share of value added in the financial sector, denoted by \( s_Y \), is equal to
\[
s_Y = \frac{P_x}{1 + zx'}, \quad (44)
\]
where \( x \) is the input of financial services per unit of human capital and \( P \) is the price of financial services. The share \( s_Y \) is a calibration target. The optimality condition for the input of financial services is
\[
\nu zx^{-1} = P. \quad (45)
\]
The production of financial services is equal to the human capital of workers employed in the financial sector \( H^F \). The demand is equal to \( xH^{NF} \). Therefore, the equilibrium condition in the market for financial services is
\[
xH^{NF} = H^F. \quad (46)
\]
Using (26), the human capital in the financial sector is equal to
\[
H^F = h_0 \left[ \frac{\omega}{1 - (1 - \omega)Eg(\lambda, \varepsilon)} \right] s_N, \quad (47)
\]
where \( s_N \) is the employment share of the financial sector, which is a calibration target and \( h_0 \) is normalized to 1.

Finally the effective human capital in the nonfinancial sector is equal to
\[
H^{NF} = h_0 \left[ 1 - \psi + \frac{\zeta\psi\omega(1 - \rho)}{1 - (1 - \omega)(1 - \rho)} \right]. \quad (48)
\]
Given the value of the parameters already pinned down, the calibration targets \( s_N, s_Y \) and \( \rho \), and the steady state value of \( \lambda \), we can solve equations (44)-(48) for \( z \), \( H^F \), \( H^{NF} \), \( x \), \( P \). In this way we have calibrated the parameter \( z \).
At this point we have all the elements (parameters and aggregate variables) needed to solve for the optimal contract in the financial and nonfinancial sectors. This allows us to derive the initial value for the investors, \( \bar{v} \), which we can use to pin down the vacancy costs \( \tau \) using the free-entry conditions

\[
\phi \bar{v} = \tau
\]  

(49)

Given the functional form of the matching function, we have that \( \phi = \frac{\eta-1}{\eta} \). Given \( \eta = 0.5 \) and the calibration target for \( \rho \), we can determine the filling probability \( \phi \). The above equation will then pins down \( \tau \).
References


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