International recessions*

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Abstract

The macro developments leading to the 2008 crisis were characterized by an unprecedented degree of international synchronization. All G7 countries experienced fast credit growth before the crisis, and right around the time of the Lehman bankruptcy, they all faced large contractions in both real and financial activity. We interpret this evidence using a two-country model with financial market frictions and conclude that the crisis was not likely driven by a US shock transmitted abroad, but rather was the consequence of a self-fulfilling shortage in global liquidity. Quantitative predictions of the model are also consistent with a number of features that are hallmarks of both the 2008 crisis and other financial crises episodes.

Keywords: Credit shocks, global liquidity, international comovement

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1 Introduction

One of the most striking features of the 2008 crisis is that in the midst of it—during the quarter following the Lehman bankruptcy— all major industrialized countries experienced extraordinarily large and synchronized contractions in both real and financial aggregates. This paper shows that the synchronization in real and financial variables is informative about the causes of the crisis. In particular it helps distinguish between two leading explanations for the global crisis. The first is based on the idea of a U.S. shock that is transmitted abroad given the high degree of economic and financial integration. The second explanation is based on the idea of a global liquidity shortage generated by self-fulfilling expectations. We contrast these two explanations using a two-country model in which firms use credit to finance hiring and investment but borrowing is constrained by the value of collateral because firms have the option of defaulting.

First we consider an exogenous tightening of credit in one country (the United States) in a financially integrated economy; we show that this depresses employment and output in both countries. The logic is that if firms are up against the credit constraint, equilibrium employment and investment is affected by the shadow cost of credit.\textsuperscript{1} Under financial integration the shadow cost of credit co-moves across borders, and so do employment and output. We also show that this shock causes a credit crunch in US, but also causes a credit boom abroad, as financial integration implies that the country where the credit is not tightened should use credit more intensively. So a US based credit shock should have caused international synchronization in real but not in financial activity (credit quantity).

We then modify our setup so that firms use their assets as collateral to obtain credit and the price of collateral is endogenous. In this setup tighter/looser credit constraints can emerge endogenously as multiple self-fulfilling equilibria. In ‘good’ equilibria, the market expects high resale prices for the assets of potentially defaulting firms, which allows for looser borrowing constraints. As a result of the high borrowing capacity, firms are not liquidity constrained and ex post there are firms with the required liquidity to purchase defaulting firms’ assets. This, in turn, keeps the price of the assets high and rationalizes, ex post, the ex ante expectation of high prices. The resulting equilibrium is characterized by a global credit boom and moderate real growth, features that are consistent with the evolution of the global economy before the 2008 crisis.

During such a path a ‘bad’ equilibrium can arise, in which the market expects low resale prices of the defaulting firms’ assets. Because of the expected low value of assets, firms face tight borrowing limits and are liquidity constrained. When firms are liquidity constrained, there are no firms capable of purchasing the assets of defaulting firms and, as a result, the resale price

\textsuperscript{1}Recent work by Greenstone and Mas 2012, Bentolilla and al. 2013, Chodorow-Reich, 2013, among others, find evidence that firms with shortage of credit do cut employment, supporting our mechanism for which credit shocks affect real economic activity.
is low. This rationalizes the expectation of low prices, leading to ‘bad’ equilibria characterized
by globally reduced credit, firms de-leveraging, and sharply depressed real activity across the
borders. Financial integration implies that the prices of collateral is equalized across countries,
and hence credit conditions are also equalized.

We view this result as suggestive that a self-fulfilling, global liquidity shortage, rather than
country-specific shocks, can be a key factor in explaining the macro events surrounding the 2008
crisis.

In the second part of the paper we develop and solve a quantitative version of our set-
up in which global liquidity shortages are recurring but occasional. The setup can also help
us understand a number of macroeconomic features that are hallmarks of financial crises. In
particular it can generate i) asymmetric behavior of real variables in credit booms (slow growth)
and credit crashes (sharp contraction), (ii) countercyclical labor productivity, (iii) crises that
are more severe when they happen after long credit booms, and (iv) high volatility of labor and
asset prices.

One important observation concerning the international dimension of the recent crisis is
that employment was hit particularly hard in the United States but not in the remaining G7
countries (see, among others, Ohanian, 2010). As a consequence, labor productivity increased
in the United States but declined in the rest of the G7 countries. Our baseline model does not
capture these cross-country differences. However, in the final section of the paper we allow for
cross-country differences in the characteristics of national labor markets (more flexibility in the
United States and less flexibility in other G7 countries). With this extension, the emergence of
a bad self-fulfilling equilibrium has the potential to explain the similar cross-country responses
of GDP and financial markets, and the heterogeneous responses of employment, productivity,
and the labor wedge.

The theory proposed in this paper is not the only explanation for the international recession.
Conceivably, one could potentially develop other theories of common global shocks in which a
credit contraction is only a consequence and not a cause of the crisis. Especially interesting
are theories based on time-varying uncertainty as in Arellano, Bai and Kehoe (2012) and on
interbank crises as in Boissay, Collard and Smets (2012). We view the comparative evaluation
of different theories of global crises as an interesting direction for future research.

The role of credit shocks for macroeconomic fluctuations has been recently investigated
primarily in closed economy models in which the shocks to credit are exogenous. In this
paper, instead, we study the international implications of these shocks and provide a micro
foundation based on self-fulfilling expectations. Our theory is in line with the idea of liquidity
crises resulting from multiple equilibria outcomes as discussed in Lucas and Stokey (2011) and

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2Examples are Christiano, Motto and Rostagno (2009), Gertler and Karadi (2009), Goldberg (2010), Guerrieri
and Lorenzoni (2010), Khan and Thomas (2010), Jermann and Quadrini (2012), and Liu, Wang and Zha (2013)
it shares some similarities with models of bubbles as in, among others, Kocherlakota (2009), Martin and Ventura (2012) and Miao and Wang (2013).

More specifically to our setup, the idea that multiple equilibria can emerge in models in which the availability of credit depends on the value of collateral assets, has been first proposed by Shleifer and Vishny (1992) and, more recently, by Benmelech and Bergman (2012) and Liu and Wang (2011). These studies, however, considers only closed economy models. Our paper shows that multiple equilibria are also important for capturing the international synchronization of recessions. In this respect, it relates to the literature studying the sources of macroeconomic comovement and international transmission of shocks, starting with Backus, Kehoe and Kydland (1992).3

The literature offers two primary explanations for international co-movement. The first is based on the existence of global or common shocks, that is, exogenous disturbances that are correlated across countries. The second explanation is based on the international transmission of country-specific shocks (for example through investment). In this paper, we show that credit shocks generate comovement for both reasons: exogenous credit shocks spill over from one country to the other, and endogenous credit shocks will appear to the econometrician like a common shock or a global factor. Recent contributions that analyze the role of financial markets for the strong international comovement during the 2007-2009 crisis include Dedola and Lombardo (2010), Devereux and Yetman (2010), Devereux and Sutherland (2011), Kollmann, Enders & Müller (2011) and Kollmann (2013). These studies, however, do not consider the possibility of endogenously correlated fluctuations in credit across countries.

Central to the multiplicity of equilibria is that financial constraints are ‘occasionally binding’. This leads to another important difference between our paper and other studies that investigate the macroeconomic impact of financial shocks. Most of these contributions limit the analysis to equilibria with always binding constraints. Mendoza (2010), Bianchi and Mendoza (2010) and Bianchi (2011) also study an economy with occasionally binding constraints but do not investigate the importance of financial shocks and the issue of international co-movement. Occasionally binding constraints are also central to Brunnermeier and Sannikov (2010) and Arellano, Bai and Kehoe (2012). Their analysis, however, is limited to productivity shocks (level and volatility) and in closed economies.

The paper is organized as follows. In Section 2 we present evidence about the recent crisis. We then describe the theoretical framework gradually, starting in Section 3 with a version of the model with fixed capital and exogenous credit shocks. After showing that exogenous credit shocks do not generate comovement in credit, and therefore, it does not capture an important

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3The idea that multiple equilibria can generate international comovement has also been proposed recently by Bacchetta and Van Wincoop (2013). Their set-up is different from ours in that multiple equilibria arise from self-fulfilling expectations about aggregate demand and not about collateral values.
feature of the 2008 crisis, we extend the model in Section 4 to allow for multiple equilibria and endogenous credit shocks. With this extension the model also generates cross-country comovement in credit, in addition to the comovement in real macroeconomic variables. Section 5 adds capital accumulation and conducts a quantitative analysis. Section 6 extends the model by allowing for cross-country heterogeneity in domestic labor markets. Section 7 concludes.

2 Macroeconomic evidence

We now present some facts about international comovement during the 2007-2009 crisis. Figure 1 plots the GDP dynamics for the G7 countries during the six most recent US recessions. In each panel we plot, for each country, the percentage deviations from the level of GDP in the quarter preceding the start of the US recession. Comparing the bottom right panel of the figure with the other panels shows that the 2007-2009 recession and, in particular, the period following the Lehman crisis, stands out in terms of both depth and macroeconomic synchronization. In none of the previous recessions did GDP fall so much and in all countries.

Figure 1: Dynamics of GDP in the G7 countries during the six most recent US recessions

Note: All series normalized to 1 in the quarter preceding the start of the US recession (NBER recession dates).
Another way to illustrate the increased international co-movement associated with the recent crisis is provided in Figure 2, which plots average bilateral correlations of 10-year rolling windows of quarterly GDP growth between all G7 countries. Two standard deviation confidence bands are also plotted. The figure shows that during the last two quarters of 2008, the average correlation jumped from 0.3 to 0.7 and the sample standard deviation fell significantly. This confirms that the 2007-2009 period stands out in the post-war era as a time of exceptional high co-movement for all developed countries, a point also emphasized by Imbs(2010), among others.

Figure 2: Bilateral rolling correlations of GDP growth for G7 countries

Note: Each correlation is computed over a 10-year window of quarterly GDP growth. The x-axis is the most recent date in the window. The vertical line denotes the third quarter of 2008 (Lehman’s bankruptcy).

The high degree of international co-movement between the United States and other major industrialized countries is also observed in other real and financial variables. Figure 3 plots GDP, consumption, investment, and employment in the period 2005-2010 for the United States and an aggregate of the other countries in the G7 group (from now on, G6). The figure highlights that, after the Lehman crisis, GDP, consumption, and investment were all hit hard in both the United States and the G6. Employment also declined in the US and abroad, even though the US decline was much larger than the decline in the G6. We will return to this difference in the last part of the paper.

Figure 4 plots the dynamics of some financial variables. The top right panel plots the growth rate of stock prices in the United States and in the G6 and it documents the massive and synchronous decline in stock prices that took place during the crisis. The top left panel

\footnote{Stock prices for the United States are the MSCI BARRA US stock market index, and stock prices for the}
Figure 3: GDP, consumption, investment and employment in US and G6: 2005-2010

Note: Data for GDP, consumption and investment are from OECD Quarterly National Accounts in PPP constant dollars. Data for employment are from OECD Main Economic Indicators. All series are normalized to 1 in the first quarter of 2007. The vertical line denotes the third quarter of 2008 (Lehman’s bankruptcy).

reports the growth in total gross debt for the nonfinancial business sector, which also dropped during the crisis.\(^5\)

Indicators of credit market conditions based on credit volumes have been criticized because they do not take into account that a credit crunch might induce firms to draw on existing credit lines, so the distress does not immediately show up in quantities (see, for example, Gao and Yun (2009)). For this reason, the bottom left panel reports a different indicator of credit market conditions. The indicator is not based on volumes of credit but on opinion surveys of senior loan officers of banks. The plotted index is the percentage of banks that relaxed the standards to approve commercial and industrial loans minus the percentage of banks that tightened the

G6 countries are computed using the MSCI BARRA EAFE+Canada index which is an average of stock prices in advanced economies except the US economy.

\(^5\)The US real debt is for the nonfinancial business sector from the Flow of Funds Accounts. The series for the G6 is the sum of net debt (in constant PPP dollars) for the corporate non-financial sector in the euro area, Japan, and Canada. Debt is defined as credit market instruments minus liquid assets (foreign deposits, checkable deposits and currency, savings deposits, money market funds, securities RPs, commercial paper, treasury securities, agency and GSE-backed securities, municipal securities, and mutual fund shares).
The key lesson we learn from Figure 4 is that, right around 2008, credit conditions moved from strongly loose to strongly tight both in the United States and in the G6 countries. This evidence will be particularly important in the second part of the paper because it allows us to identify more precisely the nature of the crisis.

A final observation relates to the asymmetry between real and financial variables in the expansion phase before the crisis and the collapse during the crisis. The top left panel of Figure 4 shows that, in the years preceding the crisis, debt experienced rapid growth (about 6% per year

The US series is from the Federal Reserve Board (Senior Loan Officers Opinions Survey). The G6 series is based on similar surveys released by the European Central Bank (ECB Bank Lending Survey), Bank of Japan (Senior Loan Officer Opinion Survey), and Bank of Canada (Senior Loan Officers Opinions Survey). It is computed as the weighted (by overall debt) average of the indices for the euro area, Japan, and Canada. Thus, the average series does not correspond exactly to the series for the G6 countries because data for the United Kingdom are not available and it includes Euro countries that are not in the G7 group. The indices are typically reported with the inverted sign (representing the percentage of officers tightening credit standards).
in the United States and 4% per year in the G6). Figure 3 shows instead that the growth in real
variables has been moderate. For example, over the same period, GDP grew about 2% per year
both in the United States and the G6. During the crisis period, however, all variables, both real
and financial, contracted sharply. This feature is not unique to the 2007-2009 financial crisis.
Several authors have observed that many historical episodes of credit booms are not associated
with much faster growth in real economic activity. However, when a credit boom experiences a
sudden stop, the reversal is often associated with sharp macroeconomic contractions. See, for
example, Reinhart and Rogoff (2009), Classens, Kose, and Terrones (2011), and Schularick and
Taylor (2012).

The facts presented in this section—high international comovement in real and financial
variables during the crisis, the large employment (for the United States) and stock market
collapse, and the asymmetry between the pre-crisis phase and post-crisis phase—cannot be
easily explained with a standard workhorse international business cycle model. In the next
sections, we propose a theoretical framework with credit shocks that helps us to understand
these facts.

3 Exogenous credit shocks

We start with a simple model without capital accumulation and with exogenous credit shocks.
This model allows us to evaluate the hypothesis that the international crisis was triggered by
a credit shock in the United States. We will show that this hypothesis is not fully consistent
with data and thus extend the setup making credit shocks endogenous as the outcome of self-
fulfilling equilibria. This extension will be able to account for both real and financial comovement
observed in the data.

There are two types of atomistic agents: investors and workers. We assume only investors
have access to the ownership of firms whereas workers can only save in the form of bonds. We
further assume that investors and workers have different discount factors: $\beta$ for investors and
$\delta > \beta$ for workers. As we will see, the different discounting implies that in equilibrium firms
borrow from workers.\footnote{Several approaches are proposed in the literature to generate a borrowing incentive for firms: tax deductability of interests, uninsurable idiosyncratic risks for lenders, bargaining of wages and so on. For our purpose, the distinction between these approaches is not important. Therefore, we simply assume different discount factors (as in Kiyotaki and Moore, 1997) which we interpret as capturing, in reduced form, all of these mechanisms.} To facilitate the presentation we first describe the closed-economy version of the model.

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3.1 Investors and firms

Investors have lifetime utility $E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$. They are the owners of firms and can trade shares with other investors. Since investors are homogeneous and they earn only capital incomes from the ownership of shares, their consumption is equal to the dividends paid by firms. The assumption that investors only hold firms' shares and cannot borrow or save in the form of bonds is without loss of generality. Borrowing and/or saving will be done on their behalf by firms. Denoting the dividends by $d_t$, the effective discount factor for investors, and thus firms, is $m_{t+1} = \beta u_c(d_{t+1})/u_c(d_t)$.

Firms operate the production function $F(h_t) = \bar{k}h_t^{\nu}$, where $\bar{k}$ is a fixed input of capital and $h_t$ is the variable input of labor. The parameter $\nu$ is smaller than 1, implying decreasing returns in the variable input. In this version of the model without capital accumulation, we can think of $\bar{k}$ as a normalizing constant.

Firms start the period with intertemporal debt $b_t$. Before producing, they choose labor input $h_t$, dividends $d_t$, and next period debt $b_{t+1}$. The budget constraint is

$$b_t + w_th_t + d_t = F(h_t) + \frac{b_{t+1}}{R_t}, \quad (1)$$

where $R_t$ is the gross interest rate.

The payments of wages, $w_th_t$, dividends, $d_t$, and current debt net of the new issue, $b_t - b_{t+1}/R_t$, are made before the realization of revenues. Thus, the firm faces a cash flow mismatch. To cover the cash mismatch, the firm contracts the intraperiod loan $l_t = w_th_t + d_t + b_t - b_{t+1}/R_t$, which is repaid at the end of the period after the realization of revenues.\(^8\) Using the budget constraint (1), we can see that the intraperiod loan is equal to the revenue, that is, $l_t = F(h_t)$.

Debt contracts are not perfectly enforceable because the firm can default. Default takes place at the end of the period before repaying the intraperiod loan. At this stage, the firm holds the revenue $F(h_t)$. The revenue represents liquid funds that can be easily diverted. Default gives the lender the right to liquidate the firm’s assets. However, after the diversion of $F(h_t)$, the only remaining asset is the physical capital $\bar{k}$. Suppose that the liquidation value of capital is $\xi_t\bar{k}$, where $\xi_t$ is stochastic. Since default arises at the end of the period, the total liabilities of the firm are $l_t + b_{t+1}/R_t$. To ensure that the firm does not default, the total liabilities must satisfy the enforcement constraint

$$l_t + \frac{b_{t+1}}{R_t} \leq \xi_t\bar{k}. \quad (2)$$

A formal derivation of this constraint is provided in Appendix A and is based on assumptions similar to those in Hart and Moore (1994).

\(^8\)As an alternative to using intraperiod loans, we could assume that firms carry cash from the previous period. The explicit consideration of cash would not change the key properties of the model but would complicate the numerical solution by adding another state variable.
Fluctuations in $\xi_t$ affect the ability to borrow, and as we will show later, they generate procyclical movements in real and financial variables. Our ultimate goal is to derive the variable $\xi_t$ endogenously from liquidity considerations, as we will do in Section 4. For the moment, however, we treat $\xi_t$ as an exogenous stochastic variable.\footnote{Movements in the liquidation price are consistent with Eisfeldt and Rampini (2006), who suggest that the liquidity of capital must be procyclical in order to match the observed reallocation of capital.}

To illustrate the role played by fluctuations in $\xi_t$, consider a pre-shock equilibrium in which the enforcement constraint is binding. Starting from this equilibrium, suppose that $\xi_t$ decreases. This forces the firm to reduce either the dividends, the input of labor or both. To see why, let’s start with the assumption that the firm does not change the input of labor $h_t$. This implies that the intraperiod loan also does not change because $l_t = w_t h_t + d_t + b_t - b_{t+1}/R_t = F(h_t)$. Consequently, the only way to satisfy the enforcement constraint (2) is by reducing the intertemporal debt $b_{t+1}$. We can then see from the budget constraint (1) that the reduction in $b_{t+1}$ requires a reduction in dividends. Thus, the firm is forced to substitute debt with equity. Alternatively, the firm could keep the dividends unchanged and reduce the intra-period loan $l_t = F(h_t)$. This would also ensure that the enforcement constraint is satisfied, but it requires the reduction in the input of labor. Therefore, after a reduction in $\xi_t$, the firm faces a trade-off: paying lower dividends or cutting employment. The optimal choice depends on the relative cost of changing these two margins, which, as we will see, depends on the stochastic discount factor $m_{t+1} = \beta u_c(d_{t+1})/u_c(d_t)$.\footnote{Recent studies have shown that there is a second margin of substitutability: in addition to equity and debt, firms also substitute bank debt with direct market borrowing (corporate bonds). See Adrian, Colla, and Shin (2012) and De Fiore and Uhlig (2012). Since we do not distinguish between the different forms of debt, what matters in our study is the total of both bank and direct market financing.}

**Firm’s problem:** The optimization problem of the firm can be written recursively as

$$V(s; b) = \max_{d,h,b'} \left\{ d + E m' V(s'; b') \right\}$$ \hspace{1cm} (3)

subject to:

$$b + d = F(h) - w h + \frac{b'}{R}$$ \hspace{1cm} (4)

$$F(h) + \frac{b_{t+1}}{R_t} \leq \xi_k,$$ \hspace{1cm} (5)

where $s$ are the aggregate states, including the shock $\xi$, and the prime denotes the next period variable. The enforcement constraint takes into account that the intraperiod loan is equal to the firm’s output, that is, $l_t = w_t h_t + d_t + b_t - b_{t+1}/R_t = F(h_t)$.\footnotetext{Movements in the liquidation price are consistent with Eisfeldt and Rampini (2006), who suggest that the liquidity of capital must be procyclical in order to match the observed reallocation of capital.}
The firm takes as given all prices, and the first order conditions are

\[ F_h(h) = \frac{w}{1 - \mu}, \]  

(6)

\[ REm' = 1 - \mu, \]  

(7)

where \( \mu \) is the Lagrange multiplier for the enforcement constraint. These conditions are derived under the assumption that dividends are always positive, which will be the case if the investors’ utility satisfies \( u_c(0) = \infty \). The detailed derivation is in Appendix B.

We can see from condition (6) that there is a wedge in the demand for labor if \( \mu > 0 \), that is, if the enforcement constraint is binding. This derives from the fact that labor needs to be financed, at least in part, with equity (lower payment of dividends). As long as the cost of equity, \( 1/Em' \), is greater than the cost of debt, \( R \), expanding the input of labor is costly in the margin because the firm needs to substitute debt with equity. It is then the return differential, \( 1/Em' - R \), that determines the labor wedge, as can be seen from equation (7).

Some partial equilibrium properties: The characterization of the firm’s problem in partial equilibrium provides helpful insights about the property of the model once extended to a general equilibrium. For partial equilibrium we mean the allocation achieved when the interest and wage rates are both exogenously given and constant.

A decrease in \( \xi \) makes the enforcement constraint tighter. Because all firms reduce the payment of dividends, the aggregate consumption of investors decreases. This reduces the discount factor \( m' = \beta u_c(d')/u_c(d) \) and raises the multiplier \( \mu \) (equation (7)). From condition (6) we can then see that the demand for labor declines. Intuitively, when the credit conditions become tighter, firms need to rely more on equity financing and less on debt. This requires investors to cut consumption (dividends) which is costly due to the concavity of the utility function. Because of this, in the short term firms do not find it convenient to raise enough equity to maintain the pre-shock production. Thus, they cut employment. If investors’ utility were linear (risk-neutrality), the discount factor would be constant and equal to \( \beta \) so, with constant interest rates, the credit shock would not affect employment. In the general equilibrium, of course, prices do change. In particular, movements in the demand of credit and labor affect the interest and wage rates. To derive the aggregate effects, we need to close the model and characterize the general equilibrium.

\[ ^{11} \] We can term the differential the “equity premium”. However, we should recognize that the premium depends not only on the price of risk (the risk premium) but also on the different discounting of shareholders and bondholders, since they are different agents.
3.2 Closing the model and general equilibrium

The representative worker maximizes the lifetime utility $E_0 \sum_{t=0}^{\infty} \delta^t U(c_t, h_t)$, where $c_t$ is consumption, $h_t$ is labor, and $\delta$ is the intertemporal discount factor. It will be convenient to assume that the period utility takes the form $U(c_t, h_t) = \ln(c_t) - \alpha h_t^{\frac{1}{\eta}}(1 + \frac{1}{\eta})$.

The worker’s budget constraint is $w_t h_t + b_t = c_t + \frac{b_{t+1}}{\delta}$, and the first order conditions for labor, $h_t$, and next period bonds, $b_{t+1}$, are

$$\alpha h_t \frac{c_t}{\delta} = w_t, \quad \text{(8)}$$

$$\delta R_t E_t \left\{ \frac{U(c_{t+1}, h_{t+1})}{U_c(c_t, h_t)} \right\} = 1. \quad \text{(9)}$$

We can now define a competitive general equilibrium. The aggregate states, denoted by $s$, are given by the credit conditions, $\xi$, and the aggregate stock of bonds, $B$.

**Definition 3.1 (Recursive equilibrium)** A recursive competitive equilibrium is defined by a set of functions for (i) workers’ policies $h_w(s), c_w(s), b(w(s); b); (ii)$ firms’ policies $h(s; b), d(s; b), b(s; b); (iii)$ firms’ value $V(s; b); (iv)$ aggregate prices $w(s), R(s), m(s'); and (v)$ law of motion for the aggregate states $s' = \Psi(s)$, such that (i) households’ policies satisfy the optimality conditions (8)-(9); (ii) firms’ policies are optimal and $V(s; b)$ satisfies the Bellman’s equation (3); (iii) the wage and interest rates are the clearing prices in the markets for labor and bonds, and the discount factor for firms is $m'(s') = \beta u_c(d_{t+1})/u_c(d_t)$; and (iv) the law of motion $\Psi(s)$ is consistent with the aggregation of individual decisions and the stochastic processes for $z$ and $\xi$.

To illustrate some of the key properties of the model, we first look at the special case without uncertainty, that is, $\xi$ is constant. In this economy, the enforcement constraint binds in a steady state equilibrium. To see this, consider the first order condition for bonds, equation (9), which in a steady state becomes $\delta R = 1$. Using this condition to eliminate $R$ in (7) and taking into account that in a steady state $Em' = \beta$, we get $\mu = 1 - \beta/\delta > 0$ (since $\delta > \beta$). The intuition is that firms would like to borrow as much as possible because the interest rate is smaller than their discount rate.

With uncertainty, however, the enforcement constraint may be binding only occasionally. In particular, it may become binding after a large and unexpected decline in $\xi$. In this event, firms will be forced to cut dividends, inducing a change in the discount factor $Em'$. Furthermore, the change in the demand for credit affects the equilibrium interest rate. Using (7) we can see that this affects the multiplier $\mu$, which in turn has an impact on the demand for labor (equation (6)). Instead, an increase in $\xi$ may leave the enforcement constraint nonbinding without direct effects on the demand of labor. Thus, the responses to credit shocks can be asymmetric: negative shocks induce large contractions in employment and output, whereas the impact of positive shocks is moderate. We will explore this asymmetry in more detail in section 5.
3.3 Financial integration

We now consider two equal countries, domestic and foreign, with the same preferences and
technology as described in the previous section. From now on we will use an asterisk to denote
variables pertaining to the foreign country. The exogenous stochastic variable $\xi_t$ is specific
to each country. We continue to assume that there is market segmentation in the ownership of
firms, that is, workers are unable to purchase shares of both domestic and foreign firms. However,
under financial integration firms borrow from a global bond market at a common interest rate
$R_t$, domestic workers can trade assets with foreign workers, and investors can purchase shares
of foreign firms.

Investors/firms: Because firms are subject to country-specific shocks, investors gain from
diversifying the cross-country ownership of shares. In particular, it is easy to show that in this
simple setup investors choose equal shares of domestic and foreign firms. This yields common
consumption for investors in both countries and thus a common stochastic discount factor
\[
m_{t+1} = m_t^* = \beta \frac{u_c((d_t + d_t^*)/2)}{u_c((d_t + d_t^*)/2)}.
\]
Investors’ consumption is the sum of dividends paid by domestic and foreign firms, $(d_t + d_t^*)/2$.

Besides the common stochastic discount factor, firms continue to solve problem (3) and the
first order conditions are given by equations (6) and (7). Let’s focus on condition (7), which we
rewrite here for both countries:
\[
R_tE_m_{t+1} = 1 - \mu_t, \\
R_tE_m^*_{t+1} = 1 - \mu^*_t.
\]

Since $E_m_{t+1} = E_m^*_{t+1}$, and there is common interest rate $R_t$, the Lagrange multipliers are
also equal, that is, $\mu_t = \mu^*_t$. Therefore, independently of which country is hit by a shock, if the
enforcement constraint is binding for domestic firms, it will also be binding for foreign firms.

The equalization of the multipliers also implies that the labor wedges in the two countries
are equal. In fact, equation (6) is still the optimality condition for the choice of labor in both

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12 Although we consider the case with only two symmetric countries, the model can be easily extended to any
number of countries and with different degrees of heterogeneity.

13 Two aspects of financial integration are important for this result. The first is stock market integration,
which implies that investors have a common stochastic discount factor. The second is bond market integration,
which implies that firms face a common interest rate. Since at this stage we want to characterize our results
concisely we only consider the case of perfect integration in both markets, but it is possible to extend the model
to consider cases in which only one market is integrated or to consider partial integration. In general, more
financial integration will result in more comovement of the Lagrange multipliers and hence of employment.
countries, that is,
\[ F_h(h_t) = w_t \left( \frac{1}{1 - \mu_t} \right), \]
\[ F_h(h^*_t) = w^*_t \left( \frac{1}{1 - \mu^*_t} \right). \]

We’ll use this property later to characterize the cross-country impact of a credit shock.

**Workers:** Although workers are still prevented from owning firms, with capital mobility they can lend to both domestic and foreign firms. Furthermore, they can engage in international financial transactions with foreign workers. In particular, we assume that domestic workers can trade state-contingent claims with foreign workers. However, firms cannot trade contingent claims with workers. This assumption is essential to maintain the relevance of financial frictions.

Denote by \( n_{t+1}(s_{t+1}) \) the units of consumption goods received at time \( t + 1 \) by domestic workers if the aggregate states are \( s_{t+1} \). These are worldwide states, and therefore, they include the aggregate states of both countries as will be made precise below. In equilibrium, the consumption units received by workers in the domestic country must be equal to the consumption units paid by foreign workers, that is, \( n_{t+1}(s_{t+1}) + n^*_{t+1}(s_{t+1}) = 0 \) for all \( s_{t+1} \).

The budget constraint of a worker in the domestic country is
\[ w_t h_t + b_t + n_t = c_t + \frac{b_{t+1}}{R_t} + \int_{s_{t+1}} n_{t+1}(s_{t+1}) q(s_{t+1})/R_t, \]
where \( q(s_{t+1})/R_t \) is the unit price for the contingent claims on foreign workers.

Given the specification of the utility function, the first order conditions for labor, \( h_t \), next period bonds, \( b_{t+1} \), and foreign claims, \( n_{t+1}(s_{t+1}) \), are
\[ \alpha h_t^\gamma c_t = w_t, \]  \hspace{1cm} (10)
\[ \delta R_t E_t \left( \frac{c_t}{c_{t+1}} \right) = 1, \]  \hspace{1cm} (11)
\[ \delta R_t \left( \frac{c_t}{c_{t+1}(s_{t+1})} \right) p(s_{t+1}) = q(s_{t+1}), \quad \text{for all } s_{t+1}, \]  \hspace{1cm} (12)
where \( p(s_{t+1}) \) is the probability (or probability density) of the aggregate states in the next period for the world economy.

Since in equilibrium the prices and probabilities of the contingent claims are the same for domestic and foreign workers, condition (12) implies that
\[ \frac{c_t}{c^*_t} = \frac{c_{t+1}(s_{t+1})}{c^*_{t+1}(s_{t+1})}. \]  \hspace{1cm} (13)
Therefore, the ratio of consumption for domestic and foreign workers remains constant over time. We denote this constant ratio by $\chi$. This is a well known property in environments with a full set of state-contingent claims.

Before continuing, we would like to clarify that the assumption of contingent claims among workers is not essential for the results of the paper. However, it greatly simplifies the characterization of the equilibrium because, as we will see, it allows us to reduce the number of sufficient state variables.

**Aggregate states and equilibrium:** The set of aggregate states $s$ includes the following variables, $s_t = (\xi_t, \xi^*_t, B_t, B^*_t, N_t)$, where $B_t$ and $B^*_t$ represent aggregate financial liabilities of firms, and $N_t$ is the aggregate foreign asset position of the domestic country.

The definition of equilibrium is analogous to the one for the closed economy except for the additional market for foreign claims, for the fact that now the bond market is a global one and the discount factor for firms is determined by the worldwide representative investor.

Although the general definition of the recursive equilibrium is based on the state variables $s_t = (\xi_t, \xi^*_t, B_t, B^*_t, N_t)$, we can use some of the properties derived above and characterize the equilibrium with a smaller set of states. Let $W_t = B_t + B^*_t$ be the worldwide wealth of households/workers. This is the sum of bonds issued by domestic firms, $B_t$, and foreign firms, $B^*_t$. Using the fact that the consumption ratio of domestic and foreign workers is constant at $\chi$ and the employment policy of firms does not depend on the individual debt, the recursive equilibrium can be characterized by the state vector $s_t = (\xi_t, \xi^*_t, W_t)$. The assumption of cross-country risk sharing among workers and investors (but not between workers and investors) allows us to reduce the number of endogenous states to only one variable, $W_t$.

Intuitively, by knowing $W_t$, we know the worldwide liability of firms, but not the distribution between domestic and foreign firms. However, to characterize the firms’ policies, we only need to know the worldwide debt, which is equal to $W_t$. Since investors own an internationally diversified portfolio of shares, effectively there is only one representative global investor. It is as if there is a representative firm with two productive units: one unit located in the domestic country and the other in the foreign country. Since both units have a common owner, it does not matter how the debt is distributed between the two units. What matters from the perspective of the investor is the total debt and the total payment of dividends.\footnote{This is similar to the problem of a multinational firm that faces demand uncertainty in different countries as studied in Goldberg and Kolstad (1995). There are also some similarities with the problem of a multinational bank with foreign subsidiaries. Cetorelli and Goldberg (2012) provide evidence that multinational banks do reallocate financial resources internally in response to country-specific shocks.}

Total workers’ wealth is also a sufficient statistic for the characterization of their policies, since the consumption ratio between domestic and foreign households remains constant at $\chi$.\footnote{This is similar to the problem of a multinational firm that faces demand uncertainty in different countries as studied in Goldberg and Kolstad (1995). There are also some similarities with the problem of a multinational bank with foreign subsidiaries. Cetorelli and Goldberg (2012) provide evidence that multinational banks do reallocate financial resources internally in response to country-specific shocks.}
This property limits the computational complexity of the model, making the use of non-linear approximation methods practical. We are now ready to characterize the impact of a country-specific credit shock.

**Proposition 3.1** An unexpected change in $\xi_t$ (domestic credit shock) has the same impact on the employment and output of domestic and foreign countries.

**Proof 3.1** We have already shown that the Lagrange multiplier $\mu_t$ is common for domestic and foreign firms. If the ratio of wages in the two countries does not change, the first order conditions imply that all firms choose the same employment. To complete the proof, we have to show that the cross-country wage ratio stays constant. Because firms in both countries have the same demand for labor and the ratio of workers’ consumption remains constant, the first order condition for the supply of labor from workers (equation (10)) implies that the wage ratio between the two countries does not change.

Thus, when the domestic country is hit by a negative credit shock, both countries experience the same decline in output (and employment), as shown in the left panel of Figure 5. This result seems to validate the hypothesis that the 2008 crisis was driven by a US-based credit shock since in the data we observe a high of international comovement of real variables (see Figure 3). However, when we look at the response of credit, the right panel of Figure 5 shows that a US based credit shock would imply a US domestic credit crunch but a boom in foreign credit. This is clearly inconsistent with the data since the 2008 crisis was also characterized by high international comovement in credit variables (see Figure 4).

To understand why a negative credit shock in one country generates a credit boom in the other, consider an initial equilibrium in which the enforcement constraint is not binding in either country. Starting from this equilibrium, suppose that only the domestic economy is hit by a credit contraction (a reduction in $\xi_t$ but not in $\xi_t^*$), inducing binding enforcement constraints in both countries. When $\xi_t$ falls in the domestic country, the shadow value of credit increases in both countries, and since for foreign firms the constraint is not tighter, they will take on more credit. In other words, foreign firms increase borrowing to pay more dividends to shareholders in both countries, in order to offset the reduction in dividends from domestic firms.

In the next section we extend the basic model presented here to provide some foundation for the time variation in credit tightness, $\xi_t$ and $\xi_t^*$. The extension provides a more appealing theory for credit shocks and generates cross-country comovement in credit flows, consistent with the hypothesis of a global self-fulfilling liquidity crisis.
4 Endogenous credit shocks

In order to make $\xi_t$ and $\xi^*_t$ endogenous, we make two key assumptions.

**Assumption I** In case of liquidation, the firm’s capital $\bar{k}$ is perfectly divisible and can be sold to households or firms. Households have the ability to transform one unit of reallocated capital in $\xi$ units of consumption goods. Firms have the ability to transform one unit of reallocated capital in $\xi > \xi$ units of consumption goods.

Thus, in the event of liquidation, the reallocation of capital to other firms is more efficient than the reallocation to households. We also assume that $\xi$ is sufficiently small so that the value of a firm is always bigger than the liquidation value of its capital, $\bar{\xi}k$.

**Assumption II** The purchase of liquidated capital requires the availability of liquid funds.

Firms can buy the capital liquidated by other firms only if they have the ability to increase borrowing. To better understand this assumption, consider the enforcement constraint,

$$\xi_t \bar{k} \geq F(h_t) + \frac{b_{t+1}}{R_t}, \quad (14)$$

where now $\xi_t$ is the expected end-of-period value of liquidated capital. If at the beginning of the period firms choose to borrow less than the limit, that is, the enforcement constraint is not

\footnote{Alternatively we could assume that in case of liquidation a fraction $\bar{\xi}$ of capital can be reinstalled in other firms. This complicates the characterization of the equilibrium but does not change the model’s key properties.}
binding, they have the option to raise additional funds at the end of the period to purchase the capital of defaulting firms. Therefore, ex-post, there will be firms that have the ability to buy the capital of a defaulting firm. In this case the market price of liquidated capital is $\xi$. However, if firms choose to borrow up to the limit at the beginning of the period, at the end of the period there will be no firms with liquidity (unused credit) needed to purchase the capital of defaulting firms. Thus, the liquidated capital can only be sold to households, and its price is $\xi$.\(^{16}\)

Under Assumptions I and II, the value of liquidated capital depends on the financial choices of firms, which in turn depend on the expected liquidation value. This interdependence allows the model to generate multiple self-fulfilling equilibria.

To see this suppose that the expected liquidation price is $\xi_t = \xi$. The low price makes the enforcement constraint (14) tighter, which may induce firms to borrow up to the limit to contain the reduction in dividends and/or employment. Then, if all firms borrow up to the limit, no firm, ex-post, has the liquidity to purchase the capital of defaulting firms and the ex-post liquidation price is $\xi$, fulfilling the market expectation. Now suppose, on the contrary, that the expected liquidation price is $\bar{\xi}$. Because the enforcement constraint (14) is not tight in the current period but could become tight in the future, firms may choose not to borrow up to the limit. But then, in case of liquidation, there will be firms with the liquidity to purchase the liquidated capital and the market price will be $\bar{\xi}$.

Whether both equilibria with tight and loose credit are possible depends on the aggregate state of the economy and on the particular capital regime (autarky versus financial integration). We first characterize equilibria in autarky.

4.1 Autarky

In this version of the model, the aggregate state of the economy is fully captured by the stock of debt, that is, $s_t \equiv B_t$. Three cases are possible:

1. The liquidation price is $\xi$ with probability 1. This arises if we are in a state $s_t$ in which firms borrow up to the limit independently of the expected price $\xi_t$.

2. The liquidation price is $\bar{\xi}$ with probability 1. This arises if we are in a state $s_t$ in which firms do not borrow up to the limit independently of the expected price $\xi_t$.

3. The liquidation price is $\xi$ with some probability $\bar{p} \in (0, 1)$. This arises if we are in a state $s_t$ in which firms borrow up to the limit when the expected price is $\xi_t = \xi$, but they do not borrow up to the limit when the expected price is $\xi_t = \bar{\xi}$.

\(^{16}\)The purchased capital cannot act as a collateral, since the firm transforms it in consumption goods, which are then sold for liquid funds. Since liquid funds are divertible, creditors have no viable means to force the borrower to pay back these funds.
The third case allows for sunspot equilibria and, therefore, fluctuations in $\xi_t$. Denote by $\varepsilon \in \{0, 1\}$ a non-fundamental shock (sunspot). This variable takes the value of zero with probability $\bar{p} \in (0, 1)$ and 1 with probability $1 - \bar{p}$, and it is serially uncorrelated. We then define $p(s_t)$ as the probability of an equilibrium with binding enforcement constraints and a low liquidation price $\xi_t = \underline{\xi}$ (tight credit equilibrium).

**Definition 4.1 (Recursive equilibria for given $\bar{p}$)** A recursive competitive equilibrium for given $\bar{p} \in (0, 1)$ is defined as a set of functions for: (i) households’ policies $h_w(s, \xi)$, $c_w(s, \xi)$, $b_w(s, \xi)$; (ii) firms’ policies $h(s, \xi; b)$, $d(s, \xi; b)$, $b(s, \xi; b)$; (iii) firms’ value $V(s, \xi; b)$; (iv) aggregate prices $w(s, \xi)$, $R(s, \xi)$, $m(s, \xi, s', \xi')$; (v) liquidation price $\xi(s, \varepsilon) \in \{\xi, \bar{\xi}\}$ and probability of low price equilibria $p(s)$ and (vi) law of motion for the aggregate states $s' = \Psi(s, \xi)$, such that (i) household’s policies satisfy the optimality conditions (8)-(12); (ii) firms’ policies are optimal and satisfy the Bellman’s equation (3); (iii) the wage and interest rate clear the labor and credit markets; (iv) the discount factor used by firms satisfies $m(s, \varepsilon, s', \varepsilon') = \beta u_c(d_{t+1})/u_c(d_t)$; (v) the probability of low price equilibria is consistent with individual firms’ policies and liquidity requirement, that is,

$$p(s) = \begin{cases} 
0, & \text{if } F(h(s, \xi; B)) + \frac{b(s, \xi; B)}{R(s, \xi)} < \xi \bar{\varepsilon} \text{ for all } \xi \in \{\xi, \bar{\xi}\} \\
\bar{p}, & \text{if } F(h(s, \bar{\xi}; B)) + \frac{b(s, \bar{\xi}; B)}{R(s, \bar{\xi})} < \xi \bar{\varepsilon} \text{ and } F(h(s, \xi; B)) + \frac{b(s, \xi; B)}{R(s, \xi)} = \xi \bar{\varepsilon} \\
1, & \text{if } F(h(s, \xi; B)) + \frac{b(s, \xi; B)}{R(s, \xi)} = \xi \bar{\varepsilon} \text{ for all } \xi \in \{\xi, \bar{\xi}\} 
\end{cases}$$

and (vi) the law of motion $\Psi(s)$ is consistent with the aggregation of individual decisions and the stochastic process for $\varepsilon$.

The next proposition establishes the existence of sunspot equilibria, that is, the existence of states for which the prices $\xi$ and $\bar{\xi}$ could both emerge in equilibrium.

**Proposition 4.1** Let $\varepsilon$ be a random variable that takes the value of 0 with probability $\bar{p} \in (0, 1)$ and 1 with probability $1 - \bar{p}$. If $\bar{\xi} - \xi$ is sufficiently large, the economy does not converges to a steady state with a unique liquidation price. In the long run it switches stochastically between equilibria with $\xi_t = \xi$ and equilibria with $\xi_t = \bar{\xi}$ with probability $\bar{p}$.

**Proof 4.1** See Appendix C.

Figure 6 illustrates informally the key property of the two equilibria and provides the intuition for the above proposition. The probability of a low price equilibrium $p_t(\xi)$ can take three values depending on the state variable $B_t$, the stock of debt. For low values of debt $B_t$, the probability of a low price equilibrium is zero, meaning that the equilibrium is unique and characterized by
This is because, even if the liquidation price is $\xi_t = \bar{\xi}$, the credit limit is greater than the debt chosen by firms, that is, $F(h) + B_{t+1}/R < \bar{\xi}k$. Why firms do not borrow up to the limit? Given the low value of $B_t$, borrowing up to the limit implies a large payment of dividends to shareholders which is not optimal since shareholders have concave utility. But then, if firms do not borrow up to the limit, $\xi_t = \bar{\xi}$ cannot be an equilibrium.

Figure 6: Probability of low price equilibria and dynamics of debt

When debt $B_t$ is big, the equilibrium is also unique and characterized by the low liquidation price $\xi_t = \bar{\xi}$. Thus, $p(\bar{\xi}) = 1$. To see why this is the only possible equilibrium, suppose that the liquidation price is $\xi_t = \bar{\xi}$ with associated borrowing limit $\bar{\xi}k$. Even if firms borrow up to the limit, the high initial debt $B_t$ implies that shareholders receive small dividend payments. Given the concavity of the utility function, firms would like to pay more dividends, but this requires additional borrowing which pushes them to the limit $\bar{\xi}k$. Since firms are constrained, the high price equilibrium is not admissible. Therefore, we conclude that multiple equilibria exist only for intermediate values of $B_t$. The dependence of the types of feasible equilibria from the debt is also a feature of the model studied in Cole and Kehoe (2000).

Next we describe why, starting from the two extreme regions where the equilibrium is unique, the economy converges to the intermediate region with multiple equilibria. If we start with a low value of $B_t$ and the borrowing limit is not currently binding, firms have an incentive to increase the stock of debt over time because the discount factor of investors is smaller than the discount factor of workers, that is, $\beta < \delta$. Eventually, they will reach the region with multiple equilibria. On the other hand, if $B_t$ is initially very high and firms are constrained, their input of labor will be inefficiently low. Thus, even if the higher discounting of investors pushes firms
toward more debt, this is counterbalanced by the fact that higher debt requires lower input of labor (remember that the enforcement constraint is $F(h_t) + B_{t+1}/R = \xi \bar{k}$). Because of this, firms will reduce their debt and return to the region with multiple equilibria (result established in the proof of the proposition).

To summarize, let $B_t$ be the initial debt. If $B_t$ is very low or very high, the equilibrium is unique for a finite number of periods and then the economy moves to the region with multiple equilibria. After that, it switches stochastically between low and high price equilibria with probability $\bar{p}$. So far we have established that sunspot equilibria can arise in the closed economy; in the next section we characterize these equilibria in the integrated economy.

4.2 Financial integration

The key result of this section is that when financial markets are integrated, the liquidation prices $\xi_t$ and $\xi_t^*$ are equalized across countries.

**Lemma 4.1** In equilibria with integrated financial markets, $\xi_t$ is always equal to $\xi_t^*$.

**Proof 4.1** Suppose that the equilibrium is characterized by $\xi_t = \xi$ and $\xi_t^* = \bar{\xi}$. To have $\xi_t = \xi$ we need $\mu_t > 0$, and to have $\xi_t^* = \bar{\xi}$ we need $\mu_t^* = 0$. However with integrated financial markets, $\mu_t = \mu_t^*$. Using the same argument, we can exclude equilibria with $\xi_t = \xi$ and $\xi_t^* = \xi^*$.

Hence, financial integration implies perfect cross-country co-movement in the liquidation prices $\xi_t$ and $\xi_t^*$, which introduces a second channel of real macroeconomic synchronization: movements in the liquidation prices (driven by sunspots) are correlated across countries. Since liquidation values drive financial flows, also the financial flows become internationally correlated.

We have not yet shown that sunspot equilibria also arise in the economy with integrated financial markets. This boils down to showing the existence of states $s$ under which the liquidation prices $\xi$ and $\bar{\xi}$ are both possible in equilibrium. As a result, the economy can switch stochastically between binding and non-binding enforcement constraints. Since the intuition of the existence of multiple equilibria in the integrated economy is similar to the one in the autarky regime, we establish this result in Appendix D.

This concludes the exposition of our main theoretical/qualitative result, i.e. in an economy with credit frictions and financial integration synchronous declines in real and financial activity (international recessions) can be triggered by global, self fulfilling liquidity crises. This result suggests that the Lehman’s default could have been the trigger that switched the world economy from an equilibrium with globally loose constraints to an equilibrium with globally tight credit. This switch induced a global contraction in both real and financial aggregates.

In the next section we will establish some quantitative properties of our setup and show that these properties can help us understand additional features of macroeconomic fluctuations.
5  Quantitative analysis

In order to bring the model to the data, we first relax the assumption that the input of capital is fixed. This introduces additional state variables that increase the computational complexity of the model. Since the enforcement constraint is only occasionally binding, we need to use global approximation techniques, which become quickly impractical with a large number of state variables. In order to reduce the set of state variables, we assume that the production function takes the form

\[ y_t = (K_t + K^*_t)^{1-\theta} k_t^{\theta} h_t^\nu, \]

where \( K_t \) and \( K^*_t \) are domestic and foreign capital, respectively. The variables \( k_t \) and \( h_t \) are, respectively, the individual inputs of capital and labor. It is further assumed that \( \theta + \nu < 1 \).

The dependence of the production function from the worldwide stock of capital, \( K_t + K^*_t \), introduces positive externalities. The purpose of this is to have constant returns in reproducible factors (AK technology), maintaining the competitive structure of the model, that is, each producer runs a production technology with non-increasing returns. This AK structure simplifies the computation of the equilibrium, which is the only motivation for making this assumption.

The stock of capital evolves according to

\[ k_{t+1} = (1 - \tau)k_t + \Upsilon \left( \frac{i_t}{k_t} \right) k_t, \]

where \( i_t \) is investment, \( \tau \) is the depreciation rate and the function \( \Upsilon(.) \) is strictly increasing and concave, capturing adjustment costs in investment. Adjustment costs are standard in international macro models in order to avoid excessive reallocation of capital across countries.

We first take advantage of the AK structure and normalize the model by the worldwide capital \( K_t + K^*_t \). Using a tilde to denote normalized variables, we rewrite the budget constraint, the law of motion for capital, and the enforcement constraint as

\[ \tilde{b}_t + \tilde{d}_t + \tilde{i}_t = \tilde{k}_t^{\theta} h_t^\nu - \tilde{w}_t h_t + \frac{g_t \tilde{b}_{t+1}}{R_t}, \]  
\[ g_t \tilde{k}_{t+1} = (1 - \tau)\tilde{k}_t + \Upsilon \left( \frac{\tilde{i}_t}{\tilde{k}_t} \right) \tilde{k}_t, \]
\[ \xi_t g_t \tilde{k}_{t+1} \geq \tilde{k}_t^{\theta} h_t^\nu + \frac{g_t \tilde{b}_{t+1}}{R_t}, \]

where \( g_t = (K_{t+1} + K^*_{t+1})/(K_t + K^*_t) \) is the gross growth rate of worldwide capital. We will denote by \( s_t = K_t/(K_t + K^*_t) \) the aggregate share of capital owned by domestic firms. Since in equilibrium \( k_t = K_t \), we have that \( \tilde{k}_t = s_t \).
As in the model without capital accumulation, investors hold an internationally diversified portfolio of shares, and firms use the common discount factor
\[ m_{t+1} = \beta[(d_{t+1} + d^{*}_{t+1})/(d_t + d^*_t)]^{-\sigma}. \]
In normalized variables, the discount factor can be rewritten as
\[ m_{t+1} = g_t^{-\sigma} \beta \left( \frac{\tilde{d}_{t+1} + \tilde{d}^*_{t+1}}{\tilde{d}_t + \tilde{d}^*_t} \right)^{-\sigma} = g_t^{-\sigma} \tilde{m}_{t+1}, \]
and the optimization problem solved by an individual firm becomes
\[ \tilde{V}(\tilde{s}; \tilde{k}, \tilde{b}) = \max_{\tilde{d}, \tilde{h}, \tilde{i}, \tilde{b}'} \left\{ \tilde{d} + g^{1-\sigma} E \tilde{m}' \tilde{V}(\tilde{s}'; \tilde{k}', \tilde{b}') \right\} \]
subject to (15), (16), (17).

The function \( \tilde{V} \) is the firm’s value normalized by aggregate worldwide capital \( K + K^* \), and \( \tilde{s} \) denotes the normalized aggregate states as specified below.

We can now see the analytical convenience of the AK structure. Thanks to this structure, we can write the firm’s value as \( V_t = (K_t + K^*_t) \cdot \tilde{V}_t \) and rescale the problem of the firm by worldwide capital. By doing so, we do not need to keep track of the aggregate stock of capital.

Appendix E derives the first order conditions for the firm, which show that the stock of capital does not enter any of these equations, which validates the conjecture that the normalized policies are independent of the worldwide stock of capital.

The properties that the Lagrange multipliers and the labor wedge \( 1/(1 - \mu_t) \) are equalized across countries also apply to this extended version of the model. In fact, from condition (24) we can see that the common discount factor and the equalization of the interest rates across countries imply \( \mu_t = \mu^*_t \). Therefore, if the enforcement constraint is binding in one country, it must also be binding in the other.

**Aggregate states and equilibrium:** Denote by \( \tilde{W}_t = s_t \tilde{B}_t + (1 - s_t) \tilde{B}^*_t \) the normalized worldwide wealth of households/workers. Because of the normalization described above, we only need to keep track of two endogenous state variables: \( \tilde{W}_t \) and \( s_t \). Therefore, compared with the simpler model considered earlier, the introduction of capital accumulation adds only one state variable, that is, the share of worldwide capital owned by domestic firms, \( s_t \).\(^{17}\) The definition of equilibria with endogenous \( \xi_t \) and \( \xi^*_t \) and the existence of sunspot equilibria are similar to the model without capital accumulation. By having only two endogenous state variables, it becomes practical to solve the model numerically using global approximation methods. Appendix E reports the list of equilibrium conditions and describes the computational procedure.

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\(^{17}\)This state is necessary because of the investment adjustment cost. Without this cost, we could also ignore \( s_t \).
5.1 Parameters

In the quantitative application, we interpret country 1 as representative of the United States and country 2 as representative of the G6 (Canada, Japan, France, Germany, Italy, and the United Kingdom). The period in the model is a quarter.

The discount factor for workers, $\delta$, and the discount factor for investors, $\beta$, are set to target, in the stochastic economy, an average yearly interest rate of 1.4% and an average yearly equity premium of 5.6%. Note that in this model the equity premium has two components: the first (which is equal to 4%) is simply due to the difference in discount factors between workers (bond holders) and investors (equity holders). The second part (which is equal to 1.6%) is due to the higher risk entailed in equity, so we label it equity risk-premium.

The utility function of workers takes the form $U(c, h) = \ln(c) - \alpha h^{1+1/\eta}/(1 + 1/\eta)$, where the Frisch elasticity of labor supply is set to $\eta = 0.75$, which is between the micro and macro estimates. The parameter $\alpha$ is set to target the average fraction of time spent working of 0.3.

Next we parameterize the production function. The parameter $\nu$ is chosen to have an average labor income share of 0.7 and the return to scale is set to $\theta + \nu = 0.9$. Then, $\theta = 0.9 - \nu$.

The stock of capital evolves according to $k' = (1 - \tau)k + \Upsilon(i/k)k$ with

\[
\Upsilon\left(\frac{i}{k}\right) = \frac{\phi_1}{1 - \zeta} \left(\frac{i}{k}\right)^{1-\zeta} + \phi_2.
\]

This functional form is widely used in the literature (see, for example, Jermann 1998). The parameters $\phi_1$ and $\phi_2$ are chosen so that in the deterministic steady state $Q = 1$ and $I = \tau K$. The depreciation rate of capital is set to 2% per quarter, and the parameter $\zeta$ is set to match a volatility of investment relative to output of around 3. At this point, we are left with the parameters that affect the liquidation price. In addition to $\xi$ and $\bar{\xi}$, we have to pin down $\bar{p}$, that is, the probability with which each country draws $\varepsilon = 0$ (which leads to pessimistic expectations $\xi = \bar{\xi}$ in states with multiple equilibria). We choose $\xi$, $\bar{\xi}$, and $\bar{p}$ jointly to match three targets: (i) an average leverage (debt over capital) of 0.5; (ii) a standard deviation for the debt-to-output (with annualized output) ratio of 0.013; (iii) and a frequency of crisis (low liquidation price equilibria) of 4%.\footnote{Although the three parameters are chosen jointly, we can identify the primary parameter that affects each of the three targets. The average leverage is primarily determined by the average $\xi$. The standard deviation of debt is primarily determined by the difference between $\bar{\xi}$ and $\xi$. The frequency of crises is primarily determined by $\bar{p}$.} Table 1 summarizes the parameter values. With this parametrization, the economy always remains in the set of states with multiple equilibria.

5.2 Results

In addition to the international comovement in real and financial variables, we outline four key properties: (i) asymmetric responses to contractions and expansions; (ii) counter-cyclicality
of labor productivity; (iii) more severe crises after long periods of credit booms; and (iv) the importance of changing credit market conditions for the volatility of labor and asset prices.

**Asymmetry** Figure 7 plots the impulse responses to a credit expansion and a credit contraction. A credit expansion is generated starting from the limiting equilibrium the economy converges after a long sequence of $\xi_t = \xi$. From this equilibrium, we consider a sequence of $\xi_t = \xi$ starting at $t = 1$. Therefore, a credit expansion is generated by a permanent switch from equilibria with $\xi = \xi$ to equilibria with $\xi = \xi$. Similarly, the impulse responses to a credit contraction are generated starting from the limiting equilibrium the economy converges after a long sequence of $\xi_t = \xi$. Starting at $t = 1$, the economy experiences a sequence of draws $\xi_t = \xi$.

Two remarks are in order. First, the impulse responses take place in a range of states that admit multiple equilibria. Therefore, the selected sequences of $\xi_t$ are possible equilibrium outcomes. Second, agents do not know in advance the actual sequence of draws. In making their choices, they take into account the uncertainty induced by the stochastic distribution of $\xi_t$.

In response to the credit expansion, we see a gradual increase in the stock of debt and a persistent but small expansion in labor and output.\(^{19}\) The response to a credit contraction, however, displays a very different pattern. The stock of debt declines more quickly, and the response of labor, output and investment are much larger at impact. Therefore, the model generates a strong asymmetric response to credit expansions and contractions.

The intuition for the asymmetry is best understood starting from a situation in which the

\(^{19}\)The macroeconomic expansion induced by the credit boom arises through the following mechanism. On impact, the firm becomes unconstrained, which eliminates the labor wedge. As firms take on more debt, they pay more dividends, increasing the discount factor $m'$. Thanks to the lower discounting, firms invest more. At the same time, the higher borrowing from firms increases the equilibrium interest rate faced by workers, which in turn increase labor supply and output.
Figure 7: Impulse responses to credit expansions and contractions.

enforcement constraint is not binding. If the constraint is relaxed, the Lagrange multiplier
cannot fall below zero and the expansionary effect on unemployment is mild (through general
equilibrium effects). If the constraint gets tighter, however, the Lagrange multiplier becomes
positive, which causes an increase in the labor wedge and, through equation (23), in employment
and output. As discussed in Section 2, this asymmetry is consistent with the macroeconomic
dynamics observed before and during the 2008 recession and other episodes of financial crises.

**Countercyclical labor productivity**  The lower right panel of Figure 7 plots the impulse
responses of labor productivity, that is, the ratio between output and hours. The panel shows how
a credit expansion causes a decline in labor productivity, whereas a credit contraction generates
an increase in labor productivity. This is important for capturing the counter-cyclical dynamics
of US labor productivity during the recent crisis documented by many (see, for example,

**Credit booms and severity of recessions**  Figure 8 plots the impulse responses to credit
expansions that later revert back to pre-expansion levels. Starting from an equilibrium to which
the economy converges after a long sequence of $\xi_t = \xi$, at time 1 the economy experiences a
switch to $\xi_t = \bar{\xi}$ (credit expansion). The value of $\xi$ stays at the higher level for several periods and then reverts back to $\xi$ permanently. Again, agents do not fully anticipate these particular sequences and form expectations based on their conditional distribution. We consider credit booms with a duration of 4 quarters (continuous line) and 20 quarters (dashed line).

![Figure 8: Duration of credit expansions and severity of contractions](image)

The key finding is that the macroeconomic impact of the credit contraction increases with the duration of the preceding credit expansion. After a protracted credit boom, the economy accumulates large volumes of debt and, when the credit reversal arrives, the required de-leveraging is bigger. This forces firms to cut more hiring and investment and generates a stronger macroeconomic contraction. In this way, the model explains why crises that arise after long periods of financial expansions are characterized by more severe macroeconomic contractions.\(^{20}\)

**Volatility of labor and asset prices**  The first column of Table 2 reports simulation statistics computed after detrending the series with a band-pass filter that preserves cycles of 1.5-8 years (Baxter and King, 1999). It is important to point out that the goal of this exercise is not to match empirical moments but only to explore the quantitative properties of the model.

Two properties are especially noticeable. First, the model can generate a high volatility

\(^{20}\)Gorton and Ordonez (2012) also obtain this result in a different model of financial frictions.
Tables 2: Business cycle statistics of key variables from detrended simulated series.

<table>
<thead>
<tr>
<th></th>
<th>Credit shocks only</th>
<th>Productivity shocks only</th>
<th>Both shocks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Standard deviations</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output</td>
<td>0.88</td>
<td>0.76</td>
<td>1.16</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.68</td>
<td>0.44</td>
<td>0.77</td>
</tr>
<tr>
<td>Labor</td>
<td>1.26</td>
<td>0.26</td>
<td>1.26</td>
</tr>
<tr>
<td>Investment</td>
<td>2.27</td>
<td>0.77</td>
<td>2.36</td>
</tr>
<tr>
<td>Tobin’s q</td>
<td>1.14</td>
<td>0.38</td>
<td>1.18</td>
</tr>
<tr>
<td>Stock market value</td>
<td>2.46</td>
<td>0.54</td>
<td>2.45</td>
</tr>
<tr>
<td>Interest rate</td>
<td>0.48</td>
<td>0.25</td>
<td>0.48</td>
</tr>
<tr>
<td>Return on equity</td>
<td>5.82</td>
<td>0.37</td>
<td>5.82</td>
</tr>
<tr>
<td><strong>Expected returns (% annualized)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interest rate</td>
<td>1.40</td>
<td>1.56</td>
<td>1.40</td>
</tr>
<tr>
<td>Return on equity</td>
<td>6.96</td>
<td>5.62</td>
<td>6.96</td>
</tr>
<tr>
<td>Equity risk premium</td>
<td>1.56</td>
<td>0.06</td>
<td>1.56</td>
</tr>
<tr>
<td>Nonbinding constraints, %</td>
<td>96.44</td>
<td>99.99</td>
<td>96.04</td>
</tr>
</tbody>
</table>

Note: The standard deviations for the returns on stocks and bonds are calculated on unfiltered data.

of labor, larger than the volatility of output. This is because fluctuations in the availability of credit cause autonomous movements in employment that, due to decreasing returns, drive smaller movements in output. Second, the model also generates high volatility of asset prices. In particular, the stock market value (equity value of firms) is almost three times more volatile than output. This can also be seen in the center (top and bottom) panels of Figure 7. Two mechanisms generate high volatility in asset prices. The first is that credit fluctuations affect the dividends (consumption) received by investors which in turn affect their stochastic discount factor. The second mechanism is through the adjustment cost of capital. Since fluctuations in credit also affect investment, they impact Tobin’s q. This finding suggests that fluctuations in the availability of credit could contribute to explaining the large volatility of stock prices observed during the recent crisis (see Figure 4).

As a result of the higher volatility in asset prices and stochastic discounting, the model can also generate a non-negligible (1.56%) risk-premium on equity as defined in Section 5.1. Notice that the model generates high volatility of equity returns together with a low volatility of interest rates. This is possible because of the assumption of market segmentation.

5.3 Productivity shocks

Before closing this section we also show how the model performs when we consider standard productivity shocks, alone (second column of Table 2) and together with credit shocks (third
column of Table 2). To add productivity shocks we specify the production function as

\[ y_t = z_t (K_t + K^{*}_t)^{1-\theta} k_t^{\theta} h_t^{1-\theta}, \]

where \( z_t \) denotes the stochastic level of productivity. The variable \( z_t \) is country specific and follows the Markov process

\[
\begin{pmatrix}
\log(z_{t+1}) \\
\log(z^{*}_{t+1})
\end{pmatrix} =
\begin{bmatrix}
\rho_z & 0 \\
0 & \rho_z
\end{bmatrix}
\begin{pmatrix}
\log(z_t) \\
\log(z^{*}_t)
\end{pmatrix} +
\begin{pmatrix}
\epsilon_{t+1} \\
\epsilon^{*}_{t+1}
\end{pmatrix},
\]

where \( \epsilon_t \) and \( \epsilon^{*}_t \) are innovations with mean 0, common standard deviation \( \sigma_\epsilon \), and correlation \( \rho_\epsilon \). We pick standard values for these parameters, that is, \( \rho_z = 0.98, \sigma_\epsilon = 0.006 \) and \( \rho_\epsilon = 0.15 \).

The second column of Table 2 shows that the model with only productivity shocks generates much lower volatilities of hours and asset prices. It is also worth noting that the enforcement constraint is basically never binding. Because of this, the labor wedge is (almost) always zero, which explains why labor and asset prices are not very volatile. The last column of Table 2 shows that the model with both shocks produces statistics very similar to those generated by the model with only credit shocks. Overall comparing table 2 and table 3 suggests that adding credit shocks improved the predictions of the model along a number of dimensions.

6 Global financial crisis with heterogeneous labor markets

In Section 2 we showed that, although US and the G6 experienced similar GDP declines, the decline in employment has been more severe in the United States. The heterogeneous dynamics of the labor market is also reflected in the dynamics of the ‘labor wedge’, that is, the difference between the marginal rate of substitution between consumption and leisure and the marginal product of labor. Formally, this is defined as \( U_h(c_t, h_t) / U_c(c_t, h_t) - F_h(k_t, h_t) \), where \( U_h \) and \( U_c \) are the marginal utilities of leisure and consumption, respectively, and \( F_h \) is the marginal product of labor. With CES utility and Cobb-Douglas production function, the labor wedge is equal to

\[
\text{Wedge} = \frac{\phi c_t}{1-h_t} - (1-\theta) \frac{y_t}{h_t}. \tag{19}
\]

Using this formula, Ohanian and Raffo (2011) find that, while the US labor wedge dropped dramatically during the recent crisis, most of the industrialized countries experienced mild declines. The goal of this section is to show that the heterogeneous dynamics of the labor market can be reconciled with the view of a global financial crisis if labor markets differ across countries.

In order to show this point we extend the model by adding two ingredients: variable labor utilization and heterogeneous labor rigidities. The role of variable labor utilization is to allow for a more powerful mechanism for endogenous fluctuations in measured labor productivity. The role of labor rigidities is to allow for the response of labor utilization to differ from the response of measured labor input. By further assuming that labor rigidities differ across countries, the model can generate heterogeneous responses of macroeconomic and labor market variables.
Let’s start with labor utilization. The production function is specified as $F(k_t, n_t)$, where $n_t$ is the effective input of labor. This results from the combination of (measured) hours, $h_t$, and (unmeasured) utilization, $e_t$, according to the function

$$n_t = \left[ \frac{\varphi}{h_t^{\varphi} + e_t^{\varphi}} \right]^{\frac{1}{\varphi-1}}.$$

The parameter $\varphi$ is the elasticity of substitution between hours spent in the workplace and actual utilization.

The utilization cost derives from workers’ disutility. Given the utility function $U(c_t, h_t + e_t)$, workers face higher disutility when they spent more hours in the workplace and when their services are utilized more intensively. Under this specification, the marginal utilization cost equals the wage rate $w_t$, and the total cost of labor is $(h_t + e_t)w_t$.

So far, the addition of labor utilization is inconsequential for the properties of the model. Since the wage rate is the price for both $h_t$ and $e_t$, the CES aggregation implies that firms choose $e_t = h_t$. Thus, we can simply focus on $h_t$ as in the original model. This equivalence no longer holds with rigidities in the choice of working hours $h_t$.

Some authors interpret labor market rigidities as constraining the extensive margin (employment) rather than the intensive margin (per-worker hours). However, since our model does not distinguish these two margins, we interpret labor market rigidities as restricting total hours $h_t$.

More specifically, we assume that firms incur the convex cost

$$\kappa(h_t - \bar{h})^2w_t,$$

where $\bar{h}$ is an exogenous fixed target.\textsuperscript{21}

The key parameter is $\kappa$. With a positive value of $\kappa$, the response of utilization $e_t$ to shocks is bigger than the response of hours $h_t$. This generates a decline in measured TFP and, potentially, a decline in measured labor productivity $y_t/h_t$. These effects increase with the value of $\kappa$. Therefore, if the first country (the United States) is characterized by lower labor market rigidities than the second country (the G6), we conjecture that the model could generate heterogeneous labor market dynamics. The assumption of heterogeneous labor rigidities between the US and other G7 countries is consistent with anecdotal as well as more systematic evidence.

In order to verify this conjecture we first assign value to the parameters specific to this version of the model. We set the elasticity of substitution between hours and utilization, $\varphi$, to 5 and set $\bar{h}$, $\kappa^1$ and $\kappa^2$ so that the model generates heterogeneous drops in labor wedges during a crisis (switch to the equilibrium with $\xi = \xi$) similar to the drops observed during the recent crisis.\textsuperscript{21}

The cost is multiplied by the wage so it does not vanish as the economy grows. Ideally, we would like to use a more standard adjustment cost, such as $\kappa(h_t - h_{t-1})^2w_t$. This alternative formulation, however, would introduce an additional state variable, $h_{t-1}$, which increases the computational complexity of the model.
crisis. Obviously, the purpose of this exercise is not to evaluate quantitatively the business cycle impact of differences in labor markets. The goal is only to show that a simple model of heterogeneous labor markets could be consistent with the qualitative dynamics of employment observed during the 2008 crisis. Given the chosen parameters, Figure 9 plots the impulse responses to a permanent credit contraction, computed as in Figure 7.

Figure 9: Impulse responses to a credit contraction with asymmetric labor markets

As can be seen from the figure, the responses of investment and output are very similar in the two countries. However, the responses of hours and the labor wedge (by construction) are significantly smaller in country 2. We also observe strong heterogeneity in the response of labor productivity which falls only slightly in country 1 but experiences a large drop in country 2. Therefore, the model could replicate the different dynamics of the labor market between the US and other G6 countries even if the dynamics of other macroeconomic variables are similar.

Before closing, we would like to clarify our interpretation of labor rigidities. Rigidities are typically thought of as consequences of institutional factors. Here, however, we give a broader interpretation. For example, it is well known that labor market rigidities are different across

\[ \text{We measure the labor wedge as if the true model was the standard RBC, which is the approach used in the literature. After simulating the model and generating the series for } c_t, h_t \text{ and } y_t, \text{ we compute the wedge by plugging the series in equation (19). The parameter values are the same as in Ohanian and Raffo (2011), that is, } \beta = 0.99, \theta = 0.36, \delta = 0.0175, g = 0.005 \text{ and } \phi \text{ is chosen so to have steady state hours of 0.33.} \]
sectors. To the extent that in certain countries the crisis has had an impact on sectors with greater labor market flexibility, we may observe larger declines in employment and hours. For instance, the construction sector is characterized by greater flexibility, because of its seasonality. Then, countries that experienced large contractions in the real estate sector were also likely to experience larger drops in employment. This is the case for Spain, which could be considered a country with a flexible labor market because the sector hit hardest by the crisis was flexible.

7 Conclusion

We would like to highlight two contributions of this paper. The first is specific to the Great Recession of 2007-2009. Although there is no disagreement on the fact that the crisis hit all developed countries, the debate has not been settled on whether the cause of the crisis was a US shock transmitted abroad, or rather a global shock. We developed a theoretical framework which can generate a global crisis either through a US based shock transmitted abroad, or through an endogenous global credit contraction, arising from self-fulfilling expectations. Both mechanisms generate a global contraction in real economic activity. However, only the global self-fulfilling shock can generate contractions in both real and financial flows of all countries. Since we documented that during the 2007-2009 crisis both real and financial flows contracted in all countries, we conclude that the crisis was likely caused by a global liquidity shock.

The second contribution is perhaps more general. The quantitative implications of our framework suggest that occasional credit/liquidity crises are important for understanding many features of business cycles and asset prices in developed economies.

Although this study focuses on the role of financial factors for generating international co-movement, it does not exclude the importance of other channels. As far as the recent crisis is concerned, we do not claim that a credit contraction was the only cause of the crisis. We have shown, however, that credit market conditions can go a long way in capturing some of the salient features of the crisis and, especially, its unprecedented international synchronization. We believe our findings are also relevant for policy. If crises are driven by global self-fulfilling liquidity shortages, policies, potentially coordinated internationally, geared toward the provision of liquidity in financial markets could avoid bad equilibria and have important real effects. We view the explicit study of how policy can affect the likelihood of crises as an interesting direction for future research.
Appendix

A Debt renegotiation

The enforcement constraint is derived from the following assumptions. Default arises at the end of the period before repaying the intratemporal loan $l_t = F(h_t)$. In case of default, the firm retains the liquidity $l_t$, and the lender confiscates the physical capital sold at the price $\xi_t$.

First define the value of the firm recursively as $V_t(b_t) = \delta_t + E_{t+1}V_{t+1}(b_{t+1})$, where $m_{t+1}$ is the discount factor, taken as given by an individual firm. Since default takes place at the end of the period, after paying dividends, the value of not defaulting is $E_{t+1}m_{t+1}V_{t+1}(b_{t+1})$.

In the event of default the parties negotiate a repayment $\tau_t$. If they reach an agreement, the firm continues operation and its value is $E_{t+1}m_{t+1}V_{t+1}(b_{t+1}) + l_t - \tau_t$. Thus, the firm retains only the divertible liquidity $l_t$ (threat value). The value of an agreement is the difference between the renegotiation value and the threat value, that is

$$E_{t+1}m_{t+1}V_{t+1}(b_{t+1}) - \tau_t. \quad (20)$$

Let’s now consider the lender. With an agreement the lender gets $\tau_t + b_{t+1}/R_t$. The intertemporal debt is discounted, since it will be repaid next period. Without an agreement the lender receives the liquidation value, $\xi_t \bar{K}$ (threat value). Thus, the net value of renegotiation is

$$\tau_t + \frac{b_{t+1}}{R_t} - \xi_t \bar{K}. \quad (21)$$

The net surplus is the sum of the values for the firm, (20), and the lender, (21),

$$S_t(b_{t+1}) = E_{t+1}m_{t+1}V_{t+1}(b_{t+1}) + \frac{b_{t+1}}{R_t} - \xi_t \bar{K}. \quad (22)$$

Under the assumption that the firm has all the bargaining power, the value of defaulting is $l_t + S_t(b_{t+1})$. Incentive compatibility requires that the value of not defaulting is (weakly) bigger than the value of defaulting, that is,

$$E_{t+1}m_{t+1}V_{t+1}(b_{t+1}) \geq l_t + S_t(b_{t+1}).$$

Substituting the definition of the net renegotiation surplus $S_t(b_{t+1})$, Equation (22), and rearranging, we obtain the enforcement constraint $\xi_t \bar{K} = l_t + \frac{b_{t+1}}{R_t}$.

B First order conditions

Consider the optimization problem (3) and let $\lambda$ and $\mu$ be the Lagrange multipliers associated with the two constraints. Taking derivatives we get

$$d: \quad 1 - \lambda = 0$$
$$h: \quad \lambda[F_k(h) - w] - \mu F_k(h) = 0$$
$$b': \quad E m' V_b(s'; b') + \frac{\lambda}{R} - \frac{\mu}{R} = 0.$$

Using the envelope condition $V_b(s; b) = -\lambda$, we obtain (6) and (7).
C  Proof of Proposition 4.1

Sunspot equilibria arise when, for a given state $s_t$, two types of equilibria are possible: the equilibrium with binding enforcement constraint and low liquidation price $\xi_t = \underline{\xi}$ (tight credit equilibrium) and the equilibrium with non-binding enforcement constraint and high liquidation price $\xi_t = \bar{\xi}$ (loose credit equilibrium). When both types of equilibria are possible, the selection is determined by the realization of the non-fundamental shock $\varepsilon \in \{0, 1\}$. Sunspot equilibria do not exist if the economy converges to one of the two types of equilibria (tight credit or loose credit) and changes in expectations about the liquidation price do not move the economy to the other type of equilibria (from tight credit to loose credit and vice versa). In what follows we show that neither tight credit equilibria nor loose credit equilibria can persist forever.

Suppose that the enforcement constraint is not initially binding. Furthermore, let’s assume that the constraint is not binding also in future periods. Therefore, we start with the assumption that the economy remains in a loose credit equilibrium and the liquidation price is $\xi_t = \underline{\xi}$ for all $t$. Since the liquidation price remains constant over time, there is not uncertainty and the economy converges to a steady state. We now show that in a steady state the enforcement constraint must be binding and, therefore, we obtain a contradiction to the assumption that the loose credit equilibrium can persist forever.

From the workers’ first order condition (9) evaluated at the steady state we obtain $\delta R = 1$. From the firms’ first order condition (7), also evaluated at the steady state, we obtain $\beta R = 1 - \mu$. Since $\delta > \beta$ by assumption, these two conditions imply that the multiplier associated with the enforcement constraint $\mu$ is strictly positive. Therefore, the enforcement constraint must be binding in a steady state. This contradicts the assumption that loose credit with $\xi_t = \underline{\xi}$ is the only possible type of equilibria.

Let’s now show that also tight credit equilibria with binding enforcement constraint and $\xi_t = \bar{\xi}$ can not persist forever. If equilibria with tight credit persist forever, the economy becomes deterministic and converges to a steady state. Using the first order conditions for households and firms evaluated at the steady state (equations (7) and (9)) we can verify that the enforcement constraint is binding in the steady state and the liquidation price must be $\xi_t = \bar{\xi}$. The next step is to show that this steady state can not persist forever, that is, the expectation of a high liquidation price $\xi_t = \bar{\xi}$ would make the enforcement constraint non-binding if $\bar{\xi} - \underline{\xi}$ is sufficiently large.

Consider the enforcement constraint evaluated at the steady state associated with $\xi_t = \underline{\xi}$

$$F(h) + \frac{B'}{R} = \underline{\xi} \bar{k},$$

where $h$, $B$ and $R$ are steady state labor, debt and interest rate. Starting from this steady state, if the expectation for the liquidation price switches to $\xi_t = \bar{\xi}$, but firms do not change $h$ and $B'$, the enforcement constraint is no longer binding. Of course, firms will also change $h$ and $B'$ in response. In particular, they increase $B'$. From the budget constraint of the firm (4) we can see that this implies a higher payment of dividends in the current period. Starting from the next period, however, the payment of dividends will be lower since a higher stock of debt implies higher interest payments. Therefore, the stochastic discount factor $m' = \beta u_c(d_{t+1})/u_c(d_t)$ increases compared to the steady state value. At the same time, using the budget constraint of workers $c + b'/R = b + wh$, we see that workers’ consumption declines in the current period (since workers save more in the form of bonds) but will be higher in the next period when workers receive more interest payments. From condition (9) we see that this generates an increase in the interest
We now consider the first order condition for the firm, equation (7). Compared to the steady state, since both $R$ and $m'$ increase, the multiplier $\mu$ must decline. If the increase in debt is sufficiently large (which will happen if $\bar{\xi} - \xi$ is sufficiently large), the multiplier $\mu$ becomes zero implying that the enforcement constraint is no longer binding. Therefore, the expectation of a high liquidation price is validated ex-post by the fact that firms become unconstrained. By assumption, this happens with probability $1 - \bar{p}$.

In the proof provided so far, we have kept $h$ constant. The input of labor also changes but a similar argument applies even if $h$ is allowed to change. We have thus proved that neither loose credit equilibria with $\xi = \bar{\xi}$ nor tight credit equilibria with $\xi = \xi$ can persist in the long-run. Thus, sunspot equilibria in which the economy switches stochastically between tight and loose credit will emerge.

D Existence of multiple equilibria in the integrated economy

The probability of tight credit equilibria with $\xi_t = \xi^*_t = \bar{\xi}$ can still be expressed as a function of the aggregate states, that is, $p(s_t)$. In an integrated economy, however, the emergence of equilibria with different liquidations prices can be induced by a change in expectations in only one of the two countries. Notice that with financial integration the aggregate state of the economy is fully captured by the worldwide stock of debt, that is, $s_t \equiv B_t + B^*_t$. Depending on $s_t$, we have three possible cases:

1. The liquidation price is $\bar{\xi}$ with probability 1. This arises if we are in a state in which firms borrow up to the limit independently of the expected price.
2. The liquidation price is $\bar{\xi}$ with probability 1. This arises if we are in a state in which firms do not borrow up to the limit independently of the expected price.
3. The liquidation price is $\bar{\xi}$ only with probability $\bar{p} \in (0, 1)$. Two cases are possible:

   (a) The liquidation price is $\bar{\xi}$ with probability $\bar{p}^2$. This arises if we are in a state $s_t$ in which firms choose to borrow up to the limit only if the liquidation prices are expected to be low in both countries, that is, $\xi_t = \bar{\xi}$ and $\xi^*_t = \bar{\xi}$. This case is associated with the draws of $\varepsilon_t = 0$ and $\varepsilon_t^* = 0$. Since this happens with probability $\bar{p}$ in each country, the probability of this event is $\bar{p}^2$.

   (b) The liquidation price is $\bar{\xi}$ with probability $2\bar{p}(1 - \bar{p}) + \bar{p}^2$. This arises if we are in a state $s_t$ in which firms borrow up to the limit even if the liquidation price is expected to be low in only one country (for example, $\xi_t = \bar{\xi}$ and $\xi^*_t = \bar{\xi}$ which is associated with $\varepsilon_t = 0$ and $\varepsilon_t^* = 1$). The probability of this event is $\bar{p}(1 - \bar{p})$. Cross country symmetry implies that this is also the case if the expectation of a low price arises in the other country. Therefore, the probability that one of the two countries obtain a low realization of $\varepsilon$ is $2\bar{p}(1 - \bar{p})$. Of course, if firms choose to borrow up to the limit whenever the price is low in one of the two countries, they will also borrow up to the limit if the price is low in both countries, that is, $\xi_t = \bar{\xi}$ and $\xi^*_t = \bar{\xi}$. The probability of this event is $\bar{p}^2$. Therefore, the total probability that firms borrow up to the limit if the liquidation price is low in at least one country is $2\bar{p}(1 - \bar{p}) + \bar{p}^2$. 

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The definition of equilibria is similar to the definition provided for the autarky regime with some small adjustments. In particular, individual policies are now functions of both liquidation prices, \( \xi \) and \( \xi^* \). For example, the borrowing policy of domestic firms will be denoted as \( b(s, \xi, \xi^*; B) \). Even if in equilibrium the liquidation prices will be equalized across countries, the use of this notation allows for (out of equilibrium) prices that are different in the two countries. Besides the use of this extended notation, the only change we need to make to Definition 4.1 is the probability of tight credit equilibria \( p(s) \). When countries are financially integrated, this probability is defined as

\[
p(s) = \begin{cases} 
0, & \text{if } \begin{cases} F\left(h(s, \xi, \xi^*; B)\right) + \frac{h(s, \xi, \xi^*; B)}{R(\xi, \xi^*)} < \xi \bar{k} \quad \text{and/or} \\
F\left(h^*(s, \xi, \xi^*; B)\right) + \frac{h^*(s, \xi, \xi^*; B)}{R(\xi, \xi^*)} < \xi^* \bar{k}, \quad \text{for all } \xi, \xi^* \in \{\xi, \bar{\xi}\} 
\end{cases} \\
\bar{p}^2, & \text{if } \begin{cases} F\left(h(s, \xi, \xi^*; B)\right) + \frac{h(s, \xi, \xi^*; B)}{R(\xi, \xi^*)} = \xi \bar{k} \quad \text{and} \\
F\left(h^*(s, \xi, \xi^*; B)\right) + \frac{h^*(s, \xi, \xi^*; B)}{R(\xi, \xi^*)} = \xi^* \bar{k}, \quad \text{for only } \xi, \xi^* = \xi 
\end{cases} \\
2\bar{p}(1 - \bar{p}) + \bar{p}^2, & \text{if } \begin{cases} F\left(h(s, \xi, \xi^*; B)\right) + \frac{h(s, \xi, \xi^*; B)}{R(\xi, \xi^*)} < \xi \bar{k} \quad \text{and/or} \\
F\left(h^*(s, \xi, \xi^*; B)\right) + \frac{h^*(s, \xi, \xi^*; B)}{R(\xi, \xi^*)} < \xi^* \bar{k}, \quad \text{for only } \xi, \xi^* = \bar{\xi} 
\end{cases} \\
1, & \text{if } \begin{cases} F\left(h(s, \xi, \xi^*; B)\right) + \frac{h(s, \xi, \xi^*; B)}{R(\xi, \xi^*)} = \xi \bar{k} \quad \text{and} \\
F\left(h^*(s, \xi, \xi^*; B)\right) + \frac{h^*(s, \xi, \xi^*; B)}{R(\xi, \xi^*)} = \xi^* \bar{k}, \quad \text{for all } \xi, \xi^* \in \{\xi, \bar{\xi}\} 
\end{cases} 
\end{cases}
\]

**Proposition D.1** Denote by \( \varepsilon \) and \( \varepsilon^* \) country-specific independent stochastic variables that take the value of 0 with probability \( \bar{p} \) and 1 with probability \( 1 - \bar{p} \). If \( \bar{\xi} - \xi \) is sufficiently large, the economy displays sunspot equilibria: it never converges to a steady state but switches between equilibria with \( \xi_t = \xi_t^* = \xi \) and equilibria with \( \xi_t = \xi_t^* = \bar{\xi} \).

**Proof D.1** The proof follows the same steps as the proof of Proposition 4.1.

If we start from a state \( s_t \) in which the liquidation price is unique, then the economy does not admit multiple equilibria initially. This is equivalent to saying that \( p(s_t) \) is initially either 0 or 1. However, after a finite number of periods, the state \( s_t \) enters a set where multiple equilibria arise, that is, the economy switches between equilibria with \( \xi_t = \xi_t^* = \xi \) and equilibria with \( \xi_t = \xi_t^* = \bar{\xi} \), with some probability \( p(s_t) \in (0, 1) \). The proposition does not establish whether this probability is \( 2\bar{p}(1 - \bar{p}) + \bar{p}^2 \) or \( \bar{p}^2 \) since this depends on parameters. The intuition why the economy reaches states with multiple equilibria is analogous to the proof of Proposition 4.1. Because of the higher discounting of investors, firms become constrained at some point in the future even if they are not constrained now. Likewise, if they are constrained, the expectation of a high liquidation price \( \xi_t = \xi_t^* = \bar{\xi} \) is sufficient to make the enforcement constraint non-binding, moving the economy to the equilibrium with \( \xi_t = \xi_t^* = \bar{\xi} \).
E The model with capital accumulation

First order conditions After imposing the equilibrium conditions $k_t = K_t$ and $\tilde{k}_t = s_t$, the optimality conditions can be written as

$$\nu s^\delta h_t^{-1} = \frac{\bar{w}_t}{1 - \mu_t},$$

(23)

$$g_t^{-\sigma} R_t E \tilde{m}_{t+1} = 1 - \mu_t,$$

(24)

$$Q_t \Upsilon' \left( \tilde{i}_t \right) = 1,$$

(25)

$$Q_t = \xi_t \mu_t + g_t^{-\sigma} E \tilde{m}_{t+1} \left\{ (1 - \mu_{t+1}) \theta s^\theta h_{t+1} - \tilde{i}_{t+1} + \left[ 1 - \tau + \Upsilon \left( \tilde{i}_{t+1} \right) \right] Q_{t+1} \right\}.$$

(26)

Here, $\mu_t$ is the Lagrange multiplier associated with the enforcement constraint and $Q_t$ is the Lagrange multiplier associated with the law of motion for capital (Tobin's $q$).

Differentiating the firm’s problem (18) with respect to $h_t$, $\tilde{b}_{t+1}$, $\tilde{i}_t$ and $\tilde{k}_{t+1}$, we get

$$\nu \tilde{k}^\theta h_t^{\nu-1} = \frac{\bar{w}_t}{1 - \mu_t},$$

$$\frac{1 - \mu_t}{R_t} + g_t^{-\sigma} E \tilde{m}_{t+1} \tilde{V}_k (\bar{s}_{t+1}; \tilde{k}_{t+1}, \tilde{b}_{t+1}) = 0,$$

$$Q_t \Upsilon' \left( \tilde{i}_t \right) \left( \tilde{k}_t \right) = 1,$$

$$Q_t = \xi_t \mu_t + g_t^{-\sigma} E \tilde{m}_{t+1} \tilde{V}_k (\bar{s}_{t+1}; \tilde{k}_{t+1}, \tilde{b}_{t+1}),$$

where $\mu_t$ is the Lagrange multiplier associated with the enforcement constraint and $Q_t$ is the lagrange multiplier associated with the law of motion of capital (Tobin’s $q$). The multiplier associated with the budget constraint is 1. For the foreign country, we have the same conditions with asterisks. The envelope conditions are

$$\tilde{V}_k (\bar{s}_t; \tilde{k}_t, \tilde{b}_t) = -1,$$

$$\tilde{V}_k = (1 - \mu_t) \theta \tilde{k}_t^{\theta-1} h_t^\nu + \left[ 1 - \tau + \Upsilon \left( \tilde{i}_t \right) \theta \tilde{k}_t^{\theta-1} h_t^\nu \right] Q_t.$$

Substituting and imposing the equilibrium conditions $k_t = K_t$ and $\tilde{k}_t = s_t$, we obtain (23)-(26).

Dynamic system and numerical solution procedure We will use the bar superscript to denote aggregate worldwide variables normalized by the worldwide stock of capital. For example, $\bar{d}_t$ is the normalized worldwide dividend, defined as $\bar{d}_t = \frac{d_t + d^*_t}{K_t + K^*_t} \equiv \bar{d}_t + \bar{d}^*_t$. The full list of equilibrium conditions are:

$$1 = \delta g_t^{-1} R_t E_t \left( \frac{\bar{c}_{t+1}}{\bar{c}_t} \right)^{-1}$$

(27)

$$\bar{c}_t = \chi \bar{c}_t$$

(28)
have a dynamic system of 16 equations in 16 unknowns. The computational procedure is based on the approximation of five functions: 

\[ \xi_t, \xi_t^*, b_t, s_t, \]

\[ g_t(x_t s_{t+1} + \xi_t^* s_{t+1}) = \begin{cases} \frac{g_t b_{t+1}}{R_t} + s_t^* h_t^* + (s_t^*)^\theta (h_t^*)^\nu & \text{if } (1 - \mu_t) \tilde{d}_{t+1}^{\sigma} \beta g_i^\sigma R_t E \tilde{d}_{t+1}^{\sigma} \\ \alpha(h_t^*)^{\frac{1}{\bar{c}}} = \frac{\tilde{w}_t}{\bar{c}_t} & \\ \tilde{g}_t s_{t+1} = (1 - \tau) s_t + \Upsilon \left( \begin{array}{c} \frac{\tilde{t}_t}{s_t} \end{array} \right) s_t & \\ \tilde{g}_t^* s_{t+1} = (1 - \tau) s_t^* + \Upsilon \left( \begin{array}{c} \frac{\tilde{t}_t}{s_t^*} \end{array} \right) s_t^* & \\ \nu s_t^* h_t^* \nu^{-1} = \frac{\tilde{w}_t}{1 - \mu_t} & \\ \nu(s_t^*)^\theta (h_t^*)^\nu^{-1} = \frac{\tilde{w}_t^*}{1 - \mu_t} & \\ Q_t^* T' \left( \begin{array}{c} \frac{\tilde{t}_t}{s_t} \end{array} \right) = 1 & \\ Q_t^* T' \left( \begin{array}{c} \frac{\tilde{t}_t}{s_t^*} \end{array} \right) = 1 \end{cases} \]

Equations (27)-(42) form a dynamic system composed of 16 equations. Given the states \( \xi_t, \xi_t^*, b_t, s_t \), the unknown variables are \( h_t, h_t^*, c_t, c_t^*, w_t, w_t^*, i_t, i_t^*, Q_t, Q_t^*, g_t, \mu_t, R_t, \tilde{d}_t, b_{t+1}, s_{t+1} \). Therefore, we have a dynamic system of 16 equations in 16 unknowns.

The computational procedure is based on the approximation of five functions:

\[ \Gamma_1(s_t+1) = \bar{c}_t^{-1} \]

\[ \Gamma_2(s_t+1) = \tilde{d}_t^{\sigma} \]

\[ \Gamma_3(s_t+1) = \tilde{d}_t^{\sigma} \left\{ (1 - \mu_t + 1) \theta s_{t+1}^{\theta - 1} h_{t+1}^\nu - \frac{\tilde{t}_t}{s_{t+1}} + \Upsilon \left( \begin{array}{c} \frac{\tilde{t}_t}{s_{t+1}} \end{array} \right) \right\} \]

\[ \Gamma_4(s_t+1) = \tilde{d}_t^{\sigma} \left\{ (1 - \mu_t + 1) \theta s_{t+1}^{\theta - 1} (h_{t+1}^*)^\nu - \frac{\tilde{t}_t}{s_{t+1}} + \Upsilon \left( \begin{array}{c} \frac{\tilde{t}_t}{s_{t+1}} \end{array} \right) \right\} \]

\[ \Gamma_5(s_t+1) = p(s_t+1) \]
The procedure starts with a guess for the approximated functions $\Gamma_1(s_{t+1})$, $\Gamma_2(s_{t+1})$, $\Gamma_3(s_{t+1})$, $\Gamma_4(s_{t+1})$ and $\Gamma_5(s_{t+1})$. We first construct a two-dimensional grid for the endogenous states $\bar{b}$ and $s$. Then for each realization of the liquidation price—$\xi_t$ and $\xi^*_t$—we guess the values taken by these functions at the grid points. Values outside the grid are obtained through bilinear interpolation. Once we know the approximated functions and probabilities for $\xi_{t+1}$ and $\xi^*_{t+1}$, we can find the values of the 16 unknowns by solving the system (27)-(42) at each grid point and for each value of $\xi_t$ and $\xi^*_t$. In finding the solutions, we check whether the enforcement constraint is binding ($\mu_t > 0$) or not binding ($\mu_t = 0$).

We then use the solutions found at each grid point to update the guesses for the five functions $\Gamma_1(s_{t+1})$, $\Gamma_2(s_{t+1})$, $\Gamma_3(s_{t+1})$, $\Gamma_4(s_{t+1})$ and $\Gamma_5(s_{t+1})$. To update these probabilities we need to check whether multiple equilibria are feasible for all possible states. Effectively, we check this on the state grid. We keep iterating until the guesses for $\Gamma_1(s_{t+1})$, $\Gamma_2(s_{t+1})$, $\Gamma_3(s_{t+1})$, $\Gamma_4(s_{t+1})$, and $\Gamma_5(s_{t+1})$, evaluated at the grid points, are equal to the values obtained by solving the dynamic system (also at the grid points).
References


