Abstract

In a machining process, product quality can be jointly affected by datum surface imperfection, fixture locator error, and machine tool error. It is widely observed that joint effects of these errors may cancel each other on certain features. Mathematical modeling and analysis of the phenomenon have much less been studied. We use the concept of equivalent fixture error (EFE) and corresponding modeling methodology to obtain better insight understanding of this fundamental phenomenon and better processes control. Based on derived process fault model, a sequential root cause identification procedure and EFE compensation methodology are developed. A case study was conducted to demonstrate the proposed diagnostic procedure. A simulation study is also given to illustrate the procedure of error compensation.

1. Introduction

For a machining process, product quality is mainly affected by fixture, datum, and machine tool errors. Fixture is a device used to locate, clamp, and support a workpiece during machining, assembly, or inspection. Fixture error is considered to be significant fixture locator deviation from specification. Machining datum are the part features that have direct contact with fixture locators. Datum error is deemed as the significant deviations of datum
surfaces and is mainly induced by imperfection of raw workpieces or improper operations from previous stages. Fixture and datum together provide the reference system for accurate cutting operations of machine tools. Machine tool error is modeled as the significant tool path deviation from its intended motion. In this paper, we mainly focus on the kinematic aspects of these three types of errors.

It has widely been observed that fixture, datum, and machine tool errors may cancel with each other, i.e., their joint effects will reduce deviations of part features. On the one hand, the phenomenon of error cancellation might conceal the fact that multiple errors occur in the process. On the other hand, we could purposely use one type of error to counteract or compensate another to reduce variation.

To our knowledge, no study has been dedicated to modeling and exploring such cancellation effect among different types of error sources for quality control of machining processes. Most research has been focusing on fixture design and machine tool error modeling.

Fixture error has been considered as one of crucial factors in the optimal fixture design and analysis. Shawki and Abdel-Aal (1965) experimentally studied the impact of fixture wear on the positional accuracy of the workpiece. Asada and By (1985) proposed kinematic modeling, analysis, and characterization of adaptable fixturing. Screw theory has been developed to estimate the locating accuracy under the rigid body assumption (Ball, 1990; and Ohwovoriole, 1981). Weil, Darel, and Laloum (1991) then developed several optimization approaches to minimize the workpiece positioning errors. A robust fixture design was proposed by Cai, Hu, and Yuan (1997) to minimize the positional error. Marin and Ferreira
(2003) analyzed the influence of dimensional locator errors on tolerance allocation problem. Researchers also considered the geometry of datum surface for the fixture design. Optimization of locating setup proposed by Weil, et al. (1991) was based on the locally linearized part geometry. Choudhuri and De Meter (1999) considered the contact geometry between the locators and workpiece to investigate the impact of fixture locator tolerance scheme on geometric error of the feature.

Machine tool error can be due to thermal effect, cutting force, and geometric error of machine tool. Various approaches have been proposed for the machine tool error modeling and compensation. A volumetric error model of a 3-axis jig boring machine is developed by Schultschik (1977) using a vector chain expression. Ferreira and Liu (1986) developed a model studying the geometric error of a three-axis machine using Homogeneous Coordinate Transformation. A general methodology for modeling the multi-axis machine was developed by Soons, Theuws, and Schellekens (1992). The volumetric error model combining geometric and thermal errors was proposed to compensate time varying error in real time (Chen et al., 1993). Other approaches, including empirical, trigonometric, and error matrix methods were summarized by Ferreira and Liu (1986).

In addition to above literature, several studies have been dedicated to modeling the error propagation in multistage manufacturing processes (Jin and Shi, 1999; Djurdjanovic, and Ni, 2001; Huang, Shi, and Yuan, 2003; and Zhou, Huang, and Shi, 2003). Homogeneous Transformation Matrix (HTM) was used to model the influence of errors on setup and cutting operations. Wang, Huang, and Katz (2005) studied a commonly observed phenomenon that fixture, datum and machine tool errors can yield the same feature deviation pattern. The
concept of Equivalent Fixture Error was proposed to model this phenomenon. An EFE process variation model was derived considering the impact of model formulation on root cause identification and measurement reduction for multistage machining process.

This paper uses EFE model to investigate the error cancellation for better understanding and better control of machining processes. Following Introduction, Section 2 reviews the concept of EFE model. Then error cancellation is studied based on EFE concept. Section 3 analyzes the theoretical implications of EFE model from perspectives of process monitoring and control, including root cause diagnosis and error compensation. In Section 4, the proposed diagnostic procedure is demonstrated by a machining experiment and the error compensation is also illustrated with a simulation study. Summary is given in Section 5.

2. Error Cancellation Modeling

Wang, et al. (2005) proposed the concept of equivalent fixture error (EFE). The equivalent amount of locator errors that can generate the same feature deviation as datum or machine tool error was defined as EFE due to datum or machine tool error. Sections 2.1 briefly introduces notations and reviews the EFE concept. Section 2.2 then models the error cancellation using EFE model.

2.1 Notations and Review of EFE Concept

Using 3-2-1 locating scheme (Fig. 1), a fixture locates the workpiece through three datum surfaces, which are known as the primary, secondary, and tertiary datum surfaces, respectively. Let \( f_i = (f_{ix}, f_{iy}, f_{iz})^T \) denote coordinate of a point on top of locator \( i, i = 1, \ldots, 6 \). Then fixture error is represented by the deviations of six locators along their axial directions,

\[
\Delta f = (\Delta f_{1z}, \Delta f_{2z}, \Delta f_{3z}, \Delta f_{4y}, \Delta f_{5y}, \Delta f_{6x})^T. \tag{1}
\]
Each surface $X_j$ is represented by its surface orientation $v_j$ and position $p_j$ (Huang et al., 2003), $j=1, 2\ldots M$, and $M$ is the number of part surfaces. Deviation of $X_j$ is composed of deviation of $v_j$ and $p_j$, i.e., $x_j=(\Delta v_j^T \Delta p_j^T)^T = (\Delta v_{jx} \Delta v_{jy} \Delta v_{jz} \Delta p_{jx} \Delta p_{jy} \Delta p_{jz})^T$. Datum error is expressed as the deviations of three datum surfaces, $x_I$, $x_{II}$, and $x_{III}$.

Machine tool error is modeled as the deviation of cutting tool path (Huang and Shi, 2003), which includes displacement error $(x_m y_m z_m)$ and rotational error pitch $\alpha_m$, roll $\beta_m$, and yaw $\gamma_m$. Using the same notation, we represent machine tool error by $\delta q_m=(x_m y_m z_m \alpha_m \beta_m, \gamma_m)^T$, which is invariant for all machined surfaces at one operation.

With a milling example, Fig. 2 shows the concept that machine tool, datum, and fixture errors could generate the same error pattern. The mathematical derivation of EFE is given in Appendix A.

Following Eq. (1), we use $\Delta d=(\Delta d_{1x} \Delta d_{2x} \Delta d_{3x} \Delta d_{4y} \Delta d_{5y} \Delta d_{6y})^T$ and $\Delta m=(\Delta m_{1x} \Delta m_{2x} \Delta m_{3x} \Delta m_{4y} \Delta m_{5y} \Delta m_{6y})^T$ to represent EFEs caused by datum and machine tool errors, respectively. By this concept, the error sources in the machining are all transformed to fixture deviations, i.e., $\Delta f$, $\Delta d$, and $\Delta m$. The relationship between EFE and feature deviation can then be derived as (see more details in Appendix B)

$$x=(\Gamma_u | \Gamma_v | \Gamma_w)(\Delta d^T | \Delta f^T | \Delta m^T)^T + \varepsilon$$  \hspace{1cm} (2)
where \( \mathbf{x} \) is the feature deviation vector (e.g., it can be \([ x_1^T \ x_2^T \ldots \ x_M^T ]^T \)). \( \mathbf{\Gamma}_u=[ \mathbf{\Gamma}_1^T \ \mathbf{\Gamma}_2^T \ldots \ \mathbf{\Gamma}_M^T ]^T \) is the mapping matrix that relates EFE to the feature deviation. The matrix \( \mathbf{\Gamma}_j, j=1, 2, \ldots, M \), is block matrix corresponding to each machined surface \( j \). The results in Appendix B shows that \( \mathbf{\Gamma}_j \) for each type of error is the same and thus three block matrices in model (2) are identical. This is consistent with the phenomenon that three types of errors can generate the same feature deviation.

2.2 Modeling of Error Cancellation

EFE can model the error cancellation and the impact of errors on feature deviation. If we group errors in Eq. (2), we have

\[
\mathbf{x} = \mathbf{\Gamma}_u(\Delta \mathbf{d} + \Delta \mathbf{f} + \Delta \mathbf{m}) + \mathbf{\varepsilon},
\]

(3)

Therefore, the cancellation effect of three types of errors can be modeled as a linear combination of mean shift of EFEs and fixture error. Their impacts on feature deviation are described by mapping matrix \( \mathbf{\Gamma}_u \) in Eq. (3). For a special case that three types of errors completely cancel each other, i.e., \( \text{E}(\Delta \mathbf{d} + \Delta \mathbf{f} + \Delta \mathbf{m}) \) is statistically insignificant, the mean of process output is within control, where \( \text{E}(\cdot) \) represents expectation of random variables in the
parentheses. It should be noticed that the variances caused by three types of errors cannot be cancelled.

In this paper, $\Delta d$, $\Delta f$ and $\Delta m$ are assumed to be independent random vectors following multivariate normal distribution. $\varepsilon$ is the random vector following the normal distribution $N(0, \sigma^2 \mathbf{I})$. $\varepsilon$ can be considered as the aggregated effects of measurement noise and inherent unmodeled terms in the machining process.

![Figure 3. Non-planar datum surfaces](image)

The modeling based on Appendices is applicable for the case where datum surfaces are all planes. When the surface is not planar, we should use tangential plane of surface at each locator point as datum surface. Figure 3 shows the setup of a 2-D part with non-planar datum surfaces. The datum surfaces are tangential planes $T_1$, $T_2$, and $T_3$. The corresponding normal vectors are $\mathbf{n}_1$, $\mathbf{n}_2$, and $\mathbf{n}_3$, respectively. If the implicit form surface equation is represented by $f_j(x_j, y_j, z_j)=0$, $\mathbf{n}_j$ and $\mathbf{p}_j$ are determined by

$$
\mathbf{n}_j = \left( \frac{\partial f_j}{\partial x_j}, \frac{\partial f_j}{\partial y_j}, \frac{\partial f_j}{\partial z_j} \right)^T, \quad f_j(p_{jx}, p_{jy}, p_{jz}) = 0, \quad j = I, II, ..., VI
$$

Then substitute Eq. (4) into the following to compute EFE ($\Delta d$ and $\Delta m$).

$$
\begin{align*}
\Delta d_{ij} (or \Delta m_{ij}) &= [n_{jx}(f_{ix} - p_{ix}) + n_{jy}(f_{iy} - p_{iy})] / n_{jx} + p_{jx} - f_{ix}, i = 1, 2, 3, \quad j = I \\
\Delta d_{ij} (or \Delta m_{ij}) &= [n_{jx}(f_{ix} - p_{ix}) + n_{jy}(f_{iy} - p_{iy})] / n_{jy} + p_{jy} - f_{iy}, i = 4, 5, \quad j = II \\
\Delta d_{ij} (or \Delta m_{ij}) &= [n_{jx}(f_{ix} - p_{ix}) + n_{jy}(f_{iy} - p_{iy})] / n_{jz} + p_{jz} - f_{iz}, i = 6, \quad j = III
\end{align*}
$$

(5)

where $(\Delta v_{ix}, \Delta v_{iy}, \Delta p_{ix}, \Delta p_{iy}, \Delta p_{iz})$ are the deviated datum surfaces, and $j = I, II, III$.
represent three datum surfaces. Eq. (5) is determined by the distance between the two points
where locators intersect the nominal datum \( X_j^0 = (v_jx, v_jy, v_jz, p_jx, p_jy, p_jz)^T \) and deviated
datum surfaces \( X_j = (v_jx, v_jy, v_jz, p_jx, p_jy, p_jz). \)

3. Theoretical Implications

Modeling of error cancellation and errors’ generating the same feature deviation have
many theoretical implications on machining process control. Wang et al. (2005) found that
EFM modeling could potentially reduce measurement in multistage machining processes. In
this paper, we further discuss the implications on three issues: diagnosability analysis, root
cause identification, and error compensation.

3.1 Diagnosability Analysis

This paper studies the diagnosability of the process that is governed by a general linear
fault model as follows, which relates the errors to the feature deviation \( x, \)
\[
\begin{align*}
x &= \Gamma \left( x_D^T \quad f^T \quad \delta q_m^T \right)^T + \varepsilon. 
\end{align*}
\]
where matrix \( \Gamma \) is determined by the part specification. Its relationship with \( \Gamma_u \) will be
discussed in Proposition 1. \( x_D = (x_i^T \quad x_{il}^T \quad x_{im}^T)^T \) is the error vector of three datum surfaces
of the raw workpiece.

If the process is diagnosable, the Least Square Estimation (LSE) can be performed, i.e.,
\[
\begin{align*}
\left( x_D^T \quad f^T \quad \delta q_m^T \right)^T &= (\Gamma^T \Gamma)^{-1} \Gamma x. 
\end{align*}
\]
The diagnosability depends on the rank of \( \Gamma \) (Zhou, et al., 2003). We can see that Eq. (7)
requires \( \Gamma^T \Gamma \) to be full rank, or equivalently, all the columns in \( \Gamma \) to be independent.
Proposition 1 addresses the structure of \( \Gamma \) for a machining process.

**Proposition 1** In model (6), block matrices in matrix \( \Gamma \) corresponding to three types of errors
are dependent and matrix $\Gamma^T \Gamma$ is always not full rank, i.e., fixture, datum, and machine tool errors cannot be distinguished by measuring the part features only.

*Proof* If we use transformation matrices $K_1$ (Eq. (A.1)) and $K_2$ (Eq. (A.2)) to map datum error $x_d$ to $\Delta d$ and machine tool error $\delta q_m$ to $\Delta m$, respectively, Eq. (2) becomes

$$\mathbf{x} = [\Gamma_u K_1 \mid \Gamma_u K_2] [\begin{bmatrix} \mathbf{x}_d^T \mid \Delta f m^T \mid \delta q m^T \end{bmatrix}]^T + \epsilon$$

Comparing Eq. (8) with Eq. (6), we get matrix $\Gamma = [\Gamma_u K_1 \mid \Gamma_u K_2]$. However, the columns corresponding to fixture and machine tool errors in matrix $\Gamma$ are dependent because columns of $\Gamma_u K_1$ and $\Gamma_u K_2$ are the linear combination of columns of $\Gamma_u$. Therefore, rank of $\Gamma$ equals the rank of $\Gamma_u$. This also implies that the system is not diagnosable.

An implication of this proposition is that LSE (7) cannot be obtained. However, the fault model (3) with error grouped eliminates the dependent columns in matrix $\Gamma$. This fact leads to sequential root cause identification in Section 3.2.

### 3.2 Root Cause Identification

Using model (3), the grouped errors $u$ can be estimated as

$$\mathbf{\hat{u}}^{(n)} = \Delta d^{(n)} + \Delta f^{(n)} + \Delta m^{(n)} = (\Gamma_u^T \Gamma_u)^{-1} \Gamma_u^T \mathbf{x}^{(n)}, \quad n=1, 2, \ldots, N$$

where $\mathbf{\hat{u}}^{(n)}$ is LSE of $u$ for the $n$th replicate of measurement. Each row of $\Gamma_u$ corresponds to output feature while each column of $\Gamma_u$ corresponds to component of error vectors. Hence, the number of rows of $\Gamma_u$ must be larger than the number of its columns to ensure that sufficient features are measured for LSE.

Denote $\Delta f^{(n)}_i$, $\Delta d^{(n)}_i$, and $\Delta m^{(n)}_i$ as the $i$th component in vector $\Delta f^{(n)}$, $\Delta d^{(n)}$, and $\Delta m^{(n)}$, respectively. We can develop the strategy for root cause identification that turns out to be a sequential fashion: (1) Necessary error information is collected first to identify the existence
of error sources using Eq. (9). The process error information can be analyzed by conducting hypothesis test on $\{\hat{u}^{(n)}\}_{n=1}^N$. Since estimated $u$ is a mixture of noise and errors, proper test statistic should be developed to detect the faults from process noise. Hypothesis test for mean and variance can then be used to find out if the faults are mean shift or large variance. (2) Additional measurement on locator deviation ($\Delta f_i^{(n)}$) and datum error ($\Delta d_i^{(n)}$) of raw workpiece is conducted (due to Proposition 1) to distinguish different types of errors. The mean shift of the errors can be estimated using the sample mean of $\Delta d_i^{(n)}$, $\Delta f_i^{(n)}$, and $\Delta m_i^{(n)}=u_i^{(n)}-\Delta d_i^{(n)}-\Delta f_i^{(n)}$. The variance can then be estimated by the sample variance for $\Delta d_i^{(n)}$, $\Delta f_i^{(n)}$, and $\Delta m_i^{(n)}$. This approach can effectively identify the machine tool errors. The detail procedures will be given in the Section 4.

3.3 Error Compensation

We can use the effect of error cancellation to compensate process errors. With the development of adjustable fixture whose locator length is changeable, it is feasible to compensate errors only by changing the length of locators. We use index $i$ to represent the $i$th adjustment period. During period $i$, $N$ part feature deviations $\{x_i^{(n)}\}_{n=1}^N$ are measured to determine the amount of locator adjustment. Such compensation is only implemented at the beginning of the period. Denote $c_i$ as the accumulative amount of locator length adjusted after the $i$th period and the beginning of period $i+1$. The compensation procedure can be illustrated with Fig. 4. One can see that a nominal machining process is disturbed by errors $\Delta d$, $\Delta f$ and $\Delta m$, and the observation noise $\epsilon$. Error sources, noise, and machining process constitute a disturbed process, as marked in the dash line block. Using the feature deviation $x_i$ for the $i$th period as input ($x_i$ can be estimated by the average of $N$ measured parts in the period $i$, i.e.,
\[ \hat{x}' = \frac{1}{N} \sum_{n=1}^{N} x^{(n)} \], a controller is introduced to generate signal \( c' \) to manipulate adjustable fixture locators to counteract the errors for the \((i+1)\)th machining period. The amount of compensation at period \( i+1 \) should be \( c' - c'^{-1} \). The error compensation model can then be
\[ x^{i+1} = S^{i+1} + \Gamma_u c' \quad \text{and} \quad S^{i+1} = \Gamma_u u^{i+1} + \varepsilon^{i+1} \tag{10} \]
where \( S^{i+1} \) is the output of the disturbed process for time \( i+1 \). This term represents the feature deviation measured without any compensation being made.

**Figure 4. Error compensation for disturbed process**

In this paper, we focus on static error because they account for the majority of overall machining errors (Zhou, Huang, and Shi, 2003). The negative value of predicted EFEs can be used to adjust locators. Thus, we derive an integral control that can minimize Mean Square Error (MSE) of the feature deviation, i.e.,
\[ c' = -(\Gamma_u^T \Gamma_u)^{-1} \Gamma_u^T \sum_{t=1}^{i} x^t = -\sum_{t=1}^{i} (\Delta \hat{d}^t + \Delta \hat{f}^t + \Delta \hat{m}^t). \tag{11} \]

Equation (11) shows that the accumulative amount of compensation for the next period is equal to the sum of the LSE of EFE of all current and previous time periods of machining. The accumulative compensation \( c' \) is helpful for evaluation of controller performance such as stability and robustness analysis. The amount of compensation for the \( i+1 \)th period is \( c' - c'^{-1} \),
\[ c' - c'^{-1} = -(\Gamma_u^T \Gamma_u)^{-1} \Gamma_u^T x^t. \tag{12} \]

The compensation accuracy can be estimated by \( \Gamma_u x - (\Gamma_u^T \Gamma_u)^{-1} \Gamma_u^T x^t = x^t - (\Gamma_u^T \Gamma_u)^{-1} \Gamma_u^T x^t \), i.e., the difference between \( x^t \) and its LSE. Denote range space of \( \Gamma_u \) as \( R(\Gamma_u) \) and null space
of $\Gamma_u^T$ as $N(\Gamma_u^T)$. Spaces $R(\Gamma_u)$ and $N(\Gamma_u^T)$ are orthogonal and constitute the whole vector space $\mathbb{R}^{q\times 1}$, where $q$ is the number of rows in $\mathbf{x}'$ (or $\Gamma_u$). By the property of LSE, we know that the estimation error vector $\mathbf{x}' - \Gamma_u(\Gamma_u^T \Gamma_u)^{-1} \Gamma_u^T \mathbf{x}'$ is orthogonal to $R(\Gamma_u)$. Therefore, the compensation accuracy of Eq. (12) can be estimated by projection of observation (feature deviation) vector $\mathbf{x}'$ onto $N(\Gamma_u^T)$. This conclusion also shows the components of observation that can be compensated. The projection of observation vector $\mathbf{x}'$ onto space $R(\Gamma_u)$ can be fully compensated with Eq. (12) while projection onto $N(\Gamma_u^T)$ cannot be compensated.

In practice, the accuracy that the adjustable locator can achieve must be considered. Suppose the standard deviation of locator’s movement is $\sigma_f$. We can set the stopping region for applying error compensation with 99.73% confidence

$$-3\sigma_f \leq \mathbf{e}' - \mathbf{e}'^{-1} \leq 3\sigma_f$$

(13)

4. Case Studies

Discussion in Section 3 implies the application of EFE concept in sequential root cause identification and error compensation. The diagnostic algorithms are proposed in this section and demonstrated with a machining experiment. EFE compensation for process control is illustrated with a simulation.

4.1 Root Cause Identification

There are several diagnostic approaches (Ceglarek and Shi, 1996; Apley and Shi, 1998; and Rong, Shi, and Ceglarek, 2001) that have achieved considerable success in fixture fault detection. The approach proposed by Apley and Shi (1998) can effectively identify multiple fixture faults. By extending this approach, we use it for sequential root cause identification:

**Step 1:** Conduct measurement on features and datum surfaces of raw workpiece to estimate
error sources $\hat{u}^{(n)}$ for each replicate by Eq. (9). The grouped error can be estimated by the average of $\hat{u}^{(n)}$ over $N$ measured workpieces, i.e., $\hat{u} = \frac{1}{N} \sum_{n=1}^{N} \hat{u}^{(n)}$, $n=1,2,\ldots,N$. As mentioned in Section 3.2, the fault vector $\mathbf{u}$ is the mixture of error sources and process noise.

**Step 2:** To detect the faults from the process noise, we can use $F$ test statistic introduced by Apley and Shi (1998):

$$F_i = \frac{\hat{S}_i^2}{\left[ (\Gamma^T_a \Gamma_a)_{ii} \right] \hat{S}_z^2}, \quad i=1,2,\ldots,6 \quad (14)$$

where $\hat{S}_i^2 = \frac{1}{N} \sum_{n=1}^{N} [\hat{u}^{(n)}_i]^2$, and $\hat{u}^{(n)}_i$ represents the $i$th component in vector $\hat{u}^{(n)}$. $(\Gamma^T_a \Gamma_a)_{ii}$ is the $i$th diagonal entry of matrix $(\Gamma^T_a \Gamma_a)^{-1}$. The estimator for variance of noise is

$$\hat{S}_z^2 = \frac{1}{N(q-6)} \sum_{n=1}^{N} [\hat{e}^{(n)}]^T \hat{e}^{(n)}, \quad \text{and} \quad \hat{e}^{(n)} = \mathbf{x}^{(n)} - \Gamma_a \hat{u}^{(n)} \text{ is for noise terms.}$$

When $F_i > F_{1-\alpha}(N, N(q-6))$, we conclude that the $i$th fault significantly occurs with confidence of 100(1-$\alpha$)%. By investigating $\{ \hat{u}^{(n)}_i \}_{n=1}^{N}$ for mean $u_i$ ($H_0: u_i=0$ vs. $H_1: u_i\neq0$), and variance $\sigma_{u_i}^2$ ($H_0: \sigma_{u_i}^2 \leq \sigma_0^2$ vs. $H_1: \sigma_{u_i}^2 > \sigma_0^2$), one can determine whether the error pattern of the faults is mean shift or variance. $\sigma_0^2$ is a small value. In the case study, we choose $\sigma_0^2 = 0.1\text{mm}^2$. By the normality assumption of EFEs ($\Delta d$, $\Delta f$, and $\Delta m$), we can use the $T$ test statistic $T = \frac{1}{\sqrt{N(N-1)}} \sum_{n=1}^{N} (u^{(n)} - u)^2$ and compare it with $t_{1-\alpha/2}(n-1)$ to test mean shift. $\chi^2 = \frac{\sum_{n=1}^{N} (u^{(n)} - u)^2}{\sigma_0^2}$ is used and compared with $\chi^2_{1-\alpha}(n-1)$ to test variance. $\alpha$ is the significance level. If $F_i < F_{1-\alpha}(N, N(q-6))$, no faults occur at the $i$th locator, or the faults cannot be distinguished from process noise.

**Step 3:** Apply the additional measurement to distinguish errors whenever faults are identified.

Locator deviation $\{\Delta f^{(n)}_j\}_{n=1}^{N}$ and datum surfaces $\{\mathbf{X}^{(n)}_j\}_{n=1}^{N}$ are measured. The EFE $\{\Delta d^{(n)}_j\}_{n=1}^{N}$ caused by datum error can be calculated by Eq. (A.1). If the errors turn out to be
mean shift ($u \neq 0$ for certain $i$), machine tool error in terms of EFE is $\Delta \mu_i = \hat{u}_i - \Delta d_i - \Delta f_i$, where $\Delta d_i$ and $\Delta f_i$ are the average EFE over all $N$ parts. Machine tool error $\delta q_m$ is then determined by the inverse of Eq. (A.2)

$$\delta q_m = K_2^{-1} \Delta m .$$

The variance of grouped error ($\sigma_{ui}^2$) can then be decomposed as

$$\sigma_{ui}^2 = \sigma_{di}^2 + \sigma_{fi}^2 + \sigma_{mi}^2 .$$

If $\sigma_{ui}^2 > \sigma_{0}^2$, variances caused by three types of errors $\sigma_{di}^2$, $\sigma_{fi}^2$, and $\sigma_{mi}^2$ can be estimated by the sample variance of $\{\Delta d_i^{(a)}\}_{n=1}^{N}$, $\{\Delta f_i^{(a)}\}_{n=1}^{N}$, and $\{\Delta m_i^{(a)}\}_{n=1}^{N}$.

The 100(1-2$\alpha$)% confidence interval (CI) of $\Delta m$ is $(\Delta \hat{m} \pm L)$, where $z_{1-\alpha}$ follows the cumulative standard normal distribution such that $\int_{-\infty}^{z_{1-\alpha}} \frac{1}{\sqrt{2\pi}} e^{-u^2/2} du = 1-\alpha$ and $L = \left[ (z_{1-\alpha} \sqrt{(\Gamma_i^T \Gamma_i)^{-1}} \sigma x, \ldots, z_{1-\alpha} \sqrt{(\Gamma_i^T \Gamma_i)^{-1}} \sigma x ) \right]$. The corresponding CI vector for $\delta q_m$ is $(K_2^{-1} \Delta m \pm K_2^{-1} L)$. The CI for $\Delta d$ and $\Delta f$ can be obtained by $(\Delta d_i \pm S_{di}(n-1)/\sqrt{n})$ and $(\Delta f_i \pm S_{fi}(n-1)/\sqrt{n})$, where $S_{di}$ and $S_{fi}$ are the sample variance for $\{\Delta d_i^{(a)}\}_{n=1}^{N}$ and $\{\Delta f_i^{(a)}\}_{n=1}^{N}$.

To demonstrate the model and the diagnostic procedure, we intentionally introduced datum and machine tool errors to mill five block workpieces. We used the same setup, raw workpiece and fixturing scheme as Wang, et al., 2005 (Fig. 5). Coordinate system $xyz$ fixed with nominal fixture is also introduced to represent the plane. Top plane $X_1$ and side plane $X_2$ are to be milled. Eight vertices are marked as 1~8 and their coordinates in the coordinate system $xyz$ are measured to help to determine $X_1$ and $X_2$. In this paper, the unit is mm for length and rad for angle. Under the coordinate system in the Fig. 5, surface specifications are $X_1 = (0 \ 0 \ 1 \ 0 \ 0 \ 15.24)^T$, and $X_2 = (0 \ 1 \ 0 \ 0 \ 96.5 \ 0)^T$. From model (3) and Eq. (A.8), we get
\[ x' = \begin{pmatrix} \Gamma_1 \\ \Gamma_2 \end{pmatrix} (\Delta d' + \Delta f' + \Delta m') + \epsilon', \]

where

\[ \Gamma_1 = \begin{pmatrix} 0 & 0.0263 & 0.0263 & 0 & 0 & 0 \\ -0.0158 & 0.0079 & 0.0079 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.1379 & 0.1379 & 1.3368 & -1.3368 & -1 \\ -0.0828 & 0.0414 & 0.0414 & -1.5 & 0.5 & 0 \\ -1.3033 & -0.8483 & 1.1517 & 0 & 0 & 0 \end{pmatrix}, \quad \Gamma_2 = \begin{pmatrix} 0 & 0 & 0 & -0.0263 & 0.0263 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.158 & -0.0079 & -0.0079 & -1.5 & 0.5 \\ 0.0828 & -0.2632 & -0.2632 & -1.2026 & 1.2026 & -1 \\ 0.118 & -0.079 & -0.079 & 1.5 & 0.5 & 0 \\ 1.3033 & 0.158 & 1.2026 & 0 & 0 & 0 \end{pmatrix} \]

The number of rows \( q \) in \( \Gamma \) is 12. We set fixture error to be zero (\( \Delta f = 0 \)). The primary datum plane I is pre-machined to be \( X_1 = (0 \ 0.018 \ -0.998 \ 0 \ 0.207 \ -1.486)^T \) and its corresponding EFE is \( \Delta d = (1.105 \ 0 \ 0 \ 0 \ 0)^T \) mm. The machine tool error is set to be \( \delta q_{m} = (0 \ 0.175 \ -1.44 \ 0.0175 \ 0)^T \) by adjusting the orientation and position of tool path. Based on coordinates of the vertices 1~8 measured, the feature deviations are given in Table 1.

![Figure 5. Nominal part, tolerance, and fixture layout (Wang, et al, 2005)](image)

<table>
<thead>
<tr>
<th>( n )</th>
<th>( \Delta v_x )</th>
<th>( \Delta v_y )</th>
<th>( \Delta v_z )</th>
<th>( \Delta p_x )</th>
<th>( \Delta p_y )</th>
<th>( \Delta p_z )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.001</td>
<td>-0.033</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>2</td>
<td>-0.000</td>
<td>-0.034</td>
<td>-0.039</td>
<td>-0.034</td>
<td>-0.035</td>
<td>-0.035</td>
</tr>
<tr>
<td>3</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.032</td>
<td>0.032</td>
<td>0.036</td>
</tr>
<tr>
<td>4</td>
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<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>5</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>( \Delta v_x )</td>
<td>0.347</td>
<td>0.253</td>
<td>0.347</td>
<td>0.579</td>
<td>0.358</td>
<td>0.479</td>
</tr>
<tr>
<td>( \Delta v_y )</td>
<td>0.379</td>
<td>0.253</td>
<td>0.379</td>
<td>0.579</td>
<td>0.358</td>
<td>0.479</td>
</tr>
<tr>
<td>( \Delta v_z )</td>
<td>0.307</td>
<td>0.268</td>
<td>0.307</td>
<td>0.539</td>
<td>0.429</td>
<td>0.539</td>
</tr>
</tbody>
</table>

Following the steps 1-3, the identified EFE faults are given in Table 2 and 3.
Choose $\alpha$ to be 0.01. The threshold value $F_{0.99}(5,5(12-6))=F_{0.99}(5,30)=3.699$. In Table 3, we can see that $F_1 > 3.699$, which indicates that fault occurs at locator 1. Using the data in the first row of Table 2 to conduct $T$ and $\chi^2$ tests for mean and variance, we find that $T > t_{1-0.01/2}(5-1)=t_{0.995}(4)=4.604$ and $\chi^2 < \chi^2_{1-0.01}(4)=13.277$. Hence, we conclude that there is significant mean shift while the variance is not large. If we make the additional measurement, by Eq. (A.2), the 98% CI for the detected mean shift of machine tool error is $\delta_{q_m}=(0.006 \ 0.167 \ -1.540 \ 0.018 \ -0.000 \ 0.000)^T \pm (0.008 \ 0.001 \ 0.000 \ 0.000 \ 0.001 \ 0.000)^T$, which is consistent with pre-introduced errors. The EFE fault model and diagnostic algorithm is experimentally validated.

**Table 2. Estimation of $u$ for five replicates (mm)**

<table>
<thead>
<tr>
<th>$\hat{u}$ (1)</th>
<th>$\hat{u}$ (2)</th>
<th>$\hat{u}$ (3)</th>
<th>$\hat{u}$ (4)</th>
<th>$\hat{u}$ (5)</th>
<th>$T$</th>
<th>$\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.937</td>
<td>2.133</td>
<td>1.775</td>
<td>2.697</td>
<td>1.902</td>
<td>2.289</td>
<td>10.119</td>
</tr>
<tr>
<td>0.050</td>
<td>0.090</td>
<td>-0.064</td>
<td>0.057</td>
<td>0.002</td>
<td>0.027</td>
<td>-</td>
</tr>
<tr>
<td>0.002</td>
<td>0.090</td>
<td>-0.0562</td>
<td>0.057</td>
<td>0.020</td>
<td>0.023</td>
<td>-</td>
</tr>
<tr>
<td>0.055</td>
<td>-0.031</td>
<td>0.003</td>
<td>0.039</td>
<td>0.015</td>
<td>0.016</td>
<td>-</td>
</tr>
<tr>
<td>0.047</td>
<td>-0.031</td>
<td>0.004</td>
<td>0.039</td>
<td>0.018</td>
<td>0.015</td>
<td>-</td>
</tr>
<tr>
<td>0.004</td>
<td>0.000</td>
<td>-0.001</td>
<td>0.000</td>
<td>-0.001</td>
<td>0.001</td>
<td>-</td>
</tr>
</tbody>
</table>

**Table 3. Additional measurement results (mm)**

<table>
<thead>
<tr>
<th>Locators</th>
<th>$\hat{u}$</th>
<th>$F_i$</th>
<th>$\Delta f$</th>
<th>$\Delta d$</th>
<th>$\Delta m$</th>
</tr>
</thead>
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<tr>
<td>1</td>
<td>2.289</td>
<td>19.525</td>
<td>0</td>
<td>1.105</td>
<td>1.184</td>
</tr>
<tr>
<td>2</td>
<td>0.027</td>
<td>0.051</td>
<td>0</td>
<td>0</td>
<td>0.027</td>
</tr>
<tr>
<td>3</td>
<td>0.023</td>
<td>0.005</td>
<td>0</td>
<td>0</td>
<td>0.023</td>
</tr>
<tr>
<td>4</td>
<td>0.016</td>
<td>0.613</td>
<td>0</td>
<td>0</td>
<td>0.016</td>
</tr>
<tr>
<td>5</td>
<td>0.015</td>
<td>0.073</td>
<td>0</td>
<td>0</td>
<td>0.015</td>
</tr>
<tr>
<td>6</td>
<td>0.001</td>
<td>0.002</td>
<td>0</td>
<td>0</td>
<td>0.003</td>
</tr>
</tbody>
</table>
Figure 6. Error compensation for each locator

4.2 Error Compensation Simulation

Using the same machining process as in Section 4.1, we can simulate error compensation for five adjustment periods. Five parts are sampled during each period. We set the fixture error to be $\Delta f=(0.276 \ 0 \ 0.276 \ 0 \ 0)^T \text{mm}$. The machine tool error is set to be $\delta q_m=(-0.075 \ -0.023 \ 0.329 \ -0.0023 \ 0.0075 \ 0)^T$ and its EFE is $\Delta m=(0 \ 0 \ 0.286 \ 0 \ 0 \ 0)^T \text{mm}$. We assume the measurement noise to follow $N(0, (0.002\text{mm})^2)$ for displacement and $N(0, (0.001\text{rad})^2)$ for orientation. The compensation values can be calculated by Eqs. (11) and (12). In this case study, the accuracy of the locator movement is assumed to be $\sigma_f=0.01\text{mm}$ and the criterion for stopping the compensation is $-0.03\leq c_i-c_{i-1}\leq 0.03\text{mm}$ (Eq. (13)). Figure 6 shows the compensation ($c_i-c_{i-1}$) for locators $f_1$~$f_4$. The values of adjustment periods 2~5 are given by the solid line in the figure. The dash dot line represents the value of $\pm 3\sigma_f$. The adjustment for locators $f_5$ and $f_6$ are all zero and not shown in the figure. One can see that the effect of compensation in the second period is dominant. The compensation for the subsequent periods is relatively small because no significant error sources are introduced for these periods.
The effect of error compensation can be illustrated with the quality improvement of two features, the plane distance along $z$ axis ($l_z$) and $y$ axis ($l_y$) as shown in Fig. 5. $l_z$ can be estimated by the mean and standard deviation of length of edges $l_{15}$, $l_{26}$, $l_{37}$ and $l_{48}$ and $l_y$ can be estimated by $l_{14}$, $l_{23}$, $l_{67}$ and $l_{58}$ for each machining period, where $l_{mn}$ is the distance between the vertices $m$ and $n$ and is estimated by the edge length of five parts in each period. Milling of planes $X_1$ and $X_2$ impacts the plane distance along $z$ and $y$ axes. The nominal part should have the same length of edges along $z$ and $y$ directions (15.24 and 96.5mm, see the dash line in Fig. 7), respectively. However, in the first adjustment period ($i=1$) without error compensation, the standard error of edge lengths are beyond specified tolerance. In the periods 2–5 when compensation algorithm has been applied, deviation of $l_z$ and $l_y$ is significantly reduced.

5. Summary

This paper investigates error cancellation among datum, fixture and machine tool errors for improving the quality control in machining process. Based on the concept of equivalent fixture error (EFE), error cancellation was modeled as linear combination of EFEs. The process fault model in terms of grouped EFEs is then derived to conduct fault diagnosis and
error compensation of machining process. EFE methodology helps to reveal the structure of matrix of fault model. We mathematically proved that a machining process with datum, fixture and machine tool errors cannot be diagnosable by only measuring the part features. To solve this problem, we develop the procedure of sequential root cause identification. First, datum error and machine tool error can be grouped with fixture error and the existence and locations of EFE can be detected. Additional measurement on process variable (locator deviation) should be implemented only if faults are detected. This procedure can detect the mean shift and variance of process faults from the process noise. A case study for a milling process of block parts has shown that the proposed approach can effectively identify the error sources. Study of error cancellation also suggests that machine tool and datum errors can be compensated by adjusting the length of fixture locators. An integral control algorithm is presented in this paper for compensation of static error. The procedure has been demonstrated with a simulation study.

Future study of EFE can be applying it to the process with dynamic disturbances. EFE can help to determine the disturbance model and find out the optimal control rule to minimize Mean Square Error.

References


Appendix A Derivation of EFE

Suppose all three datum surfaces are planar. By linearizing Eq. (5), we have derived the EFE ($\Delta d_{ix}$ $\Delta d_{iy}$ $\Delta d_{iz}$) caused by datum error as

$$
\Delta d_{ix} = -f_{ix}\Delta v_{ix} - f_{iy}\Delta v_{iy} - \Delta p_{ix}, \ i = 1,2,3, \\
\Delta d_{iy} = -f_{ix}\Delta v_{ix} - f_{iy}\Delta v_{iy} - \Delta p_{iy}, \ i = 4,5, \text{ or } \Delta d = K\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \tag{A.1}
$$

$$
\Delta d_{iz} = -f_{iz}\Delta v_{ix} - f_{iz}\Delta v_{iy} - \Delta p_{iz}, \ i = 6.
$$

The mapping matrix relating datum error to $\Delta d$ is

$$
K = \begin{pmatrix} G_1 & 0 & 0 \\ 0 & G_2 & 0 \\ 0 & 0 & G_3 \end{pmatrix}
$$

where

$$
G_1 = \begin{pmatrix} f_{ix} & f_{iy} & 0 & 0 & 0 & 1 \\ f_{ix} & f_{ix} & 0 & 0 & 1 & 0 \\ f_{ix} & f_{ix} & 0 & 0 & 0 & 1 \end{pmatrix}, \ G_2 = \begin{pmatrix} f_{ix} & 0 & f_{ix} & 0 & 1 & 0 \\ f_{ix} & 0 & f_{ix} & 0 & 1 & 0 \end{pmatrix}, \text{ and } G_3 = \begin{pmatrix} 0 & f_{ix} & f_{ix} & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}.
$$

When deriving $\Delta m$, we use the relationship between $X_j$ and machine tool error $\delta q_m$. 

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Linearization of Eq. (5) then yields

\[ \Delta \mathbf{m} = \mathbf{K}_2 \delta \mathbf{q}_m, \]  

\[ \text{where } \mathbf{K}_2 = \begin{pmatrix} 0 & 0 & -1 & -f_{1y} & f_{1x} & 0 \\ 0 & 0 & -1 & -f_{2y} & f_{2x} & 0 \\ 0 & 0 & -1 & -f_{3y} & f_{3x} & 0 \\ 0 & -1 & 0 & f_{4z} & 0 & -f_{4x} \\ 0 & -1 & 0 & f_{5z} & 0 & -f_{5x} \\ -1 & 0 & 0 & 0 & -f_{6z} & f_{6y} \end{pmatrix}. \]

Note: If the datum surfaces are not planes, datum surfaces \( \mathbf{X}_j \) become tangential planes at each locating point and there are 6 datum surfaces.

Appendix B Derivation of \( \Gamma_j \)

Huang and Shi (2003) have modeled the setup and cutting operation by Homogeneous Transformation Matrix (HTM). Feature deviation can then be expressed by (Wang, \textit{et al.} 2005):

\[ \mathbf{x}_j = \left( \mathbf{A}_{ji} \bigg| \mathbf{A}_{jf} \bigg| \mathbf{A}_{jm} \right) \delta \mathbf{q} + \mathbf{\varepsilon}_j \tag{A.3} \]

\[ \text{where } \mathbf{A}_{ji}(k) = \mathbf{A}_{jf}(k) = -\mathbf{A}_{jm}(k) = \begin{pmatrix} 0 & 0 & 0 & 0 & -2v^0_{jr} & 2v^0_{jr} \\ 0 & 0 & 2v^0_{jr} & 0 & -2v^0_{mr} & 2p^0_{kr} \\ 0 & 0 & 0 & -2v^0_{jr} & 2v^0_{mr} & 0 \\ -1 & 0 & 0 & 0 & -2p^0_{jr} & 2p^0_{mr} \\ 0 & -1 & 0 & 2p^0_{jr} & 0 & -2p^0_{mr} \\ 0 & 0 & -1 & 2p^0_{jr} & 2p^0_{mr} & 0 \end{pmatrix}, \]  

and \( \text{rank}(\mathbf{A}_{ji}) \leq 5. \)

\[ \delta \mathbf{q} = (x_d \quad y_d \quad z_d \quad \delta e_{1d} \quad \delta e_{2d} \quad \delta e_{3d} \quad x_f \quad y_f \quad z_f \quad \delta e_{1f} \quad \delta e_{2f} \quad \delta e_{3f} \quad x_m \quad y_m \quad z_m \quad \delta e_{1m} \quad \delta e_{2m} \quad \delta e_{3m})^T. \]  

\( \delta e_{1d} \quad \delta e_{2d} \quad \delta e_{3d})^T, (\delta e_{1d} \quad \delta e_{2d} \quad \delta e_{3d})^T \) and \( (\delta e_{1d} \quad \delta e_{2d} \quad \delta e_{3d})^T \) are Euler parameters of rotation caused by three types of errors respectively. Under small deviation assumption, they are half of the Euler angles, i.e., \( \delta e_1=0.5\alpha, \delta e_2=0.5\beta, \) and \( \delta e_3=0.5\gamma. \) Parameters \((x_d \quad y_d \quad z_d \quad \delta e_{1d} \quad \delta e_{2d} \quad \delta e_{3d})\) represent transformation of surface due to the faulty setup with datum error, and \((x_f \quad y_f \quad z_f \quad \delta e_{1f} \quad \delta e_{2f} \quad \delta e_{3f})\) represent transformation due to the fixture error. \( \mathbf{\varepsilon}_j \) is the noise term corresponding to the \( j \text{th} \) feature. With variational approach proposed by Cai, \textit{et
al. (1997), we may find the relationship between parameters in \( \delta q \) and error sources. This approach can be directly applied for fixture error, i.e.,

\[
(x_f \ y_f \ z_f \ \delta e_{1f} \ \delta e_{2f} \ \delta e_{3f})^T = -J^I \Phi \Delta f
\]  

(A.4)

where for generic workpiece, Jacobian Matrix \( J \) is

\[
J = \begin{bmatrix}
-\eta_x & -\eta_y & -\eta_z & 2(f_1 \ z \ y - f_1 \ x \ y) & 2(f_1 \ z \ y - f_1 \ x \ y) & 2(f_1 \ y - f_1 \ x \ y) \\
-\eta_x & -\eta_y & -\eta_z & 2(f_2 \ z \ y - f_2 \ x \ y) & 2(f_2 \ z \ y - f_2 \ x \ y) & 2(f_2 \ y - f_2 \ x \ y) \\
-\eta_x & -\eta_y & -\eta_z & 2(f_3 \ z \ y - f_3 \ x \ y) & 2(f_3 \ z \ y - f_3 \ x \ y) & 2(f_3 \ y - f_3 \ x \ y) \\
\eta_x & \eta_y & \eta_z & 2(f_4 \ z \ y + f_4 \ x \ y) & 2(f_4 \ z \ y + f_4 \ x \ y) & 2(f_4 \ y + f_4 \ x \ y) \\
\eta_x & \eta_y & \eta_z & 2(f_5 \ z \ y + f_5 \ x \ y) & 2(f_5 \ z \ y + f_5 \ x \ y) & 2(f_5 \ y + f_5 \ x \ y) \\
\eta_x & \eta_y & \eta_z & 2(f_6 \ z \ y + f_6 \ x \ y) & 2(f_6 \ z \ y + f_6 \ x \ y) & 2(f_6 \ y + f_6 \ x \ y)
\end{bmatrix}
\]  

(A.5)

\( v_j = (v_{jx} \ v_{jy} \ v_{jz})^T \) is the orientation vector of datum planes \( j = I, II, \) and III. The Jacobian matrix is definitely full rank because the workpiece is deterministically located. The inverse of Jacobian therefore exists. Matrix \( \Phi \) is \( \text{diag}(v_1^T, v_1^T, v_1^T, v_1^T, v_1^T, v_1^T) \). \( E \) is an 18 \( \times \) 6 matrix, that is, \( \text{diag}(E_1 \ E_1 \ E_1 \ E_2 \ E_2 \ E_3) \), where \( E_1 = (0 \ 0 \ 1)^T \), \( E_2 = (0 \ 1 \ 0)^T \), and \( E_3 = (1 \ 0 \ 0)^T \). We can also extend the variational approach for EFE due to datum and machine tool error, \( \Delta d \) and \( \Delta m \), respectively,

\[
(x_d \ y_d \ z_d \ \alpha_d \ \beta_d \ \gamma_d)^T = -J^I \Phi \Delta d
\]  

(A.6)

\[
(x_m \ y_m \ z_m \ \alpha_m \ \beta_m \ \gamma_m)^T = -(J^I \Phi \Delta m) = J^I \Phi \Delta m
\]  

(A.7)

Equation (A.7) has additional minus sign because the inverse transformation caused by machine tool error transform the workpiece from nominal position to its real position. Combining (A.5), (A.6), and (A.9-10), we get input matrix \( \Gamma_j \) corresponding to the machined surface \( j \):

\[
\Gamma_j = -A_{jd}J^I \Phi E
\]  

(A.8)

We can see that matrices \( \Gamma_j \) corresponding to three EFEs are the same.