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STREAM OF VARIATION MODELING AND DIAGNOSIS OF MULTI-STATION MACHINING PROCESSES

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ABSTRACT

Product dimensional quality is one of the most important topics in machining processes. However, limited research has been done on the dimensional control and root cause identification methodologies in multi-station machining processes. This paper will study the stream of variation(SOV) problem by developing a new type of model – State Space Model – to describe the dimensional deviation of the multi-station machining processes. A process level diagnostic methodology is also developed. A case study is presented to illustrate the developed modeling and diagnostics methodology.

NOMENCLATURE

N	number of stations in a machining process
Y	vector of quality characteristics
$\boldsymbol{y}$	deviation of Y
X	vector of part surfaces
$\boldsymbol{x}$	the state variables defined as the deviation of X
Z^o	nominal value of variable Z
$e(\cdot)$ *or* e_k	deviation of variable or error of station k
$v(\cdot)$ *or* v_k	variance-covariance matrix of variable or station k
d_k	datum used by station k
f_{ki}	the ith surface machined by station k
$A \xrightarrow{k} B$	A turns to be B at station k
A⇔B	surface A and B are equivalent in the sense of fault manifestation
A=B	surface A and B are same
$\oplus$	additive and /or multiplicative combination of deviation or variation
M_k	a set of surfaces measurable at station k
l	lth level in the D-Graph
$n(l)$	number of nodes at level l

1. INTRODUCTION

Product dimensional quality is one of the most important topics in machining processes. Extensive research has been done to investigate the dimensional control of machined product. Most of the research has been focused on product design and tolerance (Lee and Woo, 1989; Gao *et al.*, 1998), fixture design and analysis (Choudhuri and De Meter, 1999; Menassa and DeVries, 1991; Rong and Bai, 1996), machine tool error compensation(Chen, Yuan, Ni, and Wu., 1993). Mantripragada and Whitney(1999), Lawless *et al.*,(1999) and Agrawal *et al.*, (1999) investigate dimensional variation in assembly process. A physical model that can characterize the stage-to-stage propagation of variation is developed by Jin and Shi (1999), Ding, *et al.*, (2000). However, limited research has been done on the dimensional control and root cause identification methodologies in multi-station machining process. Statistical Process Control (SPC) has been used as the primary methodology to analyze the part dimensional data and detect the process changes during production. However, there are many inherent limitations when using SPC in the process control to identify root causes of the process variation.

One of the major challenges in multi-station machining process control is to understand, model, analyze, control and reduce the process variation, or Stream of Variation (SOV) of the process. Those challenges can be seen from the following facts:

(1) The final product deviation/variation is an accumulation or stack up of deviation/variation from all machining operations. Understanding and modeling of the deviation/variation and its propagation throughout the manufacturing process requires the information from product and process design, as well as information for quality requirements. Such kind of model currently does not exist for process control purposes.

(2) It is a challenging task to design an optimal measurement strategy with distributed sensing (allocation of measurement station along the production line and distribution of sensing points in a given measurement station). Furthermore, advanced methodologies need to be developed for the information processing to monitor the process and to trace/identify the faulty stations and their root causes based on limited measurements.

(3) Multi-station machining processes can be regarded as a complex sequential, dynamic system, where each operation is equivalent to the time index in the system analysis. Advanced methodologies are needed to ensure the control of the overall stream of variation.

Fig.1 shows the framework of SOV methodology. In section 2, SOV model is built off-line by extracting product and process design from CAD and CAPP. Based on SOV model, diagnostic methodology is proposed by integrating of SOV model and in-line statistical analysis in section 3. It includes process diagnosability assessment(Section 3.1), off-line deviation pattern generation(Section 3.2), estimation of unknown error patterns(Section 3.3) and root causes determination(Section 3.4). A case study is presented in section 4. Finally, a summary and conclusion is given in section 5.

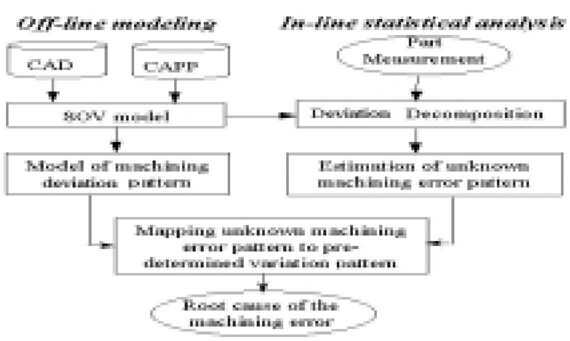


Fig. 1 Stream of Variation Theory

2. STREAM OF VARIATION MODELING

2.1 Part Model, Part Deviation and Observation

Tolerance is one of the main concerns in dimensional modeling and fault diagnosis. Vectorial surface model, proposed by Martinsen (1993), shows the advantages of vectorial tolerancing for all types of surfaces. In this paper, a modified vectorial surface model is applied.

Part Model In space, each surface of a part could be uniquely defined by its orientation vector $f_o : \vec{n} = [a,b,c]^T$, location vector $f_L : \vec{P} = [x_o, y_o, z_o]^T$, size $f_D : D = [d_1, d_2, \cdots, d_j]^T$ (Martinsen,1993) and form equation $f_F : F(x,y,z) = 0$. Therefore, surface *S* could be represented as $S = \{f_o, f_L, f_D, f_F\}$.

Suppose a part P consists of *M* surfaces: $S_1, S_2, \cdots, S_M$. SOV part model is represented by a surface set: $P = \{S_1, S_2, \cdots, S_n\}, n \le M$. The number of selected *n* surfaces can be less than total number of surfaces *M*, because for a given machining process, not all surfaces are related with variation propagation. The guidelines for choosing surfaces are: 1) surfaces to be machined; 2) design datum, including primary, secondary and tertiary datum; 3) machining datum; and 4) measurement datum.

In 3D Cartesian coordinate system, parameters of surface S_i are surface orientation vector $\vec{n} = [n_{ix}, n_{iy}, n_{iz}]^T$, location vector $\vec{p} = [p_{ix}, p_{iy}, p_{iz}]^T$, and size scalar variable $D_i = [d_{i1}, d_{i2}, \cdots, d_{ij}]^T_{j\times 1}$. If the maximum number of size parameters for all surfaces is *m* and $j < m$, then change f_D into $f_D : D_i = [d_{i1}, d_{i2}, \cdots, d_{im}]^T_{m\times 1}$, $d_{i(j+1)} = 0, \ldots, d_{im} = 0$. So the surface equation of S_i is a function of above parameters, that is, $F_i(X_i) = 0$, where $X_i = \left[n_{ix}, n_{iy}, n_{iz}, p_{ix}, p_{iy}, p_{iz}, d_{i1}, d_{i2}, \cdots, d_{im}\right]^T_{(6+m)\times 1}$.

The part is represented as:

$$X = \left[X_1^T, X_2^T, \cdots, X_n^T\right]^T \tag{1}$$

Part Deviation For a part P, the deviation of surface S_i is represented as:
$\Delta X_i = [\Delta n_{ix}, \Delta n_{iy}, \Delta n_{iz}, \Delta p_{ix}, \Delta p_{iy}, \Delta p_{iz}, \Delta d_{i1}, \Delta d_{i2}, \cdots, \Delta d_{im}]^T_{(6+m)\times 1}$. From eq.(1), the part deviation is defined as:

$$\boldsymbol{x} = \Delta X = \left[\Delta X_1^T, \Delta X_2^T, \cdots, \Delta X_n^T\right]^T \tag{2}$$

Observation of Part Deviation For part P with *p* critical dimensions(Y_i, $1 \le i \le p$), Y_i can be expressed as a function of X, that is, $Y_i = G_i(X)$. By using Taylor's series expansion, the deviation of critical dimension Y_i can be written as:

$$\Delta Y_i = C_i \Delta X + v_i \tag{3}$$

where $C_i = \left[\dfrac{dG_i}{dX^T}\right]_{1\times n(6+m)}$ and the high order term v_i is treated as noise term.

Then the part dimensional deviation $\Delta Y = \left[\Delta Y_1, \Delta Y_2, \cdots, \Delta Y_p\right]^T$ can be expressed as:

$$\Delta Y = C\Delta X + \boldsymbol{v} \tag{4}$$

where $\boldsymbol{v} = (v_1, v_2, ..., v_l)^T$ is the white noise vector and

$$C = \begin{bmatrix} -C_1- \\ -\vdots- \\ -C_p- \end{bmatrix}_{p\times n(6+m)} \tag{5}$$

which transforms the surface deviation to the dimensional deviation.

The nominal values of the Y are represented as Y^o, $Y^o = \left[Y_1^o, Y_2^o, \ldots, Y_p^o\right]$. Then from eq.(4), $Y = Y^o + C\Delta X + V$, where $Y = \left[Y_1, Y_2, \cdots, Y_p\right]^T$. If we let $\boldsymbol{y} = \Delta Y$, $\boldsymbol{y}(k)$ and $\boldsymbol{x}(k)$ to represent dimensional deviation and part deviation right after process operation *k*, eq. (4) can be rewritten as:

$$\boldsymbol{y}(k) = C(k)\boldsymbol{x}(k) + \boldsymbol{v}(k) \tag{6}$$

2.2 Deviation Propagation in Machining Processes

In last section, efforts have been devoted to describe the part deviation and observation within a given station. This section will model the deviation propagations between stations.

2.2.1 Coordinate System Definition

Two different coordinate systems are defined as follows:

- G-coordinate: G-coordinate is defined as the global machine coordinate system (x_G, y_G, z_G), in which the part is located and oriented.
- P-coordinate: P-coordinate is defined as the part coordinate system (x, y, z), in which the part is designed in CAD and the part dimensions are measured.

Let X be the part surfaces in P-coordinate and X_G be the part surfaces in G-coordinate. The relationship between the P-coordinate and G-coordinate can be expressed as

$$X_G = R_{PG} X + T \tag{7}$$

and

$$X = R_{GP}\left(X_G - T\right) \tag{8}$$

where the translation vector $T \in R^{n(6+m)\times 1}$ for X is defined as $T = \left[\underbrace{O_p, O_p, \ldots, O_p}_{n \text{ terms}}\right]^T$. O_p is defined as $O_p = \left[\underbrace{0,0,0,x_{op},y_{op},z_{op},\overbrace{0,\ldots,0}^{m \text{ terms}}}_{(6+m)\,terms}\right]$. The rotation matrix $R_{PG} \in R^{n(6+m)\times n(6+m)}$ is defined as

$$R_{PG} = diag(\underbrace{R,\cdots,R}_{n}) \tag{9}$$

where $R = diag(Rot(\alpha,\beta,\gamma)_{3\times3}, I_{3\times3}, I_{m\times m})$ and

$$Rot(\alpha,\beta,\gamma)_{3\times3} = \begin{bmatrix} c_\beta c_\gamma & -s_\gamma c_\alpha + s_\alpha s_\beta c_\gamma & s_\alpha s_\gamma + c_\alpha s_\beta c_\gamma \\ s_\gamma c_\beta & c_\alpha c_\gamma + s_\alpha s_\beta s_\gamma & c_\alpha s_\beta s_\gamma - s_\alpha c_\gamma \\ -s_\beta & s_\alpha c_\beta & c_\alpha c_\beta \end{bmatrix} \tag{10}$$

Here, c_α, s_α are abbreviations for $\cos\alpha$ and $\sin\alpha$ respectively, and similar for all other terms. $I_{3\times3}$ and $I_{m\times m}$ are identity matrices. R_{GP} is just the inverse of R_{PG}, that is, $R_{GP} = (R_{PG})^{-1}$. Specifically, *T(k)*, $R_{GP}(k)$ and $R_{PG}(k)$ are used to represent real translation vector, and rotation matrix at machining operation *k*.

2.2.2 Machining Operation and Setup Operation

Machining Operation A part can be produced by removing material from a workpiece through sequential machining operations. A mathematical expression for machining operation at station *k* can be described as follows:

$$X(k) = \left[I - B(k)\right]X(k-1) + B(k)X^u(k) \tag{11}$$

where $X^u(k)$ is the newly formed surface vector in the P-coordinate at station *k*.

$B(k)$ is a representation of process plan, defined as

$$B(k) = diag\left((I_1)_{(6+m)\times(6+m)}, \cdots (I_n)_{(6+m)\times(6+m)}\right) \tag{12}$$

where $(I_i)_{(6+m)(6+m)}$ ($1 \le i \le n$) is a *(6+m)×(6+m)* indicator matrix to the i^{th} surface S_i. I_i is defined as:

$$I_i = \begin{cases} I_{(6+m)\times(6+m)}, & \text{surface } S_i \text{ is formed in operation } k \\ 0, & \text{Otherwise} \end{cases} \tag{13}$$

where $I_{(6+m)\times(6+m)}$ is an identity matrix, $I_{(6+m)\times(6+m)} = diag(1,1,\cdots,1)_{(6+m)\times(6+m)}$.

B(k) labels all surfaces that are formed at station *k*. Similarly, *I-B(k)* indicates all other surfaces that are formed in previous stations. Matrix $A(k)$ is defined as:

$$A(k) = I - B(k) \tag{14}$$

Setup Operation Setup operation fixes a part within a fixture and puts them together onto the machine table before the machining operation. This operation can be divided into a translation operation *T(k)* and rotation operation $R_{PG}(k)$. Suppose *X(k-1)* is the part surface vector in P-coordinate right after machining operation *k-1*. By eq.(7), setup operation at station *k* is expressed as:

$$R_{PG}(k)X(k-1) + T(k) \tag{15}$$

2.2.3 Deviation Propagation Model

A deviation propagation model is used to analyze the error propagation in the machining process. Error sources at station *k*, $k = 1,2,\ldots N$, include:

1) Datum errors $e(d_k)$ and $v(d_k)$
2) Fixture failure e_k^f and v_k^f
3) Machining errors e_k^m and v_k^m
4) Noise $\boldsymbol{w}(k)$.

Setup Error Setup error e_k^s or v_k^s at station *k* is caused by datum and/or fixture errors, that is, $e(d_k) \oplus e_k^f \xrightarrow{k} e_k^s$ or $v(d_k) \oplus v_k^f \xrightarrow{k} v_k^s$. Setup error only contributes the deviation/variation of newly generated part surfaces $B(k)X^u(k)$.

Machining Error Machining error e_k^m or v_k^m causes deviation or variation of newly generated part surfaces in G-coordinate. e_k^m induced errors can be expressed as

$$B(k)\boldsymbol{x}_G^u = B(k)\left[X_G^u(k) - X_G^o(k)\right] \tag{16}$$

$$X_G^o(k) = R_{PG}^o(k)\ X^o(k) + T^o(k) \tag{17}$$

If there is no machining error, no matter setup errors exist or not, new part surfaces $B(k)X_G^u$ will be the ideal part surfaces vector in G-coordinate. For example, in Fig. 2, if setup error exits and there is no machining errors, surface S and S' have the same surface equation in G-coordinate. However, when it is transformed back to the P-coordinate, deviation caused by setup error will show up.

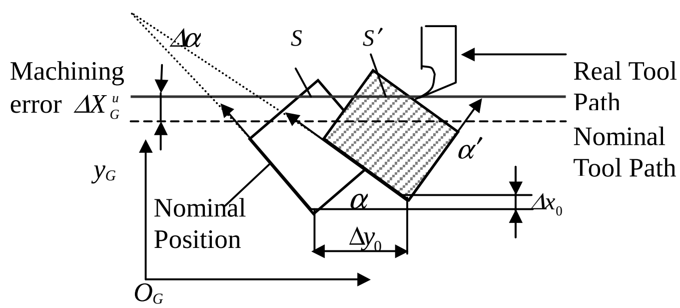


Fig.2 Error Sources Analysis for Machining Operation

Error of Station k Error of station k e_k (or v_k) is consisted of e_k^s (or v_k^s) and e_k^m (or v_k^m), that is, $e_k^s \oplus e_k^m \xrightarrow{k} e_k$ (or $v_k^s \oplus v_k^m \xrightarrow{k} v_k$).

If surface A is machined at station *k*, deviation $e(A)$ (or variation $v(A)$) is determined by e_k (or v_k) and $e(d_k)$ (or $v(d_k)$), that is, $e(d_k) \oplus e_k \xrightarrow{k} e(A)$ (or $v(d_k) \oplus v_k \xrightarrow{k} v(A)$).

Deviation Propagation Model

Theorem 1. Deviation propagation can be expressed as:

$$\boldsymbol{x}(k) = A(k)\boldsymbol{x}(k-1) + R_{GP}(k)B(k)\boldsymbol{x}_G^u(k) + \Delta R\ (k)X^o(k) - R_{GP}(k)B(k)\Delta T(k) \tag{18}$$

where rotation error $\Delta R(k) = R_{GP}(k)B(k)R^{o}_{PG}(k) - B(k)$ and translation error $\Delta T(k) = T(k) - T^{o}(k)$ (See the Appendix for proof).

This theorem indicates that part deviations at station k come from three components:

1. Deviation introduced from previous operations, $\boldsymbol{x}(k-1)$.
2. Setup error, which includes rotation error $\Delta R(k)$ and translation error $\Delta T(k)$.
3. Machining error of operation k, $B(k)\boldsymbol{x}^{u}_{G}$

From eq.(11) and eq.(14), we have $\boldsymbol{x}(k) = A(k)\boldsymbol{x}(k-1) + B(k)\boldsymbol{x}^{u}(k)$. By comparing with eq.(18), deviation vector $B(k)\boldsymbol{x}^{u}$ can be represented as:

$$B(k)\boldsymbol{x}^{u}(k) = R_{GP}(k)B(k)\boldsymbol{x}^{u}_{G}(k) + \Delta R(k)X^{o}(k) - R_{GP}(k)B(k)\Delta T(k) \quad (19)$$

As disturbance always exits in a machining process, a noise vector term $\boldsymbol{w}(k)$ is introduced into those equations. Then eq.(11) and eq.(18) are rewritten as:

$$\boldsymbol{x}(k) = A(k)\boldsymbol{x}(k-1) + B(k)\boldsymbol{x}^{u}(k) + \boldsymbol{w}(k) \quad (20)$$

Figure 3 shows how errors propagate between stations.

Now a state space model is developed to model the dimensional deviation propagation and observation for the machining process. The state equation and system observation equation are given by eq.(20) and eq.(6).

3. STREAM-OF-VARIATION DIAGNOSIS

The objective of SOV diagnosis is to develop methodology to systematically identify root causes of process variations based on part measurement. Fault diagnosis is the inverse process of fault propagation. This paper focuses on process level diagnosis: identify faulty station.

3.1 D-Graph and Process Diagnosability

The diagnostic graph(D-Graph) is proposed for the purpose of process diagnosability assessment, sensor placement strategy and other process analysis. D-Graph(Fig.4) can be generated from CAPP data. For an N station machining process, D-Graph is organized as $N+1$ levels. Level 0 has one node, denoting the datum used at the first station. All the nodes at the same level l, l=1…N, represent surfaces machined by station l. The blank node f_{ki} represents the ith surface machined by station k. Shaded node d_k represents datum used by station k. A directed arc, connecting d_k with f_{ki}, means that f_{ki} is machined at station k based on datum d_k. An undirected arc, connecting f_{ki} with $d_j, j \geq k+1$, means that surface f_{ki}, machined at station k, is to be used as the datum at station j. d_k is usually consisted of primary datum d_{k1}, secondary datum d_{k2} and tertiary datum d_{k3}, corresponding to the "3-2-1" locating scheme. d_{k1}, d_{k2} and d_{k3} are ordered from top to bottom in the D-Graph. Several surfaces could be used together as a datum. If d_k is connected by a directed arc, one surface in d_k is machined by station k-1. For instance, the primary datum of d_2 is equal to f_{13}. d_{k1}, d_{k2}, and $d_{k3}, k = 1,2,\cdots,N$, could be surfaces machined by station k-j, j=1,2, k-1, or surfaces on the raw material, like d_1.

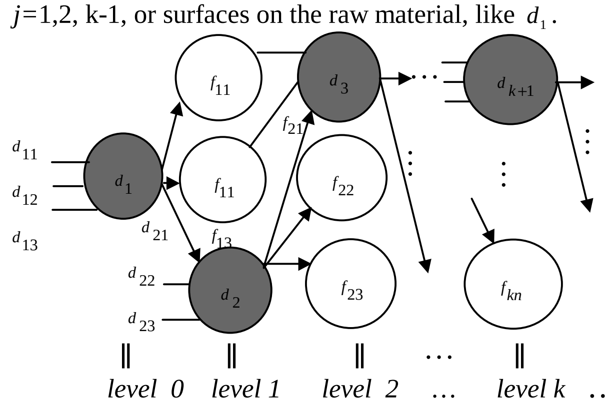


Fig.4 D-Graph

Before introducing the concept of diagnosability, a definition and a lemma are given below.

Definition 1. Equivalent surface If part surfaces A and B have the same fault manifestation in the same station, A and B are called equivalent, denoted as A⇔B.

Lemma 1. If A⇔B, then $e(A) = e(B)$ and $v(A) = v(B)$

Proof: Suppose at station k, $e(d_k) \oplus e_k \xrightarrow{k} e(A)$, $v(d_k) \oplus v_k \xrightarrow{k} v(A)$, and $e'(d_k) \oplus e'_k \xrightarrow{k} e(B)$, $v'(d_k) \oplus v'_k \xrightarrow{k} v(B)$. Suppose the datum is not changed, then $e(d_k) = e'(d_k)$, $v(d_k) = v'(d_k)$. From the definition of A⇔B, we have $e_k = e'_k$, $v_k = v'_k$. Thus, $e(A) = e(B)$ and $v(A) = v(B)$.

The concept of diagnosability is similar to that given by Danai and Chin(1991) and Hu(1997). At process level, the machining process is considered to be diagnosable if every faulty station can be uniquely identified.

Suppose we only take measurement at the end of machining process, the process is diagnosable under the following sufficient and necessary condition:

Theorem 2. The machining process is diagnosable if and only if for station k, $k = 1,2,\ldots,N$, $\exists i$, $i = 1,2,\cdots,n(k)$, $e(f_{ki})$ and $v(f_{ki})$ can be isolated into $e(d_k)$, $v(d_k)$, e_k and v_k, that is, the relation $e(d_k) \oplus e_k \xrightarrow{k} e(f_{ki})$ and $v(d_k) \oplus v_k \xrightarrow{k} v(f_{ki})$ could be reversed and has unique solution.

By the definition of diagnosability, this theorem can be easily proved by testing e_k and v_k. In this paper, we assume the

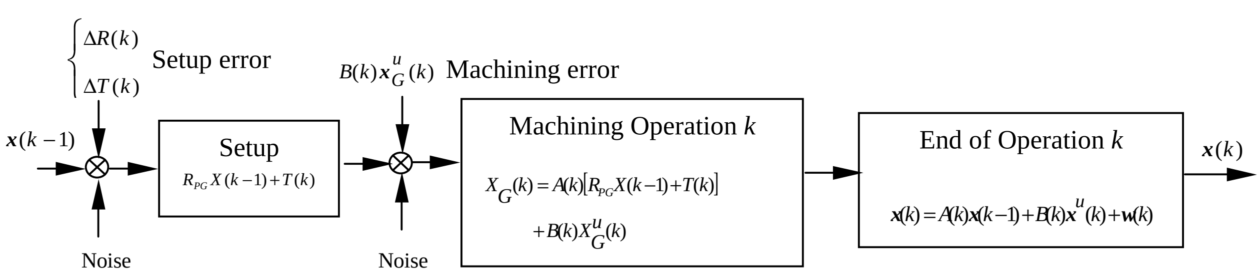


Fig.3 Error Propagation

process is diagnosable.

Lemma 2. If station k, $k=1,2,\ldots N$, $\exists i$, $i=1,2,\cdots,n(k)$, $f_{ki}\in \mathrm{M}_N$, and $d_k\in \mathrm{M}_N$, the process is diagnosable.

Proof: As $f_{ki}\in \mathrm{M}_N$, and $d_k\in \mathrm{M}_N$ for $k=1,2,\ldots N$, we have $e(d_k)$, $v(d_k)$, $e(f_{ki})$ and $v(f_{ki})$. By Theorem 1, we can get e_k and v_k. Thus, by Theorem 2, the process is diagnosable.

Lemma 3. (Datum loss) If $d_{kh}\xrightarrow{j} f_{jm}$, $d_{kh}\notin \mathrm{M}_N$, $d_{kh}=f_{(k-1)g}$, $\exists i,m,i\neq g$, $f_{(k-1)i}, f_{km}\in \mathrm{M}_N$ and $f_{(k-1)i}\Leftrightarrow d_{kh}$, $e(d_1)$ and $v(d_1)$ are negligible(or available from acceptance sampling), $1\le i,g\le n(k-1)$, $1\le m\le n(k)$, $k=2,\ldots N$, $1\le h\le 3$, $k<j\le N$, the process is diagnosable.

Proof: As $f_{(k-1)i}\Leftrightarrow d_{kh}$ and $f_{(k-1)i}\in \mathrm{M}_N$, by lemma 1, $e(d_{kh})=e(f_{(k-1)i})$ and $v(d_{kh})=v(f_{(k-1)i})$. As $f_{km}\in \mathrm{M}_N$, $e(f_{km})$ and $v(f_{km})$ are also known. By Theorem 1, we can get e_k and v_k. By Theorem 2, the process is diagnosable.

Lemma 4. (Rough cutting and finish cutting) If $f_{kg}\xrightarrow{j} f_{jm}$, $d_k\in \mathrm{M}_N$, $\exists i,i\neq g$, $f_{ki}\in \mathrm{M}_N$ and $f_{ki}\Leftrightarrow f_{kg}$, $e(d_1)$ and $v(d_1)$ are negligible(or available from acceptance sampling), $1\le i,g\le n(k)$, $k=1,\ldots N$, $1\le h\le 3$, $k<j\le N$, the process is diagnosable.

Proof: as $f_{ki}\Leftrightarrow f_{kg}$, and $f_{ki}\in \mathrm{M}_N$, by lemma 1, $e(f_{kg})=e(f_{ki})$ and $v(f_{kg})=v(f_{ki})$.

Lemma 5. If the situations of datum loss and finish cutting happen at the same time, and lemma 3 and lemma 4's conditions are all hold, the process is still diagnosable.

The proof of lemma 5 is direct extension of lemma 3 and lemma 4.

3.2 Off-line Machining Deviation Pattern Modeling

Datum change makes the operations correlated. The machined surfaces are classified into two types. Type I surface set(denoted by SI) uses perfect surfaces as datum. As variations of SI are only caused by current stations(assume surfaces are machined only once), it could easily be identified by measurement. Type II surface set(denoted by SII) uses previously machined surfaces as datum. Deviation or variations of SII are jointly affected by faults in current and previous stations.

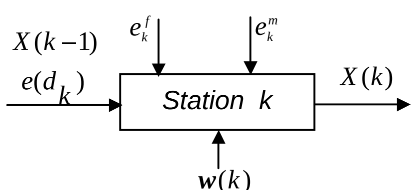


Fig.5 Errors at Real Station k

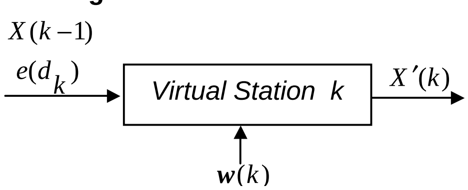


Fig.6 Errors at Virtual Station k

At station k, error sources, analyzed in Section 2.2.3, is graphically represented in Fig.5. To model the deviation pattern, fault isolation should be performed to distinguish transferred faults and newly generated faults at station k. In order to isolate faults, a virtual station k is proposed, shown in Fig.6.

Definition 2. ***Virtual Station k and virtual machining*** In virtual station k, there is no e_k^f and e_k^m. Virtual machining is performed by simulation based on SOV model.

The presentation of the diagnosis methodology follows three major steps:

(1) At real station k,
$e(d_k)\oplus\left(e_k^s\oplus e_k^m\right)\oplus e(\boldsymbol{w}(k))\xrightarrow{k} e\left(B(k)X(k)\right)$ and
$v(d_k)\oplus\left(v_k^s\oplus v_k^m\right)\oplus v(\boldsymbol{w}(k))\xrightarrow{k} v\left(B(k)X(k)\right)$.

Theorem 1. (eq.(19) and eq.(20)) is the mathematical expression of the idea above. $e\left(X(N)\right)$ and $v\left(X(N)\right)$ can be obtained by measurement.

(2) At virtual station k, $e(d_k)\oplus e(\boldsymbol{w}(k))\xrightarrow{k} e\left(B(k)X(k)\right)$ and $v(d_k)\oplus v(\boldsymbol{w}(k))\xrightarrow{k} v\left(B(k)X(k)\right)$.

In this case, eq.(20) turns to be

$$\boldsymbol{x}'(k)=A(k)\boldsymbol{x}(k-1)+B(k)\boldsymbol{x}^u(k)+\boldsymbol{w}(k) \tag{21}$$

where $B(k)\boldsymbol{x}^u(k)$ is simplified as

$$B(k)\boldsymbol{x}^u(k)=\Delta R\ (k)X^o(k)-R_{GP}(k)B(k)\Delta T(k) \tag{22}$$

Rotation error $\Delta R(k)$ and translation error $\Delta T(k)$ are only caused by $e(d_k)$, that is, $e(d_k)\xrightarrow{k} e_k^s$.

(3) (Deviation Isolation) $\left(e_k^s\oplus e_k^m\right)\xrightarrow{k} e\left(\boldsymbol{x}'(k)\right)$, where $e\left(\boldsymbol{x}'(k)\right)$ is defined as:

$$e\left(\boldsymbol{x}'(k)\right)=\boldsymbol{x}(k)-\boldsymbol{x}'(k) \tag{23}$$

From eq.(27), mean and covariance at virtual station k, denoted by $\hat{\boldsymbol{\mu}}_k$ and $\hat{\boldsymbol{K}}_k$, can be expressed as:

$$\begin{aligned}\hat{\boldsymbol{\mu}}_k&=E(\mathbf{x}'(k))\\ \hat{\mathbf{K}}_k&=Cov(\mathbf{x}'(k))\end{aligned} \tag{24}$$

The above procedure can be repeated for stations N, N-1,…,2,1. As a result, fault isolation between stations can be achieved. Fig.7 shows the concept of fault isolation between stations.

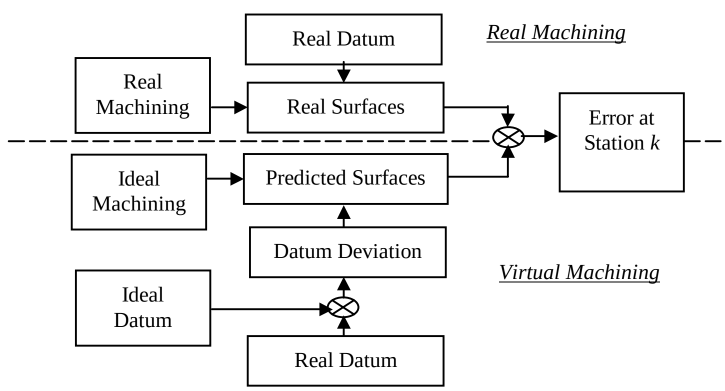


Fig. 7 Fault Isolation Between Stations

3.3 Unknown Error Pattern Estimation

Based on measurement, the mean and covariance of part surfaces at the end of the machining process are $\boldsymbol{\mu}_N=E[\boldsymbol{x}(N)]$ and $\boldsymbol{K}_N=E[(\boldsymbol{x}(N)-\boldsymbol{\mu}_N)(\boldsymbol{x}(N)-\boldsymbol{\mu}_N)^T]$. Since we assume the process is diagnosable, the mean and covariance at real machine station k can be expressed as:

$$\mathbf{x}(k)=\mathbf{x}(N)-\sum_{i=k+1}^{N}B(i)\mathbf{x}(N)$$
$$\boldsymbol{\mu}_k = E[\mathbf{x}(k)] \qquad (25)$$
$$\mathbf{K}_k = E[(\mathbf{x}(k)-\boldsymbol{\mu}_k)(\mathbf{x}(k)-\boldsymbol{\mu}_k)^T]$$

3.4 Patten Mapping and Root Cause Determination

Hypothesis testing can be made based on eq.(24) and eq.(25). Given a confidence interval, the hypothesis is as follows:

$$H_0 : \boldsymbol{\mu}_k = \hat{\boldsymbol{\mu}}_k \ \ and\ \ \boldsymbol{K}_k = \hat{\boldsymbol{K}}_k, \quad \text{no fault in station } k;$$
$$H_1 : \boldsymbol{\mu}_k \neq \hat{\boldsymbol{\mu}}_k, \quad \text{mean shift in station } k; \qquad (26)$$
$$H_2 : \boldsymbol{K}_k \neq \hat{\boldsymbol{K}}_k, \quad \text{covariance change in station } k$$

4. CASE STUDY

A case study is presented to illustrate the proposed SOV model and diagnostic methodology (Refer Fig.1 for the procedure of the case study).

A part and its specification are shown in Fig.8. Only dimensional problems are considered in this case. Planes A, B, C and D are design datum, which are also used as machining datum. Surfaces E~H are denoted for the purpose of clear description. The process plan for this part is shown in Table 1. There are four machining operations and three setups in three stations. Setup data include part orientation information on the machine table before machining operation and locating datum for the current machining operation. Fig.9 is a graphical showing of the machining process.

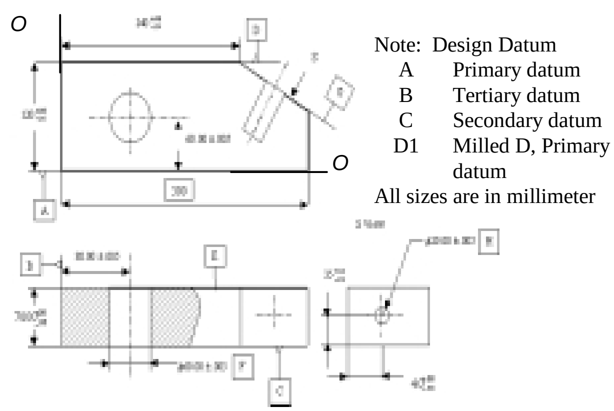


Fig.8 Part and Specification

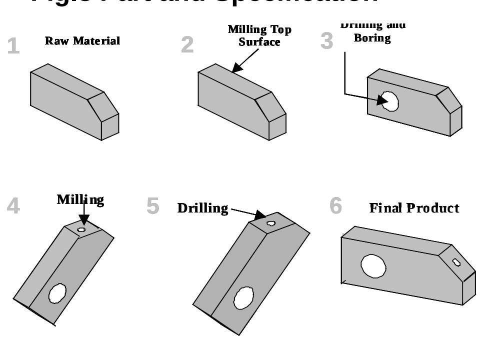
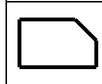
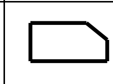
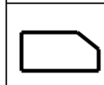
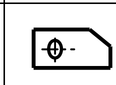
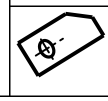
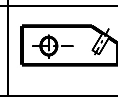


Fig.9 Machining Process

Table 1 Process Sheet

Operation			Station #	Operation Description	Locating Datum
Part Orientation	Before	After			
			1	Mill Face D1	A+C+B
			2	Drill hole F through	A+C+B
			3	a) Mill Face G b) Drill hole H	D1+C+B

4.1 Deviation Propagation Simulation

Based on eq.(20) and eq.(6), deviation propagation in the machining process is simulated. A batch of 100 parts is machined.

- Part and part deviation representation

According to the guideline in Section 2.1.1, eight surfaces are chosen. X is shown in Table 2. As only diameter of cylinder is considered as surface size, surface vector contains seven elements. The data shown in Table 2 are nominal value for operation 4, that is, $X = X^o(4)$. Before machining operation 1, components of surfaces D,F,G and H either are set to be the same as the workpiece(like D and G) or as zero (like F and H).

Table 2 Part Representation, *X*

	A	B	C	D	E	F	G	H
n_x	0	-1	0	0	0	0	0.707	0.707
n_y	0	0	-1	0	1	1	0	0
n_z	-1	0	0	1	0	0	0.707	0.707
p_x	0	0	0	0	0	80	240	268.28
p_y	0	0	0	0	70	0	0	35
p_z	0	0	0	120	0	60	120	91.72
d_1	0	0	0	0	0	60	0	20

The part deviation after station 4 is shown in Table 3.

Table 3 Part Deviation, *ΔX*

	A	B	C	D	E	F	G	H
n_x	0	0	0	0.0031	0	0.001	0.0001	0.0024
n_y	0	0	0	0.0011	0	0.0004	0.0012	-0.0004
n_z	0	0	0	0.0012	0	0.0012	0.0003	-0.0003
p_x	0	0	0	0.8732	0	0.4528	-0.126	-0.8828
p_y	0	0	0	0.9924	0	0.6830	0.057	0.1459
p_z	0	0	0	1.0488	0	1.7560	-1.367	0.432
d_1	0	0	0	0	0	0.4668	0	1.7304

- Coordinator transformation, machining operation and fault generation

The relationship between part coordinator and machine coordinator is given by eq.(7) to eq.(8). The process of generating surfaces is modeled by eq.(20).

Assume that machining errors, fixture failure and noise follow normal distribution with zero mean. The magnitude ranges of variances are: 1) noise, 0.0001~0.001; 2) error, 0.01~0.1; 3) fault, 0.5~1.

Part deviation, setup errors and machining errors are also randomly generated. Table 4 shows the nominal setup data at station 3 (setup 3), the corresponding setup error, nominal machining operation 3 (milling surface G) data and machining error.

Table 4 Process sheet for setup 3 and machining operation 3

Setup	Setup data	Setup error		Mach.	Mach. data	Mach. error
α	0	0.024		n_x	0	-0.001
β	$-\pi/4$	-0.004		n_y	0	0.002
γ	0	-0.003		n_z	1	0.0012
p_x	100	-0.88		p_x	184.85	0
p_y	60	0.1459		p_y	60	0
p_z	80	0.432		p_z	334.56	1.432

- Part observation and monitoring

Size 120, 240, 60, 20, 35 and 40 are key dimensions that should be monitored. To simplify the case study, individual control chart is used, instead of multivariate control chart required by eq.(26). The individual control charts for size 120 and 240 are shown in Fig.10($\bar{x}$ chart, $\alpha = 0.0027$, sample size n=5). Both charts show the processes are out-of-control. However, because plane D1 is used as the primary datum to mill plane G, the challenge is to determine if the out-of-control signal at station 3 is only caused by datum error, or jointly affected by datum error and machining, setup errors at station 3?

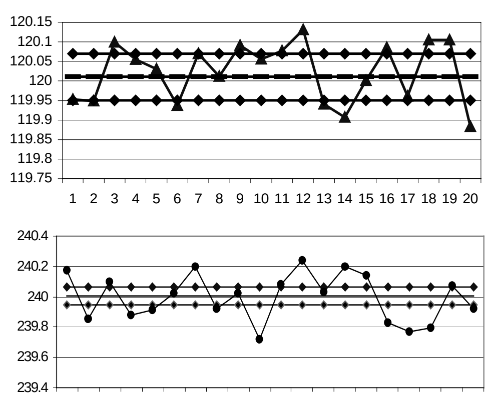


Fig.10 *x*_Bar Chart for Size 120 and 240

In section 4.2, SOV diagnosis model is used to isolate faults between stations.

4.2 Stream-of-Variation Diagnosis: Fault Isolation Between Stations

- Two type surface classification

SI surface set includes D1 and F. SII surface set includes G and H

- Measurement and identification of SI surface deviation

Deviation of the SI surfaces D1 and F can be directly obtained from measurement.

- SII surface deviation isolation between stations

Methodology developed in section 3 is used. Surface G′, which contains no machining error and fixture error, can be predicted by substituting the deviation data of D1 into eq.(21). By comparing measurement data of G and predicted surface G′, deviation caused within station 3 can be isolated.

- Deviation monitoring and root cause determination

After isolating the deviation within station 3, we assume that the datum D is perfect and substitute the isolated deviation within station 3 into eq.(20) to predict surface G. By using the predicated surface G, which contains errors in station 3, we observe part dimension 240 and monitor the process. The control chart(Fig.11) shows that station 3 is in control and the out-of-control signal in Fig.10 is caused by station 1.

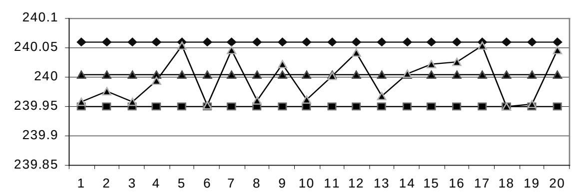


Fig.11 x_Bar Chart Based on Predicted Surface G

5. SUMMERY AND CONCLUSION

The presented Stream of Variation modeling and diagnostic methodology uses a state space model to describe the variation propagation in machining process. Virtual station and machining is applied for station level diagnosis. Further research on fixture level diagnosis should be done. The proposed SOV model and results provide a new way for dimensional control in multi-station machining processes. This research lays the foundation for implementing advanced statistics and control theory in process design, monitoring, and diagnosis for machining processes.

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APPENDIX

Theorem 1 *proof*: At station *k*, the following operations are taken in sequence:

1) Setup operation Expressed in eq.(15).
2) Machining operation During machining, new part surfaces vector $X_G(k)$ in G-coordinate turns to be:

$$X_G(k) = A(k)\left[R_{PG}(k)X(k-1) + T(k)\right] + B(k)X_G^u(k)$$

3) Finishing operation and part $X(k)$ in P-coordinate New part $X(k)$ in P-coordinate is:

$$\begin{aligned} X(k) &= R_{GP}(k)\left[X_G(k) - T(k)\right] \\ &= R_{GP}(k)\left\{A(k)\left[R_{PG}(k)X(k-1) + T(k)\right] + B(k)X_G^u(k)\right\} - R_{GP}(k)T(k) \\ &= R_{GP}(k)A(k)R_{PG}(k)X(k-1) + R_{GP}(k)B(k)X_G^u(k) + \\ &\quad R_{GP}(k)A(k)T(k) - R_{GP}(k)\left[A(k) + B(k)\right]T(k) \\ &= R_{GP}(k)A(k)R_{PG}(k)X(k-1) + R_{GP}(k)B(k)\left[X_G^u(k) - T(k)\right] \end{aligned}$$

As $R_{GP}(k)R_{PG}(k) = I$, $R_{GP}(k)A(k)R_{PG}(k) = A(k)$, *X(k)* turns to be

$$X(k) = A(k)X(k-1) + R_{GP}(k)B(k)\left[X_G^u(k) - T(k)\right]$$

The part deviation at station k is

$$\begin{aligned} \boldsymbol{x}(k) &= \Delta X(k) = X(k) - X^o(k) \\ &= A(k)X(k-1) + R_{GP}(k)B(k)\left[X_G^u(k) - T(k)\right] - X^o(k) \\ &= A(k)X(k-1) + R_{GP}(k)B(k)\left[X_G^u(k) - X_G^o(k) + X_G^o(k) - T(k)\right] - \left[A(k) + B(k)\right]X^o(k) \end{aligned}$$

As operation k does not change the part surfaces $\left[I - B(k)\right]X(k-1)$, we have

$$\left[I - B(k)\right]X^o(k) = \left[I - B(k)\right]X^o(k-1))$$

Then

$$\begin{aligned} \boldsymbol{x}(k) &= A(k)\boldsymbol{x}(k-1) + R_{GP}(k)B(k)\left[X_G^u(k) - X_G^o(k)\right] + \\ &\quad R_{GP}(k)B(k)X_G^o(k) - R_{GP}(k)B(k)T(k) - B(k)X^o(k) \end{aligned}$$

As $X_G^o(k) = R_{PG}^o(k)X^o(k) + T^o(k)$, $\Delta X_G^u(k) = X_G^u(k) - X_G^o(k)$, $\boldsymbol{x}(k)$ can be rewritten as:

$$\begin{aligned} \boldsymbol{x}(k) &= A(k)\boldsymbol{x}(k-1) + R_{GP}(k)B(k)\Delta X_G^u(k) + R_{GP}(k)B(k)\left[R_{PG}^o(k)X^o(k) + T^o(k)\right] - \\ &\quad R_{GP}(k)B(k)T(k) - B(k)X^o(k) \\ &= A(k)\boldsymbol{x}(k-1) + R_{GP}(k)B(k)\Delta X_G^u(k) + \left[R_{GP}(k)B(k)R_{PG}^o(k) - B(k)\right]X^o(k) - \\ &\quad R_{GP}(k)B(k)\left[T(k) - T^o(k)\right] \end{aligned}$$

If we define rotation error $\Delta R(k)$ as $\Delta R(k) = R_{GP}(k)B(k)R_{PG}^o(k) - B(k)$ and translation error $\Delta T(k)$ as $\Delta T(k) =$