Appetizer to Math 149S

Warm-up problems and information about the class

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Department of Mathematics
Duke University

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Warm-up through Game Playing

Game 1
Two players iteratively pick numbers from the set \{1, 2, 3, 4, 5, 6, 7, 8, 9\}. (Player 1 goes first, then player 2, then player 1 again, ...) A number can be picked at most once by any player. A player wins if out of all the numbers he/she has picked, 3 distinct numbers add up to 15. How would you play this game to maximize your wins?
Warm-up through Game Playing

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**Solution:**

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6 1 8
7 5 3
2 9 4
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Tic-tac-toe!
Warm-up through Game Playing

Game 2
There is a chocolate bar composed of $12 \times 8$ squares of chocolate. Two players take turns picking up a piece of chocolate and cutting it along the grid lines. (A player may choose to cut through and creating a new piece, or not cut through. The cuts can bend, but they must follow grid lines and must stop at a vertex of some square. One has to cut at least one link each turn.) A player wins by making the last cut to break the chocolate into 96 squares. Do you want to be player 1 or 2? How would you play this game?

Solution:
Player 1 can always win by first making a cut in the middle to create two $6 \times 8$ pieces, hence creating two symmetric "sides." For the following turns, whatever player 2 does to one side (breaking the symmetry), player 1 mirrors on the other side. Because the final configuration is symmetric, player 2 cannot win.
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Warm-up through Game Playing

Game 3 (Exercise)

There is a chocolate bar composed of \( m \times n \) squares of chocolate. (\( mn > 1 \)). Two players iteratively choose some rectangle starting from the top right corner, and remove all squares of chocolate in that rectangle. (The rectangle must be of non-zero size and must follow the grids.) The player that takes the last pieces loses. Who can always guarantee a win? Why?
What is Common?

- Mathematical puzzles
- Involves creativity
- Perhaps require a clever idea
- Develops mathematical problem solving/proof making skills.
- Fun to solve

This is what Math 149S is about!

(Although problems in Math 149S may be harder and require more mathematical theory.) We have a range of difficulty so you can learn something regardless of your current skill level (See Problem set 0.)

Goals of the course:
1. To improve your problem solving skills
2. To sharpen your proof-writing skills
3. To prepare you for the Putnam competition
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The Putnam Mathematics Competition

The Putnam Mathematics Competition is

- the premier math contest for college students.
- 6 hours 12 problems
- significant monetary prizes
- highly regarded by graduate schools in math, sciences and engineering
- highly regarded by hedge funds, and other math-literate employers

We need your help! (Traditionally Putnam winnings fund our free pizza.)

Peng Shi, Duke University Appetizer to Math 149S, Warm-up problems and information about the class 6/10
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- Meets once a week for 75 min (some weekday evening)
- Each class: 30 min powerpoint lecture, 30 min problem solving, 15 min student presentations, FREE PIZZA!
- Grade based on solving weekly problem sets: typically 3 well-written solutions a week = A
- Half-credit. Satisfies QS, S.
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Topics:

- pigeonhole principle
- invariance/extremal arguments
- induction
- recurrences
- counting
- probability
- polynomials
- number theory
- calculus
- linear algebra
- other tricks
About Math 149S

Student Instructors:

Peng Shi (Senior Math and Comp sci major)

Math Rognlie (Senior Math and Econ major)

Any questions?

If you are interested in taking the class, speak to us after and we will give you permission numbers. If you can fully solve 3-4 problems on Problem Set 0, you are in a position to, with some work, get A in our class.
Solution to Game 3

Game 3 (Exercise)

There is a chocolate bar composed of $m \times n$ squares of chocolate. ($mn > 1$). Two players iteratively choose some rectangle starting from the top right corner, and remove all squares of chocolate in that rectangle. (The rectangle must be of non-zero size and must follow the grids.) The player that takes the last pieces loses. Who can always guarantee a win? Why?

Solution:
Player 1 can always win by the following argument: Because this is a finite game with no ties, exactly one of the player must have a winning strategy. Suppose player 1 removes the top-right square, and player 2 force a win by removing squares in some rectangle $A$. Then player 1 can, instead of removing the top-right square, removed $A$ in his/her first turn, thus stealing the opponent’s winning strategy. Hence, player 2 cannot have a winning strategy. Therefore player 1 can always win! (although we are not sure how without more extensive calculations.)