The Semantics of Comparatives and Other Degree Constructions

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Abstract

(1) It is much more important than you think it is.

(1) is an example of an adjectival comparative. In it, the adjective important is flanked by more and a comparative clause headed by than. This article is a survey of recent ideas about the interpretation of comparatives, including (i) the underlying semantics based on the idea of a threshold; (ii) the interpretation of comparative clauses that include quantifiers (brighter than on many other days); (iii) remarks on differentials such as much in (1) above: what they do in the comparative and what they do elsewhere in the language; (iv) the relationship between comparatives and other Degree constructions (e.g. as important, too important); and (v) the types of phrases in which comparatives are found (adjective: tighter versus noun: more water). Given the nature and purpose of this essay, I have tried not to presuppose background in formal semantics and I have departed from standard practice in journal articles by, as much as possible, not interrupting the flow with footnotes and references. There are two appendices. The first provides more analytical detail and there I do rely on formal techniques of natural language semantics. The second covers the sources for the ideas surveyed here.

Thresholds

During a hearing test, an audiologist produces a tone at varying intensities in an attempt to find the lowest level at which the sound is detected by the listener. This is known as the absolute threshold of hearing. To say that a given sound is ‘audible to Bob’ is to say that it meets or exceeds Bob’s absolute threshold. For a given threshold, there can be many sounds that meet or exceed it, and if one sound meets or exceeds a threshold, a more intense one surely will. Conversely, a particular sound may meet or exceed many different thresholds. For this reason, one sound could be audible to both Bob and Carol even if they have different absolute thresholds. Thresholds can be ordered in such a way that if a sound meets or exceeds one of them, it meets or exceeds all lower ones. Once ordered, a method can be devised for assigning numerical values to thresholds that
reflects the ordering. This is how the decibel scale starts to come about. This way of thinking of *audible* can be extended to other gradable adjectives. Objects or events that are *painful* meet or exceed thresholds for pain. If an item is *expensive* for a wealthy man like Bill Gates, it is surely expensive for me. Speaking in terms of thresholds, Gates’s absolute threshold of expensiveness is higher than mine, and, so, if an item meets or exceeds his threshold, it surely meets or exceeds mine. For an adjective like *hot*, we may talk about temperature thresholds, for *high* height thresholds, for *heavy* weight thresholds and so on.

Comparatives

A-NOT-A ANALYSIS

Suppose we have two objects with monetary value, A and B, and some monetary threshold. Suppose that A meets or exceeds the threshold and B does not. It follows that A is more expensive than B is. Conversely, if it is true that A is more expensive than B, it follows that there must be some monetary threshold that A meets or exceeds that B does not. This type of reasoning leads to a threshold analysis of the comparative according to which the meaning of (2) is captured by (3):

(2) A is more expensive than B is.

(3) There is some expense-threshold: A meets or exceeds it and B does not meet or exceed it.

It will be useful to abbreviate (3) as in (4):

(4) $\exists \theta \text{expensive}(a, \theta) \land \neg \text{expensive}(b, \theta)$

$\theta$ is a threshold variable. ‘expensive($a, \theta$)’ should be read as ‘$a$ meets or exceeds the expense threshold $\theta$’. The other symbols are read: $\exists$, ‘there is some’; $\land$, ‘and’; and $\neg$, ‘it is not the case that’. As the adjective appears twice in (4), once with a negation, I will call this the A-not-A analysis.

The A-not-A analysis subsumes an analysis of gradable adjectives according to which each adjective denotes a relation between entities and thresholds. Whenever an entity meets or exceeds a relevant threshold, the relation denoted by the adjective holds of the pair consisting of the entity and the threshold. Earlier we said that a sound is ‘audible to Bob’ if it meets or exceeds Bob’s absolute threshold. The modifier to *Bob* signifies the choice of a particular threshold, but it is the adjective *audible* that tells us what kind of threshold it is and that the sound meets or exceeds the given threshold.
DEADJECTIVAL NOMINALIZATIONS AND THE MOTIVATION FOR THRESHOLDS

A deadjectival nominalization is a noun that is derived from an adjective. Examples include height (high), width (wide), length (long), and severity (severe). Nominalizations are of interest here because of expressions such as the height of the building or the length of the tadpole in which we appear to refer to the kinds of things we have been calling thresholds. However, the use of the definite article the shows that, unlike the corresponding adjectives, these nouns are functional, that is, they relate each individual to a unique threshold and it is that threshold that we refer to. But, if the adjective from which the noun is derived relates an individual to multiple thresholds, all the ones the individual meets or exceeds, then which one does the noun pick out? The following intuitions can help us answer this question. A longer tadpole meets or exceeds more and higher thresholds than a shorter one. To know the length of the tadpole is to know about the highest long threshold that it meets or exceeds and so that is the one the length of the tadpole refers to. Nominalizing long results in a narrowing of the relation between individuals and thresholds so that it becomes a function assigning to each entity its highest threshold. Equivalently, an extended object meets or exceeds many long thresholds, one of which is a threshold the object meets. We call that one its length.

Now if some threshold is met or exceeded by A but not by B, then the highest threshold (of the relevant kind) that is met or exceeded by A is going to exceed the highest threshold met or exceeded by B. Conversely, if A’s highest threshold exceeds B’s, then there is some threshold that A meets that B falls short of. It, therefore, follows on the proposed analyses of the comparative and of nominalization that saying:

(5) a. A is longer than B is.
   b. A is higher than B is.

is equivalent to saying:

(6) a. The length of A exceeds the length of B.
   b. The height of A exceeds the height of B.

This equivalence is the basis for a different analysis of comparatives according to which in (5a) the material following than contributes one length, the length of B, the main clause contributes another length, the length of A, and the comparative marker (er) indicates a greater-than relation. This is a viable alternative for examples like (5). But it breaks down on examples like (7) and (8) below for which there is no particular height associated with the comparative than clause:

(7) The balloon is higher today than it has been on any other day.
(8) The balloon is higher today than it was on at least one other day.
(7) would be true in the likely event that the balloon’s height differed on different past days, as long as its current height exceeds all the other heights achieved. (8) would be true in a scenario where there are many heights the balloon achieved on other days, as long as its current height exceeds one of them. In neither case is there a particular height associated with the clause following than as the alternative analysis requires.

According to the A-not-A analysis, (7) and (8) both say that there is some threshold \( \theta \) that the balloon meets or exceeds today, and (7) further says:

(9) The balloon did not meet or exceed \( \theta \) on any other day.

while (8) says:

(10) There was at least one past day on which the balloon did not meet or exceed \( \theta \).

This pair of examples reveals an important new fact about comparatives that can be appreciated by studying the position of the not in (9) and (10). It turns out that quantifiers within a than clause are sometimes included in the scope of the negation and sometimes not. In (9), any other day follows and is in the scope of the negation. In (10), at least one day is outside the scope of the negation. In general, all quantificational noun phrases and adverbs that are felicitous in a comparative clause take scope outside the negation unless they are negative polarity items such as any, ever, or in weeks.

Besides negative polarity items, there are also modal verbs that are interpreted inside the scope of the negation. To say that:

(11) The balloon is higher than it is allowed to be.

is to say that it meets or exceeds a threshold that it is NOT allowed to meet or exceed – as opposed to a threshold it is allowed to not meet or exceed. Suppose the balloon is required to be above some minimum height but below some maximum height. Thresholds between the minimum and the maximum are thresholds that the balloon is allowed to NOT meet or exceed. Thresholds above the maximum are ones that it is NOT allowed to meet or exceed. It is these above-the-maximum thresholds that (11) reports on. Curiously, although should and be allowed to have different meanings, (11) is roughly synonymous with (12):

(12) The balloon is higher than it should be.

This is because while allowed to takes scope inside negation, should takes scope outside it. (12) says that the balloon meets or exceeds a threshold that it should NOT meet or exceed and thresholds above the maximum are
precisely the thresholds that the balloon should not meet or exceed. *Had to* has a meaning roughly like *should*, if not a bit stronger (*you have to leave* versus *you should leave*), however, *had to* scopes inside negation and, so, (13) below ends up being weaker than (12):

(13) The balloon was higher than it had to be.

Thresholds above the minimum are thresholds that the balloon does not have to meet or exceed and it is these above-the-minimum thresholds that (13) reports on.

*Supposed to* is another modal and it works like *should*. The behavior of these modals with respect to the negation is a reflection of a general pattern that can be seen outside the comparative clause as well. Using \( \approx \) to mean ‘roughly synonymous with’, we can indicate the pattern for *allowed* and *supposed to* as follows:

(14) ‘allowed to X’ \( \approx \) ‘it is permitted to X’

‘not allowed to X’ \( \approx \) it is **not** permitted to X’

‘supposed to X’ \( \approx \) ‘it is required to X’

‘not supposed to X’ \( \approx \) it is required to **not** X’

Just as in the comparative clause, *allowed to* scopes inside the negation while *supposed* scopes outside it. One can go through all the modals discussed here, as well as *might*, *can*, and others and check that the pattern holds up.

Comparative clauses can include nonmodal verbs as well, as in (15) with *said*.

(15) The balloon was higher than Jack said it was.

If (15) is accurate, then Jack would have to have actually said something about the height of the balloon. This means that negation has to be below *say*, in other words, (15) is about a threshold that Jack said the balloon would not meet or exceed as opposed to a threshold that he did not say anything about. *Promise* works the same way in (16) below, intended as a complaint to a doctor after an operation:

(16) a. The scar is bigger than you promised it would be.

b. The scar meets or exceeds a size threshold that you **promised** it would not meet.

For many verbs, it is difficult to say whether the negation takes scope above or below them. This is because, for many verbs, the two options are hard to keep apart. For example, *I don’t want to leave* often falls together with *I want to not leave*. 
Summarizing now, in the comparative, two threshold statements are joined, one positive and the other negative. The negative statement may include a quantifier, modal, or sentence-taking verb. Depending on the nature of the quantifier, modal, or verb, it may scope inside or outside the negation.

Below we will discuss comparatives like 12 inches higher than. In such cases, the negation operator in our analysis has to be replaced with a scope-taking operator that captures the negation but that also takes account of the modifier 12 inches. For comparatives without such modifiers, the negation analysis is generally correct except in the special case when the complement clause includes a quantifier with exactly or only as in (17):

(17) a. Balloon A is higher than exactly two of the others are.
    b. ∃θ high(a,θ) ∧ exactly two other balloons are y such that:
       ¬ high(y,θ)

The statement in (17b) is too weak to capture the meaning of (17a). It is not easy to see this at first, but consider a situation in which, in fact, balloon A is higher than 20 other balloons and think about the bottom two. (17b) is true in that scenario but (17a) is not. What is needed is to further require that the threshold in question be the one that balloon A actually meets, in other words A’s height.

Other Degree Constructions

The comparative is often referred to as a Degree construction. The affix -er and the word more are Degree heads, they select a complement headed by than and, as in the examples discussed so far, they may appear adjacent to an adjective. There are a variety of Degree constructions that differ from the comparative in terms of the Degree head and the form of the complement. In the following sample, Degree heads are shown in small capitals and I include commonly used labels for some of these constructions:

(18) A is as high now as it was last night. EQUATIVE
(19) A is too high to reach. EXCESSIVE
(20) A is the most expensive one. SUPERLATIVE
(21) A is high enough to see.
(22) A is so expensive that even Bill can’t afford it.
(23) A is less taut than B is.

The comparative introduces a threshold that the subject of the adjective meets or exceeds and the complement is a negative statement elaborating
on that threshold. Many, if not all, of the constructions illustrated in (18)–(23) can similarly be understood as introducing a threshold that the subject meets or exceeds, but they differ from the comparative in how they elaborate on the threshold. We will take a closer look at the first two constructions in order to gain an appreciation of the tools these elaborations make use of.

**EQUATIVE: AS**

The label ‘equative’ used to describe (24) below suggests that it has the meaning of (25):

(24) A is as tall as B is.

(25) \( \exists \theta \) tall(a, \( \theta \)) \& tall(b, \( \theta \))

But this is a mistake. For any two people, there is some height threshold that they both meet or exceed, regardless of how tall each of them is. What (24) requires is that the threshold that A is said to meet or exceed is the highest one that B meets or exceeds, the threshold that B in fact meets. Any of the formulations in (27a), (27b), or (27c) captures this meaning.

(26) A is as tall as B is.

(27) a. \( \exists \theta \) tall(a, \( \theta \)) \& \( \theta \) is the highest threshold that B meets or exceeds.
   b. \( \exists \theta \) tall(a, \( \theta \)) \& \( \theta \) is the threshold that B meets.
   c. \( \exists \theta \) tall(a, \( \theta \)) \& \( \theta \) is B’s height.

The content in (27) is captured in (28), using a new operator max\(_\theta\):

(28) \( \exists \theta \) tall(a, \( \theta \)) \& (max\(_\theta\) tall(b, \( \theta \)))

where (max\(_\theta\) tall(b, \( \theta \))) is to be read ‘b meets or exceeds \( \theta \) and there is no higher tall threshold that b meets or exceeds’.

Once we introduce this new max operator what we discover is that like the negation in the comparative, it too takes variable scope with respect to quantifiers in the complement and its scope-taking practices follow the same rules as the negation in the comparative. (29) below implies that balloon A has reached its maximum permissible height.

(29) Balloon A is as high as it is allowed to be.

This interpretation is captured in the following formula:

(30) \( \exists \theta \) high(a, \( \theta \)) \& (max\(_\theta\) (allowed (high(a, \( \theta \))))
The second conjunct says that \( a \) is allowed to meet or exceed \( \theta \) and there is no higher threshold that \( a \) is allowed to meet or exceed. \( \max \), like its cousin \( \text{not} \), takes scope above \( \text{allowed} \). For an example of the other scope possibility consider (31):

(31) Balloon A is as high now as every other balloon was at six o’clock.

(31) implies that at six o’clock, all the other balloons were at the same height. One can see where this implication comes from by reflecting on the following formula:

\[
(32) \quad \exists \theta \ \text{high}(a, \theta) \land \text{for every other balloon } y: (\max_{\theta} \text{high}(y, \theta))
\]

\((\max_{\theta} \text{high}(y, \theta))\) says that \( y \) meets or exceeds the threshold \( \theta \) and that there is no higher threshold it meets or exceeds. In effect, this means that \( \theta \) is \( y \)’s height. But now, (32) says that there is some threshold that is the height of every other balloon. In other words, they all have the same height. Quantifiers like \( \text{every balloon} \) take scope outside \( \max \) just as they do with \( \text{not} \).

We have been focusing on the contribution of the complement clause to the meaning of the equative. We return now to the paraphrase of (26) given in (27c) to make a point about the main clause. (27c) says that \( A \) meets or exceeds a threshold given by \( B \)’s height. This allows that \( A \)’s height is greater than or equal to \( B \)’s height. This means \( A \)’s height is greater than or equal to \( B \)’s height. But (26) is often heard to suggest something stronger, namely, that \( A \) and \( B \) are of the same height. One way to understand this effect is that the use of the equative conversationally implicates that the corresponding comparative is false. So, (26) says that \( A \)’s height is greater than or equal to \( B \)’s, but it implicates that \( A \) is not taller than \( B \). A motivation for keeping the ‘greater-than or equal’ analysis of equatives comes from the meaning of negated equatives. (33) below normally says that \( A \) is shorter than \( B \), in other words, that \( A \) neither meets nor exceeds \( B \)’s height, and not just that they are not of equal height.

(33) \( A \) is not as tall as \( B \) is.

(This effect is ‘canceled’ in the following, where capitalization is intended to mark contrastive stress:

(34) He is not AS tall as you, he is TALLer.

Here we seem to have a negation of the kind you find in:

(35) Six is not greater than OR equal to 4, it is greater than 4.)

An important fact about the equative is that, unlike the comparative, it can be modified by the expressions in (36):
These expressions modify numerals as well and in general they presuppose a scale of increasingly stronger expressions. For the equative, the relevant stronger expression is the comparative (A is taller than B asymmetrically entails A is as tall as B).

In (37), at least informs the hearer that the speaker does not have adequate information one way or the other with respect to the stronger comparative statement; hence, the implicature discussed above does not arise. So, an utterance of (37) allows that A may very well be taller than B.

A is at least as tall as B is.

In (38), exactly points to a precise location on this scale: among the choices: as-tall < taller than, the speaker commits to just the equative being true and so what was an implicature in (26) is part of the assertion in (38).

A is exactly as tall as B is.

EXCESSIVE: TOO

The paraphrase in (40) reveals some of the novel elements employed in excessives to describe thresholds:

This fuel is too volatile to use in a car engine.

There is a volatility threshold which this fuel meets or exceeds and the fuel should not be used in a car engine, because it meets or exceeds that threshold, among other things.

In excessives, the infinitival complement, while it does not form a threshold description, it does contain an implicit negation. So, like other negative statements, excessives support let alone rejoinders:

This fuel is too volatile to use in a car engine, let alone a lawn mower engine.

This fuel should not be used in a car engine, let alone a lawn mower engine.

This fuel should be used in a car engine, let alone a lawn mower engine.

The paraphrase in (40) reflects an implicit should in (39). Other examples suggest different modals:
According to (40), the threshold description in an excessive is tied to the complement with because. In (42), this amounts to saying that the sun’s temperature makes it impossible for us to touch it. In fact, other factors prevent us from touching the sun, such as its distance from the earth. These do not detract from the claim in (42), since the excessive allows for other causes, hence, the use of among other things in the paraphrase in (40).

In the sun example, the various causes depend on different physical properties (temperature, distance) and each is a sufficient cause. But because, among other things also allows for causes that are jointly required in order to have their effect. When these various causes depend on the same physical properties we get funny examples that, while not contradictory, at first appear to be so. Consider the following case. Janet has gone to the fair with her young son and her elderly father. They get to the ticket booth and discover that there is a discount for anyone below 12 or above 65 years old. Janet’s son can get the discount, but Janet is too old to get it. Janet’s father can get the discount, but Janet is too young to get. In effect, Janet is too old and yet too young to get the discount. Given the intimate relation between the meaning of too and enough (too young ≈ not old enough), these types of situations are often described by contrasting a too statement with an enough statement: too old and yet not old enough.

The excessive includes an implicit modal as well as an implicit because. Since both concern alternative possibilities, it is sometimes hard to tease apart the contribution of the modal and the contribution of because. All of the following allow paraphrases where the modal is omitted:

(46) He was too stubborn to try the new method.
(47) She was too experienced to fall for that trick.
(48) He was too proud to admit his mistakes.
(49) He was too busy to call the plumber.

Differentials

Returning to the comparative, observe that if A is hotter than B is, there must be at least one threshold that A meets or exceeds that B does not. It is also
true that if A is hotter than B, there are likely to be many such thresholds. The greater the difference between A and B, the more thresholds there will be. The expressions in small capitals in the following examples are called differentials because they specify the extent or amount of difference:

(50) A is a bit hotter than B is.
(51) A is much hotter than B is.
(52) A is hotter than B is by a lot.

(50) says that there is only a small set of thresholds that A meets or exceeds that B does not. (51) and (52) say there is a large set of such thresholds.

Besides the expressions in small capitals in (50)–(52), possible differentials include many (many more eggs), no (no hotter), any (was not any happier), and noun phrases headed by a unit of measure (2 pounds heavier). With a few exceptions, differentials are just amount quantifiers otherwise found in noun phrases (cf. a bit of honey, no yogurt, much interest, 2 pounds of flour). In diverse and unrelated languages, one finds the words for much/a lot used both as a differentials in adjectival comparatives and in noun phrases. This dual use is somewhat surprising given the syntactic and semantic differences between the two contexts.

Among the amount quantifiers, much and many display syntactic, semantic, and morphological characteristics of adjectives, as do their cognates in other languages. And they have threshold arguments just like other adjectives. Compare (53) and (54) below:

(53) Jack’s library is big.
(54) Jack owns many books.

(53) is about the size of Jack’s library. It says that the library meets or exceeds some size threshold, the choice of which will likely depend on what the speaker expects for a man of Jack’s age, income, and interests. (54) is about the amount of books Jack owns. It says that that amount meets or exceeds a threshold, the choice of which will likely depend on the same factors. In both cases, we can use the Degree word too to bind the threshold argument:

(55) Jack’s library is too big to move in one day.
(56) Jack owns too many books to move in one day.

Like many in (56), much also combines with too. The Degree words so and as likewise combine with much and many. In some languages, much words also combine with the comparative marker. mai is the comparative marker in Romanian (mai inteligent ‘more intelligent’) and ‘more coffee’ is translated as mai multă cafea, using the much word mult-, which, like other adjectives, agrees in number and gender with the following noun.
In this section, we have learned essentially two things. Amount quantifiers can be used in comparatives to characterize the size of the set of thresholds that distinguish the entities compared. There are adjectival amount quantifiers and they denote relations between amounts and thresholds.

Nonadjectival Comparatives

Throughout this survey, the focus has been on comparatives that are formed with adjectives: more expensive, hotter, higher etc. In this section, we explore the possibility of having comparatives formed from expressions in other lexical categories (noun, verb, and adverb).

According to some dictionaries, in the phrase more coffee, the word more is a comparative adjective with a meaning that would correspond to the comparative of much. The expected form *mucher is impossible.¹ This amounts to the claim that more coffee, like its Romanian counterpart, is not, in fact, a ‘noun comparative’ but rather an adjectival comparative inside a noun phrase, like stronger coffee or more expensive coffee, but with much in place of strong/expensive.

According to this analysis, there are no noun comparatives in English. Is this an accident? The interesting semantic question is whether nouns even have threshold arguments. Coffee comes in differing amounts. So, perhaps coffee denotes a relation between stuff and amounts. For some nouns, intensity is relevant, and in those cases, thresholds seem especially apt. The modifiers in terrible smell, deep skepticism, and intense pain could be seen as descriptions of a threshold argument of the modified noun.

Adverbs like fast are semantically and grammatically like gradable adjectives as far as the issues addressed in this survey are concerned. Marc drove faster than Karen did reports on a speed threshold that Marc met or exceeded that Karen did not. Now, much can be used adverbially (he drives too much); so, if we ask about verb comparatives, we run into the same much problem we found with nouns. If more can be an instance of *mucher, then drove more than Karen did is just a verb modified by an adverbial comparative. For morphological reasons, the much problem does not arise in Chinese, and there an interesting distinction is made. With verbs like eat or read, comparatives are formed with the adverb duo ‘much’ while with verbs like hate or understand, the comparative is formed directly on the verb. Intuitively, I eat more than you do is a comparison of amounts while I hate it more than you do is a comparison of intensities.

In summary, there are certainly adverbial and adjectival comparatives. Noun and verb comparatives may also be possible, but English is not the best language to use to find out.

Appendix I. Semantic Composition

The meanings of the sentences discussed above come about by combining the meanings of the words in a particular way. This appendix provides
some detail on how that works. I presuppose familiarity with model-theoretic techniques used in formulating semantic theories. The presentation is cast in the context of a grammar in which expressions have silent indices. These indices play the role of connecting argument positions with quantifiers that bind them, on the model of predicate logic variables. The format is inspired by file change semantics (Heim 1983), event semantics and discussion in Higginbotham (1985). I make use of silent operators and scope is syntactically represented.

I assume following much of the syntax literature (e.g. Kennedy 2002; Lechner 2004), that in all the comparatives discussed here, there is ellipsis in the complement clause. For example, the unelided counterpart of (2) is (57).

(2) A is more expensive than B is.

(57) A is more expensive than B is expensive.

The structural relations between the Degree head and its complements is a complicated and much investigated topic (see Grosu and Horvath 2006 and references therein). The semantic issues discussed here do not decide among the alternatives. For concreteness, I adopt the structure in (58) as it requires the least manipulation to arrive at the string in (57).

(58) Syntax of Comparative

\[
\text{DegP} \quad \text{Deg'} \\
\quad \text{Deg'} \quad \text{PP} \\
\quad \text{more} \quad \text{AdjP} \\
\quad \quad \text{Adj} \\
\quad \quad \text{expensive} \quad \text{than} \\
\quad \quad \quad \text{B is expensive}
\]

In synthetic comparatives such as higher, the adjective and the affix -er combine at a later stage (Embick 2007).

Predicates are subscripted with bracketed sequences of argument indices and quantifiers (including names) receive binding indices:

(59) \(A_\chi\text{ is more}_{\theta} \text{ expensive}_{<\chi,\theta>}\text{ than }B_\gamma\text{ is expensive}_{<\gamma,\theta>}\).

I assume, following Chomsky (1977), that, as in a relative clause, a silent (and semantically vacuous) operator is fronted within the complement clause. That operator plays a role in the coindexation of \(\text{more}_\theta\) with \(\text{expensive}_{<\gamma,\theta>}\) in (59). Finally, there is a silent operator generated adjacent to the adjective (possibly in Q°, see Corver (1997)).
(60) $A_x$ is more$_\theta$ expensive$_{<x, y>}$ than $B_y$ is not expensive$_{<y, z>}$.

(60) is the logical form for the simplest kinds of examples discussed above. Before we are done, there will be two amendments in which two more operators will be introduced.

To interpret this syntactic structure, we appeal, among other things, to the following semantics rules. In these rules, I use bold variables for metalanguage values of the corresponding unbolded object-level variables:

(61) $\sem_{\text{Deg}}(\text{more}_\theta \varphi \psi) = 1 \iff \exists \theta \left[ \varphi^{\theta/\theta} = \psi^{\theta/\theta} = 1 \right]$

(62) $\sem_{\text{not}} \varphi = 1 \iff \varphi = 0$

The silent not operator may undergo LF movement yielding the following strings for the examples discussed earlier (traces, if there are any, are not interpreted and so they are kept invisible):

(63) a. $A$ is taller than every boy is.
    b. $A_x$ is more$_{\theta}$ tall$_{<x, y>}$ than every$_y$ boy$_{<y>}$ is not tall$_{<y, z>}$.
    ‘There is some height threshold that $A$ meets or exceeds and that every boy does not meet or exceed.’

(64) a. $A$ is taller than any boy is.
    b. $A_x$ is more$_{\theta}$ tall$_{<x, y>}$ than not any$_y$ boy$_{<y>}$ is tall$_{<y, z>}$.
    ‘There is some height threshold that $A$ meets or exceeds such that there is not any boy who meets or exceeds it’

For the examples with modals in them, it is necessary to indicate all implicit world arguments as well as an argument of the modal corresponding to a contextually supplied accessibility relation. $@$ and $w$ are world indices. Modals take world and accessibility arguments but they are also world quantifiers, so they have both argument indices and binding indices:

(65) a. $A$ is higher than it is allowed to be.
    b. $A_x$ is er$_{\theta}$ high$_{<x, y, @>}$ than not it$_x$ is allowed$_{<@, \text{Acc}, w>}$ to be high$_{<x, y, w>}$.

(66) a. $A$ is higher than it should be.
    b. $A_x$ is er$_{\theta}$ high$_{<x, y, @>}$ than it$_x$ should$_{<@, \text{Acc}, w>}$ be not high$_{<x, y, w>}$.

Here are meanings for the modals:

(67) $\sem_{\text{allowed}_{<@, \text{Acc}, w}} \varphi = 1 \iff \exists w (g(\text{Acc}(w)(g(@))) \land \varphi^{w/w} = 1)$

(68) $\sem_{\text{should}_{<@, \text{Acc}, w}} \varphi = 1 \iff \forall w (g(\text{Acc}(w)(g(@))) \rightarrow \varphi^{w/w} = 1)$

$g(\text{Acc}(w)(g(@)))$ says that world $w$ is accessible to world $g(@)$ under the relation given by $g(\text{Acc})$. 

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Next, we move to the equative which has the same syntax as the comparative, with *more* and *than* both replaced by *as*. In addition, recall, we posited a silent operator in the equative clause, which again we take to be generated adjacent to the elided adjective.

(69) A is as expensive as B is.

(70) \(A_x \text{ is } as_{\theta} \text{ expensive}_{<x,\theta>} \text{ as } B_y \text{ is } \text{max}_{\theta} \text{ expensive}_{<y,\theta>}\).

\(\text{max}_{\theta}\) is understood as a threshold modifier and so it receives a binding index for the argument it comments on. Its semantics is given below:

(71) \[\text{max}_{\theta} \varphi \rceil^e = 1 \iff [\varphi] \rceil^e = 1 \land \forall \theta (\theta > g(\theta) \rightarrow [\varphi] \rceil^{g(\theta)} = 0)\].

Other types of threshold modifiers are discussed in Rett (2006, forthcoming). The semantics of *as*\(\theta\) is the same as that of *more*\(\theta\).

What follows now are two modifications to the semantics of the comparative. First, we note that the logical form in (72) below will be true if A is more expensive than several differently priced hats were (think of a price between that of the cheapest hat and the second cheapest hat). The corresponding ‘surface’ string in (73) would be false in that case.

(72) A is more\(\theta\) expensive\(\theta\) than [exactly 1 of the hats]\(\theta\)

was NOT expensive\(\theta\).

(73) A is more expensive than exactly 1 of the hats was.

This motivates an occurrence of the max operator in the main clause of comparatives (cf. Heim 2001):

(74) A is more\(\theta\) max\(\theta\) expensive\(\theta\) than [exactly 1 of the hats]\(\theta\)

was NOT expensive\(\theta\).

With this correction of the logical form of comparatives, it now follows that the sentences in (75) below are truth conditionally equivalent:

(75) a. A is not as high as B is.

b. B is higher than A is.

(75a) has an overt negation in the main clause and a covert max in the subordinate clause, while (75b) has a covert negation in the subordinate clause and a covert max in the main clause. More interestingly, in the context of the present theory, the intuitive equivalence within the pairs in (76) and (77) shows that modals arrange themselves with respect to overt negation the same way they do with respect to covert negation.
Finally, we come to differential comparatives:

(78) a. A is at least 4 inches higher than B is.
    b. A is higher than B is by at least 4 inches.

For (78a), I adopt the structure in (79):

(79) **Comparative with differential**

Differentials are optional arguments of the comparative, as they are for a host of target-missing predicates:

(80) a. Jack failed the exam by 4 points.
    b. The candidate fell 4 points short of the electoral threshold.
    c. The candidate fell short of the threshold by 4 points.
    d. Jack missed the target by 4 inches.

*over-, out-, and under-* create other differential taking verbs (*overcook, undercount* and *outspent*).

(80b) and (80c) suggest a way to understand what is going on in the differential comparative. The idea would be that (78) says not only that B fell short of the threshold that is A's height but it also tells us by how much. In effect, we need to replace our negation operator with a **fall-short** operator. In order to see how the semantics of **fall-short** will go, it helps to think about an example like (80a). If the passing grade on Jack's exam was 65, then 65 is a threshold that Jack hoped to meet or even better exceed. Unfortunately, he failed the exam by 4 points, which means he got a 61. '4 points' here corresponds to the fact that there are 4 grades equal to or below 65 that Jack did not meet or exceed. They are 65, 64, 63 and 62. So, *fail* and any of the other expressions in (80),
rely on two arguments, one is a target threshold and the other is a measure of difference. With this in mind, we propose the following semantic rule that uses \( \delta \) to stand for a measure of difference:

\[
\text{if} \theta \text{ is a binding index in (81), because it ‘manipulates’ the values for } \theta \text{ in } \phi, \delta \text{ measures the size of the set of thresholds that are ‘missed’. By-phrases and specifiers of DegP are used to establish a particular measure. In effect, they are names of measures and so as names, they receive a binding index. This is captured in the following semantic rule:}
\]

\[
\text{(81) } \text{[fall-short}_{<\delta, \theta} \phi]^e = 1 \text{ iff } \psi(\theta) \land [\phi|_{\theta}^{\theta} = 0]
\]

\( \theta \) is a binding index in (81), because it ‘manipulates’ the values for \( \theta \) in \( \phi \). \( \delta \) measures the size of the set of thresholds that are ‘missed’. By-phrases and specifiers of DegP are used to establish a particular measure. In effect, they are names of measures and so as names, they receive a binding index. This is captured in the following semantic rule:

\[
\text{(82) } \text{[at least 4 inches}_{\delta} \phi]^e = 1 \text{ iff } \psi|_{\delta} = 1 \text{ where } \delta \text{ is the set measure corresponding to at least 4 inches.}
\]

Just as ‘4 points’ describes a set of grades \{65, 64, 63, and 62\}, \textit{at least 4 inches} names a measure that holds, for example, of the set of height thresholds met or exceeded by a 65 inch pole but not by 61 inch pole.

\[
\text{(83) } A_x \text{ is } \text{[at least 4 inches}_{\delta} \text{ er}_\theta \text{ max}_\theta \text{ high}_{<\delta, \theta}} \text{ than } B_y \text{ is fall-short}_{<\delta, \theta} \text{ high}_{<\delta, \theta} \text{ ‘There is a threshold which is A's height and B falls short of that threshold by at least 4 inches’}
\]

The \text{fall-short} operator will take scope just like \text{not} and \text{max}. In particular, if we take \textit{some amount} to name a measure that holds of any nonempty set, we get equivalence among the pair in (84) below (using \text{fall-short} in a and \text{not} in b, both with scope below \textit{should}):

\[
\text{(84) a. Jack is some amount taller than he should be.}
\]

\[
\text{b. Jack is taller than he should be.}
\]

And if we take \textit{no} to name a differential that holds only of the empty set, (85a) turns out equivalent to (85b) and, therefore, to (85c), assuming that \text{fall-short} scopes above \textit{had to}, as expected.

\[
\text{(85) a. Jack is no taller than he has to be.}
\]

\[
\text{b. There are no thresholds at or below Jack’s height that he doesn’t have to meet or exceed.}
\]

\[
\text{c. Every threshold at or below Jack’s height is one he has to meet or exceed.}
\]

Next, we turn to excessives whose syntax is given in the tree below:
On the paraphrase proposed earlier, there is a causal relation claimed to hold between a threshold proposition and the proposition given by the infinitive. We will, therefore, need a ‘because’ operator in the semantics of too. I assume a ‘because’ operator that connects two propositions and a possible world as follows:

\[ \text{because}(w)(p)(q) \approx 'p \text{ and q are true in world w and p is a reason for q in w}' \]

Our meaning for too is now:

\[ \mathbb{L}((\text{too}_{\text{Acc,}a,\theta,w} \varphi) \psi) \equiv 1 \iff \exists \theta \text{ because (g(@)) } (\lambda w \varphi[w/\theta/w,\theta]) (\lambda w w' (g(\text{ACC})(w')(w) \land \mathbb{L}[w/\theta/w]) \land \mathbb{L}[w/\theta/w]) \]

Our example sentence is now indexed as follows:

\[ A_x \text{ is } ((\text{too}_{\text{Acc,}a,\theta,w} \text{ volatile}_{x,\theta,w}) \text{ PRO}_u \text{ to use}_{u,x,w}) \]

Given the interpretation for too, (89) now amounts to (90) roughly paraphrased in (91):

\[ \exists \theta \text{ because (g(@)) } (\lambda w \text{ volatile}_{x,\theta,w}) (\lambda w w' (g(\text{ACC})(w')(w) \land \mathbb{L}[w/\theta/w]) \land \mathbb{L}[u,x,w]) \]

\[ \text{there is some volatility-threshold } \theta: A \text{ meets or exceeds it and because of that one should not use A.} \]

Above we noted that different examples with too appear to have different modalizations. Following Meier (2003), this is now understood in terms of different contextually supplied values for the accessibility relation argument of too in (88).

Appendix II. Sources and Further References

The A–not-A analysis was first offered as an explanation for the occurrence of negative polarity items in comparative clauses. This explanation
is found in Ross (1969), Seuren (1973, 1984), and in a number of earlier sources, including Small (1924) and Joly (1967), who further argued that the English comparative clause derives historically from a negation containing clause and that the negation played a role in the development of \textit{than}. McConnell-Ginet (1973: Chapter II) summarizes this work. The other source for A-not-A is a suggestion of David Kaplan’s reported in Lewis’s (1970) discussion of vagueness. For Lewis, thresholds, or what he called ‘delineation coordinates’, are parameters relative to which one could assign extensions to vague predicates. For a given threshold, the extension of \textit{warm} would include all and only those things whose temperatures meet or exceed the threshold. Lewis uses this as a basis to assign degrees of truth to positive sentences (e.g. \textit{The soup is warm}) as well as to describe how vagueness projects in various syntactic contexts. In that setting, the intuition behind the A-not-A analysis of \textit{Jack is taller than Jill} is that there is a possible way to fix a boundary for \textit{tall} so that Jack comes out tall but Jill does not. This approach is further developed in Kamp (1975), Klein (1980, 1982), McConnell-Ginet (1973), and Landman (1991: Section 3.5).

The chief alternatives to A-not-A presuppose an ordered set of degrees with adjectives denoting relations between individuals and degrees and with the comparative encoding the greater-than relation (for summaries and comparisons, see Klein 1991 and Kennedy 2005). This type of approach is carefully laid out in Cresswell (1976) where the adjective \textit{tall} is taken to relate an individual to his height. In part because of the issues discussed here but for other reasons as well (see Rullmann 1995: Chapter 2), \textit{tall} was subsequently taken to relate an individual to his height and to any lower height. This revised view breeds confusion. We now find locutions such as ‘the degrees to which Jack is tall’ or worse ‘the heights that Jack has’ or ‘Jack’s ages’. For this reason, I recommend the technical vocabulary ‘threshold’ and its natural companion ‘meet or exceed’ even if it does create some discontinuity with previous work in semantics and with the syntactic category label ‘Degree’.

Within the A-not-A tradition, Larson (1988) observed that Klein’s (1980: 82) account failed when complement clauses contain certain quantifiers. He proposed a solution that, in effect, rearranged the position of the negation relative to other parts of the clause. Larson’s analysis improved on Klein’s with respect to quantifiers and verbs that take scope above negation but it fell short of Klein’s account with respect to quantifiers and verbs that take scope below negation. Almost the same history repeated itself in the greater-than tradition with von Stechow (1984), covering only the under-negation quantifiers (and noting the problem for the others) followed by Schwarzschild and Wilkinson (2002), which covers only the above-negation quantifiers. Schwarzschild and Wilkinson have a comprehensive discussion of quantificational DPs and adverbs but almost no discussion of modals or attitude verbs. In an antecedent to that paper, the two types of modals (e.g. \textit{allowed} versus \textit{should}) and their
differential behavior with auxiliary negation was noted, but this was omitted from the published paper because the proposed theory had nothing to say about the scope dichotomy. What was missing was an operator that could take variable scope within the complement clause. In the interim, Heim argued for a variable scope operator within the complement clause of less comparatives to explain facts brought to light in Rullmann (1994, 1995). Heim’s analysis appears in Heim (2006) and it represents part of a larger project to understand how Degree operators in general interact with other kinds of quantifiers (Heim 2001; Kennedy 1999; Larson 1988; Rullmann 1995; Seuren 1984; Sharvit and Stateva 2002; Stateva 2000, 2003; von Stechow 1984). The presence of a variable scope operator inside the complement clause allows us to produce correct logical forms for the sentences in question, but it leaves open why the operator obeys the scope rules it does. The solution to this problem surely has to do with other interactions between modals and negation (von Fintel and Iatridou 2007) as well as modals and threshold-based numerical quantifiers such as at least 3 (Hackl 2000: Chapter 3). Our increasing understanding of negative polarity and perhaps more importantly positive polarity items will also help to fill out this complex scope-taking picture (Szabolcsi 2004).

One final note on the two traditions in analyzing comparatives is in order. I have focused here on one particular comparative construction. There are a variety of others in the world’s languages (Stassen 1985, 2005). Even the English she’s smarter than him is syntactically and possibly semantically distinct from the clausal comparatives discussed here (Hoeksemia 1983; Lechner 2001, 2004; Pancheva 2006; Pinkal 1990). A-not-A is unlikely to be correct for all comparative constructions and likely some make no use of thresholds at all (Beck et al. 2004; Li forthcoming).

The account of the equative presented here is based on the one sketched in McCawley (1981), although the implicature based on the comparative goes back to Horn (1972). The analysis of the as complement clause is straight out of von Stechow (1984) except that in that paper max has fixed scope over the entire clause under as.

My discussion of excessive too descends chiefly from Nelson (1980). For alternative analysis and discussion of further properties of this and related constructions, see Meier (2003) and Hacquard (2006). The let alone facts are discussed in Fillmore et al. (1988) and Sawada (2003).

McConnell-Ginet (1973) is my source for the idea that differentials describe the size of the set of thresholds that make a comparative statement true. Schwarzschild (2005) further argues that a preadjectival measure phrase (5 feet tall) also describes the size of a set thresholds as against the popular idea that it names a point on a scale and serves as an argument for tall. Schwarzschild (2006) provides details on how we use the same expressions as amount quantifiers and as differentials.

Cresswell (1976) introduces the idea of amount-threshold arguments of nouns. For intensity-threshold arguments of nouns, see Rodríguez
Ramalle (2001), Morzycki (2005), Matushansky and Spector (2005), and Baker (2003: Section 4.3).


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**Short Biography**

Roger Schwarzschild is a Professor of Linguistics at Rutgers University. He has published one book and about a dozen articles. The topics include the semantics of plurals, intonation, and Degree constructions. His PhD is from the University of Massachusetts at Amherst.

**Notes**

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1 A similar thing happens with *many-er*. Likewise, in *enough bagels* and *enough coffee*, the word *enough* is said to have the meaning of the ungrammatical *many enough* and *much enough*, respectively. See, for example, ‘more, a 2 (comparative of much), 3 (comparative of many)’. The Oxford English Dictionary, 2nd ed. 1989. OED Online. Oxford University Press. December 2000 <http://dictionary.oed.com/cgi/entry/00315707>.

2 The analogy with relative clauses suggests a way to understand the deviance of comparative clauses that contain overt negative elements (Lees 1961: 175):

(i). #I know him better than nobody does.

On the current view, what follows than in (i) is like a relative clause with two negations in it, one of them covert, and it seems to have the same status as (iia) as opposed to (iib).

(ii). a. #That is the way that nobody does not eat it.

b. That is the way that everybody eats it.

Another option would be to assign max an argument index and a binding index, as was done with the modals. This may accord better with its role as a modifier. Its semantics would have to be modified accordingly.

See Büring (forthcoming), for a dissenting view.

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