Packet Routing over Parallel Time-Varying Queues with Application to Satellite and Wireless Networks

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Consider a constant service rate routing problem:
(heterogeneous service rates \( \{\mu_1, \mu_2, ..., \mu_n\} \) )

2 Natural Routing Strategies:

Greedy: \( \pi_{greedy} \)

Choose queue \( k \) such that
\[
k = \arg\min_{j \in \{1, ..., n\}} \left\{ \frac{L_i + U_j(t)}{\mu_j} \right\}.
\]

Work Conserving: \( \pi_{WC} \)

Choose queue \( k \) such that
\[
k = \arg\min_{j \in \{1, ..., n\}} \left\{ \frac{U_j(t)}{\mu_j} \right\}.
\]

\( U_{greedy}(t) \) can be arbitrarily larger than \( U_{WC}(t) \). However, \( U_{WC}(t) \) stays within a fixed upper bound from any other strategy.
Multiplexing Inequality:

\[ U_{\text{single}}(t) \leq U_{\text{multi}}(t) \]

(For any routing strategy over the parallel queues)

However, for the work conserving strategy \( \pi_{WC} \), we also have an upper bound:

\[ U_{\text{single}}(t) \leq U_{\text{WC}}(t) \leq U_{\text{single}}(t) + (n - 1)L_{\text{max}} \]

Comparing \( \pi_{WC} \) to any other routing strategy \( \pi \):

\[ U_{\text{WC}}(t) \leq U_{\pi}(t) + (n - 1)L_{\text{max}} \]

...and it can be shown that \((n-1)L_{\text{max}}\) is the best bound possible for non-predictive, non-preemptive routing schemes, hence \( \pi_{WC} \) is minimax optimal.
The $\pi_{WC}$ routing algorithm uses a pre-queue to achieve work conservation in systems with time-varying server speeds (route to a server immediately when it empties).

How do we route when no pre-queue is available? (Ex: Queues are in different physical locations)

Input process $X(t)$ --- rate ergodic, rate $\lambda$.
Processing rates $\{\mu_i(t)\}$ --- ergodic, time average rates $\{\mu_i^{av}\}$.

How do we stabilize the system without knowing the input stream, and without knowing future processing rates?

Consider Join-the-Shortest-Queue strategy: $\pi_{JSQ}$
($JSQ =$ Route the incoming packet to the queue $j$ with the smallest unfinished work $U_j(t)$ ).
New notion of stability useful for understanding stability issues in systems with general ergodic inputs:

Consider a single server queue with a finite buffer of size $M$:

$$X(t) \sim \text{rate } \lambda$$

Define $DR(M) = \text{Packet drop rate when buffer size is } M \text{ bits}$. (clearly $DR(M)$ is a non-increasing function of $M$).

**Definition:**
A system is *loss rate stable* if $DR(M) \to 0$ as $M \to \infty$.

This definition is closely related to the existing notion of stability defined in terms of a vanishing complementary occupancy distribution $Pr[U > m] \to 0$ as $m \to \infty$. It can be shown:

$$\lambda \leq \mu_{av} : \text{ necessary condition for stability.}$$

$$\lambda < \mu_{av} : \text{ sufficient condition if inputs and linespeeds are Markov Modulated.}$$
Compare drop rate under JSQ policy to a single-server queue:

Let $DR_{JSQ}(M+nL_{max})$ represent the packet drop rate in the multi-queue system under the JSQ routing policy when all queues have buffer size $M$.

**Theorem:**

\[ DR_{JSQ}(M+nL_{max}) \leq DR_{single-queue}(M) \]

Thus, the system under $\pi_{JSQ}$ is loss rate stable iff the single queue system is loss rate stable. (Hence, it is stable whenever the system is stabilizable).

Joint routing and Power Allocation:
**Power Allocation**--Processing rates depend on power allocation $p_i(t)$ and time varying channel state $c_i(t)$: $\mu_i(p_i(t), c_i(t))$.

Each satellite $s$ has multiple beams and a fixed power resource $P_{tot}(s)$.

Must jointly route packets and allocate power to the different queues subject to a fixed power resource $\sum p_i(t) \leq P_{tot}$.

**Decoupled Policy:**
- Routing: JSQ
- Power Allocation:

Maximize $\sum \mu_i(p_i, c_i(t))$ subject to $\sum p_i = P_{tot}$
Example: Poisson arrival process, fixed length packets (size $L$).

Assume, for the simplicity of the example, that the time varying linespeeds $\mu_i(t)$ are arbitrary but sum to a constant rate $\mu$.

Let $N_i(t) =$ Number of packets in queue $i$ at time $t$.

Translate unfinished work into number of packets: $N = \lceil U / L \rceil$

$$DR_{JSQ}(M) \leq DR_{Single}(M - k) \leq Pr[N_{M/D/1} > n]$$
**Theorem:**

\[ DR_{JSQ}(M+nL_{\text{max}}) \leq DR_{\text{single-queue}}(M) \]

Proof outline: Let \( G(t) \) represent packet drops during \([0, t]\).

We show \( G_{JSQ}(t) \leq G_{\text{single}}(t) \) for all time \( t \).

Prove claim over “completely busy periods”:

Let: \( a = \) arrivals during \([t_B, t]\).
\( d = \) departures during \([t_B, t]\).

1. Packet Conservation equalities:

\[ U_{JSQ}(\tau) = U_{JSQ}(t_B) + a \quad d_{JSQ} - g_{JSQ} \]
\[ U_{\text{single}}(\tau) = U_{\text{single}}(t_B) + a \quad d_{\text{single}} - g_{\text{single}} \]

2. \( d_{JSQ} \geq d_{\text{single}} \):

\[ d_{JSQ} \geq d_{\text{single}} \]
3. Just before c.b.p., at least one queue of multi-server system is empty:

\[ U_{JSQ}(t_B) \leq (n - 1)[M + nL_{max}] \]

4. JSQ Strategy: When a packet is dropped at time \( \tau \), all queues must have more than \([M + (n-1)L_{max}]\) unfinished work:

\[ U_{JSQ}(\tau) > n[M + (n - 1)L_{max}] \]

These facts plus algebra yield the result. \( \square \)