Quiz 1 — EE 549
Wednesday, Feb. 27, 2008

INSTRUCTIONS

- This quiz lasts for 85 minutes.
- This quiz is closed book and closed notes. No Calculators or laptops allowed.
- Put your name at the top corner of this page. If you are a DEN student, put "DEN" in parentheses.

QUESTIONS

There are three questions, with point values given below:

1) Calculating Rates (30 points)

2) Network Calculus (35 points)

3) Scheduling for Rate Stability (35 points)

FORMULAS

1) I-O Equation:
   - For any $t_1 \leq t_2$: $U(t_2) = U(t_1^-) + X[t_1, t_2] - Y[t_1, t_2]$

2) Leaky Bucket Formulas:
   $X(t) \sim (r, \sigma)$, $\mu(t) \sim (\overline{\mu}, \gamma)$. If $r \leq \overline{\mu}$ then:
   - $U(t) \leq \sigma + \gamma$
   - $Y(t) \sim (r, \sigma + \gamma)$
   - $Delay_{FIFO} \leq \frac{\sigma+\gamma}{\overline{\mu}}$

3) Two leaky bucket inputs $X_1(t), X_2(t)$ (splitting property):
   $X_1(t) \sim (r_1, \sigma_1), X_2(t) \sim (r_2, \sigma_2)$, $\mu(t) \sim (\overline{\mu}, \gamma)$. If $r_1 + r_2 \leq \overline{\mu}$, then:
   - $Y_1(t) \sim (r_1, \sigma_1 + \sigma_2 + \gamma)$
   - $Y_2(t) \sim (r_2, \sigma_1 + \sigma_2 + \gamma)$
   - $Y(t) = Y_1(t) + Y_2(t) \sim (r_1 + r_2, \sigma_1 + \sigma_2 + \gamma)$

I. Calculating Rates (30 Points)

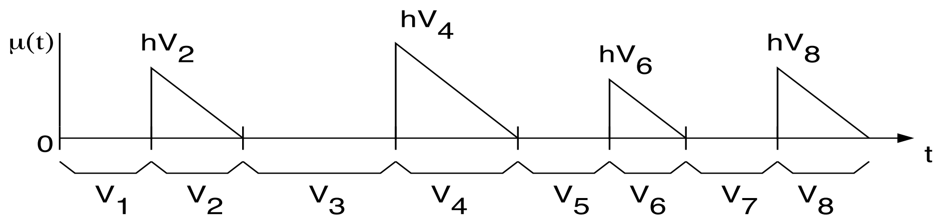

Fig. 1. An illustration of the $\mu(t)$ process for Problem I. The parameter h is a constant that determines the slope of the triangles.

Let $\mu(t)$ be a stochastic transmission rate process with alternating ON/OFF behavior that takes place over time intervals $\{V_1, V_2, V_3, \ldots\}$, as shown in Fig. 1. Specifically:

1) Odd Intervals: $\{V_1, V_3, V_5, \ldots\}$
   - Odd interval durations have units of seconds and are given by $\{V_1, V_3, V_5, \ldots\}$, which are *i.i.d. and exponentially distributed with distribution* $p_V(t) = \alpha e^{-\alpha t}$.
   - $\mu(t) = 0$ for times t that are within odd intervals.
2) Even Intervals: $\{V_2, V_4, V_6, \ldots\}$
   - Even interval durations have units of seconds, are independent of the odd intervals, and are given by $\{V_2, V_4, V_6, \ldots\}$, which are *i.i.d. and uniformly distributed* over the interval $[2.5, 3.5]$.
   - $\mu(t) > 0$ for times within even intervals. Specifically, at the beginning of the ith even interval (which has duration V_{2i}), the transmission rate is given by hV_{2i} (where h is a constant scaling factor), and the transmission rate decreases linearly, reaching zero at the end of the interval (see Fig. 1).

We want to calculate the time average rate $\overline{\mu}$ associated with this $\mu(t)$ process (from first principles).

a) Define the notation that you will use.

b) Write an equation (*without using or taking any limit yet*) that will be useful to compute the time average rate.

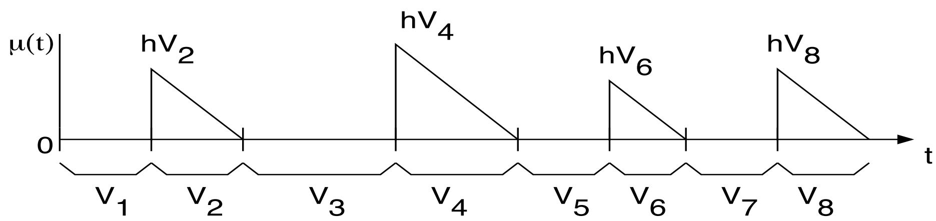

Fig. 2. The same illustration of the $\mu(t)$ process from the previous page (shown again here for convenience). The parameter h is a constant that determines the slope of the triangles.

c) Take limits and perform the necessary calculations to compute $\overline{\mu}$. Please provide explanation/justification for your steps. *Recall that:*

- Odd Intervals $\{V_1, V_3, V_5, \ldots\}$ are i.i.d. with *exponential distribution* $p_V(t) = \alpha e^{-\alpha t}$.
- Even Intervals $\{V_2, V_4, V_6, \ldots\}$ are i.i.d. and *uniformly distributed over the interval* $[2.5, 3.5]$.

II. Network Calculus (35 Points)

a) Give the definition of a leaky bucket arrival process $X(t)$ with rate r bits/sec and burst σ bits, i.e., $X(t) \sim (r, \sigma)$.

b) Let $X(t) \sim (r, \sigma)$, and suppose all packets have fixed size B kilobits (where $B \leq \sigma$). Suppose we split the process into two streams by *independent probabilistic routing*, so that every packet is independently placed to stream 1 with probability $1/2$, and else it is placed in stream 2 (see Fig. 3).

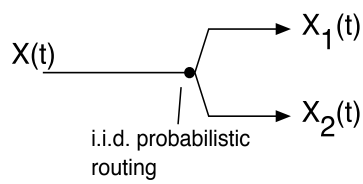

Fig. 3. An illustration of the i.i.d. probabilistic splitting for Problem IIb.

Let $X_1(t)$ and $X_2(t)$ represent the resulting processes consisting of packets placed to streams 1 and 2, respectively, so that $X_1(t) + X_2(t) = X(t)$. Give the tightest possible leaky bucket parameters (r_1, σ_1) you can for the $X_1(t)$ process. Give two or three sentences (possibly containing equations) that explain your answer.

c) Consider a single-server work conserving queue with input $X(t) \sim (r, \sigma)$ and transmission rate $\mu(t) \sim (\overline{\mu}, \gamma)$. Prove that if $r \leq \overline{\mu}$, then the $Y(t)$ bit departure process is leaky bucket with parameters $(r, \sigma + \gamma)$. That is, prove that $Y(t) \sim (r, \sigma + \gamma)$.

d) Consider the multi-hop network with leaky bucket inputs as shown in Fig. 4.

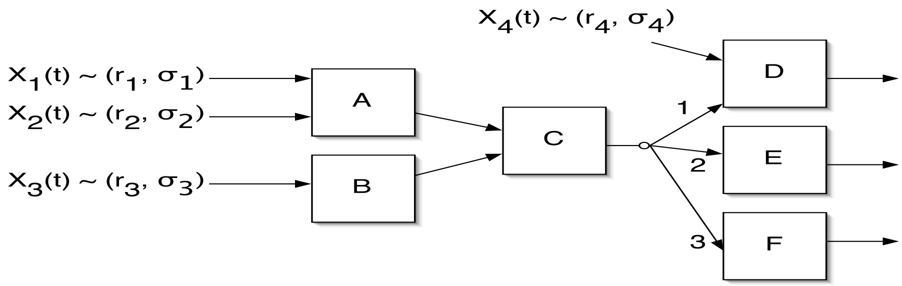

Fig. 4. The Multi-Hop Network (with FIFO) for Problem IId.

Assume service is FIFO and that we have a fluid departure model. Assume arrival processes are given by $X_i(t) \sim (r_i, \sigma_i)$ for $i \in \{1, 2, 3, 4\}$. The data streams $X_1(t), X_2(t), X_3(t)$ each take different routes after node C:

- Stream 1 data enters node D.
- Stream 2 data enters node E.
- Stream 3 data enters node F.

The fourth process $X_4(t)$ enters node D (along with the stream 1 data that exits node C). Assume processing rates are constant at each node and given by $\mu_A, \mu_B, \mu_C, \mu_D, \mu_E, \mu_F$. Further assume that:

$$r_1 + r_2 \leq \mu_A \ , \ r_3 \leq \mu_B \ , \ r_1 + r_2 + r_3 \leq \mu_C \ , \ r_1 + r_4 \leq \mu_D \ , \ r_2 \leq \mu_E \ , \ r_3 \leq \mu_F$$

Compute the tightest possible worst-case queueing bounds for $U_A(t)$, $U_B(t)$, $U_C(t)$, $U_D(t)$, $U_E(t)$, and $U_F(t)$ using the network calculus results from this course. *Justify your answers where appropriate.*

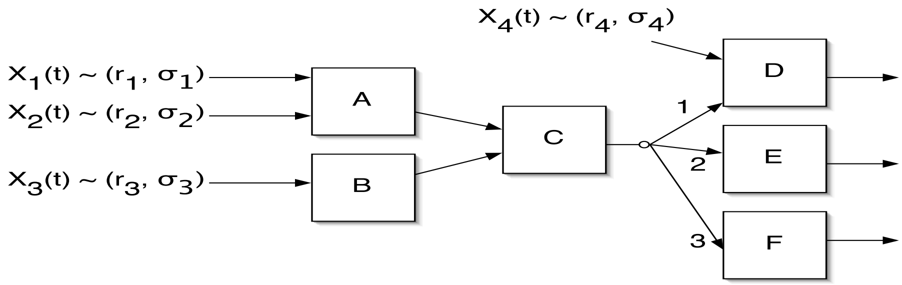

Fig. 5. The same Multi-Hop Network (with FIFO) from part (d) on previous page.

e) For the same multi-hop network with FIFO as in part (d) (illustrated again above), compute the tightest possible worst case end-to-end delay for data of stream $X_1(t)$ (including the delay in all nodes that it visits).

III. Scheduling for Rate Stability (35 Points)

a) Consider the network of Fig. 6, with input processes $X_1(t)$, $X_2(t)$, and $X_3(t)$ with well defined time average rates r_1, r_2, r_3. There are 5 nodes in the network, and the routing and scheduling in the network works according to some unknown policy. There are only 2 possible output ports for the data, with constant transmission rates μ_A and μ_B, respectively.

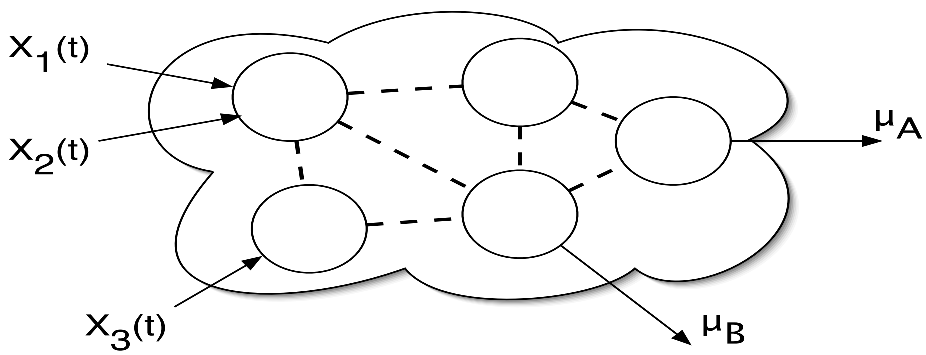

Fig. 6. The network of Problem IIIa with 3 inputs and two output ports.

Suppose that $r_1 + r_2 + r_3 > \mu_A + \mu_B$. Prove (using a limit argument) that total backlog in the network goes to infinity with probability 1. (*Your proof must be mathematically rigorous — just saying "the total input rate is greater than the max output rate" will receive zero credit.*)

b) Consider the wireless system with opportunistic scheduling shown in Fig. 7. The system is timeslotted, and there is a single server that must be scheduled to one of the two top queues (queues 1 and 2) every slot. We can observe the ON/OFF channel conditions before making a scheduling decision. Assume all packets are size 1 kilobit, and the server can serve one packet (of size 1kb) over link i on any timeslot when the server is allocated to channel i and that channel is ON (for $i \in \{1, 2\}$). Assume channels are independent of each other and i.i.d. over slots with ON probabilities p_1, p_2, respectively.

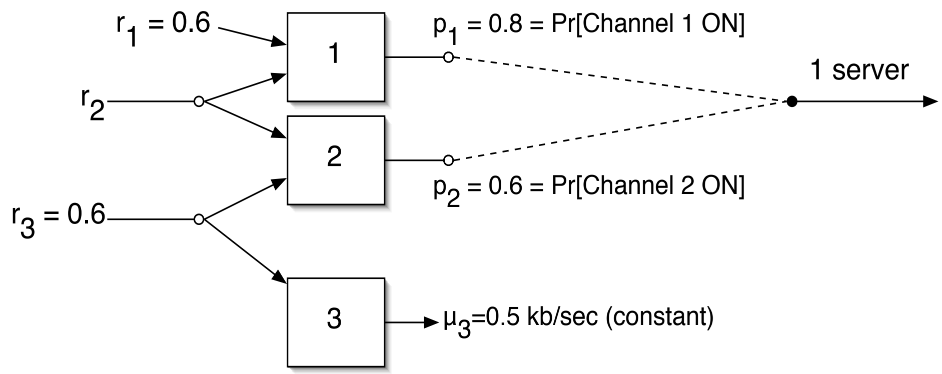

Fig. 7. A wireless system with opportunistic scheduling and one server.

There are three input streams with rates r_1, r_2, r_3, respectively (all packets are size 1 kilobit). The stream 2 data can be routed to either queue 1 or queue 2, and stream 3 data can be routed to either queue 2 or queue 3, as shown in Fig. 7. All routing must be done immediately upon packet arrival. Queue 3 is *not part of the wireless system, and has a constant server rate of* $\mu_3 = 0.5$ *kb/sec for all time*. Assume that:

- $r_1 = 0.6$ kb/sec , $r_3 = 0.6$ kb/sec
- $p_1 = 0.8$, $p_2 = 0.6$ (where $p_i = Pr[\text{Channel } i \text{ is ON}]$ for $i \in \{1, 2\}$).

Design a scheduling and routing policy that maximize the rate r_2 that can be stably supported in this network (so that all queues must be rate stable under your algorithm). You must:

- Specify the scheduling and routing policy.
- Show that it supports a certain rate r_2 (compute the value r_2).
- Show that your value r_2 is the maximum possible rate the network can support for stream 2.

(you can use the next page if needed, the picture is repeated on the next page for convenience)

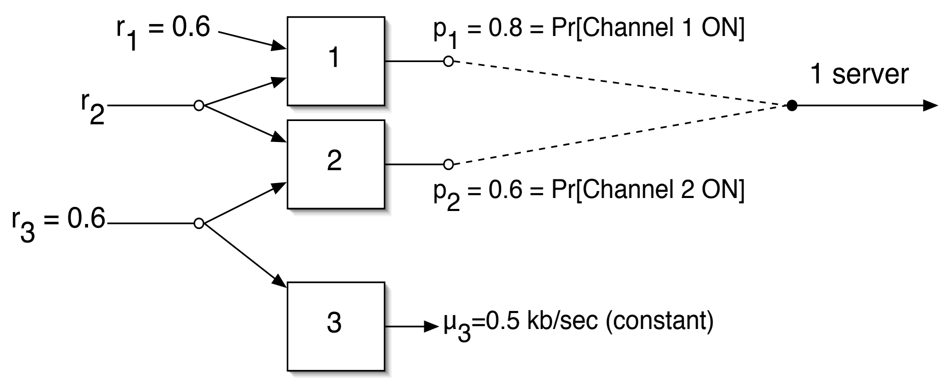

Fig. 8. The same figure of the previous page, repeated here for convenience.

- $r_1 = 0.6$ kb/sec , $r_3 = 0.6$ kb/sec
- $p_1 = 0.8$, $p_2 = 0.6$ (where $p_i = Pr$[Channel i is ON]).

(The picture and question from the previous page are repeated above for convenience)