EE549: Problem Set #8
Due: Monday April 14, 2008

I. A TANDEM OF QUEUES WITH BERNOULLI SERVICE

Consider a $B/B/1$ queue with input rate λ and service rate μ_1. The output enters another queue that operates in slotted time and has i.i.d. Bernoulli service (that is independent of the first queue) with service rate μ_2.

a) Suppose that $\lambda < \mu_1$ and $\lambda < \mu_2$. Compute the average number of packets in each queue.

b) Suppose the output of the second queue enters a third queue with independent i.i.d. Bernoulli service of rate μ_3. However, an additional independent arrival process that is Bernoulli of rate λ_2 also enters that third queue. What is the average number of packets in this third queue?

II. RANDOMIZED WIRELESS SCHEDULING

Consider a tandem of two discrete time queues with a single input stream that is Bernoulli of rate λ. The output of the first queue enters the second. This is a wireless system and, because of interference constraints, only one queueing station can transmit on any given slot. Thus, there is only one server in the system. We consider the very simple server scheduling rule as follows: the server is randomly placed to queue 1 with probability p_1 and to queue 2 with probability p_2, independently of the input process and i.i.d. over slots (where $p_1 + p_2 = 1$). If a server is placed to a queue during a slot, exactly one packet can be served on that slot. If the server is placed to queue 1, then a single packet can be served and shifted to queue 2. If the server is placed to queue 2, then a single packet can be transmitted out of queue 2 and thus exits the system.

a) Assume that $\lambda < p_1$ and $\lambda < p_2$. Suppose the system is in steady state, so that the output of the first queue is Bernoulli. Draw the Markov chain for the number of packets in the second queue, and compute the stationary probabilities. (Hint: The second queue is *not* a $B/B/1$ queue).

b) Compute the average delay in the system.

c) Optimize the probabilities p_1 and p_2 to minimize the average delay expression in part (b). What is the maximum possible rate λ that leads to finite average delay?

III. ON/OFF ARRIVALS

Let $L(t)$ represent the number of packets in a discrete time queue. Assume all packet sizes are fixed, and the queue evolves according to the following equation:

$$L(t+1) = \max[L(t) - \mu(t), 0] + A(t)$$

Assume that the service rate process $\mu(t)$ is i.i.d. Bernoulli with $Pr[\mu(t) = 1] = \mu$. However, assume that arrivals $A(t)$ are *time-correlated* in the following way: Let $Z(t)$ be a 2-state discrete time Markov chain with states ON and OFF, as shown in Fig. 1.

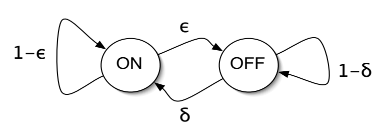

Fig. 1. A 2-state ON/OFF Markov chain for the arrival process $A(t)$.

The arrival process $A(t)$ is determined from $Z(t)$ as follows:

$$A(t) = \begin{cases} 1 & \text{if } Z(t) = ON \\ 0 & \text{if } Z(t) = OFF \end{cases}$$

Recall that $\lambda = \delta/(\epsilon + \delta)$. Assume that $\lambda < \mu$. We want to find the steady state probabilities for the Markov chain. It can be shown that a steady state exists (so that there is a unique probability vector solution to the stationary equation) whenever $\lambda < \mu$. Assume t is a time when the system is in steady state.

a) Compute $Pr[L(t) = 0$ AND $Z(t) = OFF] + Pr[L(t) = 0$ AND $Z(t) = ON]$ as a function of ϵ, δ, μ. Assume $\epsilon = 0.3$, $\delta = 0.1$, $\mu = 0.5$, and give a numeric value for $Pr[L(t) = 0$ AND $Z(t) = OFF] + Pr[L(t) = 0$ AND $Z(t) = ON]$ that is accurate to 10 decimal places.

b) Compute $Pr[L(t) = 0$ AND $Z(t) = ON]$ as a function of ϵ, δ, μ. Assume $\epsilon = 0.3$, $\delta = 0.1$, $\mu = 0.5$, and give a numeric value for $Pr[L(t) = 0$ AND $Z(t) = ON]$ that is accurate to 10 decimal places.

c) Compute $Pr[L(t) = i$ AND $Z(t) = ON]$ and $Pr[L(t) = i$ AND $Z(t) = OFF]$ for all $i \in \{0, 1, 2, \ldots\}$. Assume $\epsilon = 0.3$, $\delta = 0.1$, $\mu = 0.5$, and give a numeric value for $Pr[L(t) = 7]$ that is accurate to 10 decimal places.