EE549: Problem Set #4 part 2
Due Monday Feb. 25

This is part 2 of the problem set 4, and is worth 50 points.

I. INTEGRATING A PERIODIC FUNCTION

Let $\mu(t)$ be a periodic function of period P, so that:

$$\mu(t) = \mu(t-P) \quad \text{for all time } t$$

Define the constant A as follows: $A \triangleq \int_0^P \mu(\tau)d\tau$. Prove the following:

$$\int_t^{t+P} \mu(\tau)d\tau = A \quad \text{for all time } t$$

That is, this problem asks you to prove that the integral of a periodic function is the same when integrated over any interval that has duration equal to the period.

II. LEAKY BUCKET INPUTS INTO TREES

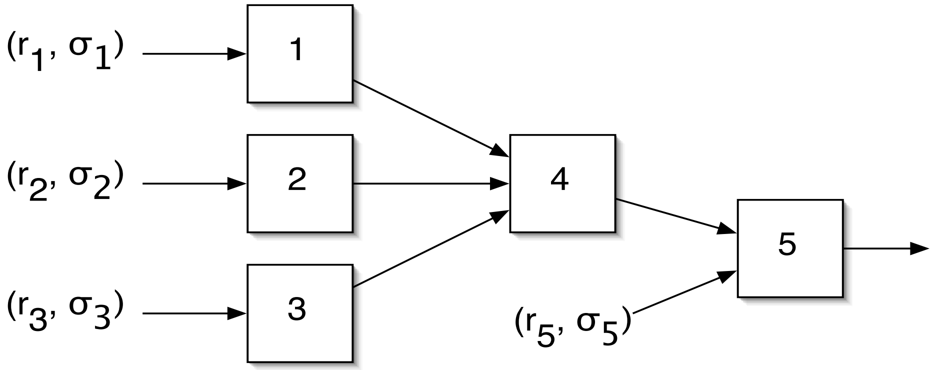

Fig. 1. A 5-node tree for with constant service rates $\mu_1, \ldots, \mu_5$.

Consider the 5-node tree of Fig. 1 with four leaky bucket inputs $X_i(t)$ with parameters (r_i, σ_i) for $i \in \{1, 2, 3, 5\}$. Suppose that service is work conserving and FIFO in all nodes, and that each node i has a constant processing rate μ_i for $i \in \{1, 2, 3, 4, 5\}$. Assume that the following 5 inequality constraints hold:

$$r_1 \leq \mu_1 \ , \ r_2 \leq \mu_2 \ , \ r_3 \leq \mu_3 \quad (1)$$

$$r_1 + r_2 + r_3 \leq \mu_4 \quad (2)$$

$$r_1 + r_2 + r_4 + r_5 \leq \mu_5 \quad (3)$$

a) Assuming a fluid departure model, find the worst case backlog bounds at all nodes. That is, prove that $U_i(t) \leq U_i^{max}$, and compute explicit values for the upper bounds U_i^{max}.

b) Prove that $Y_4(t) \sim (r, \gamma)$ for some rate r and some burst parameter γ (that is, you should compute r and γ).

c) What is the worst case end-to-end network delay of packets from stream 1 (this includes delay in node 1, node 4, and node 5).

d) Repeat (a)-(c) under a packet-based departure model with maximum packet size B_{max} (that is, find bounds on $U_i(t)$ for all i, find leaky bucket parameters for $\tilde{Y}_4(t)$ (the packet-based departure process of node 4), and find the worst case end-to-end delay of stream 1).

e) Repeat (a)-(c) under a packet-based departure model with constant packet sizes of size B bits.

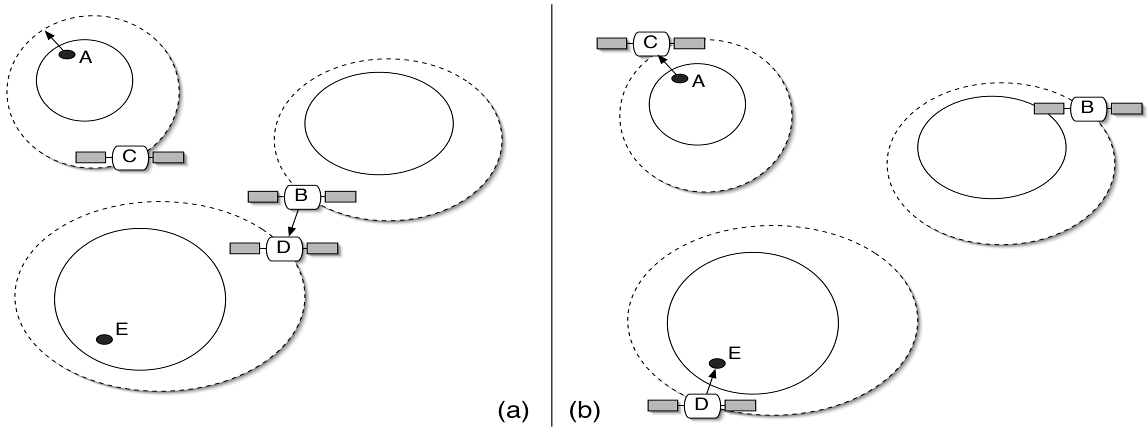

Fig. 2. (a) A periodic space network with satellites in a particular location defining connectivity. (b) A different connectivity arrangement.

III. NETWORK CALCULUS WITH SPACE NETWORKS

Consider the space network in Fig. 2, with 3 "planets" and 3 periodically orbiting satellites.

- Planet A has an orbiting satellite C with periodic connectivity, with 1 hour of connectivity followed by 4 hours of disconnection. Planet A can transmit to its satellite C at a rate of x_{AC} bits/second during the hour of connectivity, and at a rate of 0 during the next 4 hours of disconnectivity.
- Satellite C can connect to Satellite D periodically for 1 hour out of every 24 hours, with a rate of x_{CD} when connected, and 0 when not connected.
- Satellite B can connect to Satellite D periodically for 2 hours out of every 15 hours, with a transmission rate of x_{BD} during connectivity, and 0 when not connected.
- Satellite D can connect to the base on planet E periodically for 1 hour out of every 12 hours, with transmission rate of x_{DE} during connectivity, and 0 when not connected.

Suppose that planet A has data given by a leaky bucket stream $X_A(t)$ with rate and burst parameters (r_A, σ_A) (in units of bits/sec and bits, respectively). Suppose that Satellite B has data given by leaky bucket stream $X_B(t)$ with rate and burst (r_B, σ_B). All data must be delivered to the base on planet E. Note that Satellite D will store all data delivered to it in a single queue. Assume all service is FIFO, and assume a fluid flow departure model (which is a very good model for these timescales).

a) Draw a corresponding queueing model with 4 single-server queues and two input processes $X_A(t)$ and $X_B(t)$. Label the leaky bucket parameters of each server process, giving their precise rate and lag parameters $(\overline{\mu}, \gamma)$.

b) State a sufficient condition for stability in terms of r_A, r_B, and your leaky bucket parameters.

c) Give a bound on worst case backlog for each queue of the network.

d) Give an upper bound on the delay for a packet from Planet A to reach the base at Planet E.

IV. MULTI-INPUT MULTI-OUTPUT FEEDFORWARD NETWORKS

Consider the 4-node network of Fig. 3. There are four leaky bucket inputs $X_i(t)$ with parameters (r_i, σ_i) for $i \in \{1, 2, 3, 4\}$. Suppose that each stream has a fixed path defined as follows:

- Stream 1: Uses Nodes A, B.
- Stream 2: Uses Nodes A, B, D.
- Stream 3: Uses Nodes A, C, D.
- Stream 4: Uses Nodes C, D

Suppose all four nodes A, B, C, D have constant transmission rates $\mu_A, \mu_B, \mu_C, \mu_D$. Assume that:

$$\begin{aligned} r_1 + r_2 + r_3 &\leq \mu_A \\ r_1 + r_2 &\leq \mu_B \\ r_3 + r_4 &\leq \mu_C \\ r_2 + r_3 + r_4 &\leq \mu_D \end{aligned}$$

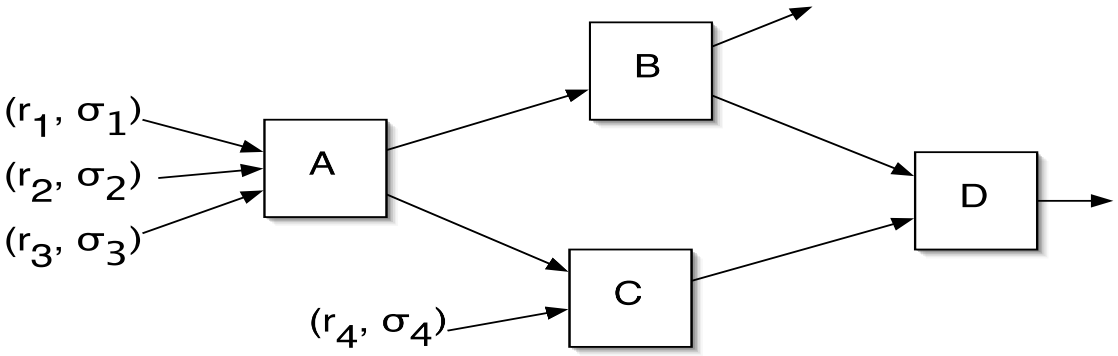

Fig. 3. A 4-node network with four input streams and two output ports.

Suppose that service is FIFO and a fluid flow departure process is used.

a) Compute bounds on $U_A(t)$, $U_B(t)$, $U_C(t)$, $U_D(t)$. Hint: Be careful to apply the network calculus correctly! You will find that the bounds are larger than you might at first think. You should likely define inter-mediate steps where you compute individual leaky bucket parameters of the appropriate output streams.

b) Compute bounds on the worst-case end-to-end delay for Stream 1, Stream 2, Stream 3, Stream 4.