Control rules for dispatching trains on general networks with multiple train speeds

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Abstract

Nowadays passenger trains are able to travel at a much faster speed compared to freight trains. However, the limitation of the rail infrastructure makes it more practical and cost effective to allow the passenger trains to share some portions of the railway tracks with slower freight trains. Previous research shows that a flexible switchable dispatching policy which can significantly reduce the delay of the fast trains over a dedicated track policy by reducing the likelihood of the fast trains catching up with the slow trains. The previous analysis assumes two possible speeds of trains. In this paper, we propose dispatching rules to specifically address the situation where trains travel at multiple levels of speed. We develop three dispatching policies for double-track segments. The best switchable policy reduces the average train delay by 21% over a dedicated track policy. With the existence of crossovers in the middle of the double-track segment, the switchable policy can reduce the average train delay by as high as 41.9%. Three triple-track dispatching rules are also studied.
1 Introduction

Railway has always been an effective mode to transport both people and goods. Freight trains are about four times more fuel efficient than trucks and passenger trains are popular because they can comfortably transport people to their destinations on time at a low cost. Nowadays, with the advancement of technology, passenger trains are able to travel at a much faster speed compared to freight trains. However the limitation of the rail infrastructure makes it more practical and cost effective to allow passenger trains to share some portions of the railway tracks with slower freight trains. If a faster passenger train catches up with a freight train on the railway track and there are no crossover junctions, the nature of the railway transportation determines that the passenger train has to follow the freight train at the speed of the freight train while keeping a safety headway from the freight train.

The detailed schedules of the trains are not always available before their movement (i.e. it is common to see freight trains travel without a pre-determined schedule). Also, normally the schedules only contain rough time windows for train departures and arrivals and the detailed movements of the trains need to be determined by the dispatchers in real time. In this case, we need simple dispatching rules to efficiently guide the trains through the network and a method to accurately predict the delay time of trains under such dispatching rules.

Mu and Dessouky (2013) propose a flexible switchable dispatching policy which can significantly reduce the delay of the fast trains over a dedicated track dispatching policy by reducing the likelihood of the fast trains catching up with the slow trains. In a dedicated track policy, the tracks can only be used by trains traveling in a designated direction. In a switchable policy, the tracks can be used by trains traveling in both directions. In order to make their analytical delay functions computationally tractable, they only consider two possible train speeds in the network (fast and slow trains) and ignore the effects of train length and safety headways between the trains. However in reality, trains can travel at multiple levels of speed in
the network (i.e., the passenger train company might provide train service with different travel times for the same train route at different train fares). This paper proposes dispatching rules to specifically address the situation where trains travel at multiple levels of speed. Simulation models are used to accurately predict the delay of our proposed policies. The simulation models also consider the train lengths and safety headways so that the predicted results match more closely with real world operations. Besides double-track railway segments, this paper also studies similar switchable dispatching policies for triple-track segments.

Two major techniques have been applied to study the delay of railway operations in the literature: simulation model and queueing theory. Frank (1966) studies delay on a single track. He assumes a single train speed and deterministic travel times. He also studies the cycle time of trains and number of trains needed to meet a certain level of demand for transportation. Greenberg et al. (1988) use queueing models to predict dispatching delays on single-track, low speed rail segments. They calculate the train delay based on a simple dispatching rule on the single-track segment. They extend the analysis on one single-track segment to the entire railway network. Chen et al. (1990) study the delay for different types of trains on a single track. The mean and variance of train delay are approximated by solving a system of nonlinear equations. Harker et al. (1990) extend Chen’s model to partially double-track railway segments. Ozekici et al. (1994) apply Markov chain techniques to study the knock-on delays. They study different arrival patterns of passengers and dispatching policies. The average passenger waiting time is derived explicitly. Huisman et al. (2001) study the delay of railway comprising heterogeneous trains. The delay times of trains are obtained by solving a system of differential equations. Huisman et al. (2002) propose a solvable queueing network model to predict network-wide train delays. Their analytical results closely match with real delay data of the Netherlands railway system. Yuan et al. (2007) optimize the capacity utilization of stations by using a new stochastic model of train delay propagation, which is associated with pre-determined timetables, whereas the delay in this paper is referring to simple knock-on delay.

Dessouky et al. (1995) and Lu et al. (2004) model train movements through
complex railway networks. Their model simulates multiple train speeds, headways, train lengths and acceleration and deceleration rates of trains. Hooghiemstra et al. (1998) build a simulation model for transportation planners to study network level dynamics under different timetables. Yalçinkaya and Bayhan (2009) use a simulation modeling and response surface methodology approach to determine the optimal settings such as headway to minimize passenger waiting times. Medanic and Dorfman (2002) propose a discrete event model using a travel-advance strategy (TAS) to schedule trains on a single line and is extended by Dorfman and Medanic (2004) for general networks and incorporated in a simulation model of an actual real system. Li et al. (2008) present an alternative simulation model that implements an algorithm that makes use of global information of the system state to schedule trains. The simulation models can be at different scales and closely imitate real situations.

In this paper, we use simulation models to study three dispatching rules for double-track railway segments with heterogeneous traffic. The first switchable policy is similar to the policy proposed in Mu and Dessouky (2013). This policy switches the fast train, if it has potential delay on its designated track, to the designated track for the opposite direction if that track is empty. The earlier paper only considered the case of two possible train speeds while in this paper we generalize to consider multiple speeds the train can travel. The second policy has a smarter condition to dispatch the trains by considering the speed of the attempting switching train. The last policy also considers the blocking time in the dispatching rule. That is, fast trains are switched to the opposite direction designated track if they do not extend the current busy period on the opposite direction designated track for too long. We also study the improvement of the efficiency of the dispatching rules obtained by placing crossovers in the segment. Three dispatching rules are proposed for triple-track segments. The first dispatching policy is considered to be conservative in terms of switching fast trains, whereas the second one is aggressive in terms of switching tracks. The third dispatching rule, which performs the best, is the hybrid of the first two policies.

The rest of the paper is organized as follows. Section 2 introduces the dispatching polices for double-track segments with trains travelling at multiple speeds.
Section 3 extends the dispatching policy to better utilize crossovers in the railway segment. Section 4 studies the dispatching policies for a typical triple-track railway segment. Finally, Section 5 summarizes the findings of this paper.

2 Dispatching rules for double-track segments with multiple train speeds

2.1 Descriptions of dispatching policies

Mu and Dessouky (2013) study the dispatching policies for double-track segment when there are only two train speeds. They focused on developing an analytical model to predict the expected delay with a switchable policy. To keep the model analytical model tractable they assumed the train length and safety headways were both zero. In this paper, we extend the study on double-track to the case where trains are travelling at multiple speeds and consider the train length and safety headways in our analysis.

Figure 1 shows a typical double-track railway segment between two major intersections. The length of the track segment is denoted by D. There are multiple types of trains travelling on the track segments. Each type of train is identified by its speed and we assume that the arrival of each train type at each end of the double-track segment is an independent Poisson Process. The upper and lower tracks of the segment can be traveled in both directions. The free running time of the train is defined to be the minimum traveling time of the train assuming there is no other traffic in the network. The delay time of the train can be calculated as:

\[ \text{Delay} = \text{Completion time} - \text{Arrival time at the segment} - \text{Free running time} \]
A delay can occur
1. when a train tries to keep the required headway length with another train traveling ahead of it;
2. when a faster train catches up with a slower train traveling in the same direction so that the faster train has to travel at the speed of the slower train, while keeping the required headway;
3. when a train arrives at the track segment and it has to wait for the track to be cleared from being occupied by trains traveling in the reverse direction.

Probably the easiest dispatching policy for a double-track segment is to dedicate one track of traffic to one direction (i.e., all eastbound trains travel on the lower track while all westbound trains travel on the upper track). We refer to this policy as a dedicated policy. The drawback of the dedicated policy is that it can be likely for a fast train to catch up with a slower train on its dedicated track. If the fast train catches up with a slower train, it has to keep a safety distance between the slower train and travel at the speed of the slower train. Thus, the fast train can experience a significant delay if there is a large difference in train speeds.

Next we introduce three dispatching rules that allow trains to travel in either track segments. Without loss of generality, for the following dispatching rules, let the lower track be the designated track for trains traveling eastbound and let the upper track be the designated track for trains travelling westbound. We refer to the three dispatching rules as Switchable2-I, Switchable2-II and Switchable2-III policy. The Switchable2-I policy considers the potential delay of the arriving train when deciding to switch the train. Besides the potential delay, the Switchable2-II policy also considers the speed of the arriving train when deciding to switch the train. The faster the speed is,
the more likely the train will switch. To increase the switching frequency, the Switchable2-III policy switches the train to the other track in cases where the other track is not completely empty of trains.

Switchable2-I policy (Figure 2)

1. Upon arrival, if the designated track is occupied by trains travelling in the opposite direction, the arriving train waits for the opposing moving trains to finish traveling on the segment before proceeding on its designated track.
2. Upon arrival, if the designated track is not occupied by trains travelling in the opposite direction, and if the potential length of delay of the arriving train traveling on its designated track is less than $\omega$, the arriving train will start traveling on its designated track.
3. Upon arrival, if the designated track is not occupied by trains travelling in the opposite direction, and if the potential length of delay of the arriving train traveling on its designated track is greater than $\omega$, the arriving train will attempt to switch to the other track. The arriving train will use the reverse direction track only if it is empty. If the reverse direction track is occupied by trains travelling in either direction, the arriving train will use its designated track.

The Switchable2-I policy is similar to the switchable policy described in Mu and Dessouky (2013), where they derive analytical equations to measure the delay assuming a no headway requirement since they assume the trains are infinitesimally small. The Switchable2-I policy will attempt to switch an arriving train if it will catch up with a slower train on its designated track and its potential delay is greater than $\omega$. The optimal value of $\omega$ ranges from 0 to the time difference between the free running times of the slowest and fastest train. If a fast train catches up with a slower train near the end of its designated track, the potential delay of the fast train might not be significant enough for the policy to switch the fast train to the other track, since usage of the other track might block the traffic in the other direction. When a train attempts to
switch, the policy only allows it to switch when the reverse direction track is empty. If the reverse direction track is occupied by another switched train which is traveling in the same direction as the train attempting to switch, the policy prohibits the train from switching so not to extend the reverse direction busy period for the other track.

![Diagram](image)

**Figure 2: Switchable2-I policy**

With many different train speeds, even a slow train can catch up with another slower train and experience delay on its designated track. If we only consider the potential delay on the designated track as the criterion to switch the train, we might tend to switch some slow trains. The switched slow trains will block the other track for a long time, which is not desired for the traffic in the other direction. Intuitively, if both a relatively fast and a relatively slow train have the same potential delay on the
designated track. The relatively fast train should be switched because the fast train can finish traveling on the reverse direction track in a shorter amount of time. Thus a higher potential delay and a higher train speed should lead to a higher chance to switch.

Let \( D_p \) denote the potential length of delay an arriving train will experience on its designated track. Let \( S_{ar} \) denote the speed of the arriving train. In the Switchable2-II policy, instead of having \( D_p \geq \omega \) as the criterion to switch the arriving train, a more complicated criterion \( \alpha D_p + \beta S_{ar} \geq \delta \) is used, where \( \alpha \), \( \beta \) and \( \delta \) are parameters.

A good assignment of the values of \( \alpha \), \( \beta \) and \( \delta \) can be obtained by discretizing them and enumerating the parameters in multiple simulation runs.

**Switchable2-II policy (Figure 3)**

1. Upon arrival, if the designated track is occupied by trains travelling in the opposite direction, the arriving train waits for the opposing moving trains to finish traveling on the segment before proceeding on its designated track.

2. Upon arrival, if the designated track is not occupied by trains travelling in the opposite direction, and if \( \alpha D_p + \beta S_{ar} < \delta \), the arriving train will start traveling on its designated track.

3. Upon arrival, if the designated track is not occupied by trains travelling in the opposite direction, and if \( \alpha D_p + \beta S_{ar} \geq \delta \), the arriving train will attempt to switch to the other track. The arriving train will use the reverse direction track only if it is empty. If the reverse direction track is occupied by trains travelling in either direction, the arriving train will use its designated track.
Switchable2-III is based on the Switchable2-II policy. For the first two policies, the attempted switching trains will switch if the reverse direction track is empty. The idea being that if another train is allowed to travel on the reverse direction track when one is already traveling on it may cause a significant amount of time that segment is blocked for a train traveling on its designated direction. However, in the case of multiple speeds, if the reverse direction track is occupied by a switched train, it will do no significant harm if we switch another faster train to the reverse direction track, given the latter switched train can catch up with the former switched train. To extend this idea, in the Switchable2-III policy, if the reverse direction track is occupied by a switched train, a newly arriving train can switch to the reverse direction track only if
the arriving train will extend the busy period on the reverse direction track by no longer than $\mu$ time units. A good value of $\mu$ can be found by discretizing it and enumerating in multiple simulation experiments.

Switchable2-III policy (Figure 4)

1. Upon arrival, if the designated track is occupied by trains travelling in the opposite direction, the arriving train waits for the opposing moving trains to finish traveling on the segment before proceeding on its designated track.

2. Upon arrival, if the designated track is not occupied by trains travelling in the opposite direction, and if $\alpha D_p + \beta S_{ar} < \delta$, the arriving train will start traveling on its designated track.

3. Upon arrival, if the designated track is not occupied by trains travelling in the opposite direction, and if $\alpha D_p + \beta S_{ar} \geq \delta$, the arriving train will attempt to switch to the other track. The arriving train will use the reverse direction track if it is empty. If the reverse direction track is occupied by trains travelling in the same direction as the arriving train and if the arriving train extends the current busy period on the reverse direction by no longer than $\mu$ time units, the arriving train will switch to the reverse direction track. Otherwise, the arriving train travels on its designated track.
2.2 Numerical experiments

Numerical experiments are conducted to compare the performance of the three
dispatching policies. Simulation is used to compute the average delay of the various dispatching policies. The simulation model is built using Arena (Kelton et al., 2009). In the basic settings of the experiments, the arrival rate of the trains in one direction is 0.16 trains per minute and the length of the track segment is eight miles. There are five possible speeds for the trains. The speed of each arriving train is equally likely to be 50 m/h, 70 m/h, 90 m/h, 120 m/h and 140 m/h. The lengths of the trains travelling at speed of 50 m/h and 70 m/h are 5,000 feet and 6000 feet, respectively. The trains travelling at speed of 90 m/h, 120 m/h, and 140 m/h have lengths of 1000 feet. The reason the faster trains are shorter is that they are more likely to be passenger trains instead of freight trains. The safety headway between two consecutive trains is set to be one mile. For the numerical experiments, all the distance units are in miles and the time units are in minutes.

For the Switchable2-I policy, the optimal value of \( \omega \) needs to be determined for each scenario. The possible values for \( \omega \) range from 0 to \( D/S_{sl} - D/S_{fa} \), where \( S_{sl} \) and \( S_{fa} \) denote the slowest and fastest train speeds possible on the track, respectively. In the numerical experiment, we discretize the value of \( \omega \) into steps of 0.1 and enumerate all the possible values. The value of \( \omega \) which gives the smallest average delay for all the trains is used in the Switchable2-I policy.

For the Switchable2-II policy, the values of \( \alpha \), \( \beta \) and \( \delta \) need to be determined. Without loss of generality, the value of \( \alpha \) can be fixed at 1 and a good assignment of values of \( \beta \) and \( \delta \) can be obtained by discretization and enumeration. There are no obvious upper bounds for \( \beta \) and \( \delta \). In the experiment, the upper bound of \( \beta \) is set to 2 and the upper bound of \( \delta \) is set to \( 1(D/S_{sl} - D/S_{fa}) + 2S_{fa} \). The best values of \( \beta \) and \( \delta \) are always found to be far below their upper bounds. The best values of \( \beta \) and \( \delta \) which produce the lowest average train delay are used by the Switchable2-II policy.

The values of \( \alpha \), \( \beta \) and \( \delta \) in the Switchable2-III policy are determined the
same way as in the Switchable2-II policy. The extra parameter $\mu$ has an upper bound of $(D+1.136)/S_D$ (where 1.136 accounts for the longest train length in unit of miles) and a lower bound of 0. The parameter $\mu$ is also discretized and enumerated together with the other two parameters.

Table 1 shows the average delays of the four policies when the arrival rate is 0.16 trains per minute. The average is based on 10 simulation runs. By choosing a good switching threshold value, the Switchable2-I policy is able to significantly reduce the average delay from the dedicated policy. However, the more complex switching condition function of Switchable2-II policy reduces the delay some more and the Switchable2-III policy is able to further reduce the average train delay. Compared to the dedicated policy, the best switchable policy, the Switchable2-III policy reduces the average train delay by 21%.

<table>
<thead>
<tr>
<th>Arrival rate = 0.16</th>
<th>Average train delay</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dedicated policy</td>
<td>0.9666</td>
<td>0.0013</td>
</tr>
<tr>
<td>Switchable2-I</td>
<td>0.8202</td>
<td>0.0011</td>
</tr>
<tr>
<td>Switchable2-II</td>
<td>0.7923</td>
<td>0.0012</td>
</tr>
<tr>
<td>Switchable2-III</td>
<td>0.7623</td>
<td>0.0012</td>
</tr>
</tbody>
</table>

Table 2 shows the delay of the fastest trains (e.g. high priority passenger trains) under different dispatching policies on the 8-mile long track segment. The Switchable2-III policy is able to reduce the delay under the dedicated policy by 0.484 minutes. Considering that the normal route length of the passenger train is much longer than 8 miles (e.g., in downtown Los Angeles area, the route length of passenger trains can be as high as 40 miles), the potential reduction of delay for passenger trains over
their entire routes could be significant.

Table 2: Comparisons of four policies (fastest train delay)

<table>
<thead>
<tr>
<th>Arrival rate = 0.16</th>
<th>Average train delay</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dedicated policy</td>
<td>1.7178</td>
<td>0.0013</td>
</tr>
<tr>
<td>Switchable2-I</td>
<td>1.3855</td>
<td>0.0011</td>
</tr>
<tr>
<td>Switchable2-II</td>
<td>1.3313</td>
<td>0.0014</td>
</tr>
<tr>
<td>Switchable2-III</td>
<td>1.2335</td>
<td>0.0013</td>
</tr>
</tbody>
</table>

Figure 5 shows the performance of the four policies as the arrival rate varies. The relative performances between the four policies remain the same as seen in Table 1. As expected, the delay increases as the arrival rates increase but the gap between the dedicated policy and the switchable policy reduces at the increased rates since there is less opportunity for switching when there are more trains in the network. Figure 6 shows the performance of the four policies as the track length varies. As the figure shows, the average delay relationship with the track segment length is similar to the arrival rates since there is less opportunity for switching with longer segments assuming no crossovers within the segment.

Figure 5: Varying arrival rates
3 Dispatching rules for double-track segments with crossovers

3.1 Description of dispatching rule

Now suppose the double-track segment has a crossover in the middle of the segment. The introduction of the crossover at the middle can significantly increase the effectiveness of the switchable policy. With the help of the crossover, trains can switch to the other track at the beginning of the track and then switch back in the middle. Also, trains can switch to the other track in the middle of the track. In both ways, the switched trains do not have to travel through the entire segment of the other track. Thus the double track segment could be better utilized. Next, we are going to describe a switchable policy (namely, Switchable2-w/cross) which is based on the Switchable2-III policy. Treating the crossover in the middle of the segment as a station connecting two halves of the segment, the Switchable2-w/cross policy dispatches trains almost the same as what the Switchable2-III policy will do for those two double-track segments connected together. The Switchable2-w/cross policy is designed to dispatch trains with multiple speeds on a double-track segment with crossovers in

Figure 6: Varying track lengths
the middle. The description of the Switchable2-w/cross policy below focuses on the eastbound trains and uses the notations in Figure 7. Let $D_p^{EB1}$ denote the potential delay of the arriving train on track segment EB1. Let $D_p^{EB2}$ denote the potential delay of the train on track segment EB2 as it arrives at EB2.

![Double-track railroad segment with crossovers](image)

**Switchable2-w/cross policy**

1. Upon arrival, if EB1 is occupied by trains travelling in the opposite direction, the arriving train waits for the opposing moving trains to finish traveling on the segment before proceeding on EB1.

2. Upon arrival, if EB1 is not occupied by trains travelling in the opposite direction, and if $\alpha_i D_p^{EB1} + \beta_i S_{ar} < \delta_i$, the arriving train will start traveling on EB1.

3. Upon arrival, if EB1 is not occupied by trains travelling in the opposite direction, and if $\alpha_i D_p^{EB1} + \beta_i S_{ar} \geq \delta_i$, the arriving train will attempt to switch to WB1. The arriving train will use WB1 if it is empty and if EB2 is not occupied by westbound trains. If WB1 is occupied by eastbound trains and if the arriving train extends the current busy period on WB1 by no longer than $\mu_i$ time units, the arriving train will switch to WB1. Otherwise, the arriving train travels on EB1.

4. When an eastbound train reaches the end of track EB1, if EB2 is occupied by trains travelling in the opposite direction, the train at the end of EB1 waits for the opposing moving trains to finish traveling on EB2 before proceeding on
EB2. When an eastbound train reaches the end of track EB1, if EB2 is not occupied by trains travelling in the opposite direction, and if \( \alpha_2 D_{p}^{EB2} + \beta_2 S_{ar} < \delta_2 \), the eastbound train will start traveling on EB2. But if \( \alpha_2 D_{p}^{EB2} + \beta_2 S_{ar} \geq \delta_2 \), the eastbound train will attempt to switch to WB2. The eastbound train will use WB2 if it is empty. If WB2 is occupied by eastbound trains and if the arriving train extends the current busy period on WB2 by no longer than \( \mu_2 \) time units, the train at the end of EB1 will switch to WB2. Otherwise, the train travels on EB2.

5. When an eastbound train reaches the end of track WB1, if \( \alpha_2 D_{p}^{EB2} + \beta_2 S_{ar} < \delta_2 \), the eastbound train will start traveling on EB2. But if \( \alpha_2 D_{p}^{EB2} + \beta_2 S_{ar} \geq \delta_2 \), the eastbound train will attempt to continue to WB2. The eastbound train will use WB2 if it is empty. If WB2 is occupied by eastbound trains and if the train at the end of WB1 extends the current busy period on WB2 by no longer than \( \mu_2 \) time units, the train at the end of WB1 will continue to WB2. Otherwise, the train travels on EB2.

### 3.2 Numerical experiments

In this numerical experiment, the length of the track is set to eight miles. The speed and length of each arriving train have the same characteristics as in Section 2.2. The crossover is located in the middle of the eight-mile long track segment. Suitable values of the parameters \( \alpha_1 \), \( \beta_1 \), \( \delta_1 \), \( \mu_1 \), \( \alpha_2 \), \( \beta_2 \), \( \delta_2 \) and \( \mu_2 \) in the Switchable2-w/cross policy can be obtained as in the Switchable2-I-II policy.

Figure 8 shows the average delay under both the dedicated policy and the Switchable2-w/cross policy. The results clearly show that the Switchable2-w/cross policy dominates the dedicated policy as the arrival rates vary. In this numerical experiment, the Switchable2-w/cross policy can reduce the average train delay by as high as 41.9%.
4 Dispatching rules for triple-track segments

4.1 Description of dispatching rules

Triple track rail segments are common in metropolitan areas and busy railway corridors. In this section, we extend our switchable policy on double-track segments to triple-track segments. For simplicity, in this section we study the case where trains travel at only two speeds (fast and slow). However, the proposed dispatching policies can be easily extended for the case of multiple train speeds. A typical triple-track segment is illustrated in Figure 9.
Similar to the switchable policy on double-track segments, let the upper track be the designated track for westbound trains and let the lower track be the designated track for eastbound trains. All the trains that switch their tracks will be traveling on the middle track. When a fast train arrives, if it will catch up with a slow train on its designated track, it will attempt to switch to the middle track. For the triple-track segment, the criterion on deciding whether the newly arriving fast train should switch changes from the criterion for double-track segments. For double-track segments, the fast train can only switch when the other track is empty. When the other track is occupied by another switched fast train, the arriving fast train is not allowed to switch, since there might be some trains traveling in the opposite direction waiting to use the other track. However since the middle track is only traveled by fast trains for the triple-track case, intuitively, we should increase the number of switching fast trains. Next we will introduce three switchable dispatching rules for triple-track segments. These three dispatching rules differ in the criterion for switching fast trains. The first policy, Switchable3-I policy, switches the fast trains to the middle track only when the fast trains can start travelling immediately on the middle track. The second policy, Switchable3-II policy, switches the fast trains to the middle track as long as the fast trains have potential delay on their designated tracks. The last policy is the hybrid of the Switchable3-I and Switchable2-II policy. It compares the potential delay of the fast train on its designated track and the middle track to determine whether to switch the fast train.

Switchable3-I policy (Figure 10)

1. All slow trains travel on their designated track.
2. When a fast train arrives, if its potential delay on its designated track is less than $\omega$, the fast train starts travelling on the designated track.
3. When a fast train arrives, if its potential delay on its designated track is greater than $\omega$, then the fast train will try to use the middle track. The fast train will use the middle track only if it is empty or occupied by trains traveling in the same direction as the arriving fast train. If the middle track is occupied by
trains travelling in the opposite direction of the arriving fast train, the arriving fast train will use its designated track.

**Switchable3-II policy (Figure 11)**

1. All slow trains travel on their designated track.
2. When a fast train arrives, if its potential delay on its designated track is less than \( \omega \), the fast train starts travelling on the designated track.
3. When a fast train arrives, if its potential delay on its designated track is greater than \( \omega \), then the fast train will use the middle track. The fast train will start traveling on the middle track if the track is empty or occupied by trains traveling in the same direction as the arriving fast train. If the middle track is...
occupied by trains travelling in the opposite direction of the arriving fast train, the arriving fast train will wait at the beginning of the middle track for the busy period of the opposite direction to finish and then start traveling on the middle track.

The Switchable3-II policy dispatches more fast trains to use the middle track than the Switchable3-I policy. As long as the arriving fast train will have potential delay on its designated track, the Switchable3-II dispatching policy switches the train to the middle track, regardless of the status of the middle track. The intuition behind the Switchable3-II policy is that, given that the middle track is traversed by only fast trains, even if the arriving train has to wait for the track, it will probably not wait for a long time.

Figure 11: Switchable3-II policy
Switchable 3-III policy (Figure 12)

1. All slow trains travel on their designated track.

2. When a fast train arrives, if its potential delay on its designated track is less than $\omega$, the fast train starts travelling on the designated track.

3. When a fast train arrives, if its potential delay on its designated track is greater than $\omega$, then the fast train will attempt to use the middle track. The fast train will start traveling on the middle track if the track is empty or occupied by trains traveling in the same direction as the arriving fast train. If the middle track is occupied by trains travelling in the opposite direction of the arriving fast train and the potential delay on its designated track is smaller than the waiting time at the middle track assuming no future fast trains travelling in the opposite direction will arrive to the middle track, the arriving fast train will use its designated track. If the potential delay on its designated track is larger than the waiting time at the middle track assuming no future fast trains travelling in the opposite direction will arrive to the middle track, the arriving fast train will wait at the beginning of the middle track for the busy period of the opposite direction to finish and then start traveling on the middle track.
Switchable3-III policy is a hybrid of the Switchable3-I and Switchable3-II policies. It switches fast trains more smartly than both polices. Switchable3-II policy switches the arriving fast train to the middle track regardless of the waiting time at the middle track. Whereas Switchable3-III policy switches the arriving fast train to the middle track when the waiting time at the middle track might be shorter than the potential delay of the fast train on its designated track.
4.2 Numerical Experiments

In order to compare the performance of all three policies, the following numerical experiments have been conducted. Our base settings are:

- Length of track segment, 8 miles
- Speed of a fast train, 140 mph
- Speed of a slow train, 50 mph
- Length of a fast train, 1000 feet
- Length of a slow train, 6000 feet
- Headway between consecutive trains, 1 mile

Triple-track segments can handle more trains than a double-track segment. Thus for a triple-track segment, trains arrive at a higher arrival rate than for a double-track segment. Each arriving train is equally likely to be a slow or a fast train. The optimal values of the threshold parameter $\omega$ of the three dispatching policies are found by enumeration.

In Figure 13, it is clear that when the speed of the fast train decreases from 140 mph to 90 mph, under the Switchable3-I policy, the average train delay decreases. The intuitive explanation being that since fast train delays are mainly caused by catching up with slow trains on their designated tracks and when the speed difference between slow and fast train gets smaller, the delay of a fast train should decrease. However, for the Switchable3-II policy, such a relationship would not be true as we can see in Figure 14. To explain this, recall that under the Switchable3-II policy, fast trains will switch to the middle track regardless of the status of the middle track. If the speed of the fast train decreases, it takes more time for a fast train to traverse the middle track. Thus switched fast trains have to wait for a longer time if the middle track is occupied by trains in the opposite direction. This extra waiting time at the middle track increases the total expected delay of trains. Figure 15 shows the results of the Switchable3-III policy. Since the Switchable3-III policy is a hybrid of the Switchable3-I and Switchable3-II policy, the increasing and decreasing effects on the average train delay, as the speed of the fast train decreases, have been neutralized. Thus, for the Switchable3-III policy, the
average train delay does not change much as the speed of the fast train decreases.

Figure 13: Switchable3-I policy

Figure 14: Switchable3-II policy
Figures 16 and 17 show the comparisons of the three policies as the arrival rates vary. The only difference of the experiment settings in the two figures is that in Figure 16 the fast trains travel at 140 mph and in Figure 17 the fast trains travel at a speed of 90 mph. When the speed difference between fast and slow trains is large (e.g., speed of fast train is 140 mph), the Switchable3-II policy performs better than the Switchable3-I policy, whereas when the speed difference is smaller (e.g. speed of fast train is 90 mph), the Switchable3-I policy outperforms the Switchable3-II policy. Thus there is no absolute dominance between the Switchable3-I and Switchable3-II policies. The Switchable3-III policy dominates the other two policies, as expected.
5 Conclusions

In this paper, we focus on developing dispatching policies for double-track and triple-track segments. As opposed to a regular dedicated policy, we propose several dispatching policies based on the idea of switching faster trains, that would catch up with slower trains, to the other track. We first study the dispatching policies for the
situation where trains travel at multiple speeds. We develop three dispatching policies that are extensions of the switchable policy of Mu and Dessouky (2013). The three dispatching policies are compared using simulation. In the simulation, we have considered the train lengths and safety headway length. The best switchable policy reduces the average train delay by 21%. With the existence of crossovers in the middle of the double-track segment, the Switchable2-w/cross policy can reduce the average train delay by as high as 41.9%. Three triple-track dispatching rules are also studied. The last of the three proposed dispatching rules dominates the other two rules, since the last dispatching rule combines the advantages of the first two rules.
References


