Coordinated Freight Routing with Individual Incentives for Participation

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\begin{abstract}
The sharp increase of e-commerce over the last few years has led to an increase in the volume of trucks both in ports and in commercial areas. Truck traffic has a negative impact on traffic flow in general due to the size of trucks and their slower dynamics. The continuously increasing use of navigation apps has led drivers to make their routing decisions in an independent manner in an effort to minimize their own individual travel time, with possible significant deviation from a socially optimum solution. In this paper, we consider the use of coordinated routing in order to achieve load balancing. Users send their OD matrices as well as their preferred departure time to the coordinator who gives them routing instructions based on a socially optimum cost. This design enables us to derive sufficient conditions under which we prove the existence of truthful in equilibrium and budget balanced on average mechanisms, which are able to create individual incentives for voluntary participation of the truck drivers. Subsequently, we design our mechanism in a way that it only uses a minimal set of sufficient conditions in order to guarantee the existence of a solution and maximize its efficiency. Finally, the extensive simulation results of our approach in the Braess and the Sioux Falls networks demonstrate that the proposed mechanism can approach the system optimum solution.

\end{abstract}

\textbf{Index Terms}—Freight, Load Balancing, Mechanism Design, Incentive Compatibility, User Equilibrium, Routing.

\section{Introduction}

The transportation sector constitutes an integral part of the global economy. For example, in the United States (U.S.), it comprises 8.9\% of the nation's economic activity as measured by the gross domestic product (GDP) \cite{1} while in Europe the corresponding percentage is about 5\% \cite{2}. However, traffic congestion on the U.S. highways annually costs the trucking industry more than $63 billion in operating expenses, including 996 million hours of lost productivity \cite{3}. These reports together with the fact that the impact of trucks on traffic flow is much higher due to their slower dynamics \cite{4}, \cite{5} make the truck routing problem of major importance.

In the absence of cooperation, the users of the network act independently in an effort to minimize their own individual cost \cite{6}, \cite{7}. This behavior of the network users is known in the transportation literature as User Equilibrium (UE) \cite{8}, \cite{9}. On the other hand, the situation where the network users act in a manner which contributes to the minimization of a social welfare cost is known as System Optimum (SO). The inefficiency of the UE compared to the SO has been addressed to the literature as the Price of Anarchy (PoA) \cite{10}. Bounds on the PoA of general networks for different types of latency functions have been calculated \cite{11}, \cite{12}. Recently, the PoA of realistic transportation networks using traffic data has been also estimated \cite{13}, showing the necessity for its reduction.

Several research efforts have been made in order to reduce the PoA in transportation networks where most of them use the idea of congestion pricing \cite{14}, \cite{15}, in which each agent pays a fee corresponding to the additional cost its presence causes to the total system cost. Positive results on the willingness of drivers to participate in such policies have been presented in \cite{16}. Additional design criteria concerning equity and fairness considerations of congestion pricing applications have been studied in \cite{17}--\cite{21}. Related work on the sensitivity of the users with respect to pricing schemes due to their heterogeneity can be found in \cite{22}--\cite{24}.

The analysis of congestion pricing techniques and other relevant monetary schemes is based on traffic assignment models which can be either static or dynamic. Various static equilibrium models are presented in \cite{25} and methods for solving them in \cite{26}. Several extensions of these models include traffic models with stochastic demands \cite{27}, capacity uncertainties \cite{28} or models with incomplete information for the network users \cite{29}.

In this paper, we consider a non-atomic game theoretic model (with a continuum of users) with stochastic demands for the different origin-destination (OD) pairs presented in \cite{30}, but we further generalize it to the case where the traffic conditions vary with time. The users (players) of this model (game) are considered to be the truck drivers and their actions consist of the declaration of their desired OD pair as well as their preferred departure time.

After proving the existence of a UE for the defined model, we design a coordination mechanism whose purpose is the optimum assignment of the trucks into the network in a way that a social welfare cost is minimized. In contrast with the vast majority of literature studying pricing schemes in the form of tolls applied to the drivers, for instance \cite{31}--\cite{33}, our mechanism differs significantly since it also allows that some of the players (truck drivers) are getting paid in order to follow routes that may be costly to them but beneficial for the social optimum. This design allows us to create individual incentives...
for voluntary participation [34] of the truck drivers. Moreover, since the proposed mechanism also takes into account the different traffic conditions during the day, it may suggest to some players to receive a payment in order to change their trip time in an effort to alleviate traffic congestion problems during peak hours. However, the fact that the players know that they may get paid for scheduling their trip during a different time than the one they actually wanted, may lead them in a situation where they declare a different departure time than their preferred one in order to get a reimbursement while concurrently traveling during their desired time interval. To avoid this kind of exploitability of the mechanism, we derive sufficient conditions which prove the existence of truthful in equilibrium and budget balanced on average mechanisms [35], which are able to create individual incentives for voluntary participation of the truck drivers. Furthermore, we show that removing the budget balance on average property, these conditions guarantee the maximum possible earnings of a company willing to apply such a mechanism while concurrently satisfying the voluntary participation and the truthfulness criteria. Subsequently, in an effort to maximize the efficiency of our design, we propose an algorithmic procedure which generates those conditions which are necessary for the existence of monetary schemes with the aforementioned characteristics. Simulations on the benchmark network of Sioux Falls show that the proposed mechanism can lead to a SO solution.

The rest of the paper is organized as follows. In Section II, we describe mathematically the network model used and we prove the existence of a UE. The proposed game-theoretic mechanism is presented in Section III and simulation results of our approach are provided in Section IV. Section V presents the conclusions. Finally, the mathematical proofs are presented in the Appendix.

II. MATHEMATICAL FORMULATION

A. Network Model

The notation used throughout the paper is summarized in Table I. As mentioned earlier, we use a non-atomic game theoretic model with demand uncertainties for the OD pairs in which we take into account that the traffic conditions vary with time. The players of the game are considered to be the truck drivers whose actions include both the OD pair and the departure time selection. To capture the time-varying behavior of traffic, we assume that the coordinator defines a planning horizon and splits it into non-overlapping time intervals such that it guarantees that every truck driver will have sufficient time to complete his/her trip during one time interval. This assumption makes the notion of time and time interval to have the same meaning in our analysis. Let the variable \( t = 1, \ldots, N \) denote the time intervals of the planning horizon. For example, if the planning horizon is defined to be a day, \( t = 1, 2, 3, 4 \) represents morning, noon, afternoon and night respectively. Then, \( d_j^t \) is a random variable with finite support representing the demand of OD pair \( j \) during time interval \( t \). We further assume that each truck driver knows the distribution of all the random variables \( d_j^t \) (symmetric information model).

The transportation network is described by a graph \( G = (V, L) \) where \( V \) is the set of nodes and \( L = \{1, \ldots, m\} \) is the set of links where each link corresponds to a road segment. Each road segment is allowed to serve both passenger and freight transport vehicles. Hence, \( X_{lt}^p \) represents the number of passenger vehicles traversing road segment \( l \) during time interval \( t \) and \( X_{lt}^r \) stands for the corresponding number of trucks. In our model, we assume that the truck drivers make their routing decisions while knowing the number of passenger vehicles at each road segment. This assumption is not restrictive since passenger traffic has a repetitive behavior during the same day and time of the week in the absence of unexpected incidents. Note that in the case where they made their routing decisions based on the distribution of \( X_{lt}^p \), we could write the cost of each link as:

\[
E \left[ \sum_{l \in r} C_{lt}(X_{lt}^p, X_{lt}^r) \right] = E \left[ \sum_{l \in r} \hat{C}_{lt}(X_{lt}^r) \right] \tag{1}
\]
which is a function of $X_{lT}^t$ only and where $\tilde{C}_{IT}(X_{lT}^t)$ is defined as:

$$\tilde{C}_{IT}(X_{lT}^t) = E[C_{IT}(X_{lT}^t, X_{lT}^t)|X_{lT}^t]$$

(2)

Note that $C_{IT}(X_{lT}^t, X_{lT}^t)$ is assumed to be a known nonlinear function which represents the operation cost of a truck driver who traverses road segment $l$ during time interval $t$. It is assumed to be strictly increasing in both its arguments, has continuous first partial derivatives and $\partial C_{IT}/\partial X_{lT}^t$ is bounded away from zero.

Let $\alpha_{j,r}^t$ be the fraction of truck drivers with OD pair $j$, intended departure time interval $\bar{t}$, actual departure time interval $t$ who follow route $r \in R_j$. Then, the number of trucks in link $l$ during time interval $t$ can be expressed as:

$$X_{lT}^t = \sum_{j=1}^{v} \sum_{r \in R_j} \sum_{t \in r} d_{j}^{t} \alpha_{j,r}^t$$

(3)

where $v$ is the total number of OD pairs of the network, $N$ is the total number of time intervals of the planning horizon and $d_{j}^{t}$ is the demand of OD pair $j$ during time interval $t$.

In the following subsection, we consider the case where each truck driver makes individual routing decisions. This enables us to define the notion of the User Equilibrium and prove its existence for the network model used.

B. Routing without Coordination: User Equilibrium

In the absence of coordination, each truck driver selfishly routes himself/herself in an effort to minimize her/his own individual cost leading to the so-called User Equilibrium (UE) case [8] where no user (truck driver) has an incentive to unilaterally change his/her route and departure time interval selections as such a change will lead to a higher individual cost. Let us first define:

$$\alpha = \{\alpha_{j,r}^t : t, \bar{t} = 1, \ldots, N, j = 1, \ldots, v, r \in R_j\}$$

(4)

Using (4), let us denote by $F_{j,r}^t(\alpha)$ the expected cost of a truck driver in the OD pair $j$ with intended departure time interval $\bar{t}$, actual departure time interval $t$ who follows route $r$. Then, $F_{j,r}^t(\alpha)$ is given by the following equation:

$$F_{j,r}^t(\alpha) = E \left[ \sum_{l \in r} C_{IT}(X_{lT}^t, X_{lT}^t(\alpha)) + D(t, \bar{t}) \right]$$

(5)

where $X_{lT}^t(\alpha)$ is given by (3) and $D(t, \bar{t})$ represents the delay cost of not departing at the desired time interval $\bar{t}$.

The condition for an $\alpha$ to be a UE is that for every OD pair $j$, any desired departure time interval $\bar{t}$ and for any actual departure time interval $t$ and choice of route $r$ for which $\alpha_{j,r}^t > 0$, it holds:

$$F_{j,r}^t(\alpha) \leq F_{j,r}^{t',r}(\alpha)$$

(6)

for any alternative route $r' \in R_j$ and any other departure time interval $t'$, where $F_{j,r}^{t',r}(\alpha)$ is given by (5). The existence of a UE for the model defined in subsection A is proved by transforming the time dependent network into a static network as explained below.

**Network Transformation:** Given the transportation network, we first consider $N$ copies where each copy corresponds to the traffic condition of the network during a specific time interval. Then, for each OD pair $j$ and each desired departure time $\bar{t}$, we introduce an additional node $O_{j,\bar{t}}$. Finally, we add a link connecting node $O_{j,\bar{t}}$ to the origin of the OD pair $j$ having a constant cost $D(t, \bar{t})$. Note that $D(t, \bar{t}) = 0$ for $t = \bar{t}$.

In Fig. 1, the network transformation described above has been applied to a network with 2 OD pairs where we have assumed that the planning horizon consists of 2 time intervals.

Using this network transformation, the resulting network is no longer time dependent and hence, the proof of the existence of a UE is established by using the already existing results in [27] and [30].

Having proved the existence of a UE, we know that each truck driver will minimize his/her own expected individual cost. However, it is well known (see e.g. [8]) that these individual routing decisions do not result to the minimum expected total system cost defined by the following equation:

$$E[T_{\alpha}(\alpha)] = E[T_{p}(\alpha)] + E[T_{tr}(\alpha)]$$

(7)

where $E[T_{p}(\alpha)]$ represents the expected total cost (e.g. total travel time) for passenger vehicle drivers defined as:

$$E[T_{p}(\alpha)] = E \left[ \sum_{t=1}^{N} \sum_{i=1}^{m} X_{lT}^t C_{lp}(X_{lT}^t, X_{lT}^t(\alpha)) \right]$$

(8)

where $m$ represents the number of road segments of the transportation network and $C_{lp}(X_{lT}^t, X_{lT}^t(\alpha))$ is a nonlinear function denoting the operation cost of each passenger vehicle traversing road segment $l$ during time interval $t$. On the other hand, $E[T_{tr}(\alpha)]$ in (7) corresponds to the expected total non-fee truck cost (operation cost + delay cost) and can be written as:

$$E[T_{tr}(\alpha)] = E \left[ \sum_{t=1}^{N} \sum_{i=1}^{m} X_{lT}^t C_{IT}(X_{lT}^t, X_{lT}^t(\alpha)) \right] +$$

$$+ E \left[ \sum_{j=1}^{v} \sum_{r \in R_j} \sum_{t \in r} d_{j}^{t} \alpha_{j,r}^t D(t, \bar{t}) \right]$$

(9)

In the following section, we will examine if and under what conditions we can design mechanisms which minimize the expected total system cost as given by (7), by appropriately choosing the decision variable $\alpha$ while concurrently creating individual incentives for voluntary participation of the truck drivers.

III. ROUTING WITH COORDINATION: MECHANISM DESIGN

A. Overview

In this section we assume that all truck drivers submit their desired OD pair to a central coordinator who issues individual routes such that the expected total system cost given by (7) is minimized. Since the routes that minimize this cost do not correspond to optimum individual routes, some truck drivers will benefit and some others will lose. As a result, there is not strong incentive for voluntary participation. In order to address this issue, we design coordinated mechanisms to address
the issue of minimizing the total system cost while making sure that no truck driver incurs a cost higher than his/her corresponding cost at the UE (i.e. without coordination). To achieve this, the mechanism involves the use of fees for those who benefit and payments to those that lose compared to the UE.

The overall design of the mechanism consists of two optimization problems. In the first problem, the coordinator of the mechanism calculates the optimum fractions $\alpha$ according to which he/she should distribute the truck drivers into the different routes. The calculation involves the solution of an optimization problem whose objective is the minimization of the expected total cost of the network while simultaneously the expected total non-fee truck cost is less than in the case where the truck drivers follow a UE. Note at this point that we use the UE as a benchmark for our design since we may expect that it is an optimistic version of the real world traffic conditions where all truck drivers have updated information of the traffic conditions through the use of navigation apps. This is an assumption that is verified numerically in [36] where it is shown that the travel time of the users of the simulated network of Los Angeles area is increased whenever a part of them is not routed and does not have updated information of the traffic conditions. Hence, enforcing our mechanism to provide a better solution than the UE, we may expect that this solution will be even better compared to the real world traffic conditions.

Subsequently, after having calculated the fractions $\alpha$, we use them into the second optimization problem through which the coordinator calculates the optimum payments $\tau$ by minimizing a defined measure of unfairness. Furthermore, the constraints of the second optimization problem are written in a way that are equivalent to requiring that the optimum payments $\tau$ lead to a truthful and budget balanced on average mechanism which is able to create individual incentives for voluntary participation of the truck drivers. Hence, sufficient conditions to guarantee the feasibility of the second optimization problem are derived. These conditions are written as linear functions of the fractions $\alpha$ and thus, they can be added as constraints in the first optimization problem. However, since these conditions happen to be only sufficient for the existence of monetary schemes with the desired criteria, we propose an algorithm which progressively adds as constraints in the first optimization problem only those conditions which appear to be necessary for guaranteeing the feasibility of the second, in an effort to maximize the efficiency of the overall mechanism and drive its solution as close as possible to the SO.

B. Main Design

In the previous section, we proved the existence of a UE in the traffic model used. Moreover, it is generally known that a traffic network may have multiple UEs. Therefore, in order to ensure that the designed mechanism is going to provide a lower cost than any UE, it is considered necessary to compare its efficiency with the UE presenting the lowest total truck cost.

We consider a mechanism which promotes voluntary participation in which all the players (truck drivers) prefer the application of the mechanism compared to the particular UE. If the total truck cost under the mechanism suggestions is higher than in the UE case, then we may not expect voluntary participation. According to the aforementioned observations, let us define the following optimization problem:

$$
\begin{align*}
\text{minimize} & \quad E[T_n(\alpha)] \\
\text{subject to} & \quad E[T_{UE}(\alpha)] \leq E[T_{tr}^{UE}] \\
& \quad \sum_{t=1}^{N} \sum_{r \in R} \alpha_{c,t,r} = 1, \forall c,j, \bar{t} \\
& \quad \alpha_{c,t,r} \geq 0
\end{align*}
$$

(10)

where the index $c$ is used in the fractions $\alpha$ in order to denote a particular realization $c$ of the demand $d_{r,j}$. The cost $E[T_n(\alpha)]$ is given by (7)-(9). Moreover, the first constraint of (10) guarantees that the expected total cost of the truck drivers will be less under the mechanism suggestions than in the UE, the second constraint guarantees that the proportions of truck drivers which are routed at every realization, every OD pair and every desired time interval $\bar{t}$ sum up to one while the third constraint guarantees that each fraction is going to be greater or equal to zero. Overall, the optimization problem (10) is clearly feasible since $\alpha^{UE}$ satisfies the constraints. In [30], the authors proved its convexity under the assumption that the functions $C_{IT}(X_{rpt}^{\bar{t}})$ and $C_{lp}(X_{rpt}^{\bar{t}})$ are convex. Note also that the solution of (10) is a function of the demand indicating that the coordinator of the mechanism is able to make a decision having full information of the traffic conditions i.e., knowing the exact realization of the demand $d_{r,j}$.

The solution of the optimization problem (10) gives us the optimum fractions according to which the coordinator will route the truck drivers into the network. However, (10) can only create incentives for voluntary participation on the collective level, meaning that it can only guarantee that the expected total non-fee truck cost is lower for the routes generated by the coordinator than those generated individually based on the UE assumption. In order to create individual incentives, the mechanism should be able to guarantee that
each individual truck driver is guaranteed to have a lower cost if he/she follows the mechanism suggestions than making independent routing decisions. Therefore, we need to find ways in order to take into account individual incentives. In addition, we need to consider the issue of truthfulness i.e., the mechanism should take into account the possibility of truck drivers manipulating the system and prevent it.

Since the mechanism allows the truck drivers to declare their preferred departure time interval, it should guarantee that no player (truck driver) will have an incentive to declare a different time interval than the one he/she actually wants in order to gain a benefit. Finally, since the mechanism allows that some players pay a fee while some others may receive a payment as part of the incentive for participation and fairness, an important step in the overall design is to ensure that the mechanism will be at least weakly budget balanced on average i.e., the expected total payments made and received are greater or equal to zero.

Before presenting the optimization problem through which the optimum payments are calculated, let us first define the expected total benefits that the truck drivers earn under the mechanism suggestions compared to the UE expressed by:

$$B(\tau) = E \left[ \sum_{j=1}^{v} \sum_{t=1}^{N} \sum_{\tau_{t,r} \in \tau_{t,r}} (E_{\tau_{t,r}}(J_{j,t}^{U,E,D}) - E_{\tau_{t,r}}(J_{j,t}^{M,D})) \right]$$

(11)

where $J_{j,t}^{U,E,D}$ is the expected cost of a truck driver in OD pair $j$ with preferred departure time interval $\bar{t}$ at the UE, $J_{j,t}^{M,D}(\tau)$ is the total cost of a truck driver (operation cost + delay cost + fees) in OD pair $j$ following route $r$, intended departure time interval $\hat{t}$ and actual departure time interval $t$ under mechanism M suggestions and the symbol “bar” above $\alpha$ denotes the optimum fractions. Using (11), a measure of unfairness representing a weighted variance of the distribution of the benefits is defined as:

$$U(\tau) = E \left[ \sum_{j=1}^{v} \sum_{t=1}^{N} \sum_{\tau_{t,r} \in \tau_{t,r}} (E_{\tau_{t,r}}(J_{j,t}^{U,E,D} \cdot J_{j,t}^{M,D}(\tau)) - \frac{E[d_{\alpha}^{t,k}J_{j,t}^{M,D}(\tau)]}{E[d_{\alpha}^{t,k}J_{j,t}^{M,D}(\tau)]} \sum_{j=1}^{v} \sum_{t=1}^{N} \sum_{\tau_{t,r} \in \tau_{t,r}} (E_{\tau_{t,r}}(J_{j,t}^{U,E,D} - J_{j,t}^{M,D}(\tau))) \right]^{2}$$

(12)

where the term $E[T_{\tau_{t,r}}^{M}(\tau)]$ is the expected total cost of the truck drivers (operation cost + delay cost + fees) given by:

$$E[T_{\tau_{t,r}}^{M}(\tau)] = E \left[ \sum_{t=1}^{N} \sum_{l=1}^{m} X_{l,r}^{T} C_{TT}(X_{l,r}^{T}, X_{l,r}^{T}(\bar{\alpha})) + \sum_{t=1}^{N} \sum_{l=1}^{m} \sum_{l=1}^{n} \sum_{l=1}^{m} d_{j,t}^{l} \beta_{j,t,r}^{l} (D(t, \bar{t} + \tau_{j,t,r}(\bar{\alpha})) \right]$$

(13)

and $A_{\alpha}^{t,k}$ is the average cost (operation cost + delay cost + fees) of a truck driver in OD pair $j$ and preferred departure time interval $\bar{t}$. For more details on this measure of unfairness, see [30]. Taking under consideration the aforementioned definitions, we can formulate the following optimization problem:

$$\min_{\tau(\cdot)} U(\tau)$$

s.t. $$\sum_{t=1}^{N} \sum_{\tau_{t,r} \in \tau_{t,r}} p_{c_{\alpha}^{t,k}J_{j,t}^{M,D}(\tau)} \leq G_{j,i}, \forall j \geq i$$

$$f_{j,(i,k)}^{T}(\tau_{j,(i,k)}) \leq E[J_{j,i,k}^{M,D}] - E[J_{j,i,k}^{M,D}], \forall j, i, k \geq i$$

$$-f_{j,(i,k)}^{T}(\tau_{j,(i,k)}) \leq E[J_{j,i,k}^{M,D}] - E[J_{j,i,k}^{M,D}], \forall j, i, k \geq i$$

$$\sum_{t=1}^{N} \sum_{\tau_{t,r} \in \tau_{t,r}} p_{c_{\alpha}^{t,k}J_{j,t}^{M,D}(\tau)} \geq 0$$

(14)

where the function $G_{j,i}$ and the inner product $f_{j,(i,k)}^{T}(\tau_{j,(i,k)})$ are given by the following equations:

$$G_{j,i} = \sum_{t=1}^{N} \sum_{\tau_{t,r} \in \tau_{t,r}} p_{c_{\alpha}^{t,k}J_{j,t}^{M,D}(\tau)}$$

$$f_{j,(i,k)}^{T}(\tau_{j,(i,k)}) = \sum_{t=1}^{N} \sum_{\tau_{t,r} \in \tau_{t,r}} p_{c_{\alpha}^{t,k}J_{j,t}^{M,D}(\tau)}$$

(15)

respectively and $p_{c_{\alpha}^{t,k}J_{j,t}^{M,D}(\tau)}$ expresses the probabilities of each possible realization of the demands. Moreover, the quantity $E[J_{j,i,k}^{M,D}]$ denotes the expected non-fee cost (operation cost + delay cost) of a truck driver with preferred departure time interval $i$, but declared departure time interval $k$. Therefore, the first constraint of (14) together with (15) express that the expected average cost of each player is going to be lower under the mechanism suggestions than in the UE, while the second and third constraints together with (16) express that no player will have an incentive to declare that he/she wants to make his/her trip during a different time interval than the one he/she actually wants since his/her expected cost is going to be higher. Last, the fourth constraint of (14) expresses that the expected total sum of the payments made and received by the mechanism is greater or equal to zero (weakly budget balanced on average mechanism).

The optimum payments $\tau_{c_{\alpha}^{t,k}J_{j,t}^{M,D}(\tau)}$ calculated by solving the optimization problem (14) will define, if it exists, a truthful and weakly budget balanced on average mechanism which is able to create individual incentives for voluntary participation of the truck drivers. In order to address the question of the existence of such payment schemes, let us first define the following two inequalities:

$$J_{j,i}^{U,E,D} \leq J_{j,i}^{U,E,D} + \sum_{t=1}^{N} \sum_{\tau_{t,r} \in \tau_{t,r}} p_{c_{\alpha}^{t,k}J_{j,t}^{M,D}(\tau)}$$

(17)

$$J_{j,i}^{U,E,D} \leq J_{j,i}^{U,E,D} + \sum_{t=1}^{N} \sum_{\tau_{t,r} \in \tau_{t,r}} p_{c_{\alpha}^{t,k}J_{j,t}^{M,D}(\tau)}$$

(18)
where $J^t_{j,i}^{E,D}$ expresses the expected cost of a truck driver traversing OD pair $j$ with intended departure time interval $i$ at the UE. Now, we are ready to state the following theorem.

**Theorem 1:** There exists a truthful in equilibrium and weakly budget balanced on average mechanism which creates individual incentives for voluntary participation of the truck drivers if (17) and (18) hold.

**Proof:** See Appendix A. □

Theorem 1 proves that (17) and (18) are sufficient conditions to guarantee the feasibility of (14). However, observe that in (14), we have imposed the last constraint to be greater or equal to zero which only guarantees that the mechanism will be weakly budget balanced on average. Moreover, in this case, $U(\tau)$ is a nonconvex function of $\tau$ due to the fact that the variable $\tau$ appears in the denominator through the term $E[T^M_{tr}(\tau)]$ which is clearly undesirable. As it can be seen from the proof of Theorem 1, we could enforce (10) to provide us with a solution at which $E[T^M_{tr}(\tilde{\alpha})] = E[T^M_{tr}^E]$ and thus, we could guarantee the budget balance on average property. However, we expect that this conversion will reduce significantly the efficiency of our mechanism since it restricts the feasible space of (10) and thus, we will avoid it. Therefore, let us convert (14) into the following optimization problem:

\[
\begin{align*}
\min_{\tau(\cdot)} & \quad U(\tau) \\
\text{s.t.} & \quad \sum_{c=1}^{N} \sum_{r \in R_j} \sum_{t_i=t_r}^{t_{\text{end}}} p_c \alpha_{c,j,t,r}^{\tau} \leq G_{j,t,r}, \forall j, t, r \\
& \quad f_{j,(i,k)}^{T} \tau_{j,(i,k)} \leq E[J_{i,k}^{M,D}] - E[J_{i,t}^{M,D}], \forall j, i, k \geq i \\
& \quad -f_{j,(i,k)}^{T} \tau_{j,(i,k)} \leq E[J_{k,i}^{M,D}] - E[J_{k,t}^{M,D}], \forall j, i, k \geq i \\
& \quad \sum_{c=1}^{N} \sum_{j=1}^{N} \sum_{t_i=t_r}^{t_{\text{end}}} \sum_{t_i=t_r}^{t_{\text{end}}} p_c \delta^{c,j}_{t_i,t_r} \alpha_{c,j,t,r}^{\tau} \geq 0
\end{align*}
\]

(19)

where the only difference between (14) and (19) is that we have enforced the last constraint to hold as an equality (budget balance on average property).

**Theorem 2:** The optimization problem (19) is feasible if (17) and (18) hold. Additionally, it is a convex (quadratic) optimization problem.

**Proof:** See Appendix B. □

Theorem 2 proves that (17) and (18) are sufficient conditions to guarantee the existence of truthful in equilibrium and budget balanced on average mechanisms which are also able to create individual incentives for voluntary participation of the truck drivers. Furthermore, enforcing the budget balance on average property to hold, converts the problem of the calculation of the optimum payment schemes into a convex (quadratic) optimization problem.

**Remark 1:** We need to mention at this point that in both (14) and (19), we could convert the voluntary participation constraint to hold at each individual realization of the demands and without any need to average the cost of the truck drivers. Moreover, (17) and (18) would still remain sufficient conditions. However, even if these incentives appear to be stronger than the one provided, we avoided their use since the number of constraints increases significantly.

**Remark 2:** The formulation of the optimization problem (19) where the payment schemes satisfy the budget balance on average property, is intended for a non-profit organization, a government agency etc. On the other hand, in the case where a private company whose objective is the maximization of its earnings, decides to implement the above monetary schemes into the truck drivers of a transportation network, then by removing the last constraint of (19), conditions (17) and (18) guarantee the maximum possible earnings for the company while concurrently the derived payment schemes satisfy both the individual voluntary participation and the truthfulness criteria. More specifically, the company only has to calculate the optimum fractions $\hat{\alpha}$ according to which it is going to distribute the trucks into the network ensuring that (17) and (18) hold. Then, the optimum payment schemes applied to the drivers will be given by (27). Furthermore, by the way of construction of the proof of Theorem 1, its earnings can be easily calculated to be $E[T^E_{tr}^E] = E[T^E_{tr}^E]$.

It has become clear from the results of Theorem 1 and Theorem 2 that if we are able to ensure that (17) and (18) hold true, then we could design a mechanism satisfying the desired criteria. Observe that (17) and (18) are linear inequalities of the fractions $\alpha$ and thus, we can enforce (10) to provide us with a solution that will satisfy these conditions without an important effect on its complexity. To this end, let us modify (10) as follows:

\[
\begin{align*}
\text{minimize} & \quad E[T_{tr}(\alpha)] \\
\text{subject to} & \quad E[T^E_{tr}(\alpha)] \leq E[T^E_{tr}^E] \\
& \quad \sum_{t_i=t_r}^{t_{\text{end}}} \alpha_{c,j,t,r}^{\tau} = 1, \forall c, j, t, r \quad (20)
\end{align*}
\]

where the difference between (10) and (20) is that we have added inequalities (17) and (18) as constraints. The question that needs to be answered at this point is whether (20) is feasible and the answer to this question comes from the following Lemma.

**Lemma 1:** The optimization problem (20) is feasible.

**Proof:** See Appendix C. □

Lemma 1 together with the results of Theorem 1 and Theorem 2 allows us to design our mechanism. More specifically, the coordinator can solve the optimization problem (20) in order to calculate the optimum fractions $\hat{\alpha}$ according to which he/she should distribute the truck drivers into the network. Then, he/she solves the optimization problem (19) in order to calculate the optimum payments $\tau^*$ that will be made or received by the truck drivers. However, in what follows, we will present a modification of this algorithm which is going to improve the efficiency of the overall mechanism.

**Remark 3:** At this point, the necessity of the derivation of (17) and (18) must be noted. As also mentioned before, (17) and (18) are sufficient conditions to guarantee the feasibility
of (19). However, an intuitive question to ask is whether it is necessary to satisfy some specific conditions in order to guarantee the feasibility of (19) and the answer to this question is positive. To see this, observe that the second and the third constraints of (19) have opposite left-hand side parts. Hence, a necessary condition in order for these two constraints to define a nonempty polyhedral set is to require that the sum of their right-hand sides is greater or equal to zero i.e.,

$$\sum_{c} \sum_{t=1}^{N} \sum_{r \in R_{j}} p_{c}(\alpha_{c,j,r}^{t,k} - \alpha_{j,r}^{c,t}) (D(t,i) - D(t,k)) \geq 0,$$

$$\forall j,i,k \geq i \tag{21}$$

It is straightforward to see that the sum of (17) and (18) satisfy (21).

C. Algorithmic Procedure of the Mechanism

Due to the way we construct the proof of Theorem 1, inequalities (17) and (18) are only sufficient conditions to guarantee the feasibility of (19). This means that (19) could also be feasible even in the case where (17) and (18) do not hold. Hence, if we solve (20) instead of (10) in an effort to guarantee the feasibility of (19), we may lose some of the efficiency of our mechanism. To this end, we further modify the algorithm to follow the steps mentioned below:

Iteration 1

Step 1: Solve the optimization problem (10) in order to calculate the optimum fractions $\tilde{\alpha}^{1}$.

Step 2: Solve the optimization problem (19) in order to calculate the optimum payments $\bar{\tau}$. If (19) is solved, STOP; The optimum solution of the mechanism is given by the pair $(\tilde{\alpha}^{1}, \bar{\tau})$. Else, proceed to Step 3.

Step 3: Choose $\tau$ as given by (27) and substitute it to the second and the third constraints of (19). For those constraints that are violated after this substitution, take the corresponding inequalities which will be a subset of (17) and (18) and add them as constraints in the optimization problem (10), call it (10.1). Proceed to the next iteration.

Iteration k

Step 1: Solve the optimization problem (10.k-1) in order to calculate the optimum fractions $\tilde{\alpha}^{k}$.

Step 2: Solve the optimization problem (19) in order to calculate the optimum payments $\bar{\tau}$. If (19) is solved, STOP; The optimum solution of the mechanism is given by the pair $(\tilde{\alpha}^{k}, \bar{\tau})$. Else, proceed to Step 3.

Step 3: Choose $\tau$ as given by (27) and substitute it to the subset of the second and the third constraints of (19) which had never been violated in any of the previous k-1 iterations. For those constraints that are violated after this substitution, take the corresponding inequalities which will be a subset of (17) and (18) and add them as constraints in the optimization problem (10.k-1), call it (10.k). Proceed to the next iteration.

IV. Simulation Results

In the first two subsections, we present the numerical methods we used in order to calculate the UE and the SO solutions while in the last two subsections, we apply the proposed mechanism into the Braess and the Sioux Falls networks and we make a comparison with the UE and the SO solutions in terms of the expected total system cost and the expected total truck cost.

A. Numerical calculation of the User Equilibrium

The UE which presents the minimum total truck cost can be calculated by solving the following nonlinear optimization problem with complementarity constraints [37]:

$$\text{minimize} \quad E[T_{tr}(\alpha)]$$

$$\text{subject to} \quad 0 \leq \alpha_{j,r}^{t} \perp F_{j,r}^{t}(\alpha) - \delta_{j,r}^{t} \geq 0, \forall j,t,r$$

$$\sum_{t=1}^{N} \sum_{r \in R_{j}} \alpha_{j,r}^{t} = 1, \forall j, \bar{t} \tag{22}$$

where $\delta_{j,r}^{t}$ is a set of free variables and $F_{j,r}^{t}(\alpha)$ is given by (5). The optimal objective value of (22) is denoted by $E[T_{tr}^{UE}]$ and will be used in the right hand side of the first constraint of (10).

B. Numerical calculation of the System Optimum

The System Optimum (SO) solution is the total cost of the network if routing was centrally planned and is given from the solution of the following optimization problem:

$$\text{minimize} \quad E[T_{s}(\alpha)]$$

$$\text{subject to} \quad \sum_{t=1}^{N} \sum_{r \in R_{j}} \alpha_{c,j,r}^{t,\bar{t}} = 1, \forall c,j, \bar{t}$$

$$\alpha_{c,j,r}^{t,\bar{t}} \geq 0 \tag{23}$$
C. Braess network

In order to validate the theoretical results and show the performance of the proposed mechanism compared to the UE and the SO solutions, we ran some simulations in a small traffic network known as the Braess network [38] which is illustrated in Figure 2. This network consists of 2 OD pairs which have three and two possible routes, respectively.

![Fig. 2. The Braess network.](image)

For the simulations purpose, we considered two different scenarios. In the first scenario, we split the planning horizon into 2 time intervals and we let the demand to take one of the following 4 equiprobable values:

\[
\begin{align*}
  d_1 &= \begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix},
  d_2 &= \begin{bmatrix} 3.4 \\ 2.9 \\ 2.5 \end{bmatrix},
  d_3 &= \begin{bmatrix} 2.8 \\ 3.7 \\ 5.1 \end{bmatrix},
  d_4 &= \begin{bmatrix} 4.3 \\ 3.3 \\ 4.6 \end{bmatrix},
\end{align*}
\]

where each column of \(d_1,d_2,d_3\) and \(d_4\) corresponds to an OD pair and each row corresponds to a particular time interval.

The link cost functions \(C_{lp}\) and \(C_{IT}\) are given by the following relations:

\[
\begin{align*}
  C_{1p} &= C_{1T} = 1 + (X_{1p} + X_{1T}) + (X_{1p} + X_{1T})^2 \\
  C_{2p} &= C_{2T} = 1 + (X_{2p} + X_{2T}) + (X_{2p} + X_{2T})^2 \\
  C_{3p} &= C_{3T} = 0.5(X_{3p} + X_{3T})^2 \\
  C_{4p} &= C_{4T} = 2 + 0.5(X_{4p} + X_{4T})^2 \\
  C_{5p} &= C_{5T} = (X_{5p} + X_{5T})^2
\end{align*}
\]

while the number of passenger vehicles at each link of the network was assumed to be constant across all time intervals and possible demand realizations in order to simplify the simulations and was equal to \((X_{1p}, X_{2p}, X_{3p}, X_{4p}, X_{5p}) = (4, 4, 1, 3, 2)\).

The delay cost function \(D(t, \bar{t})\) was considered to have the following form:

\[
D(t, \bar{t}) = \gamma |t - \bar{t}|
\]

where \(\gamma > 0\) is a constant which is assumed to be common for all the truck drivers and was chosen to be \(\gamma = 0.8\). Note that a delay cost function with the form of (24), also adopted in [39], indicates that a driver is equally harmed whether he/she departs earlier or later than his/her desired departure time interval \(\bar{t}\).

The simulation results are summarized in Table II. Table II shows that our mechanism was able to reach the same total truck cost and total system cost values as the SO solution. More specifically, the mechanism provided a total truck cost value 1.2% lower than the corresponding UE value. Moreover, the total system cost was also decreased by 0.6% compared to the UE solution. Note that during the simulation run, our algorithm did not need to add any feasibility cuts to the optimization problem (10) in order to guarantee the feasibility of (19) and thus, it terminated in just one iteration.

For the second scenario, we ran a simulation in the same network by splitting the planning horizon into 6 time intervals. The variable \(\gamma\) took the value \(\gamma = 0.5\) and we let the demand to take one of the following 4 equiprobable values:

\[
\begin{align*}
  d_1^T &= \begin{bmatrix} 3 & 2 & 4.8 & 3.6 & 2.7 & 3.3 \\ 2 & 1 & 3.5 & 4.4 & 3.1 & 3.9 \end{bmatrix},
  d_2^T &= \begin{bmatrix} 3.4 & 2.9 & 2 & 3.5 & 4.7 & 3.2 \\ 2.5 & 4 & 3.6 & 3.5 & 3.1 & 4.1 \end{bmatrix},
  d_3^T &= \begin{bmatrix} 2.8 & 3.7 & 2.7 & 3.1 & 2.8 & 3.9 \\ 5.1 & 3.8 & 2.6 & 4.2 & 3.8 & 2.9 \end{bmatrix},
  d_4^T &= \begin{bmatrix} 4.3 & 3.3 & 2.4 & 3.4 & 3 & 3.4 \\ 3.7 & 4.6 & 3.6 & 3.9 & 2.5 & 3.2 \end{bmatrix}
\end{align*}
\]

The results presented in Table III show that the designed mechanism was able to reach the SO solution by decreasing the expected total truck cost and the expected total system cost by 3.4% and 1.7% respectively compared to the UE. Again, the proposed algorithm did not need to add any feasibility cuts and it terminated in just one iteration.

D. Sioux Falls network

In this section, the simulation results obtained from the application of the proposed mechanism in the Sioux Falls network [40] are presented. The Sioux Falls network consisting of 24 nodes and 76 links constitutes a benchmark problem in the transportation research field.
In our simulation, we assume that the cost of each road segment corresponds to travel time and is given by a Bureau of Public Roads (BPR) function [41] of the following form:

\[ C_{IIP}(X_{IIP}, X_{IIT}) = C_{IT}(X_{IIP}, X_{IIT}) = v_a + v_b \left( \frac{X_{IIP}}{v_k} + 3X_{IIT} \right)^4 \]  

(25)

Furthermore, in order to retain computational tractability, we assumed that there are only 6 available OD pairs for the truck drivers, namely \( j_1 = (n_1, n_7), j_2 = (n_1, n_1), j_3 = (n_{10}, n_{11}), j_4 = (n_{10}, n_{20}), j_5 = (n_{15}, n_{5}), j_6 = (n_{24}, n_{10}) \) and moreover, we assumed that for each OD pair, truck drivers will choose between the 10 least congested routes. The planning horizon of the simulation model was split into two time intervals. The number of passenger vehicles at each link was considered to remain constant in the two time intervals and the values \( v_a, v_b, v_k \) were chosen similar to the ones adopted in [30]. The demands of the OD pairs at each time interval were chosen to take one of the following two equiprobable values:

\[
d = \begin{cases} 
  d_1 & \text{w.p. 0.5} \\
  d_2 & \text{w.p. 0.5}
\end{cases}
\]

where

\[
d_1 = \begin{bmatrix} 
  3 & 4.5 & 6 & 3 & 14 & 3.6 \\
  1 & 2.8 & 5.4 & 7 & 9 & 2 
\end{bmatrix}
\]

\[
d_2 = \begin{bmatrix} 
  5 & 1.8 & 3.9 & 15 & 6.4 & 2.4 \\
  6 & 5.5 & 1.8 & 6.5 & 11 & 6 
\end{bmatrix}
\]

where each column of \( d_1 \) and \( d_2 \) corresponds to an OD pair and each row corresponds to a particular time interval. The simulation results are summarized in Table IV.

**TABLE IV**
**Simulation Results of the Sioux Falls Network**

<table>
<thead>
<tr>
<th>( E[I_T] )</th>
<th>( E[I_s] )</th>
<th>( E[I_r] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>15100.3</td>
<td>13372.1</td>
<td>13372.1</td>
</tr>
<tr>
<td>83924.8</td>
<td>79245.2</td>
<td>79245.2</td>
</tr>
</tbody>
</table>

Table IV shows that our mechanism reached the same total truck cost and total system cost values as the SO solution. More specifically, it provided a total truck cost value 11.4% lower than the corresponding UE value while concurrently reducing the total system cost by 5.6% compared to the UE solution. Note also that during this simulation run, the proposed algorithm terminated again in just one iteration.

**V. Conclusion**

In this paper, a coordination mechanism for the freight routing problem is designed. The proposed mechanism uses monetary schemes in order to satisfy some specific criteria. To this end, we derive sufficient conditions which ensure that the truck drivers will have an incentive to participate in the mechanism compared to the UE and will additionally be truthful during their declaration of their preferred departure time. Then, we prove that the overall mechanism can be budget balanced on average and we subsequently propose an algorithm which only uses a minimal set of sufficient conditions in order to guarantee the existence of a solution and to maximize the efficiency of the proposed mechanism. Last, the application of our approach to the Sioux Falls network demonstrates that our mechanism can reach the SO solution showing an improvement of more than 10% on the expected total cost of the truck drivers and more than 5% on the expected total system cost compared to the UE solution.

There are several possible extensions of the current work. At first, studying the case where the time intervals are narrow and some drivers do not have enough time to complete their trip is of practical importance. Additionally, taking into account the heterogeneity of the users is a very important issue. For example, different users may value differently their travel time or on the other hand, some drivers may be more sensitive on a pricing scheme than some others due to different income levels. Last, the extension of the current approach such that it can be applied in real-time is certainly of major importance.

**Appendix**

**A. Proof of Theorem 1**

The statement of the theorem is equivalent to studying the feasibility of the optimization problem (14). Hence, we will first make the first constraint of (14) to hold as an equality and then we will examine if and under what conditions the second, third and fourth constraints also hold true. Let us define:

\[
\sum_{c=1}^{N} \sum_{r=1}^{R} \sum_{t_{x,r}^{c,t}} p_{c}a_{c,t}^{c,t} + \sum_{c=1}^{N} \sum_{r=1}^{R} \sum_{t_{x,r}^{c,t}} \left( - \left( J_{c,j,i}^{M} + D(t,i) \right) + A_{c,j,i}^{UE,D} \right)
\]

At this point, choose \( \tau_{c,j,i,t,r} \) such that:

\[
\tau_{c,j,i,t,r} = -\left( J_{c,j,i,r}^{M} + D(t,i) \right) + A_{c,j,i}^{UE,D}
\]

(27)

Now, substituting (27) to the second constraint of (14) and using (16), we get:

\[
\sum_{c=1}^{N} \sum_{r=1}^{R} \sum_{t_{x,r}^{c,t}} p_{c}a_{c,t}^{c,t} \left[ -\left( J_{c,j,i,r}^{M} + D(t,i) \right) + A_{c,j,i}^{UE,D} \right] + \\
\sum_{c=1}^{N} \sum_{r=1}^{R} \sum_{t_{x,r}^{c,t}} p_{c}a_{c,t}^{c,t} \left[ J_{c,j,i,r}^{M} + D(t,i) \right] \leq \\
\sum_{c=1}^{N} \sum_{r=1}^{R} \sum_{t_{x,r}^{c,t}} p_{c}a_{c,t}^{c,t} \left[ -\left( J_{c,j,t,k}^{M} + D(t,k) \right) + A_{c,j,k}^{UE,D} \right] + \\
\sum_{c=1}^{N} \sum_{r=1}^{R} \sum_{t_{x,r}^{c,t}} p_{c}a_{c,t}^{c,t} \left[ J_{c,j,t,k}^{M} + D(t,i) \right]
\]

(28)
\[
\sum_{c} \sum_{t=1}^{N} \sum_{r \in R_j} p_c \sum_{t=1}^{N} \sum_{r \in R_j} p_c a_{c,t,i}^j A_{c,j,t}^{U,E,D} \leq \sum_{c} \sum_{t=1}^{N} \sum_{r \in R_j} p_c a_{c,t,i}^j A_{c,j,t}^{U,E,D} + \n\]
\[
+ \sum_{c} \sum_{t=1}^{N} \sum_{r \in R_j} p_c a_{c,t,i}^j [D(t, i) - D(t, k)] \Leftrightarrow \sum_{c} \sum_{t=1}^{N} \sum_{r \in R_j} p_c A_{c,j,t}^{U,E,D} \sum_{t=1}^{N} \sum_{r \in R_j} a_{c,t,i}^j \leq \sum_{c} \sum_{t=1}^{N} \sum_{r \in R_j} p_c A_{c,j,t}^{U,E,D} \sum_{t=1}^{N} \sum_{r \in R_j} a_{c,t,i}^j + \n\]
\[
+ \sum_{c} \sum_{t=1}^{N} \sum_{r \in R_j} p_c a_{c,t,i}^j [D(t, i) - D(t, k)] \Leftrightarrow J_{c,j,t}^{U,E,D} \leq J_{c,j,t}^{U,E,D} + \n\]
\[
+ \sum_{c} \sum_{t=1}^{N} \sum_{r \in R_j} p_c a_{c,t,i}^j (D(t, k) - D(t, i)) \tag{29}\n\]

where in the last induction, we made use of the second constraint of (10). Working in the same manner and substituting (27) to the third constraint of (14) and using (16), we get:

\[
\sum_{c} \sum_{t=1}^{N} \sum_{r \in R_j} p_c a_{c,t,i}^j [-(\bar{J}_{c,j,t,k,r}^M + D(t, k))] \leq \sum_{c} \sum_{t=1}^{N} \sum_{r \in R_j} p_c A_{c,j,t}^{U,E,D} + \n\]
\[
+ \sum_{c} \sum_{t=1}^{N} \sum_{r \in R_j} p_c a_{c,t,i}^j [\bar{J}_{c,j,t,k,r}^M + D(t, k)] \leq \sum_{c} \sum_{t=1}^{N} \sum_{r \in R_j} p_c A_{c,j,t}^{U,E,D} + \n\]
\[
+ \sum_{c} \sum_{t=1}^{N} \sum_{r \in R_j} p_c a_{c,t,i}^j (D(t, k) - D(t, i)) \tag{30}\n\]

which can be equivalently written as:

\[
\sum_{c} \sum_{t=1}^{N} \sum_{r \in R_j} p_c a_{c,t,i}^j A_{c,j,t}^{U,E,D} \leq \sum_{c} \sum_{t=1}^{N} \sum_{r \in R_j} p_c a_{c,t,i}^j A_{c,j,t}^{U,E,D} + \n\]
\[
+ \sum_{c} \sum_{t=1}^{N} \sum_{r \in R_j} p_c A_{c,j,t}^{U,E,D} \sum_{t=1}^{N} \sum_{r \in R_j} a_{c,t,i}^j \leq \sum_{c} \sum_{t=1}^{N} \sum_{r \in R_j} p_c A_{c,j,t}^{U,E,D} \sum_{t=1}^{N} \sum_{r \in R_j} a_{c,t,i}^j + \n\]
\[
+ \sum_{c} \sum_{t=1}^{N} \sum_{r \in R_j} p_c a_{c,t,i}^j [D(t, k) - D(t, i)] \Leftrightarrow J_{c,j,t}^{U,E,D} \leq J_{c,j,t}^{U,E,D} + \n\]
\[
+ \sum_{c} \sum_{t=1}^{N} \sum_{r \in R_j} p_c a_{c,t,i}^j (D(t, k) - D(t, i)) \tag{31}\n\]

Regarding the fourth constraint of (14), using (27) we get:

\[
\sum_{c} \sum_{j=1}^{v} \sum_{t=1}^{N} \sum_{r \in R_j} p_c d_{c,j,t}^j a_{c,j,t,r}^j \]
\[
= \sum_{c} \sum_{j=1}^{v} \sum_{t=1}^{N} \sum_{r \in R_j} p_c d_{c,j,t}^j (\bar{J}_{c,j,t,k,r}^M + D(t, k)) + \n\]
\[
+ A_{c,j,t}^{U,E,D} = \]
\[
= -E[\bar{T}_{tr}^{M}(\bar{\alpha})] + \sum_{c} \sum_{j=1}^{v} \sum_{t=1}^{N} \sum_{r \in R_j} p_c d_{c,j,t}^j A_{c,j,t}^{U,E,D} \sum_{t=1}^{N} \sum_{r \in R_j} a_{c,j,t,r}^j = \]
\[
= -E[\bar{T}_{tr}^{M}(\bar{\alpha})] + \sum_{c} \sum_{j=1}^{v} \sum_{t=1}^{N} \sum_{r \in R_j} p_c d_{c,j,t}^j A_{c,j,t}^{U,E,D} \sum_{t=1}^{N} \sum_{r \in R_j} a_{c,j,t,r}^j = \]
\[
= -E[\bar{T}_{tr}^{M}(\bar{\alpha})] + E[T_{tr}^{UE}] \geq 0 \]

where in the last equality we used the second constraint of (10) and the last inequality holds due to the first constraint of (10). Note at this point that (29) and (31) are identical to (17) and (18) and this concludes the proof.

### B. Proof of Theorem 2

In Theorem 1, we proved the feasibility of (14) if (17) and (18) hold. Hence, now, we only need to prove that even if we enforce the last constraint of (14) to hold as an equality, the problem will remain feasible. So, suppose we solve (14) and we calculate $\tau^*$ which gives us an optimal objective value $U^*$. In this case, if the expected total payments sum up to zero, we are done. So, let us assume:

\[
\sum_{c} \sum_{j=1}^{v} \sum_{t=1}^{N} \sum_{r \in R_j} p_c d_{c,j,t}^j a_{c,j,t,r}^j = T^* > 0 \tag{32}\n\]

Then, define:

\[
\tau_{c,j,t,r}^{*} = \tau_{c,j,t,r} - \frac{T^*}{\sum_{c} \sum_{j=1}^{v} \sum_{t=1}^{N} \sum_{r \in R_j} d_{c,j,t}^j} \forall c, j, t, r \tag{33}\n\]

Substituting (33) into the last constraint of (14) and using (32), we get:

\[
\sum_{c} \sum_{j=1}^{v} \sum_{t=1}^{N} \sum_{r \in R_j} p_c d_{c,j,t}^j a_{c,j,t,r}^j \tau_{c,j,t,r}^{*} = \]
\[
= \sum_{c} \sum_{j=1}^{v} \sum_{t=1}^{N} \sum_{r \in R_j} p_c d_{c,j,t}^j a_{c,j,t,r}^j \left[\tau_{c,j,t,r}^{*} - \frac{T^*}{\sum_{c} \sum_{j=1}^{v} \sum_{t=1}^{N} \sum_{r \in R_j} d_{c,j,t}^j}\right] = 0 \]

Now, observe that the second term of (33) is positive and hence $\tau_{c,j,t,r}^{*} < \tau_{c,j,t,r}^{*}$, $\forall c, j, t, r$. So, since $\tau_{c,j,t,r}^{*}$ satisfied the first constraint of (14), $\tau_{c,j,t,r}^{*}$ will also satisfy it.

Regarding the first truthfulness constraint, substituting (33)
into the second constraint of (14) and using (15) and (16), we get:

\[
\sum_{c \in C} \sum_{t=1}^{N} \sum_{r \in R_j} p_c (\bar{\alpha}^{c,t,k}_{j,r} \bar{r}_{c,j,t,i,r} - \alpha^{c,t,k}_{j,r} \bar{r}_{c,j,t,k,r}) - \bar{\xi}_j^c, r \leq 0, \forall j, k \geq i
\]

where in the inequality part we used the fact that \( \tau^* \) is feasible in (14) and in the last equality we made use of the second constraint of (10). Now, working in the same manner and substituting (33) into the third constraint of (14) and using (15) and (16), we get:

\[
\sum_{c \in C} \sum_{t=1}^{N} \sum_{r \in R_j} p_c \left( \frac{\bar{\alpha}^{c,t,k}_{j,r} \bar{r}_{c,j,t,k,r} - \alpha^{c,t,i}_{j,r} \bar{r}_{c,j,t,i,r}}{1} \right) - \bar{\xi}_j^c, r \leq 0, \forall j, k \geq i
\]

Up to now, we have proved that if at optimality of (14) the expected total payments are greater than zero, then we can always find another \( \tau \) given by (33) which is also feasible in (14). This proves that the optimization problem (14) will remain feasible even if we impose its last constraint to hold as an equality and equivalently proves the feasibility of the optimization problem (19).

At this point, observe that \( U(\tau) \) is a nonconvex function due to the fact that the variable \( \tau \) appears in the denominator through the term \( E[1/\tau] \). However, due to the last constraint of (19), \( E[1/\tau] \) will no longer be a function of \( \tau \) converting \( U(\tau) \) into a convex (quadratic) function and this concludes the proof.

C. Proof of Lemma 1

First, it can be easily seen that the UE satisfies the first three constraints of (20). Regarding constraints (17) and (18), observe at first, that they are equivalent to (28) and (30) respectively. Now, at the UE, (28) and (30) can be rewritten as:

\[
\sum_{c \in C} \sum_{t=1}^{N} \sum_{r \in R_j} p_c \alpha_{c,t,k}^{U,E,c,t} [\bar{J}_{c,j,t,k,r}^U + D(t,i)] \leq \sum_{c \in C} \sum_{t=1}^{N} \sum_{r \in R_j} p_c \alpha_{c,t,k}^{U,E,c,t} [\bar{J}_{c,j,t,k,r}^D + D(t,i)] \leftrightarrow E[\bar{J}_{c,j,t,k,r}^U] \leq E[\bar{J}_{c,j,t,k,r}^D]
\]

and

\[
\sum_{c \in C} \sum_{t=1}^{N} \sum_{r \in R_j} p_c \alpha_{c,t,k}^{U,E,c,t} [\bar{J}_{c,j,t,k,r}^U + D(t,k)] \leq \sum_{c \in C} \sum_{t=1}^{N} \sum_{r \in R_j} p_c \alpha_{c,t,k}^{U,E,c,t} [\bar{J}_{c,j,t,k,r}^D + D(t,k)] \leftrightarrow E[\bar{J}_{c,j,t,k,r}^U] \leq E[\bar{J}_{c,j,t,k,r}^D]
\]

respectively. Inequalities (34) and (35) express that at the UE, no player (truck driver) has an incentive to pretend that he/she wants to travel during a different time interval than the one he/she actually wants since his/her expected cost is going to be higher. This is a property that holds true at the UE and this concludes the proof.

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