

You are free to use any programming language or environment and any help you want.

PLEASE, USE AT LEAST A 10PT FONT, AND DO NOT SUBMIT MORE THAN 12 PAGES OF PRINTOUTS FOR THIS ASSIGNMENT. POINTS CAN BE TAKEN OFF FOR USING VERY SMALL LETTERS OR PRODUCING TOO MANY PAGES OF OUTPUT.

The objective of this assignment is to see how implicit numerical schemes work for parabolic and hyperbolic equations.

Problem 1. Consider the heat equation

$$u_t(x, t) = 0.25u_{xx}(x, t), \quad 0 < t \leq 2, \quad 0 < x < 1,$$

with $u(0, t) = u(1, t) = 0$ and

$$u(x, 0) = \begin{cases} 20x, & 0 \leq x \leq 1/2 \\ 20(1-x), & 1/2 \leq x \leq 1. \end{cases}$$

Solve it numerically by the Crank-Nicholson method taking $h = k = 0.1$. Plot a 3-D graph of the result. Then compare the result with the Fourier series solution (use your judgement as to how many terms to keep in the Fourier series: this is your only chance to get to the exact solution as close as possible).

What you will turn in:

- (1) The graph $(x, t, \bar{u}(x, t))$, where \bar{u} is the numerical solution you got.
- (2) The graph $(x, t, |\bar{u}(x, t) - u(x, t)|)$, where u is the Fourier series solution.
- (3) Printout of the program you used to generate the graphs. Please indicate the language (C, C++, etc.) or environment (Matlab, Mathematica, etc.) you used.

Each page you turn in must have your name and date *printed* on it. The graph must have a title, labelled axes, and the scale along each axis.

Problem 2. Consider the wave equation

$$u_{tt}(x, t) = u_{xx}(x, t), \quad 0 < t \leq 2, \quad 0 < x < 1,$$

with $u(0, t) = u(1, t) = u_t(x, 0) = 0$, $u(x, 0) = x(1-x)$.

Solve it numerically by the implicit method taking $h = k = 0.1$. Plot a 3-D graph of the result. Then compare the result with the exact solution (unlike the heat equation, you can get the exact solution without the Fourier series.)

What you will turn in:

- (1) The graph $(x, t, \bar{u}(x, t))$, where \bar{u} is the numerical solution you got.
- (2) The graph $(x, t, |\bar{u}(x, t) - u(x, t)|)$, where u is the exact solution.
- (3) Printout of the program you used to generate the graphs. Please indicate the language (C, C++, etc.) or environment (Matlab, Mathematica, etc.) you used.

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