

You are free to use any programming language or environment and any help you want.

PLEASE, USE AT LEAST A 10PT FONT, AND DO NOT SUBMIT MORE THAN 10 PAGES OF PRINTOUTS FOR THIS ASSIGNMENT. POINTS CAN BE TAKEN OFF FOR USING VERY SMALL LETTERS OR PRODUCING TOO MANY PAGES OF OUTPUT.

Problem 1. The main objective is to practice plotting graphs.

PART 1. Plot the graph of the function

$$h_{10}(x) = \sum_{k=1}^{10} \frac{\sin((k!)^2 x)}{k!}$$

for $x \in [0, 1]$, $x \in [0, 0.1]$, $x \in [0, 0.01]$, $x \in [0, 0.001]$.

What you will turn in:

- (1) Four separate graphs.
- (2) Printout of the program you used to generate the graphs. Please indicate which language (C,C++,Fortran, etc.) or environment (Matlab, Mathematica, etc.) you used.

Each page you turn in must have your name and date *printed* on it. Each graph must have a title, labelled axes, and the scale along each axis.

PART 2. Consider the function

$$h(x) = \sum_{k=1}^{\infty} \frac{\sin((k!)^2 x)}{k!}.$$

For what $x \in [0, 1]$ will this function be

- (1) defined?
- (2) continuous?
- (3) differentiable?

Provide a one-sentence explanation for each of your answers.

Problem 2. The objective is to compute Fourier coefficients numerically and to analyze the Gibbs phenomenon.

Let $f = f(x)$ be a 2π -periodic function defined for $x \in (-\pi, \pi]$ by $f(x) = x$. Let $S_n(x) = \sum_{k=1}^n b_k \sin(kx)$, where $b_k = (1/\pi) \int_{-\pi}^{\pi} f(x) \sin(kx) dx$.

Do the following:

- (1) Plot the graphs of $S_n(x)$ for $n = 10, 50, 100$ and $x \in [-\pi, \pi]$. The choice of the procedure to compute b_k is up to you. Keep in mind that if you divide the interval $[-\pi, \pi]$ to approximate the integral, your step size must be small enough to "see" the oscillations of the sines.
- (2) Estimate $\max_{x \in [-\pi, \pi]} S_n(x)$ for $n = 10, 50, 100$.
- (3) Compute $\lim_{n \rightarrow \infty} \frac{S_n(\pi - \pi/n)}{\pi}$ (either compute analytically or guess from the graphs). This is a quantitative measure of the Gibbs phenomenon.

What you will turn in:

- (1) Three separate graphs with the corresponding values of $\max_{x \in [-\pi, \pi]} S_n(x)$.
- (2) Printout of the program you used to generate the graphs. Please indicate the language (C,C++, etc.) or environment (Matlab, Mathematica, etc.) you used, and the procedure you used to compute the Fourier coefficients b_k .
- (3) The numerical value of $\lim_{n \rightarrow \infty} \frac{S_n(\pi - \pi/n)}{\pi}$ and the corresponding explanations.

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