Procurement contracts: Theory vs. practice

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A B S T R A C T

Laffont and Tirole's [Laffont, J., Tirole, J., 1986. Using cost observation to regulate firms. Journal of Political Economy 94, 614–641.] classic model of procurement under asymmetric information predicts that optimal contracts will always entail some cost sharing and that payments will be a convex function of realized cost. In contrast, pure cost-reimbursement contracts are common in practice, as are contracts in which payments are a concave function of realized cost. We consider a straightforward extension of Laffont and Tirole's model that admits optimal contracts of the forms that prevail in practice. The extension simply allows the supplier to be able to reduce production costs more easily when costs are initially high than when they are initially low.

1. Introduction

Real-world procurement contracts take on a variety of forms. Rogerson (1992) describes four distinct types of contracts commonly employed by the U.S. Department of Defense: (1) pure fixed price (PFP) contracts, in which the supplier receives a single, fixed payment for the procured item, regardless of the supplier's realized cost; (2) pure cost reimbursement (PCR) contracts, in which the payment made to the supplier is precisely the supplier's realized cost of producing the item; (3) incentive fixed price (IFP) contracts, in which payment to the supplier increases with realized cost up to a threshold cost level, and is capped at this threshold level; and (4) incentive cost reimbursement (ICR) contracts, in which payment to the supplier again increases with realized cost up to a threshold, and then reflects realized cost exactly above the threshold.1

These four types of contracts are illustrated in Figs. 1–4. Notice from Figs. 3 and 4 that payment is a concave function of realized cost under an IFP contract, while payment is a convex function of realized cost under an ICR contract.

Under the procurement contract illustrated in Fig. 5, payment is a concave function of realized cost in some regions and a convex function of cost in other regions. This structure parallels the reward structure regulated utilities commonly face when they operate under “incentive regulation.” Extremely high profit (corresponding to unit cost below c1) and extremely low profit (corresponding to unit cost above c2) typically are not politically acceptable in regulated industries. Consequently, allowed profit is bounded above and below, corresponding to payments (revenues) that vary dollar for dollar with costs as costs decline below c1 and as costs increase above c2 in Fig. 5. In contrast, to motivate the firm to reduce its operating costs, incentive regulation implements profit sharing for intermediate profit realizations (corresponding to cost realizations c1 ∈ (c1, c2) in Fig. 5). By allowing the firm to retain some, but not all, of the cost reduction it implements in the form of higher profit, incentive regulation can secure benefits for the regulated firm and consumers alike.2

Despite the rich variety of procurement and regulatory contracts that are observed in practice, the classic (and still the standard) economic model of procurement under asymmetric information admits only convex contracts like the ICR contracts illustrated in Fig. 4. Except in trivial cases of limited interest, Laffont and Tirole's (1986) classic model of procurement does not admit PFP or PCR contracts. The primary model on which LT focus their analysis also does not admit concave contracts like the IFP contract illustrated in Fig. 3. This model also does not produce contracts with both concave and convex regions, like the contract depicted in Fig. 5.

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1 The Department of Defense (DOD) assigns different names to these four types of contracts. As Rogerson (1992, p. 11) reports, the DOD refers to: (1) PFP contracts as firm fixed price (FFP) contracts; (2) PCR contracts as cost plus fixed fee (CPFF) contracts; (3) IFP contracts as cost plus incentive fee (CPIF) contracts that revert to FFP contracts above a threshold cost level; and (4) ICR contracts as CPIF contracts that revert to CPFF contracts above a threshold cost level.

2 Different incentive regulation plans incorporate different forms of profit sharing for intermediate profit realizations. See Sappington (2002), for example, for a discussion of the different types of incentive regulation plans that have been implemented in the telecommunications industry.
The purpose of this research is to consider a simple generalization of LT's classic procurement model that admits a broader class of contract forms, including those commonly observed in practice. The generalization has an intuitive interpretation: the supplier can reduce production costs more easily when initial cost (the component of cost beyond the supplier's control) is high than when it is low. In this sense, we allow initial cost and the supplier's cost-reducing effort to be substitutes in reducing final production costs. Intuitively, one might envision a feasible range of final cost realizations. If costs are initially close to the lower bound of this range, further cost reductions are relatively difficult to achieve. In contrast, if costs are initially quite high, some cost reduction is not difficult to achieve. Chalkley and Malcomson (CM) (2002) analyze a model of this type in a health care setting. The effort that a health care provider devotes to reducing treatment costs in CM's model is more effective at reducing these costs the more severely ill the patient is (and thus the more costly the patient would be to care for in the absence of cost-reducing effort).

CM focus on the welfare gains that an optimal contract can secure relative to the simple payment structures that are commonly employed in the health care industry. In contrast, we focus on the different contract structures that can be optimal in settings where high initial costs are associated with increased potential for cost reduction. We find that this simple – and arguably reasonable and intuitive – generalization of LT's model admits optimal contracts that are concave (like IFP contracts) and that have both concave and convex regions (as in Fig. 5). The generalization also allows PCR contracts (which induce no cost-reducing effort from the supplier) to be optimal, even in settings where cost-reducing effort is efficient, and so would always be secured absent asymmetric information about innate cost.

Our presentation of these findings proceeds as follows. Section 2 describes the key features of the formal model we analyze. Section 3 reviews the key technical details of our analysis. Section 4 illustrates the variety of contracts that can arise in our model, and identifies the factors that influence the shape of optimal procurement contracts. Section 5 provides a concluding discussion. The proofs of our formal conclusions are outlined in the Appendix. A more detailed technical appendix is available upon request.

2. The Model

A buyer seeks to minimize the expected cost of procuring a single unit of a commodity from a monopoly supplier. The supplier will only deliver the commodity if he derives non-negative utility from doing so. The buyer is privately informed about his innate cost of production \( \beta \) from the outset of his interaction with the buyer. The buyer's beliefs about \( \beta \) are captured by the density function \( f(\beta) \), which has support on the interval \( [\beta^- , \beta^+] \). For expositional ease, we assume \( 1-F(\beta^+)>0 \) for all \( \beta \in [\beta^- , \beta^+] \), where \( F(\beta) \) is the corresponding

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\(^{3}\) More fundamental modifications of LT's model also can admit optimal contracts that include those commonly observed in practice. These modifications include the introduction of transactions costs and renegotiation costs (e.g., Bajari and Tadelis, 2001) and preferences for simple contracts (e.g., Rogerson, 2003; Chu and Sappington, 2007a).

\(^{4}\) In both our model and LT's model, a PFP contract is optimal only if the buyer shares the supplier's knowledge of initial production cost.
cumulative distribution function. In contrast to innate cost, which is observed only by the supplier, the supplier’s final production cost is observed publicly. Final cost (c) is the difference between innate cost and the supplier’s cost-reducing effort, e. Formally, c = β̂ - e. The supplier’s utility, u(·), is the difference between the payment (p(c)) and the personal cost he incurs in delivering cost-reducing effort.

C(eβ) will denote the supplier’s personal (unobserved) cost of delivering effort e when innate cost is β. To reflect diminishing returns to cost-reducing effort, we follow LT in assuming C(eβ) is a strictly increasing, strictly convex function of e for e > 0, with C(0) = 0. In contrast to LT, we allow for the possibility that the supplier may also imply that incremental cost reductions will be more onerous for the supplier, for example. Chu and Sappington (2007b) analyze a setting in which the supplier’s ability to in saving innovations discovered in earlier research projects, for example.

Expression (4) defines the effort the supplier with innate cost β must supply to secure final cost β̂ - e(β̂).

3. Technical analysis

The purpose of this section is two-fold: (1) to outline the key steps in solving [BP]; and (2) to explain how the distinguishing assumption of our model – that it is less onerous for the supplier to implement cost reductions when costs are initially high – complicates the technical analysis. The reader who is primarily interested in our qualitative findings may prefer to skip much of the technical analysis in this section.

To solve [BP], we follow LT by replacing the global incentive compatibility (GIC) constraints (3) with local first-order conditions w(β, e) = 0 for all β ∈ [β̂, ̂β] that call the local first-order incentive compatibility (LIC) constraints. If the corresponding local second-order conditions are satisfied (i.e., if w(β, e) = 0 for all β ∈ [β̂, ̂β]), then the LFIC constraints ensure that the supplier will not exaggerate or understate his innate cost slightly (i.e., locally). However, the LFIC constraints do not preclude pronounced misrepresentation of β, even when the local second-order conditions are satisfied. We will denote by [BP⁺] the buyer’s problem where the LFIC constraints replace the GIC constraints. To solve [BP], we first solve [BP⁺]. Then we determine if the GIC constraints are satisfied at the identified solution to [BP⁺]. If the GIC constraints are satisfied (as they will be when the sufficient condition presented in Lemma 4 is satisfied), then the identified solution to [BP⁺] is the solution to [BP]. If the GIC constraints are not satisfied, then the identified solution to [BP⁺] will not constitute the solution to [BP], and so the solution must be identified by other means (such as those employed in the setting of Finding 2 below).

The LFIC constraints ensure w(β, e) = 0 for all β ∈ [β̂, ̂β]. Furthermore, Eq. (4) implies w(β̂, ̂β) = 0. Therefore, Eq. (3) implies that when the LFIC constraints are satisfied, the supplier’s utility must increase with β at the rate:

\[ \frac{\partial u(\hat{\beta}|\hat{\beta})}{\partial \beta} |_{\hat{\beta}} = u'(|\hat{\beta}) = -[C_1(\epsilon(\hat{\beta}), \hat{\beta}) + C_2(\epsilon(\hat{\beta}), \hat{\beta})] \text{ for all } \beta \in [\hat{\beta}, \bar{\beta}]. \] (5)

When u'(|\hat{\beta}) ≤ 0 for all β ∈ [β̂, ̂β], the buyer’s participation constraints in Eq. (2) will be satisfied for all β ∈ [β̂, ̂β] as long as u(β̂) = 0. Consequently, the buyer’s utility given innate cost β can be expressed as u(β) = - ∫ β̂ β e(ξ)|dξ in this case. Therefore, from expressions (2) and (5), the payment from the buyer to the supplier with innate cost β can be written as:

\[ p(\beta) = \beta - e(\beta) + C(\epsilon(\beta), \beta) + \int_{\beta}^{\hat{\beta}} \left[ C_1(\epsilon(\xi), \xi) + C_2(\epsilon(\xi), \xi) \right] d\xi. \] (6)

Substituting the expression in Eq. (6) into Eq. (1) and integrating by parts reveals that [BP⁺] can be written as:

Minimize \[ p(\beta) = \beta - e(\beta) + C(\epsilon(\beta), \beta) + \int_{\beta}^{\hat{\beta}} \left[ C_1(\epsilon(\xi), \xi) + C_2(\epsilon(\xi), \xi) \right] d\xi. \] (7)

Thus, the supplier’s reservation utility is normalized to zero.

From the revelation principle (e.g., Myerson, 1979), this formulation is without loss of generality.

The key qualitative conclusions drawn below also hold if C(·) = 1 for some β ∈ [β̂, ̂β], but the proofs are more tedious in this case.

Thus, we assume the supplier pays for his final production costs. LT assume the buyer pays these costs. The particular convention that is adopted is inconsequential.

Negative effort (e < 0) is feasible. As LT do implicitly, we assume C(0) = 0 for e < 0, so the supplier is able to inflate his innate cost at will, but enjoys no direct increase in utility from doing so. Laffont and Tirole (1992) allow the supplier to benefit directly from cost inflation (so negative effort entails perquisites, for example). Chu and Sappington (2007b) analyze a setting in which the supplier’s ability to inflate his innate cost is limited.

Thus, the supplier’s reservation utility is normalized to zero.

From the revelation principle (e.g., Myerson, 1979), this formulation is without loss of generality.
Assumption 1.

\[ C(\beta, \beta) = K \left\{ \frac{\beta - \beta^*}{\hat{\beta} - \beta^*} \right\}^2 \] where \( K > 0 \).

Assumption 2.

\[ K \geq \left\{ \frac{\beta^*}{\hat{\beta} - \beta^*} \right\}^{-1} \]

Assumption 3.

\[ \gamma \in [-2, -1] \]

Assumption 1 simplifies the ensuing analysis by assuming \( C(\beta, \beta) \) to be a quadratic function of effort (\( \epsilon \)). Assumption 2 ensures that effort costs are relatively large. Assumption 3 requires \( \gamma \) to be negative, and so lower innate costs are associated with higher total and marginal costs of effort. The bounds that Assumption 3 imposes on \( \gamma \) ensure that the supplier’s effort costs decline substantially as his innate cost increases, but that the decline is not so pronounced as to cause the supplier’s equilibrium utility to increase. Therefore, the supplier has a systematic incentive to exaggerate his innate cost when Assumptions 1–3 hold, as he does in LT’s model.12 Unless otherwise noted, Assumptions 1–3 will be maintained throughout the ensuing analysis.

Lemma 1 confirms that the supplier’s equilibrium utility declines as his innate cost increases at the solution to [BP]. When Assumptions 1–3 hold, just as in LT’s basic model, Lemma 1 also identifies an upper bound on the supplier’s equilibrium level of cost-reducing effort.

**Lemma 1.** \( u_1(\beta) \leq 0 \) and \( e(\beta) \leq \hat{\beta} - \beta^* \) for all \( \beta = [\hat{\beta}, \hat{\beta}] \) at the solution to [BP].

**Lemma 2.** \( e(\beta) = \frac{1}{2K} \left( \frac{\beta - \hat{\beta}}{\hat{\beta} - \beta} \right) \) for all \( \beta = [\hat{\beta}, \hat{\beta}] \).

**Lemma 3.**

\[ e(\beta) \leq \epsilon^* = \frac{1}{2\gamma} \left( \gamma - 1 \right) \left( \beta - \beta^* \right) \]

Lemmas 1–3 characterize the solution to [BP], which imposes only the local first-order incentive compatibility (LIFC) constraints. However, our ultimate concern is with the solution to [BP], which imposes the relevant global incentive compatibility (GIC) constraints. The GIC constraints will be satisfied at the solution to [BP] if:

\[ \frac{\partial u(\hat{\beta})}{\partial \beta} \geq 0 \text{ for all } \hat{\beta} < \beta \text{ and } \frac{\partial u(\hat{\beta})}{\partial \beta} \leq 0 \text{ for all } \hat{\beta} > \beta. \]

The first and second inequalities in Eq. (10), respectively, ensure that the agent with innate cost \( \beta \) will not gain by understating or overstating \( \beta \). Lemma 4 provides a sufficient condition for the inequalities in Eq. (10) to hold at the solution to [BP].

**Lemma 4.** If \( e_2(\beta) \leq 1 \) for all \( \beta = [\hat{\beta}, \hat{\beta}] \), then \( e_2(\beta) \) is the effort supply at the solution to [BP].

**Lemma 4** reveals that if realized production costs \( (\beta - e_2(\beta)) \) increase with \( \beta \) at the solution to [BP], then this solution will constitute the solution to [BP].13 The analysis of Finding 4 in Section 4 reveals how the solution to [BP] is identified when production costs do not increase with \( \beta \) systematically (because \( e_2(\beta) > 1 \) or some \( \beta \)) at the solution to [BP].

12 If a reduction in \( \beta \) were to increase \( C(\beta, \beta) \) more substantially, countervailing incentives (e.g., Lewis and Sappington, 1989; Maggi and Rodriguez-Clare, 1995; Jullien, 2000) could arise in that the supplier might wish to underestimate his innate cost in order to exaggerate his effort costs. We impose assumptions that are sufficient to preclude countervailing incentives in order to simplify the technical analysis and to ensure that the basic structure in our model parallels the structure in LT’s basic model.

13 The fact that \( \frac{\partial u(\hat{\beta})}{\partial \beta} \leq 0 \) for all \( \beta = [\hat{\beta}, \hat{\beta}] \) is instrumental in ensuring that \( e_2(\beta) \leq 1 \) and so realized production costs increase with \( \hat{\beta} \) in LT’s model. This monotone inverse hazard rate condition does not ensure that \( e_2(\beta) \leq 1 \) in our model, as the analysis of Finding 2 reveals.
4. Findings

We now illustrate some of the rich variety of optimal procurement contracts that can arise even under the restrictions imposed by Assumptions 1–3. Finding 1 provides one set of conditions under which a pure cost reimbursement (PCR) contract is optimal. Under the specified conditions, the buyer optimally induces no cost-reducing effort from the supplier even though the effort would systematically reduce total production costs.

Finding 1. Suppose $f(\beta) = \frac{1}{\beta}$, $\gamma = -2$, and $K = \frac{1}{2} \left[ \beta - \beta \right]^{-1}$. Then $p(c) = c$ for all cost realizations, $c$, and so $e(\beta) = 0$ for all $\beta \in [\beta, \beta]$ at the solution to [BP].

Notice from Lemma 3 that the first-best effort, $e^*(\beta) = \frac{2[\gamma, \beta]}{\beta}$, is strictly positive for all $\beta$ in the setting of Finding 1. Yet the buyer optimally implements a PCR contract, which induces no cost-reducing effort from the supplier. This decision reflects the well-known trade-off between rent and efficiency, modified to account for the cost structure under consideration here.

The buyer can always reduce her procurement costs when innate costs ($\beta$) are high by inducing the supplier to deliver some cost-reducing effort ($e$). Effort is motivated by sharing with the supplier some of the cost savings that his effort secures. This sharing is implemented via a payment in excess of realized cost over a range of $\beta$'s. Of course, payment in excess of cost over a range of high $\beta$'s provides rent to the supplier when the smaller innate costs are realized. The rent arises because the supplier can always substitute his good fortune (low innate cost) for the costly effort that the supplier must deliver when $\beta$ is high to achieve any specified final cost level. To avoid awarding the supplier excessive rent when $\beta$ is low, the buyer may choose to induce less than the first-best effort from the supplier when $\beta$ is high.

Indeed, the buyer may optimally induce no cost-reducing effort from the supplier for the highest innate cost realizations. The buyer can do so by implementing a cost reimbursement contract that pays the supplier exactly his realized production costs when relatively high production costs are observed. Because it provides the supplier with no incentive to deliver cost-reducing effort for the highest $\beta$ realizations, such a contract promotes relatively high procurement costs for these innate cost realizations. However, the contract can reduce overall expected procurement costs by limiting the rent that the supplier commands when the smaller realizations of $\beta$ arise.

In LT’s model, it is never optimal to implement cost reimbursement over the entire range of $\beta$ realizations. This is the case because the first-best effort supply does not vary with $\beta$ in LT’s model, since $C(\cdot)$ is independent of $\beta$. Consequently, a policy that never induced any cost-reducing effort from the supplier would incur substantial efficiency losses (i.e., $e^* - C(e^*)$) for all $\beta$ realizations while providing very limited gains for the smaller $\beta$ realizations. These gains would be limited because as the supplier’s innate cost declines toward $\beta$, the likelihood of even lower innate cost realizations becomes small, and so the reduction in the supplier’s expected rent that is secured by implementing cost reimbursement becomes small relative to the efficiency losses associated with no cost-reducing effort.

In contrast to LT’s model, the first-best effort supply declines as $\beta$ declines in our model. As the supplier’s innate cost declines, it becomes more difficult for him to implement further cost reductions (since $C_2'(\beta) < 0$ and $C_1(\cdot) < 0$), and so the efficient level of cost reduction declines. Therefore, the increase in procurement cost that arises when the supplier delivers no cost-reducing effort becomes less pronounced as $\beta$ declines. Consequently, even though the reduction in the supplier's expected rent that is secured by inducing no cost-reducing effort from the supplier declines as $\beta$ declines, the loss from inducing this effort distortion (i.e., the increase in the sum of production and effort costs above the efficient level) also declines as $\beta$ declines. In the setting of Finding 1 where all smaller realizations of $\beta$ are equally likely and where $|\gamma|$ is relatively large, the reduction in expected rent that is secured by inducing no cost-reducing effort continues to exceed the (diminishing) losses from the inefficiently small effort supply as $\beta$ declines toward its lower bound. Consequently, the supplier optimally implements a pure cost reimbursement contract (as illustrated in Fig. 2) in this setting even though such a contract is never optimal in the basic setting considered by LT.  

Finding 2 reveals another contract form – an incentive fixed price contract like the one depicted in Fig. 3 – that can be an optimal contract here, but not in LT’s basic model. Finding 2 considers a setting where the higher innate cost realizations are relatively likely and the lower innate cost realizations are relatively unlikely. In such a setting, the supplier will optimally induce the supplier to deliver cost-reducing effort when such effort is least costly for the supplier (i.e., for the higher $\beta$ realizations). Although the effort provides rent to the supplier when the lower $\beta$'s arise, these realizations are not very likely, and so the corresponding expected rent is limited.

Finding 2. Suppose $f(\beta) = \frac{1}{\beta}$, $\gamma = -2$, and $K = \frac{1}{2} \left[ \beta - \beta \right]^{-1}$. Then $p(c) = c + b$ for all $c \leq c_0$ and $p(c) = c + b$ for $c > c_0$ at the solution to [BP], where $c_0 = \frac{1}{2} \left[ 3 - \sqrt{7} \right] \beta + \frac{1}{2} \sqrt{7} - 1$ and $b$ is a strictly positive constant.

Consequently, $e(\beta) = 0$ for $\beta = c_0$, $c < c_0$ for $\beta = c_0$, and $c > c_0$ at the solution to [BP].

Notice that the buyer optimally induces the supplier to realize the same production cost, $c_0$, for all innate costs in excess of this level (i.e., for all $\beta \in [\beta, \beta]$) in the setting of Finding 4. This policy reflects a compromise between the buyer’s preferred outcome and feasible outcomes. Ideally, the buyer would like to induce substantial cost-reducing effort for the highest innate cost realizations and relatively little effort for the intermediate realizations in the setting of Finding 2. This is the case because the highest $\beta$ realizations are relatively likely compared to the intermediate realizations and because cost-reducing effort becomes substantially less onerous for the supplier as $\beta$ increases due to the relatively large value of $|\gamma|$. To induce her preferred pattern of cost-reducing effort, the buyer would have to offer payments that are relatively insensitive to realized cost for the highest $\beta$ realizations but that track realized costs more closely for the intermediate $\beta$ realizations. This payment structure, coupled with effort supply costs that decline substantially as $\beta$ increases, would induce effort that increases so rapidly with $\beta$ that final cost ($\beta - e(\beta)$) would decline as $\beta$ increases over a range of the higher innate cost realizations at the solution to [BP]. Such a reward structure would create an incentive for a supplier with an intermediate $\beta$ realization (who can secure a low final cost with little or no effort) to exaggerate his true innate cost realization.

To prevent such innate cost exaggeration, the buyer would have to induce less cost-reducing effort for the highest $\beta$ realizations (by making payments track costs more closely) and induce more cost-reducing effort for the intermediate $\beta$ realizations (by making payments track costs less closely). To eliminate all incentives to exaggerate $\beta$, this process would have to continue until the reward structure induced the

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14 As the proof of Finding 1 reveals, this is the case even when the solution to [BP] is not the solution to [BP] as in the analysis of Guennerie and Lafont (1984), for example.

15 Technically, the local second-order conditions are not satisfied for all $\beta \in [\beta, \beta]$ at the solution to [BP], i.e., $\frac{\delta^2 u}{\delta \beta^2} < 0$ for some $\beta$. This is the case because, as noted in the Appendix A, $\frac{\delta u}{\delta \beta} < 0$ if and only if $[C_2(\beta) + C_1(\beta)]e(\beta) = 1$. Furthermore, Assumption 1 implies that $C_2(\beta) + C_1(\beta) > 0$ at the solution to [BP]. Therefore, the local second-order conditions require $\beta < 0$, so final cost cannot decline with $\beta$. However, it can be shown that $e(\beta) > 0$ for the higher realizations of $\beta$ at the solution to [BP] in the setting. Consequently, although the LFC constraints ensure $\frac{\delta u}{\delta \beta} < 0$ for all $\beta \in [\beta, \beta]$, truthful reporting of $\beta$ provides a local minimum rather than a local maximum of the supplier’s utility.
same final cost realization for the intermediate and the highest $\beta$ realizations, as it does under the optimal policy described in Finding 2.

The lowest innate cost realizations are relatively unlikely in the setting of Finding 4. Therefore, the losses that arise when the supplier is induced to deliver no cost-reducing effort for the lowest $\beta$ realizations are relatively unlikely to be incurred. For this reason, and because the first-best effort level declines rapidly as $\beta$ declines since $\gamma$ is relatively large, the expected gains from inducing no cost-reducing effort from the supplier for all of the lowest $\beta$ realizations outweigh the corresponding expected costs. Therefore, the buyer optimally implements cost reimbursement (which induces no cost-reducing effort) for all of the smaller innate cost realizations in the setting of Finding 2.

If the lowest innate cost realizations were less likely than they are in the setting of Finding 2, the buyer would be less concerned about the rent that the supplier secures when these smallest $\beta$ realizations occur. Consequently, the buyer would optimally induce some cost-reducing effort from the supplier when the intermediate innate cost realizations arise. As a result, the disparity in induced effort supply (and corresponding compensation) between the intermediate and the highest $\beta$ realizations would be less pronounced than it is under the buyer’s ideal reward structure in the setting of Finding 2. Consequently, the supplier’s incentive to exaggerate intermediate $\beta$ realizations would be eliminated, and so the optimal contract would specify payments that: (i) increase with costs dollar for dollar for the lower cost realizations; and (ii) increase with costs at a decreasing rate for the higher cost realizations. It is readily shown that the optimal procurement contract assumes this concave structure if, for example, $\gamma=-2$, $K=\frac{1}{4}(\bar{\beta} - \underline{\beta})^{-1}$, and $f(\beta) = \frac{\underline{\beta} - \beta}{d+1}$.

The pattern of cost-reducing effort described in Finding 2 stands in contrast to the effort supply induced in LT’s basic model. In that model, cost-reducing effort declines as $\beta$ increases, and no effort may be induced over a range of the higher innate cost realizations. In the setting of Finding 2, effort increases with $\beta$ for the largest innate cost realizations (i.e., for $\beta \in [\bar{\beta}, \bar{\beta}_2]$) and no effort is induced for the smallest innate cost realizations. These differences, of course, reflect the fact that effort costs increase as $\beta$ declines here, while effort costs are independent of $\beta$ in LT’s model.

Finding 3 considers a setting where intermediate innate cost realizations are relatively likely while the lowest and highest $\beta$ realizations are relatively unlikely. This setting gives rise to an optimal procurement contract like the one depicted in Fig. 5, where payments increase with costs dollar for dollar for the extreme cost realizations. This cost sharing is introduced for intermediate cost realizations.

**Finding 3.** Suppose $\gamma=-2$, $K=[\beta - \bar{\beta}]^{-1}$, and $f(\beta) = \frac{20\beta^2 + 2\beta - \bar{\beta}}{\beta^2 - \bar{\beta}}$.

Then at the solution to [BP]: $c(\beta) = \begin{cases} 0 & \text{for } \beta_0 \leq \beta_1 \\ e_0(\beta) & \text{for } \beta_1 < \beta \leq \beta_2 \\ 0 & \text{for } \beta > \beta_2 \end{cases}$

where $\beta_0 = \frac{5+\sqrt{10}}{2} \, [10 - \sqrt{10}]$ and $\beta_2 = \beta_1 + 2 \sqrt{\frac{30}{101} (\bar{\beta} - \underline{\beta})}$.15

The supplier’s effort varies continuously with his innate cost under the optimal contract in the setting of Finding 3. The supplier delivers no cost-reducing effort for the highest innate cost realizations ($\beta = \beta_2, \beta_3$) in this setting. Although such effort would reduce procurement costs for the highest innate cost realizations, it would increase procurement costs for the intermediate $\beta$ realizations by allowing the supplier to secure rent. Because the intermediate $\beta$’s are relatively likely in the setting of Finding 3, the buyer optimally reduces rent for these realizations by implementing cost reimbursement (and thus inducing no cost-reducing effort) for the highest $\beta$ realizations. Thus, as depicted in the upper linear segment of the contract in Fig. 5, the payment to the agent increases at the same rate that his final cost increases for the highest cost realizations.

The buyer also induces no cost-reducing effort for the lowest innate costs ($\beta \in [\beta_0, \beta_1]$) by implementing payments that increase at the same rate that the supplier’s final cost increases for the smallest realized cost levels, as illustrated in the lower linear segment of the contract in Fig. 5. Effort is relatively costly for the supplier when these lower innate costs are realized, and so the costs of effort (including the rent generated by expanded effort) outweigh its benefits in this region.

In contrast, the buyer induces cost-reducing effort by implementing cost sharing for the intermediate innate cost realizations ($\beta \in [\beta_1, \beta_2]$) in the setting of Finding 3. Although the cost sharing for intermediate $\beta$’s admits rent for the lower $\beta$’s, these innate cost realizations are relatively unlikely in the setting of Finding 3, and so expected rent is relatively limited. Consequently, the expected reduction in procurement cost introduced by cost sharing for intermediate $\beta$’s outweighs the corresponding expected increase in procurement cost for the lower $\beta$’s, and so such cost sharing is optimal. The cost sharing is implemented by payments that increase less rapidly than realized costs for the intermediate values of realized costs, as in the middle (non-linear) segment of the contract in Fig. 5.

Of course, when $\gamma<0$ is sufficiently close to zero (or when $\gamma>0$), convex contracts of the type identified by LT and illustrated in Fig. 4 also can be optimal. In this case, effort costs do not increase much as $\beta$ declines toward $\bar{\beta}$. Consequently, for the reasons identified by LT, it will often be optimal for the buyer to induce the supplier to deliver considerable cost-reducing effort when the lowest $\beta$’s arise. This pattern of effort supply is optimally induced by a convex payment structure in which payments vary little with observed costs for the lowest cost realizations and track costs more closely for the highest cost realizations.

In summary, optimal procurement contracts may be convex, as LT’s basic model predicts. However, as Findings 1–3 illustrate, optimal procurement contracts can take on a variety of other forms (such as those depicted in Figs. 2, 3, and 5) when high initial costs are associated with sufficiently rich opportunities for cost reduction.17

**5. Conclusion**

We have demonstrated that a simple extension of LT’s classic procurement model admits a variety of contract forms that are employed in practice. The extension considers settings in which the supplier finds it less onerous to reduce production costs when his innate cost ($\beta$) is high than when it is low. In these settings, the buyer would induce more cost-reducing effort from the supplier as $\beta$ increases if $\beta$ were observed publicly. However, in standard fashion, asymmetric knowledge of $\beta$ makes it optimal to induce less effort as $\beta$ increases, *ceteris paribus*, in order to reduce the supplier’s rent. These conflicting forces give rise to a wide variety of optimal procurement contracts.

To illustrate, when the highest realizations of $\beta$ are particularly likely and when it becomes substantially less onerous for the supplier to reduce his production costs as $\beta$ increases, the buyer will optimally induce the supplier to deliver substantial cost-reducing effort for the highest $\beta$ realizations and little or no effort for the smallest $\beta$ realizations. This effort supply is optimally induced with a concave payment structure, in contrast to the convex contracts that are always optimal in LT’s basic model (where the supplier’s effort costs do not vary with $\beta$). Under conditions such as those identified in Finding 2, the optimal contract can be a simple piecewise linear contract of the form illustrated in Fig. 3.

More generally, optimal procurement contracts may be concave but not piecewise linear, they may be convex, or they may contain both convex and concave regions. The optimal procurement contract

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15 It is readily verified that $e_0(\beta) = \begin{cases} \frac{145 + 20\beta - 10\beta_1}{20 - 4\beta_1} \, (\beta - \bar{\beta}) & \text{for } \beta \in [\beta_1, \beta_2] \end{cases}$ in the setting of Finding 3, where $x = \frac{\beta - \underline{\beta}}{\bar{\beta} - \underline{\beta}}$.

17 As noted above, a pure fixed price contract like the one illustrated in Fig. 1 will only be optimal (both here and in LT’s model) if the buyer shares the supplier’s knowledge of $\beta$ from the outset of their relationship.
may also be linear, as in a pure cost reimbursement contract. The form of the optimal contract reflects the intuitive and classic trade-off between rent and efficiency, adjusted to account for the possibility that cost reduction becomes more onerous as initial costs decline.

Although the extension of LIT’s model that we have considered is a simple one, the extension complicates considerably the characterization of optimal procurement contracts. We have analyzed optimal contract design in structured settings that ensure strong parallels with LIT’s basic model. Future research might analyze alternative settings, including those in which the supplier’s effort costs decline more rapidly as innate cost (β) increases, so that the supplier’s equilibrium utility increases with β over some or all relevant ranges.

Future research also might proceed beyond the highly structured settings that we have analyzed to characterize optimal procurement contracts more generally. The examples we analyzed suggest that a pure cost reimbursement contract (as illustrated in Fig. 2) will tend to be optimal when the potential for further cost reduction declines fairly rapidly as innate cost declines while the likelihood of the lowest innate cost realizations is relatively pronounced. In contrast, an incentive fixed price contract like the one depicted in Fig. 3 will tend to be optimal when the higher innate cost realizations are substantially more likely than the lower innate cost realizations. In addition, a contract with linear segments for the highest and lowest cost realizations and a concave structure for intermediate cost realizations (as illustrated in Fig. 5) may be optimal when the intermediate β realizations are particularly likely. General conditions under which these conclusions hold remain to be specified. A more general specification of how the properties of optimal procurement contracts are affected by the interaction between the buyer’s initial beliefs and the intensity of the inverse relationship between innate costs and the potential for cost reduction also awaits future research.

Appendix

A.1. Sketch of Proof of Lemma 1

From Eq. (5) in the text, u(β) ≤ 0 for all β = [β, β] if C(1(β) + C2(β) ≥ 0 for all β = [β, β]. It is readily shown that when Assumption 1 holds:

\[ C_1(β) + C_2(β) ≥ 0 \Rightarrow e(β) ≤ 2(β - β) |γ| \]

(A1)

The Lemma is then proved by showing that for any feasible contract, an alternative contract with e(β) ≤ β ≤ 2(β - β) by Assumption 3) for all β = [β, β] can be constructed that has lower expected payment than the original contract. The alternative contract is of the form ([(β', β'), β', β]) U (Cp, β', β], C(β, β, i = j) where (Cp, β, i = j) is the set of cost-payment pairs that constitute the original contract. Although the alternative contract may induce a higher realized production cost by reducing the supplier’s equilibrium effort supply, the corresponding reduction in payment reduces expected procurement costs.

Recall that the buyer’s task in problem [BP] is to minimize P = ∫ R(e(β))dF(β), where:

\[ R(e(β)) = β - e(β) + C(e(β), β) + [C_1(β, β) + C_2(β, β)] \frac{F(β)}{F(β)} \]

(A2)

When Assumption 1 holds:

\[ \frac{∂R(·)}{∂e} = -1 + 2K \frac{(β - β)^γ}{F(β)} + e2K \frac{(β - β)^γ}{F(β)} \left( 1 + \left( \frac{γ}{β - β} \right) \frac{F(β)}{F(β)} \right) \]

(A3)

\[ \frac{∂^2R(·)}{∂e^2} = -2K \frac{(β - β)^γ}{F(β)} \left( 1 + \left( \frac{γ}{β - β} \right) \frac{F(β)}{F(β)} \right) \]

(A4)

These expressions are useful in proving Observation 1 which, in turn, is useful in proving Lemma 2.

Observation 1. If β ∈ [β, β] at the solution to [BP], then if and only if

\[-1 + 2K \frac{(β - β)^γ}{F(β)} ≥ 0 \]

Sketch of Proof. Recall from Lemma 1 that u(β) ≤ 0 and e(β) ≤ β for all β = [β, β] at the solution to [BP] when Assumptions 1–3 hold. For a fixed β, the term in Eq. (A4) is independent of γ. Because γ < 0, this term can be negative, in which case expected payment is a concave function of e under the optimal contract. Consequently, the optimal e will be at a boundary: either 0 or β - β. (Although e = 0 is feasible, the supplier always delivers non-negative effort. Negative effort increases final production cost at least as rapidly as it increases the payment from the buyer, and so is not advantageous for the supplier).

It is readily shown that \[ \frac{dR}{de} (β) = 0 \] at e = \[ \frac{1 - 2K \frac{(β - β)^γ}{F(β)}}{1 + \left( \frac{γ}{β - β} \right) \frac{F(β)}{F(β)} } \] greater than \[ \frac{β - β}{F(β)} \]. Therefore, since R(·) attains its critical point on \[ \frac{(β - β)^γ}{F(β)} \] rather than on \[ \frac{β - β}{F(β)} \], the symmetry of the quadratic function R(·) implies that \[ \frac{K_1 e}{F(β)} ] ≤ \frac{R_k}{F(β)} \] under the specified condition. Since e = 0 under this condition, Eq. (A3) implies \[ \frac{dR}{de} (β) = -1 + 2K \frac{(β - β)^γ}{F(β)} \geq 0 \].

If the term in Eq. (A4) is positive, expected payment is a convex function of e under the optimal contract. In this case, if the expression in Eq. (A3) is non-negative at e = 0, then e is optimal. Again, then, \[ \frac{dR}{de} (β) = -1 + 2K \frac{(β - β)^γ}{F(β)} \geq 0 \] from Eq. (A3). In contrast, if the expression in Eq. (A3) is negative at e = 0 (so \[ -1 + 2K \frac{(β - β)^γ}{F(β)} ≤ 0 \]), then effort is optimal positive. Therefore, effort is optimally 0 if and only if \( -1 + 2K \frac{(β - β)^γ}{F(β)} = 0 \).

A.2. Sketch of Proof of Lemma 2

From Observation 1, e = 0 if \( -1 + 2K \frac{(β - β)^γ}{F(β)} = 0 \). Observation 1 also implies that if \( -1 + 2K \frac{(β - β)^γ}{F(β)} = 0 \), then the optimal e is the value of e at which the expression in Eq. (A3) is zero. This value is as specified in the Lemma.

A.3. Sketch of Proof of Lemma 3

From Observation 1, e(β) = 0 if \( -1 + 2K \frac{(β - β)^γ}{F(β)} = 0 \). Therefore, e(β) ≤ e(β) for e(β) ≥ 0 when \( -1 + 2K \frac{(β - β)^γ}{F(β)} = 0 \).

It is readily shown that when \( -1 + 2K \frac{(β - β)^γ}{F(β)} = 0 \):

\[ e(β) = e(β) = 0 \]

(A5)

The inequality in Eq. (A5) holds because: (1) K ≥ (β - β) −1 by Assumption 2, and so \[ \frac{K}{β - β} ≥ \frac{(β - β)^γ}{F(β)} \]; and (2) \[ \frac{γ}{β - β} ≥ 1 + \frac{γ}{β - β} \] by Assumption 3.

A.4. Sketch of Proof of Lemma 4

The proof proceeds by showing that under the maintained conditions:

\[ \frac{∂u(β)}{∂β} ≥ 0 \text{ for } β < β' \text{ and } \frac{∂u(β)}{∂β} ≤ 0 \text{ for } β > β' \]

(A6)

at the solution to [BP]. When Eq. (A6) holds, the global incentive compatibility (GIC) constraints will be satisfied at the solution to [BP], and so this solution will constitute the solution to [BP].
It is readily shown that when $\hat{\beta} < \beta$:

$$\frac{\partial u(\hat{\beta})}{\partial \beta} = \left[1 - e^{-\left(\frac{\beta - \gamma}{\beta - \beta}\right)}\right]C_1(\hat{\beta}(\beta; p_0) - \beta)$$.

(A7)

Because $e_0(\beta) \leq 1$ (by assumption), $1 - e^{-\left(\frac{\beta}{\beta - \beta}\right)} \geq 0$. Therefore, to prove that $\frac{\partial u(\hat{\beta})}{\partial \beta} \geq 0$, Eq. (A7) implies that it will suffice to show $C_1(\hat{\beta}(\beta; p_0), (\beta; p_0)) \leq 0$. From Assumption 1:

$$C_1(\hat{\beta}(\beta; p_0), (\beta; p_0)) = 2K\left(\frac{\beta - \beta}{\beta - \beta}\right) - \frac{\beta - \beta}{\beta - \beta} \geq 0$$.

(A8)

Eq. (A9) follows from Eq. (A8) because $e(\hat{\beta}, \beta) = e(\hat{\beta} + \beta - \hat{\beta})$ and so $e(\hat{\beta}; \beta) = e(\hat{\beta})$ from Eq. (4) in the text.

Because $\hat{\beta} < \beta$ and $\gamma > 0$, the expression in Eq. (A9) is decreasing in $e(\hat{\beta})$. Since $e(\hat{\beta}) < \epsilon(\beta)$, Eq. (A9) will hold if:

$$\left[\frac{\beta - \beta}{\beta - \beta}\right] e^*(\hat{\beta}) - \left[\frac{\beta - \beta}{\beta - \beta}\right] e^*(\hat{\beta}) + \left[\frac{\beta - \beta}{\beta - \beta}\right] \geq 0$$.

(A10)

$$\left[\frac{\hat{\beta} - \beta}{\hat{\beta} - \beta}\right] e^*(\hat{\beta}) - 1 + 2K\left[\frac{\hat{\beta} - \beta}{\hat{\beta} - \beta}\right] \geq 0$$.

(A11)

Eq. (A11) is derived from Eq. (A10) by dividing all terms by $\left(\frac{\beta - \beta}{\beta - \beta}\right)$. Because $\hat{\beta} < \beta$ and $K \geq 0$, Eq. (A11) will hold if:

$$\left[\frac{\beta - \beta}{\beta - \beta}\right] e^*(\hat{\beta}) + \left[\frac{\beta - \beta}{\beta - \beta}\right] \geq 0$$.

(A12)

Notice that $f(\hat{\beta}) = 0$. Furthermore:

$$f(\hat{\beta}) = \left[\frac{\beta - \beta}{\beta - \beta}\right] - 2\left[\frac{\beta - \beta}{\beta - \beta}\right] \leq 0$$.

(A13)

The inequality in Eq. (A13) holds because $|\gamma| \leq 2$, $\beta - \beta \leq \beta - \beta$, and $|\gamma| \geq 1$. Because $f(\hat{\beta}) < 0$ and $f(\hat{\beta}) = 0$, we know that Eq. (A12) holds for all $\hat{\beta} < \beta$. Therefore, $C_1(\hat{\beta}(\beta; p_0), (\beta; p_0)) \geq 0$.

The proof proceeds by demonstrating that the following two conclusions hold in the present setting:

**Conclusion 1.** For any feasible solution to [BP], there is a solution to [AF] that ensures lower expected payment for the buyer.

**Conclusion 2.** The solution to [AF] satisfies the GIC constraints and is of the form identified in the Finding. Consequently, the solution identified in the Finding is the solution to [BP] in this setting.

Conclusion 1 is proved as follows. For any effort function $\hat{e}(\beta)$ that satisfies the GIC constraints, define $e(\beta)$ such that $\hat{e}(\beta) = \max\{e(\beta) - e(\beta_0), \mu p_{\mu\mu} < \mu \} < e(\beta)$. It is readily verified that $e(\beta)$ is weakly increasing in $\beta$. Furthermore, it can be shown that $|\hat{e}(\beta) - e(\beta)| \leq e(\beta) - e(\beta_0)$ for all $\beta$ because $e(\beta)$ satisfies the GIC constraints. Consequently, since expected payment $E(R(\beta))$ is a convex, quadratic function of $e$ that attains its minimum value at $e_{\text{opt}}$, $R(\hat{e}(\beta)) \leq R(\hat{e}(\beta))$ for all $\beta$. Conclusion 1 is proved as follows. For any feasible solution to [AF], $\hat{e}(\beta)$, define $\beta_0 = \frac{e_{\text{opt}}}{2} + \sqrt{\frac{e_{\text{opt}}}{2} - \hat{e}(\beta)}$ as the realization of $\beta$ for which:

$$\beta - \hat{e}(\beta) > \beta_0 - \hat{e}(\beta) > \beta - e_0(\beta)$$.

Therefore, from Eq. (A4), expected payment is a concave function of $e$. Consequently, as shown in the proof of Observation 1, $e(\beta)$ is optimally 0 for all $\beta$ $\in [\hat{\beta}, \beta]$. Furthermore, it is readily verified that the GIC constraints are satisfied at this solution, so it is indeed the solution to [BP].
Such a $\beta_0$ exists as long as $\hat{c}(\beta) \leq e_0(\beta)$ because $\hat{c}(\beta) = \beta - \hat{e}(\beta)$ is (weakly) increasing in $\beta$ (recall $\hat{e}(\cdot)$ is a feasible solution to [AF]) and $e_0(\beta) = \beta - e_0(\beta)$ is a decreasing function of $\beta$ for $\beta = \left( \frac{\beta_1}{2} + \frac{\beta_2}{2} \right)$ (since $e_0(\beta) \geq 1$ in this region). If $\hat{c}(\beta) > e_0(\beta)$ (so $e_0(\beta) = \beta - e_0(\beta)$), define $\beta_0$ as $\beta$.

Now consider the following feasible solution to [AF]:

$$\hat{e}(\beta) = \begin{cases} 0 & \text{for } \beta < \beta_0 - e(\beta_0) \\ \beta - [\beta_0 - e(\beta_0)] & \text{for } \beta \geq \beta_0 - e(\beta_0). \end{cases}$$

It can be shown that $[\hat{c}(\beta) - e_0(\beta)] \geq [\hat{e}(\beta) - e_0(\beta)]$ for all $\beta \in [\beta_1, \beta_2]$. Because $R(e(\cdot))$ is a convex, quadratic function of $e(\cdot)$ that attains its minimum value at $e_0(\beta)$, $\hat{e}(\cdot)$ secures a lower value of $R(\cdot)$ than $e(\cdot)$.

Finally, the identified solution to [AF] can be shown to satisfy the GIC constraints using an approach similar to the approach employed in the proof of Lemma 4.  

A.7. Sketch of Proof of Finding 3

It is readily shown that $1 - 2k \left[ \frac{\hat{e} - \hat{e}}{\hat{e} - \hat{e}} \right] \geq 0$ in this setting if and only if $\beta \in (\beta_1, \beta_2)$. Therefore, from Observation 1, no effort is optimally induced for the lowest and the highest innate cost realizations at the solution to [BP]. Lemma 2 also implies that $e_0(\beta) = \left[ \frac{-15e^2 + 20e - 16}{28e^2 - 40e + 3} \right] [\beta - \hat{e}]$ for $\beta \in (\beta_1, \beta_2)$, where $x = \frac{\hat{e} - \hat{e}}{\hat{e} - \hat{e}}$. It is tedious but straightforward to verify that $e_0(\beta) < 1$. Therefore, by Lemmas 3 and 4, the GIC constraints are satisfied, and $e_0(\beta)$ is the solution to [BP].

References


