Implementing high-powered contracts to motivate intertemporal effort supply

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and

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We characterize the optimal contract between a principal and a risk-neutral, wealth-constrained agent when an adverse selection problem follows a moral hazard problem. The optimal contract in this setting often is more steeply sloped for the largest output levels than is the optimal contract in either the standard moral hazard setting or the standard adverse selection setting. The large incremental rewards for exceptional performance motivate the agent to deliver substantial effort both before and after he acquires privileged information about the production environment.

1. Introduction

In the standard moral hazard problem (e.g., Holmstrom, 1979), the principal and the agent are symmetrically informed about the environment in which they operate throughout their relationship. In contrast, in the standard adverse selection problem (e.g., Laffont and Tirole, 1986), the agent has superior knowledge of the environment from the outset of his relationship with the principal. Although these canonical models of moral hazard and adverse selection provide useful insight, they do not always provide the most accurate description of important agency relationships. In practice, critical information often arrives over time, and initial periods of largely symmetric information are followed by periods of asymmetric information.

Consider, for example, settings in which a principal such as a franchisor, an automobile dealership, or an investment banking firm contracts with an agent, such as a franchisee, a sales representative, or an investment banker. The contract might specify the fraction of the realized sales revenue the franchisee must pay to the franchisor, the salesperson’s compensation as a function of the number of automobiles he sells, or the share of the fees charged to his clients that the investment banker will be awarded. In all of these settings, the principal and the agent may initially be symmetrically informed about the potential profitability of their interaction, as in the standard moral hazard model. However, after signing the contract, the agent may acquire superior information about the needs and preferences of potential customers/clients and/or about local economic conditions, as in the standard adverse selection problem.

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The purpose of this research is to characterize the optimal agency contract in a setting where a principal and an agent initially are symmetrically informed about the stochastic environment in which they operate, but it is common knowledge that the agent will ultimately acquire superior knowledge about the environment. The agent is risk neutral, but has limited wealth. Consequently, the principal cannot resolve all contracting frictions simply by selling the operation to the agent before he becomes privately informed about its value.

We find that when an initial moral hazard problem is subsequently complicated by an adverse selection problem, optimal contracts can exhibit features that do not arise in the presence of either moral hazard or adverse selection alone. In particular, optimal contracts often provide incremental rewards for exceptional performance that exceed the incremental value of the agent’s performance. Such high-powered payment structures generally are not optimal in a pure adverse selection setting because they afford excessive rent to the privately informed agent. 1 (A high-powered payment structure is one that induces the agent to deliver more than the efficient [surplus-maximizing] level of output-enhancing effort for some realizations of his private information by paying the agent more than the incremental value he generates over some range of equilibrium performance levels.) However, large incremental rewards for exceptional performance are optimal in the present setting because they induce substantial output-increasing effort from the agent both before and after he acquires privileged knowledge of the environment. Early effort (before acquiring information) and late effort (after acquiring information) are both produced with increasing marginal cost. Therefore, early effort enables the agent to deliver exceptional performance at relatively modest incremental cost when the environment turns out to be highly productive. In contrast, early shirking can make it unduly costly for the agent to achieve the exceptional performance for which the particularly pronounced compensation is paid. In essence, high-powered payment structures increase the opportunity cost of early shirking, and thereby can motivate the agent to work diligently before he acquires precise information about the operating environment.

The principal values early effort in part because it serves as a bond that the agent puts at risk before he becomes privately informed about the operating environment. If the environment turns out to be particularly unproductive, the agent does not earn enough to recover the bond and so incurs negative ex post utility, even though both the payment he receives from the principal and his ex ante expected utility are nonnegative. Thus, the bond that the agent delivers in the form of early effort limits the rent that he ultimately commands from his private information.

The optimal contract tends to provide particularly generous incremental rewards for superior performance and to induce expected effort furthest above efficient levels when the magnitude and the duration of the ultimate information asymmetry are moderate. This is the case because when the magnitude and/or the duration of the asymmetry are pronounced, the pressing adverse selection concerns compel the principal to implement a relatively low powered contract in order to limit the rent of the privately informed agent. (A “low-powered contract” here denotes a payment structure under which payments to the agent increase sufficiently slowly with his output that he never supplies more than the efficient level of effort.) In contrast, when the magnitude and/or duration of the asymmetry are limited, adverse selection concerns are not very constraining and the principal can both induce the agent to deliver the efficient effort supply and eliminate his rent (i.e., achieve the first-best outcome) without employing a high-powered payment structure. Thus, high-powered reward structures are of greatest value when the magnitude and the duration of the information asymmetry are moderate.

In addition to producing new qualitative conclusions about the properties of an optimal reward structure, the linking of a moral hazard problem and an adverse selection problem introduces a technical complication that generally does not arise in either problem alone. The standard approach of characterizing the agent’s effort with the first-order condition for an interior

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1 Bower (1993) analyzes a two-period model in which the agent’s first-period effort reduces his production costs in both periods. The principal implements a high-powered reward structure in the second period in this model to limit the agent’s incentive to understate his ability and then reduce first-period effort (which increases second-period costs) to conceal the understatement.
optimum may not identify the global optimum in the present setting. This is the case because the agent may prefer to deliver little or no effort in order to avoid putting a large effort bond at risk. Thus, an alternative solution procedure is required to identify the optimal reward structure.

We develop and present these findings as follows. Section 2 describes the basic model on which our analysis focuses. Section 3 characterizes the optimal agency contract in benchmark settings. Section 4 analyzes the optimal contract in the setting of primary interest. Section 5 concludes and considers extensions of our analysis. The proofs of all findings are presented in the Appendix.

Before proceeding, we explain how our analysis relates to others in the literature. Like our model, models of the “rat race” (e.g., Akerlof, 1976; Miyazaki, 1977; Holmstrom, 1999) predict early effort supply in excess of the efficient level. Fully informed agents initially exert inefficiently high effort in these models in order to signal their superior capabilities and thereby secure higher competitive wages in subsequent periods. No such signalling is present in our model because the agent’s performance is not observed until the end of the interaction between the principal and agent. Furthermore, effort above efficient levels can arise both early and late in our model.

Several studies (e.g., Lewis and Sappington, 1997; Crémer, Khalil, and Rochet, 1998; Finkle, 2005) demonstrate how a set of contracts that includes a high-powered payment structure can induce a principal or an agent to acquire better information about the prevailing environment. Payments that vary sharply with observed performance can be designed to be profitable for parties that are well informed about the prevailing environment but unprofitable for parties that are not so informed. The high-powered payment structures that we identify do not serve to motivate information acquisition, as the information structure is exogenous in our model.3

The rationale for the high-powered payment structure in our model may be most similar to the rationale for the steeply sloped wages that arise in moral hazard models of intertemporal effort supply. Lazear (1979, 1981), for example, documents the value of paying an employee a wage below his marginal product early in his career and a wage above his marginal product later in his career. The generous late wage limits the employee’s incentive to shirk toward the end of his tenure with the firm. The low early wage limits the employee’s lifetime rent by effectively requiring him to post a bond that he forfeits if he shirks early in his career. Our analysis differs from these analyses in part because the agent in our model acquires private information about his productivity. The interplay between the moral hazard and adverse selection problems is central in our analysis, and underlies our conclusions regarding the relationship between the power of the optimal contract and both the magnitude and the duration of the information asymmetry between the principal and the agent.

As in our model, the posting of a bond plays an important role in Laux’s (2001) model, where multiple tasks are optimally assigned to a single agent. When he operates multiple tasks, the agent stands to forfeit the rent from all tasks if he shirks on a single task. The prospect of losing this large bond serves to motivate the wealth-constrained agent while limiting his rent. In contrast to Laux’s consideration of a pure moral hazard setting, we focus on the interplay between moral hazard and adverse selection concerns in a setting with a single task.

Other studies (e.g., Innes, 1990; Kim, 1997; Oyer, 2000) demonstrate that large bonuses for performance above a threshold can help to motivate a risk-neutral, wealth-constrained agent to supply unobserved effort. Zhou and Swan (2003) show that corresponding bonuses also can be optimal for a principal that uses piecewise linear contracts to motivate a risk-averse agent.4

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2 Chu and Sappington (2008) consider a corresponding more general model.
3 Laffont and Martimort (LM) (2002) analyze a model where a binary adverse selection problem follows a binary moral hazard problem and where first-period effort increases the likelihood of a more favorable second-period environment. To induce the agent to supply first-period effort, the principal may increase the agent’s rent in the more favorable environment by increasing the output he produces in the less favorable environment toward the efficient output.
4 Kahn and Huberman (1988) demonstrate the value of discontinuous reward structures in settings where performance is not contractible. (Also see Waldman, 1990 and O’Flaherty and Siow, 1992.) The agent’s final performance is fully contractible in our model.
The bonus payments in these static, pure moral hazard models do not induce effort in excess of efficient levels and typically do not secure the first-best outcome. In contrast, the optimal payment schedule in our model (under which the agent’s compensation increases rapidly but continuously with his performance over an extended range of the highest output levels) induces effort above the efficient levels when the magnitude and duration of the information asymmetry are moderate. The principal secures the first-best outcome in our model when the magnitude and duration of the information asymmetry are limited.

The high-powered payment structure that we identify also contrasts with the low-powered contracts that are optimal in Holmstrom and Milgrom’s (1991) multitasking model. In their model, it is optimal to implement a low-powered contract on performance dimensions that are readily measured in order to limit undue diversion of effort from other performance dimensions that are more difficult to measure. In our model, it is optimal to implement a high-powered payment structure on the sole observable performance measure in order to motivate multiple sources of effort that enhance performance and limit the agent’s rent by effectively increasing the bond he posts.

Holmstrom and Milgrom (1987) examine how best to motivate a risk-averse agent to deliver unobserved effort in a dynamic setting where the agent produces output and acquires updated information over time. HM show that when the agent has unlimited wealth and exhibits constant absolute risk aversion, payments to the agent optimally increase linearly with his aggregate output. Thus, it is as though the moral hazard problems that the principal faces in each period are independent. In contrast, the moral hazard problem and the adverse selection problem are intimately linked in our model, where the agent’s limited wealth precludes the posting of a financial bond. The principal offers large incremental payments for exceptional aggregate performance in order to induce the agent to both deliver substantial late effort and post an early effort bond in lieu of a financial bond.

2. Elements of the model

We consider the interaction between two risk-neutral parties, a principal and an agent. The principal offers the agent a contract at time 0 that will govern their relationship throughout the [0, 1] time interval. The agent will only accept the binding contract if it ensures him nonnegative expected utility during this interval. The contract specifies a payment \( P(\cdot) \) from the principal to the agent at time 1. This payment must be nonnegative because the agent has no wealth.\(^5\) The payment can vary with the agent’s final output, \( x \), which is observed at time 1. Final output is the sum of innate output \( (\beta \in [\underline{\beta}, \overline{\beta}]) \) and the total effort that the agent supplies during the \([0, 1]\) time interval.\(^6\) Final output is observed publicly, but only the agent observes his effort and the realization of \( \beta \).

The [0, 1] time interval is divided into two periods. During the first period, \([0, t)\), the agent shares the principal’s imperfect knowledge of innate output \( (\beta) \). We follow Laffont and Tirole (1986) (LT) in presuming that the principal’s initial beliefs about innate output \( (\beta) \) are captured by the uniform density function \( f(\beta) = \frac{1}{\Delta} \) on \([\underline{\beta}, \overline{\beta}]\), where \( \Delta \equiv \overline{\beta} - \underline{\beta} \). At time \( t \in [0, 1] \), the agent alone learns the exact value of \( \beta \). This information asymmetry persists throughout the second period, \([t, 1] \).\(^7\) Thus, the principal effectively faces a standard moral hazard problem in the first period and a standard adverse selection problem in the second period. However, the two problems are linked because it is common knowledge from the outset that the adverse selection problem will follow the initial moral hazard problem.

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\(^5\) If the agent’s wealth were not limited, the principal could simply sell the project to the agent at a price that reflects the expected value of the project, given the efficient effort supply throughout the \([0, 1]\) time interval (e.g., Shavell, 1979).
\(^6\) Innate output is the output that would arise if the agent delivered no effort.
\(^7\) Chu and Sappington (2008) consider alternative distributions of \( \beta \) and settings in which the agent’s information improves more gradually throughout the \([0, 1]\) time interval.
The agent’s utility is the difference between the payment he receives and the costs he incurs in delivering output-enhancing effort. \( r(\tau) \) will denote the rate at which the agent supplies effort at time \( \tau \in [0, 1] \). The agent’s instantaneous cost of supplying effort at rate \( r(\tau) \) is assumed to be \( \frac{[r(\tau)]^2}{4k} \), where \( k > 0 \) is a constant.\(^8\) Lemma 1 characterizes the agent’s minimum possible cost of supplying effort under the presumed quadratic cost structure. The lemma refers to two arbitrary points in time, \( t_1, t_2 \), where \( 0 \leq t_1 < t_2 \leq 1 \).

**Lemma 1.** The minimum cost of delivering total effort \( e \) during time interval \([t_1, t_2]\) is \( \frac{e^2}{4k[t_2-t_1]} \), because the agent’s effort cost is minimized when he delivers effort at the constant rate \( \frac{e}{t_2-t_1} \) throughout the period.

Lemma 1 implies that the agent’s minimum cost of delivering total effort \( e_1 \geq 0 \) in period 1 is \( \frac{[e_1]^2}{4k} \) and that his minimum cost of delivering total effort \( e_2 \geq 0 \) in period 2 is \( \frac{[e_2]^2}{4k(1-t)} \).\(^9\) Lemma 1 also implies that the surplus-maximizing (efficient) effort supply entails the delivery of effort at the constant rate \( 2k \) throughout the \([0, 1]\) time interval.\(^10\)

The principal seeks to maximize the difference between expected output and expected payment to the agent. The agent seeks to maximize the difference between the expected payment he receives from the principal and the costs he expects to incur in delivering output-enhancing effort. When he has delivered total effort \( e_1 \) in period 1, the agent’s second-period utility when innate output is \( \beta \) and he delivers total second-period effort \( e_2 \) is \( P(\beta + e_1 + e_2) - \frac{[e_2]^2}{4k(1-t)} \). Let \( U_2(e_1 | \beta) \) denote the maximum utility the agent can secure in the second period when innate output is \( \beta \) and the agent has delivered effort \( e_1 \) in the first period. Formally, \( U_2(e_1 | \beta) = \max_e \{ P(\beta + e_1 + e) - \frac{[e]^2}{4k(1-t)} \} \).

Let \([PP]\) denote the principal’s problem in this setting. Using Lemma 1, \([PP]\) can be written as

\[
\max_{\beta, e_1, e_2} \int_\beta [\beta + e_1 + e_2(\beta; e_1) - P(\beta + e_1 + e_2(\beta; e_1))] \frac{1}{\Delta} d\beta
\]

subject to:

\[
P(\cdot) \geq 0; \tag{2}
\]

\[
e_1 \in \arg\max_e \left\{ \int_\beta U_2(e | \beta) \frac{1}{\Delta} d\beta - \frac{e^2}{4kt} \right\}; \quad \text{and} \tag{3}
\]

\[
e_2(\beta; e_1) \in \arg\max_e \left\{ P(\beta + e_1 + e) - \frac{e^2}{4k(1-t)} \right\}. \tag{4}
\]

(1) reflects the principal’s desire to maximize the expected difference between output and payments to the agent. (2) ensures that all payments to the agent are nonnegative. (3) characterizes the agent’s effort choice in the first period. (4) captures the agent’s effort choice in the second period, given his private information and his first-period effort choice.\(^11\)

### 3. Benchmark solutions

Before characterizing the solution to \([PP]\), we describe the solutions to three benchmark problems.

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\(^8\) The presumed quadratic cost structure simplifies the ensuing technical analysis. Chu and Sappington (2008) consider more general cost structures.

\(^9\) Lemma 1 implies that the agent will deliver effort at a constant rate throughout the first period. He will also deliver effort at a (possibly different) constant rate throughout the second period.

\(^10\) The efficient rate of effort supply, \( r^* \), is the value of \( r \) that maximizes \( r - \frac{r^2}{4k} \). Therefore, \( r^* = 2k \).

\(^11\) \([PP]\) does not include an explicit participation constraint for the agent. However, the agent can always secure nonnegative expected utility by delivering no effort, because the payments he receives from the principal must be nonnegative.
The first-best solution. First consider the solution to the principal’s problem when the agent’s effort is observable and contractible. Let \( e_2(\beta) \) denote the agent’s effort supply in the second period when innate output is \( \beta \). Then, using Lemma 1, the principal’s problem in this first-best setting can be written as

\[
\text{Maximize } e_1 - \frac{e_1^2}{4kt} + \int_{\beta}^\beta \left( e_2(\beta) - \frac{[e_2(\beta)]^2}{4k[1 - t]} \right) \frac{1}{\Delta} d\beta.
\]

Performing the maximization in (5) provides

\[
e_1^* = 2kt; \quad \text{and } e_2^*(\beta) = 2k[1 - t] \quad \text{for all } \beta \in [\beta, \bar{\beta}].
\]

(6) implies that in the absence of moral hazard and adverse selection concerns, the principal will instruct the agent to supply effort at the constant (efficient) rate, \( 2k \), throughout the [0, 1] time interval.  

The solution when \( t = 0 \). Now consider the setting where \( t = 0 \), so the agent knows the innate output from the outset of his relationship with the principal. The principal faces a standard adverse selection problem in this setting. The solution to this problem, which is well known from LT’s analysis, is summarized in Lemma 2.

**Lemma 2.** Suppose \( t = 0 \). Then \( e_2(\beta) = e^*(\beta) \equiv \max[0, 2k - (\beta - \beta)] \) for all \( \beta \in [\beta, \bar{\beta}] \) at the solution to \( [P] \).

Lemma 2 reflects standard considerations. The principal optimally induces less than the first-best level of effort \( (2k) \) for all \( \beta < \bar{\beta} \) in order to limit the agent’s ability to substitute innate output for effort and thereby secure rent from his private knowledge of \( \beta \). The induced effort distortion is more pronounced the smaller is \( \beta \).

The solution when \( t = 1 \). Finally, suppose \( t = 1 \), so the first period lasts for the entire [0, 1] time interval and the agent never learns the realization of \( \beta \) before supplying effort. The principal faces a pure moral hazard problem in this setting. As Lemma 3 reveals, the principal can secure the first-best solution in this setting despite the agent’s limited wealth. The lemma refers to \( \chi \equiv \bar{\beta} + \max[0, \frac{1}{2}(2k - \Delta)] \).

**Lemma 3.** Suppose \( t = 1 \). Then the principal can ensure the first-best solution with the contract:

\[
P(x) = \begin{cases} 
0 & \text{for } x < \chi \\
\max \left\{ 1, \frac{\Delta}{2k} \right\} \left[ x - \chi \right] & \text{for } x \geq \chi.
\end{cases}
\]

When \( \Delta < 2k \), the magnitude of the information asymmetry (\( \Delta \)) is relatively small. In this case, the principal can secure the first-best solution when \( t = 1 \) by awarding to the agent all of the output he produces in excess of \( \chi = k + \frac{1}{2}[\beta + \bar{\beta}] \). When the agent supplies the efficient effort \( 2k \), he is certain to achieve an output of at least \( \beta + 2k > \chi \), and so he is assured of a nonnegative payment. The principal secures the entire expected surplus from efficient production \( (\chi) \) by awarding the agent only the incremental output above the efficient surplus level.  

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12 When the agent’s first-period effort, \( e_1 \), and final output, \( \beta + e_1 + e_2 \), are both observable, the principal may find it optimal to increase \( e_1 \) above its first-best level \( (e^*_1) \). Doing so increases the effort bond \( (\frac{e^2}{2kt}) \) that the agent forfeits when final output is sufficiently small, and thereby reduces the agent’s utility for all \( \beta \) realizations. (See Laffont and Martimort, 2002 for a related analysis.)

13 Following LT, we assume that the agent can deliver negative effort (which reduces output) at no personal cost. Chu and Sappington (2007) analyze the changes that arise in other settings.

14 The expected surplus from efficient production is \( E[\beta] + e^* - \frac{e^*^2}{4kt} = \frac{1}{2}[\beta + \bar{\beta}] + 2k - k = \frac{1}{2}[2\bar{\beta} - \Delta] + k = \bar{\beta} + \frac{1}{2}(2k - \Delta) = \chi \).
proof of Lemma 3 reveals, the prospect of retaining the entire output above \( x \) is sufficient to motivate the agent to deliver the efficient effort supply.

When \( \Delta > 2k \), the magnitude of the information asymmetry (\( \Delta \)) is more pronounced. In this case, the principal induces the agent to deliver the efficient effort supply by awarding him more than the incremental output he generates above \( \bar{\beta} \). Despite the large incremental reward for exceptional performance, this payment structure provides no rent to the agent because as \( \Delta \) increases above \( 2k \), the probability that the realized output exceeds \( \bar{\beta} \) when the agent delivers effort \( 2k \) (and thus the probability that the agent receives a positive payment from the principal) declines.\(^{15}\)

The principal is able to secure the first-best outcome when she faces only a moral hazard problem here because although payments to the agent must be nonnegative, the agent’s \( \text{ex post} \) utility can be negative. Given the presumed lower bound on \( \beta \), the agent can ensure that he always receives a nonnegative payment by delivering a sufficiently large level of effort. However, when the realized \( \beta \) is sufficiently close to \( \beta \), the agent secures only a negative level of utility because his effort cost exceeds the payment he receives. In essence, when realized performance is poor, the agent forfeits the effort bond that he has posted. The agent is willing to post this bond, though, because he recovers more than the bond when favorable realizations of \( \beta \) lead to output above the threshold. Through appropriate choice of the threshold output level and the incremental payment for output above the threshold, the principal can induce the agent to deliver the efficient effort supply while limiting his expected rent to zero.

4. The optimal contract

Now consider the setting of primary interest, where the agent’s effort is not observable and the principal faces both a moral hazard problem and an adverse selection problem (so \( t \in (0, 1) \)).

The agent’s first-period effort and second-period effort are perfect substitutes in increasing final output. Therefore, in order to minimize his effort costs, the agent will expand his first-period effort supply to the point where the associated marginal cost is equal to the corresponding expected marginal cost of supplying second-period effort, that is,

\[
\frac{e_1}{2kt} = \int_{\beta}^{\bar{\beta}} \left\{ \frac{e_2(\beta)}{2k[1 - t]} \right\} \frac{1}{\Delta} d\beta. \quad (7)
\]

(7) implies that the agent will supply effort in the first period at the same rate that he expects to supply effort in the second period, that is,

\[
\frac{e_1}{e^*_2} = \frac{E\{e_2(\beta)\}}{e^*_2} \quad \text{and so} \quad \frac{e_1}{t} = \frac{E\{e_2(\beta)\}}{1 - t}, \quad (8)
\]

where \( E\{\cdot\} \) denotes the agent’s first-period expectation about the prevailing innate output, \( \beta \).\(^{16}\)

(8) can be viewed as the first-order condition for the agent’s optimal choice of first-period effort. When the agent’s objective in (3) is a concave function of \( e_1 \) at the solution to \([PP]\), the agent’s first-period effort choice is accurately characterized by this first-order condition, which can be written as \( e_1 = \left[ \frac{t}{1 - t} \right] E\{e_2(\beta)\} \). In this case, the principal’s problem, \([PP]\), can be written

\(^{15}\) This probability is \( \int_{\beta - 2k}^{\bar{\beta}} \frac{1}{\Delta} d\beta = \frac{2k}{\Delta} \).

\(^{16}\) The equality in (8) reflects the presumed quadratic cost structure. More generally, if \( c(r(\tau)) \) denotes the agent’s instantaneous cost of supplying effort at rate \( r(\tau) \) at time \( \tau \), the agent will minimize his effort cost by delivering effort in the first period at a higher (respectively, lower) rate than he expects to deliver effort in the second period when \( c''(\cdot) > 0 \) (respectively, \( c''(\cdot) < 0 \)). Intuitively, when the agent’s instantaneous marginal cost of supplying effort increases at an increasing rate, the agent will deliver effort at a relatively high rate in the first period in order to limit the need to deliver effort at a particularly high rate (which entails substantial marginal cost) in the second period when the highest \( \beta \)’s are realized.
as the following problem, labeled \([PP]\)\(^{17}\):

\[
\begin{align*}
\text{Maximize} & \quad \int_{\beta}^{\bar{\beta}} \left\{ \left[ \frac{1}{1-t} \right] e_2(\beta; e_1) - P(\beta + e_1 + e_2(\beta; e_1)) \right\} \frac{1}{\Delta} d\beta \\
\text{subject to:} & \quad P(\cdot) \geq 0;
\end{align*}
\]

subject to:

\[P(\cdot) \geq 0;\]

and

\[
e_2(\beta; e_1) \in \operatorname{argmax}_e \left\{ P(\beta + e_1 + e) - \frac{e^2}{4k[1-t]} \right\},
\]

where \(e_1 = \left[ \frac{t}{1-t} \right] \int_{\beta}^{\bar{\beta}} e_2(\beta) \frac{1}{\Delta} d\beta.\)

The solution to \([PP]\) is derived using standard techniques. Let \(u(\beta) \equiv U_2(e_1 | \beta)\) denote the agent’s utility net of first-period effort costs under the optimal contract when innate output \(\beta\) is realized. Following LT, the agent’s objective is readily shown to be a concave function of his second-period effort. Furthermore, at any feasible solution to \([PP]\), the agent’s utility will increase with \(\beta\) at the rate

\[
u'(\beta) = \frac{e_2(\beta)}{2k[1-t]} \Rightarrow u(\beta) = u(\beta) + \int_{\beta}^{\bar{\beta}} \left[ \frac{e_2(\beta)}{2k[1-t]} \right] d\bar{\beta}.\]

Because the agent’s utility is the difference between the payment he receives from the principal and his effort cost, (10) implies

\[
P(\beta + e_1 + e_2(\beta; e_1)) = u(\beta) + \int_{\beta}^{\bar{\beta}} \left[ \frac{e_2(\beta)}{2k[1-t]} \right] d\bar{\beta} + \frac{[e_2(\beta)]^2}{4k[1-t]}.\] \hspace{1cm} (11)

Furthermore, integration by parts reveals

\[
\int_{\beta}^{\bar{\beta}} \int_{\beta}^{\bar{\beta}} \left[ \frac{e_2(\beta)}{2k[1-t]} \right] d\bar{\beta} \frac{1}{\Delta} d\beta = \int_{\beta}^{\bar{\beta}} \frac{e_2(\beta)}{2k[1-t]} \left[ \beta - \beta \right] \frac{1}{\Delta} d\beta.\] \hspace{1cm} (12)

Substituting from (11) and (12) into (9) implies that the principal can be viewed as choosing \(e_2(\beta) \geq 0\) to maximize

\[
\begin{align*}
\int_{\beta}^{\bar{\beta}} \left\{ \left[ \frac{1}{1-t} \right] e_2(\beta) - u(\beta) - \frac{e_2(\beta)}{2k[1-t]} \left[ \beta - \beta \right] - \frac{[e_2(\beta)]^2}{4k[1-t]} \right\} \frac{1}{\Delta} d\beta \\
= \left[ \frac{1}{1-t} \right] \int_{\beta}^{\bar{\beta}} \left\{ e_2(\beta) - \frac{[e_2(\beta)]^2}{4k} - \frac{e_2(\beta)}{2k} \left[ \beta - \beta \right] \right\} \frac{1}{\Delta} d\beta - u(\beta).
\end{align*}
\]

Pointwise optimization of (13) reveals that as long as the agent’s objective in (3) is a concave function of his first-period effort supply, the agent’s induced second-period effort supply at the solution to \([PP]\) is the standard effort supply, \(e^s(\beta)\), identified in Lemma 2. Thus, when the agent’s effort costs are quadratic, the principal will induce the same second-period effort supply in the present setting that she would induce in a standard, static adverse selection setting where the agent is privately informed about \(\beta\) from the outset of his relationship with the principal. This conclusion reflects the following considerations.

\(^{17}\)The coefficient on \(e_2(\cdot)\) in (9) reflects the fact that \(E[\beta + e_1] = E[\beta] + E[e_1] = E[\beta + e_1].\)

\(^{18}\)Define \(u(\beta | \beta) = P(\beta + e_1 + e_2(\beta)) - \frac{e_2(\beta | \beta)}{[e_2(\beta | \beta)]^2},\) where \(e_2(\beta | \beta) = \beta + e_2(\beta | \beta).\) Then the revelation principle implies that at any feasible solution to \([PP]\), \(u(\beta) = \frac{[e_2(\beta | \beta)]^2}{2k[1-t]} - \frac{e_2(\beta | \beta)}{2k} \left[ \beta - \beta \right] \frac{1}{\Delta} d\beta - u(\beta).\)

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If the principal were concerned only with second-period performance, she would induce the following second-period effort supply:\(^{19}\):

$$e_2(\beta) = \max\{0, 2k[1 - t] - [\beta - \bar{\beta}]\} \quad \text{for all } \beta \in [\beta, \bar{\beta}].$$  \hspace{1cm} (14)

The effort levels in (14) are less than the surplus-maximizing level \((2k[1 - t])\) for all \(\beta \in [\beta, \bar{\beta}]\). These distortions reflect the optimal balancing of rent and efficiency, given the agent’s cost \((\frac{\kappa^2}{4t(1-t)})\) of delivering second-period effort \((e_2)\). For each \(\beta\), the identified effort supply equates the marginal increase in output from expanding effort and the associated marginal increase in the sum of the agent’s effort cost and expected rent.

Of course, when \(t > 0\), the principal is concerned with both first-period and second-period performance. The principal recognizes that when the agent’s effort cost is quadratic, he will supply effort in the first period at the same rate that he expects to deliver effort in the second period (as (8) indicates). Consequently, by increasing second-period effort above the levels in (14), the principal can induce increased first-period effort. The principal values first-period effort in part because the cost that the agent incurs in delivering this effort \((\frac{\kappa^2}{4t})\) serves as a bond that the agent puts at risk before he learns the realization of \(\beta\). When the agent posts such a bond, his ex post utility can be reduced below 0 for the smallest \(\beta\) realizations even though the payment he receives from the principal and his ex ante expected utility are both nonnegative.\(^{20}\) The agent will experience negative utility when the maximum second-period utility \((P(\cdot) - \frac{\kappa^2}{4t(1-t)})\) he can secure is less than the effort cost he incurs in the first period. By reducing the agent’s utility below 0 for the smallest \(\beta\) realizations, the principal limits the rent that the agent commands from his private information about \(\beta\).\(^{21}\)

When the first period is short, the agent will post only a modest effort bond even if he supplies effort during the first period at a relatively high rate. Therefore, when the first period is short, the agent is willing to supply effort throughout the \([0, 1]\) time interval at the rate required to deliver the standard effort supply in the second period. The principal’s problem when she anticipates a constant expected rate of effort supply throughout the \([0, 1]\) time interval parallels her problem when she faces an adverse selection problem throughout this period (as reflected in (13)). Consequently, the principal optimally induces the standard effort supply in the second period, as Proposition 1 reports.

**Proposition 1.** Suppose \(t \in (0, \frac{\Delta}{\Delta + 2k})\). Then at the solution to \([PP], e_2(\beta) = e^*(\beta)\) for all \(\beta \in [\beta, \bar{\beta}]\).\(^{22}\) Consequently,

$$\frac{e_1}{e^*_1} = \frac{E[e_2(\beta)]}{e^*_2} \begin{cases} 1 & \text{if } t < \frac{\Delta}{4k} \\ \frac{\Delta}{4k} & \text{if } t > \frac{\Delta}{4k}. \end{cases}$$

Proposition 1 reveals that although the induced effort is less than the efficient effort for all \(\beta < \bar{\beta}\) in a standard adverse selection setting, the same effort levels can constitute effort in excess of the efficient level when they are induced during a relatively short period of time. This is the

\(^{19}\) This conclusion follows directly from LT’s analysis. The effort supply in (14) parallels the standard effort supply in Lemma 2 except that \(2k\) is multiplied by \(1 - t\). This scaling factor reflects the fact that when the duration of the information asymmetry is \(1 - t\) rather than 1, the agent’s effort cost is \(\frac{\kappa^2}{4t(1-t)}\) rather than \(\frac{\kappa^2}{4t}\), and so the surplus-maximizing effort supply is \(2k[1 - t]\) rather than \(2k\).

\(^{20}\) The agent will experience the lowest ex post utility for the smallest \(\beta\) realizations because, from (10), the agent’s utility is nondecreasing in \(\beta\) under any feasible payment schedule.

\(^{21}\) In the setting described in footnote 12 where first-period effort is observable, the principal can simply direct the agent to increase the first-period effort bond that he puts at risk. In the present setting, the principal must increase the induced second-period effort supply in order to induce the agent to post a larger first-period effort bond.

\(^{22}\) Furthermore, \(e_1 = [\frac{1}{\sqrt{\Delta}}][\frac{\kappa^2}{4k}]\) if \(\Delta < 2k\), whereas \(e_1 = [\frac{1}{\sqrt{\Delta}}]\frac{\kappa^2}{4k}\) if \(\Delta > 2k\) at the solution to \([PP]\) when \(t \in (0, \frac{\Delta}{\Delta + 2k})\).
case because the cost of delivering any fixed amount of effort is greater the shorter is the time period during which the effort is delivered. Therefore, when the second period is sufficiently short (i.e., when \( t > \frac{\Delta}{4k} \))), the induced second-period expected effort exceeds the efficient second-period effort \((2k[1 - t])\). First-period effort will similarly exceed efficient first-period effort because the agent finds it most profitable to deliver effort during the first period at the same rate that he expects to deliver effort during the second period (as (8) indicates).

The standard effort levels introduce a complication as the duration of the initial period of symmetric information increases (i.e., when \( t \in (\frac{\Delta}{4k}, \min(\frac{\Delta}{4k}, 1)) \)). Total first-period effort would be relatively large in this case if the agent delivered effort during the first period at the same rate that he expects to deliver effort in the second period when he supplies the standard effort levels in that period. Rather than deliver this large first-period effort and consequently experience negative ex post utility for a broad range of \( \beta \) realizations, the agent prefers to deliver little or no effort in the first period. Thus, the agent will not deliver first-period effort \( e_1 = \frac{1}{\Delta}E(e^2(\beta)) \) and the solution to \([PP]^{24}\) is not the solution to \([PP]^{24}\).

The principal must reduce the induced second-period effort supply below the standard effort supply in order to induce the agent to deliver a substantial effort bond when the first period is relatively long. Given the convexity of the agent’s effort costs, the requisite reduction in expected second-period effort is achieved most efficiently by reducing the largest effort levels. This reduction is achieved by reducing the rate at which payment increases with output for the highest output levels. In response to the reduced compensation for incremental output above a threshold, the agent caps his effort supply at \([1 - t] \Delta \). This reduction is achieved by reducing the rate at which payment increases with output for the highest output levels. In response to the reduced compensation for incremental output above a threshold, the agent caps his effort supply at \([1 - t] \Delta \) rather than continue to supply additional effort as \( \beta \) increases above a threshold, \( \beta \equiv \beta - 2k + \frac{\Delta}{4k} \). The associated reduction in the expected second-period effort rate renders the agent willing to deliver first-period effort at the same, lower rate, because doing so entails the posting of a smaller bond. \( \beta \) These considerations underlie Proposition 2.

**Proposition 2.** Suppose \( t \in (\frac{\Delta}{4k}, \min(\frac{\Delta}{4k}, 1)) \). Then at the solution to \([PP]^{24}\):

\[
e_2(\beta) = \begin{cases} 
e^2(\beta) & \text{for } \beta \in [\beta, \tilde{\beta}] \\ \frac{1 - t}{t} \Delta & \text{for } \beta \in (\tilde{\beta}, \beta) \end{cases}
\]

Hence, \( \frac{e_1}{e_2} = \frac{E(e_2(\beta))}{E(e_1(\beta))} \) if \( t < \frac{\Delta}{4k} \) and \( t > \frac{\Delta}{4k} \).

When the duration of the information asymmetry is sufficiently limited relative to its magnitude (so \( t \geq \frac{\Delta}{2} \)), the principal optimally induces the agent to supply effort at the constant, efficient rate throughout the second period and thereby secures the first-best outcome, just as she does when an adverse selection problem never arises. (Recall Lemma 3.) The principal can achieve this ideal outcome because the agent faces a high opportunity cost of delivering little effort throughout a long first period. If he has delivered little first-period effort, the agent will be compelled to supply effort at a high (and therefore costly) rate during the short second period if he is to deliver the substantial total effort required to secure the largest payments offered by the principal. Anticipating the high cost of delivering substantial effort during the short second period, the agent decides to supply considerable effort during the long first period, because he can do so at a relatively modest (and therefore less costly) rate. With a reduced concern about

\[23^{24}\] Because the effort supply induced at the solution to \([PP]^{24}\) differs from the effort supply identified in (14), the principal has an incentive to renegotiate the contract at the start of period 2. Of course, the agent would anticipate such renegotiation if it were feasible, and so the identified outcomes would not arise.

\[24^{25}\] Technically, this is the case because the agent’s objective is not a concave function of his first-period effort when the standard effort supply is induced in the second period.

\[25^{26}\] As the proof of Proposition 2 demonstrates, the diminished level of induced second-period effort renders the agent’s objective a concave function of his first-period effort.

\[26^{26}\] Furthermore, \( e_1 = 2k - \frac{1}{4k} \Delta - \frac{1}{4k}[2k - \Delta] \) if \( \Delta \leq 2k \), whereas \( e_1 = 2k - \frac{1}{4k} \Delta \) if \( \Delta > 2k \) at the solution to \([PP]^{24}\) when \( t \in (\frac{\Delta}{4k}, \min(\frac{\Delta}{4k}, 1)) \).

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the agent withholding first-period effort, the principal can maximize total expected surplus by awarding the agent the entire increment in output he generates above a threshold. The associated steep rise in the agent’s utility as $\beta$ increases can be offset by the associated large bond that the agent posts in the first period.\(^{27}\)

The principal cannot ensure the first-best outcome when the information asymmetry is more pronounced (i.e., when $\Delta > 2k$). To induce the efficient effort supply for all realizations of $\beta$, the principal must pay the agent the entire incremental output that he produces above a threshold. This generous reward provides rent to the agent when the information asymmetry during the period of adverse selection is sufficiently pronounced. However, as the length of the second period declines to 0, the optimal payment schedule converges to the schedule identified in Lemma 3, and both the agent’s rent and the optimal effort distortions become negligible. These conclusions are summarized in Proposition 3.

\textbf{Proposition 3.} When $\Delta \leq 2k$ and $t \in \left[\frac{\Delta}{2k}, 1\right]$, the solution to [PP] is the payment schedule specified in Lemma 3, which secures the first-best outcome, and so $\frac{e_1}{e^*_1} = \frac{E\{e^*_2(\beta)\}}{e^*_2} = 1$. When $\Delta > 2k$, the first-best solution is not a feasible solution to [PP]. However, as $t \to 1$, the payment schedule that solves [PP] converges to the payment schedule identified in Lemma 3. Furthermore, $e_1 \to e^*_1$, $E\{e_2(\beta)\} \to e^*_2$, and the principal’s expected net return approaches the entire expected surplus from efficient production.

The effort supply described in Propositions 1–3 is illustrated in Figure 1. The figure illustrates how the ratio of actual to efficient effort ($\frac{E\{e_2(\beta)\}}{e^*_2} = \frac{e_1}{e^*_1}$) that is induced by the optimal payment schedule varies with the magnitude and the duration of the information asymmetry. When this asymmetry is sufficiently limited in duration and magnitude (so $t$ is close to 1 and $\frac{\Delta}{2k}$ is small), the principal can secure the first-best outcome, as Proposition 3 reports. The resulting efficient effort supply is reflected in the flat plateau near the top left-hand corner of Figure 1. In contrast, when the information asymmetry is pronounced both in magnitude and duration, the optimal contract induces effort below efficient levels, as Proposition 1 reports. These effort distortions are reflected in the lower, right-hand portion of Figure 1. Effort above efficient levels is induced when

\[^{27}\] This bond is $\frac{1}{2k}[(\frac{1}{\alpha})E\{e_1(\beta)\}]^2 = \frac{\alpha^2(t_1 - t)^2}{4k(t_1 - 1)} = kt$, which is the agent’s cost of delivering effort $e_1 = [\frac{1}{\alpha}]E\{e_1(\beta)\}$ in the first period.
the duration and the magnitude of the information asymmetry are intermediate, as Proposition 2 reports. These effort distortions are reflected in the peak that rises above the plateau in the top, middle portion of Figure 1.

Corollary 1 identifies more precisely the (intermediate) values of $t$ and $\Delta$ at which the ratio of induced to efficient effort is most pronounced under the optimal payment schedule. The corollary also reports that first-period effort and expected second-period effort can be as much as 117% of the corresponding efficient effort.

**Corollary 1.** At the solution to $[PP]$, $\max_{\Delta \in [0,1]} \{ e_1^*(\beta) \} = \max_{\Delta \in [\sqrt{2k},2\sqrt{2k}]} \{ e_2^*(\beta) \}$ is nondecreasing in $t$ for $t \in (0, \frac{1}{2})$ and nonincreasing in $t$ for $t \in (\frac{1}{2}, 1)$. At $t = \frac{1}{2}$, $\max_{\Delta \in [0,1]} \{ e_1^*(\beta) \}$ is nondecreasing in $\Delta$ for $\Delta \in (0, \sqrt{2k})$ and nonincreasing in $\Delta$ for $\Delta > \sqrt{2k}$. The maximum value of $\frac{e_1^*(\beta)}{e_2^*(\beta)}$ is $4 - 2\sqrt{2} \approx 1.17$.

It remains to further characterize the properties of the optimal payment schedule that induce the effort patterns identified in Propositions 1–3 and Corollary 1. To do so, notice that after he learns the realization of $\beta$, the agent delivers the level of effort that equates the marginal benefit and the marginal cost of additional effort. The marginal benefit is the incremental payment he receives, which is given by the slope of the optimal payment schedule. The agent’s marginal cost of effort $e_2(\beta)$ is $\frac{e_1(\beta)}{e_2(\beta)}$. Therefore, as (10) implies, the agent’s compensation optimally increases with his output at the rate $\frac{e_2(\beta)}{e_2^*(\beta)}$. Consequently, because the agent’s equilibrium second-period effort is nondecreasing in $\beta$, payments optimally increase with output at a nondecreasing rate, and so the optimal payment schedule is a convex function of output that is most steeply sloped at the highest equilibrium output level. The slope of the payment schedule at this point is $m = \frac{e_2(\beta)}{e_2^*(\beta)}$. Proposition 4 explains how the maximum slope of the optimal payment schedule varies with the magnitude and the duration of the information asymmetry.

**Proposition 4.** At the solution to $[PP]$: (i) $m = \frac{e_2(\beta)}{e_2^*(\beta)} \geq 1$, with strict inequality unless $\Delta \leq 2k$ and $t \in [\frac{\Delta}{k}, 1]$; (ii) $m$ is a nondecreasing function of $\Delta$ for all $t \in [0,1]$; and (iii) for any given $\Delta > 0$, $m$ is increasing in $t$ for $t \in [0, \frac{\Delta}{\Delta k+1}]$ and nonincreasing in $t$ for $t \in (\frac{\Delta}{\Delta k+1}, 1]$.

Figure 2 helps to explain Proposition 4 and to characterize the solution to $[PP]$ more generally. The figure illustrates the optimal payment schedule for selected values of $\Delta \in \{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7} \}$ and $t \in \{0.1, 0.5, 0.9\}$. To facilitate comparisons across the nine panels in Figure 2, the variable $x - x_0$ is placed on the horizontal axis, where $x_0 = k + \frac{1}{k} [\beta + \overline{\beta}]$ is the expected surplus from efficient production. The payment schedule is drawn with dashes in regions where its slope is less than or equal to 1. The schedule appears as a solid line in regions where its slope exceeds 1.29

Conclusion (i) of Proposition 4 reports that whenever the first-best outcome is not feasible, the maximum slope of the optimal payment schedule in the present setting exceeds the maximum slope of the optimal payment schedule in the standard adverse selection problem.30 The more pronounced incremental reward for the highest output level in the present setting reflects the extra benefit that expanded second-period effort secures here, that is, the increase in the first-period effort bond it can induce, and the associated reduction in the agent’s rent. This extra benefit of expanded second-period effort leads the principal to secure second-period effort in excess of the levels she would induce in a standard adverse selection setting of duration $1 - t$, that is, in excess of the levels identified in (14).

---

28 The expected surplus from efficient production $(x_0 + e^* - \frac{e^*}{e}) = \frac{1}{2} [\beta + \overline{\beta}] + 2k - k = k + \frac{1}{2} [\beta + \overline{\beta}] + \frac{1}{2} x_0$ can change as $\Delta$ changes, as can equilibrium output levels. Subtracting $x_0$ from $x$ in Figure 2 helps to control for these potential sources of variation in the optimal payment schedule.

29 The dotted lines in Figure 2 represent extrapolations of the optimal payment schedule to output levels that do not arise in equilibrium. The extrapolations are provided to facilitate comparisons of the optimal payment schedules across panels in Figure 2.

30 From Lemma 2, the maximum slope of the optimal payment schedule in the standard adverse selection problem is $\frac{\Delta \bar{\beta}}{e^*} = 1$. 

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Conclusion (ii) of Proposition 4 reports that for a fixed duration of the first period (\(t\)), \(\bar{m}\) increases as the magnitude of the second-period information asymmetry (\(\Delta\)) increases. This conclusion reflects the findings in Propositions 1–3. Recall from Proposition 3 that when \(\Delta\) is sufficiently small relative to \(t\), the principal can secure the first-best outcome by paying the agent the full increment in output he generates above a threshold (so \(P'(x) = 1\)). Such an
optimal payment schedule is depicted in panel b1 in Figure 2 for the case where $\Delta = 0.5k$ and $t = 0.5$.

As the magnitude of the information asymmetry increases, holding $t$ constant, the agent will command rent from his private information. As Proposition 2 reports, the best the principal can do is induce the standard effort supply for the smaller $\beta$ realizations and a constant effort supply ($\frac{1-t}{\Delta} \Delta$) for the larger $\beta$ realizations when $t \in (\frac{\Delta}{\Delta+2k}, \min(\Delta, 1))$. As illustrated in panel b2 in Figure 2 for the case where $t = 0.5$ and $\Delta = 1.5k$, this second-period effort supply is optimally induced with a payment structure that increases at an increasing rate for the smaller outputs and at a constant rate ($\frac{eS}{c^2} = \frac{\Delta}{2(1-t)}$) for the larger output realizations.

As $\Delta$ increases further (so $t < \frac{\Delta}{\Delta+2k}$), the principal will implement the standard effort supply in the second period because doing so will not induce the agent to withhold first-period effort. (Recall Proposition 1.) The principal induces the standard effort supply with a convex payment schedule of the form illustrated in panel b3 in Figure 2 for the case where $t = 0.5$ and $\Delta = 2.5k$. The optimal payment schedule provides no compensation to the agent ($P(\cdot) = 0$) for the smaller output levels in order to reduce the agent’s second-period effort to zero for the smaller $\beta$ realizations and thereby limit the agent’s rent. Payments increase with output at an increasing rate for the larger outputs. The slope of the payment schedule increases to $\bar{\mu} = \frac{eS}{c^2} = \frac{2k}{\Delta(1-t)} = 2$ at the highest equilibrium output level.

Conclusion (iii) of Proposition 4 indicates that for fixed $\Delta$, the maximum slope of the optimal contract ($\bar{\mu}$) is greatest when the periods of symmetric and asymmetric information are intermediate in length. To understand why, recall from Proposition 3 that when the length of the second period is close to 1 (so $t$ is close to 0), the principal largely faces an adverse selection problem and so induces the standard effort supply in the second period. She does so with a payment schedule that has maximum slope $\frac{eS}{c^2} = \frac{2k}{2(1-t)} = \frac{1-t}{t}$. This maximum slope increases as $t$ increases above 0. The higher payment rate induces an increased rate of second-period effort supply and thus, from (8), an increased rate of first-period effort supply, which secures a larger effort bond as the length of the first period increases. As $t$ increases above $\frac{\Delta}{\Delta+2k}$, the principal induces less than the standard effort for the highest $\beta$ realizations to prevent the agent from withholding first-period effort in order to reduce the effort bond he puts at risk. As Proposition 2 reports, induced second-period effort is capped at $\frac{1-t}{\Delta} \Delta$. This maximum effort is secured with a payment schedule that has maximum slope $\frac{eS}{c^2} = \frac{1-t}{\Delta} \Delta = \frac{\Delta}{2t}$. This slope declines as $t$ increases.

These findings are illustrated in panels a2, b2, and c2 in Figure 2 for the case where $\Delta = 1.5k$. When $t = 0.1$, the principal implements the standard effort supply with the strictly convex payment schedule depicted in panel a2. The optimal payment schedule becomes linear for the higher output levels when the length of the first period is between $\frac{1}{t}$ and $\frac{3}{4}$. One such schedule is drawn in panel b2 for the case where $t = 0.5$. When the first period is sufficiently long in this setting (so $t > \frac{3}{4}$), the principal can secure the first-best outcome. The payment schedule that induces this ideal outcome is illustrated in panel c2 for the case where $t = 0.9$.

Figure 2 also helps to illustrate the primary practical implications of our findings. For example, panels b1 and c1 in the figure suggest that a linear compensation structure in which the agent is the residual claimant for output above a threshold often is optimal when the uncertainty about the environment ($\Delta$) is relatively limited. This may be the case, for example, when a fast-food franchisor and a potential franchisee are initially unsure of the revenue that a new restaurant will generate in a local neighborhood due to uncertainty about the tastes and disposable incomes of local residents. However, the well-documented experience of established franchisees may limit the relevant uncertainty about the potential variation in revenue. In such a setting, a franchisor can motivate a wealth-constrained franchisee to work diligently without affording him excessive rent by awarding him all of the revenues he generates above a threshold.

Different payment structures are optimal when the uncertainty about the potential profitability of the environment ($\Delta$) is more pronounced. Furthermore, when this uncertainty is large,
the nature of the optimal payment schedule can vary substantially according to the duration of the initial period of symmetric information. When this initial period is short, the principal primarily faces a severe adverse selection problem for an extended period of time, and so is not greatly concerned with motivating effort supply during the short period of symmetric information. Consequently, the principal will institute payments that do not increase rapidly with performance (as illustrated in panel a3 in Figure 2, for example), much as she would in a standard adverse selection setting. Such a payment structure may be optimal, for example, in a setting where an automobile sales representative learns fairly soon after the start of a new sales period how able and eager local residents are to purchase a new automobile.

In contrast, an investment banker may spend a great deal of time learning his clients’ needs and developing his portfolio of services before he must implement the services for his clients as the appropriate economic conditions arise. In such a case, the investment banking firm will want to motivate its employee to work diligently during the fairly lengthy initial phase of service to his clients. To do so, the firm can implement a payment schedule like the one in panel b2 in Figure 2. This payment schedule presents the investment banker with a high opportunity cost of withholding effort throughout the fairly lengthy initial phase, and thereby induces him to deliver a substantial effort bond during this phase of his employment.31

5. Conclusions

We have examined the optimal design of contracts in a dynamic setting where, in contrast to both the standard moral hazard problem and the standard adverse selection problem, the agent acquires private information ($\beta$) about the environment after a period of time has elapsed. We have shown that pronounced incremental rewards for exceptional performance can be optimal in this setting. High-powered payment schedules can induce the agent to deliver substantial effort before he learns the realization of $\beta$ in order to minimize his total cost of producing the large outputs that secure the particularly generous payments. The principal values this first-period effort in part because it serves as a bond that the agent puts at risk before learning the realization of $\beta$. The associated possibility of forfeiting this effort bond limits the agent’s rent in a setting where his limited wealth precludes the posting of a financial bond.

We found that the large incremental rewards for exceptional performance tend to be particularly pronounced when the agent’s ultimate information advantage is moderate and when the period of information asymmetry is intermediate in length. When the magnitude and the duration of the asymmetry are pronounced, payments that do not rise steeply with performance best limit the agent’s rent, as in standard adverse selection settings. In contrast, when the magnitude and the duration of the information asymmetry are limited, the principal can both eliminate the agent’s rent and motivate him to deliver the efficient effort supply (i.e., secure the first-best solution) without resorting to particularly steeply sloped payment schedules.

These conclusions were derived in a stylized setting where the agent’s effort in the first period is a perfect substitute for effort in the second period in increasing final output. Our key qualitative conclusions hold more generally. For example, suppose final output is $\beta + \alpha_1 e_1 + \alpha_2 e_2$, where $\alpha_1$ and $\alpha_2$ are strictly positive parameters. An increase in $\alpha_1$ has two effects in this setting. First, output increases more rapidly with the effort supplied in period $i$. The corresponding increased importance of motivating effort in period $i$ has the same impact as an increase in the length of period $i$. Second, the sensitivity of output to effort in period $i$ increases relative to the sensitivity of output to variations in $\beta$. In this respect, an increase in $\alpha_1$ functions like a decrease in the magnitude of the second-period information asymmetry. Formally, an increase in $\alpha_1$ gives rise to the same qualitative changes as an increase in $t$ and a decrease in $\Delta_1$. Similarly, an increase in $\alpha_2$ gives rise to the same qualitative changes as an increase in $1 - t$ and a decrease in $\Delta_2$.

31 Of course, considerations that are not included in our model can affect the properties of contracts that are observed in practice. Some of these considerations are described in the concluding section.
We have focused on a two-period setting with quadratic effort costs and a uniform distribution of the underlying uncertainty for analytic ease. However, the key forces that operate in our model arise more generally (Chu and Sappington, 2008). Therefore, our key qualitative conclusions will persist more generally. Of course, the properties of optimal reward schedules in practice may differ from those derived above because of additional real-world considerations that are not present in our model. For example, the optimal contract will likely be less steeply sloped if the agent is risk averse because steeply sloped contracts can impose considerable risk on the agent. Similarly, increased agent wealth may reduce the slope of the optimal contract because a wealthy agent can post a financial bond in lieu of the effort bond that he is induced to post in our model. Furthermore, more proportionate sharing of the realized returns may better motivate valuable effort from both the principal and the agent in settings with double moral hazard concerns (e.g., in the franchise setting, where realized revenues reflect in part the product quality supplied by the franchisor).

Additional extensions of our analysis await further research. For example, the agent’s early and late efforts might be complements rather than substitutes, and effort costs might not be identical and separable over time. Furthermore, additional imperfect signals of the agent’s effort supply might be available before the end of the agency relationship. Early signals endow the principal with expanded ability to implement separate financial rewards for the supply of early and late efforts. Consequently, the principal may need to rely less on steeply sloped payment schedules to motivate the delivery of both early and late effort, particularly if she can credibly threaten to contract with a different agent for late effort supply when the performance of the original agent is deemed to be inadequate. Future research also might analyze the incentives

Appendix

Proof of Lemma 1. The agent will minimize his personal cost of delivering any chosen effort level $e$, and so will choose effort rate $r(\tau)$ over time period $[t_1, t_2]$ to

$$\text{Minimize } \int_{t_1}^{t_2} \frac{1}{4k} r(\tau)^2 d\tau \quad \text{subject to } \int_{t_1}^{t_2} r(\tau) d\tau = e. \quad (A1)$$

The relevant Lagrangian function is $\int_{t_1}^{t_2} \left[ \frac{1}{2} r(\tau)^2 - yr(\tau) \right] d\tau + ye$ (where $y$ is the Lagrange multiplier). Setting the partial derivative of this function with respect to $r(\tau)$ equal to 0 reveals that $r(\tau) = 2k$. Integrating this expression for $r(\tau)$ over the $[t_1, t_2]$ interval and using the equality in (A1) provides $y = \frac{e}{2k(t_2 - t_1)}$. Therefore, the agent’s minimum cost of supplying effort $e$ over the time interval $[t_1, t_2]$ is

$$\int_{t_1}^{t_2} \frac{1}{4k} r(\tau)^2 d\tau = \frac{y^2}{4k} \int_{t_1}^{t_2} [2k]^2 d\tau = \frac{e^2}{4k(t_2 - t_1)}. \quad (A2)$$

Proof of Lemma 2. The proof follows directly from Laffont and Tirole (1986).

Proof of Lemma 3. The proof for the case where $\Delta \leq 2k$ follows from the proof of Proposition 3. The proof for the case where $\Delta > 2k$ is analogous.

Proof of Proposition 1. (13) and Lemma 2 imply that as long as the second-order conditions are satisfied at the identified solution, the solution to $[PP]$ entails $u(\beta) = 0$ and

$$e_2(\beta) = \begin{cases} 2k - [\overline{\beta} - \beta] & \text{for } \beta > \overline{\beta} - 2k \\ 0 & \text{for } \beta \leq \overline{\beta} - 2k. \end{cases} \quad (A3)$$

32 See Innes (1990) for a discussion of these issues.

33 To define the complete reward contract for any output, we can set $P(x) = P(\overline{\beta} + e_1 + e_2(\overline{\beta}))$ for $x > \overline{\beta} + e_1 + e_2(\overline{\beta})$. This implies that $e_2(\beta) = \max(0, e_2(\overline{\beta}) + \overline{\beta} - \beta)$ for $\beta > \overline{\beta}$.
From (8), the optimal value of \( e_1 \) is readily shown to be

\[
e_1 = \left[ \frac{t}{1 - t} \right] E [e_2(\beta)] = \begin{cases} \left[ \frac{t}{1 - t} \right] \frac{2k^2}{\Delta} & \text{for } \Delta \geq 2k \\ \left[ \frac{t}{1 - t} \right] \frac{4k - \Delta}{2} & \text{for } \Delta \leq 2k. \end{cases}
\]  

(A4)

The agent’s second-period objective function in (4) is a concave function of \( e \) for the reasons specified in LT.

To analyze the concavity of the agent’s first-period objective function, let \( C_i(e) = \frac{d}{d\beta} \) and \( C_i(e) = \frac{d}{d\beta} \) here and throughout the ensuing analysis. Then the second partial derivative of the agent’s objective function in (3) is

\[
\int_2^7 u''(\beta + e - e_1) \frac{1}{\Delta} d\beta - C_i(e_1)
\]

\[
= \int_2^7 C_i(e_2(\beta + e - e_1)) e_2'(\beta + e - e_1) \frac{1}{\Delta} d\beta - C_i(e_1) \quad \text{(A5)}
\]

\[
= \left[ \frac{1}{1 - t} \right] \frac{1}{2k} \frac{1}{\Delta} \left[ e_2(\beta) - e_1 \right] e_2(\beta + e_1) - e_1(\beta + e_1)] - \frac{1}{2kt}.
\]

(A6)

The equality in (A5) follows from (10). Because \( e_2(\beta + e - e_1) \leq 2k \) from (A3) and \( e_2(\beta + e - e_1) \) is nonnegative, the expression in (A6) is at most

\[
\left[ \frac{1}{1 - t} \right] \frac{1}{\Delta} - \left[ \frac{1}{1 - t} \right] \frac{1}{2k} \frac{2k - \Delta[1 - t]}{2k \Delta[1 - t]} = \frac{[2k + \Delta][1 - t] - \Delta}{2k \Delta[1 - t]}
\]

(A7)

This expression in (A7) is nonpositive when \( t \leq \frac{\Delta}{2k \Delta} \). Therefore, \( \int_2^7 [U_i(e | \beta) - C_i(e)] \frac{1}{\Delta} d\beta \) is a concave function of \( e \) when \( t \leq \frac{\Delta}{2k \Delta} \).

From (A4), when \( \Delta \leq 2k, e_1 \geq e_1' \geq \frac{1}{1 - t} \left[ \frac{2k^2}{\Delta} \right] \geq 2k \Rightarrow 4kt \geq \Delta \). (A4) also implies that when \( \Delta > 2k, e_1 < e_1' \geq \frac{1}{1 - t} \left[ \frac{2k^2}{\Delta} \right] < 2k \Rightarrow \frac{1}{1 - t} < \frac{2k}{\Delta} \Rightarrow \frac{1}{1 - t} < \frac{2k}{\Delta} \). If \( \frac{1}{1 - t} \leq \frac{2k}{\Delta} \), then \( 1 - t \geq \frac{1}{2} \) and so \( \frac{1}{1 - t} \leq 2k \leq \Delta \). If \( \frac{1}{1 - t} < \frac{2k}{\Delta} \), then \( \frac{1}{1 - t} < \frac{2k}{\Delta} \) because \( [1 - t] \Delta \geq 2k \Rightarrow t \leq \frac{2k}{\Delta} \). Therefore, \( \frac{1}{1 - t} < 2k \Delta \) when \( t \leq \frac{2k}{\Delta} \) and \( \Delta > 2k \).

Proof of Proposition 2. We will show that the solution to [PP] when \( \frac{2k}{\Delta^2} < t < \frac{1}{2} \) entails

\[
e_2(\beta) = \begin{cases} 0 & \text{for } \beta \leq \beta_1 - 2k \\ 2k + \beta - \beta_1 & \text{for } \beta \in \left[ \beta_1 - 2k, \beta_2 - 2k + \left[ \frac{1 - t}{t} \right] \Delta \right) \\ \left[ \frac{1 - t}{t} \right] \Delta & \text{for } \beta \in \left[ \beta_2 - 2k + \left[ \frac{1 - t}{t} \right] \Delta, \beta_1 \right) \end{cases}
\]

(A8)

and

\[ u(\beta) = 0. \]

(A9)

Given (A8), (8) implies

\[
e_1 = \left[ \frac{t}{1 - t} \right] E [e_2(\beta)] = \begin{cases} 2k - \frac{1}{2} \left[ \frac{1 - t}{t} \right] \Delta & \text{for } \Delta \geq 2k \\ 2k - \frac{1}{2} \left[ \frac{1 - t}{t} \right] \Delta - \frac{1}{2} \left[ \frac{1}{1 - t} \right] \Delta \left[ 2k - \Delta \right]^2 & \text{for } \Delta \leq 2k. \end{cases}
\]

(A10)

The solution identified in (A8) and (A9) is feasible because no output pooling is induced and the second partial derivative of the objective function in (3) with respect to \( e \) is, from (A6),

\[
\left[ \frac{1}{1 - t} \right] \frac{1}{2k} \left[ \frac{1}{\Delta} \right] \left[ e_2(\beta + e - e_1) - e_2(\beta + e - e_1) \right] - \frac{1}{t} \left[ \frac{1}{2k} \right] \leq \left[ \frac{1}{1 - t} \right] \frac{1}{2k} \left[ \frac{1}{\Delta} \right] \left[ \frac{1 - t}{t} \right] \Delta - \frac{1}{t} \left[ \frac{1}{2k} \right] = 0.
\]

(A11)

The inequality in (A11) holds because, from (A8), \( e_2(\beta + e - e_1) - e_2(\beta + e - e_1) \leq \left[ \frac{1 - t}{t} \right] \Delta \). (A11) implies that \( \int_2^7 [U_i(e | \beta) - C_i(e)] \frac{1}{\Delta} d\beta \) is a concave function of \( e \) and so the relevant second-order condition is satisfied.

To determine the principal’s expected payoff under the identified contract, recall from (13) that the principal seeks to maximize
\[ \left[ \frac{1}{1-t} \right] \int_\beta \left[ e_2(\beta) - \frac{e_2(\beta)}{4k} - \frac{e_2(\beta)}{2k} [\beta - \beta] \right] \frac{1}{\Delta} d\beta - u(\beta) \]

\[ = - \left[ \frac{1}{4k(1-t)} \right] \int_\beta \left[ e_2(\beta) - (2k + \beta - \beta) \right] \frac{1}{\Delta} d\beta \]

\[ + \left[ \frac{1}{4k(1-t)} \right] \int_\beta \left[ 2k + \beta - \beta \right] \frac{1}{\Delta} d\beta - u(\beta). \quad (A12) \]

(A12) implies that under the feasible solution identified in (A8) and (A9), the principal’s expected gain (i.e., her payoff net of \( \frac{1}{\Delta} \[\beta + \beta\] \)) when \( \Delta \geq 2k \) is

\[ \left[ \frac{1}{4k(1-t)} \right] \int_{\beta-2k} \left[ 2k + \beta - \beta \right] \frac{1}{\Delta} d\beta - \int_{\beta-2k} \left[ e_2(\beta) - (2k + \beta - \beta) \right] \frac{1}{\Delta} d\beta \]

\[ = \left[ \frac{1}{4k(1-t)} \right] \int_\beta x^2 dx - \int_0 \int_{2k-\beta} x^2 dx \]

\[ = \frac{1}{12kt} \left[ 12k^2 - 6k \left[ 1 - \frac{t}{t} \right] \Delta + \left( \left[ 1 - \frac{t}{t} \right] \Delta \right)^2 \right]. \quad (A13) \]

The principal’s corresponding expected gain when \( \Delta \leq 2k \) is

\[ \left[ \frac{1}{4k(1-t)} \right] \int_{\beta-\Delta} \left[ 2k + \beta - \beta \right] \frac{1}{\Delta} d\beta - \int_{\beta-\Delta} \left[ e_2(\beta) - (2k + \beta - \beta) \right] \frac{1}{\Delta} d\beta \]

\[ = \left[ \frac{1}{4k(1-t)} \right] \int_\beta x^2 dx - \int_0 \int_{2k-\beta} x^2 dx \]

\[ = \frac{1}{12kt} \left[ 12k^2 - 6k \left[ 1 - \frac{t}{t} \right] \Delta + \left( \left[ 1 - \frac{t}{t} \right] \Delta \right)^2 \right] - \frac{[2k - \Delta]^2}{12k[1-t]} \Delta. \quad (A14) \]

The optimality of the identified solution follows from the following four lemmas, which are proved in Chu and Sappington (2008).

Lemma A1. \( e_1 < \Delta \) at the solution to \([PP]\) when \( \frac{1}{e_1} < t < \frac{1}{\Delta} \).

Lemma A2. \( u(\beta) = 0 \) at the solution to \([PP]\).

Lemma A3. Final output \( \beta + e_1 + e_2(\beta) \) is a nondecreasing function of \( \beta \).

Lemma A4. \( e_2(\beta) \) is nondecreasing on \([\beta, \beta]\).

Lemma A4 implies that \( e_2(\beta) \leq \left[ \frac{1-t}{t} \right] \Delta \) for \( \beta \in [\beta, \beta] \). Consider the following relaxed version of \([PP]\):

\[ \text{Maximize} \left[ \frac{1}{1-t} \right] \int_\beta \left[ e_2(\beta) - \frac{e_2(\beta)}{4k} - \frac{e_2(\beta)}{2k} [\beta - \beta] \right] \frac{1}{\Delta} d\beta \]

subject to \( e_2(\beta) \leq \left[ \frac{1-t}{t} \right] \Delta \).

The solution to this problem is min \( \{ e^3(\beta), \left[ \frac{1-t}{t} \right] \Delta \} \), where \( e^3(\beta) = \max \{ 0, 2k - [\beta - \beta] \} \) from Lemma 3. We have shown that this proposed solution is a feasible solution to \([PP]\), and so is optimal. From (A10), when \( \Delta \leq 2k \)

\[ e_1 > e_1' \Leftrightarrow 2k - \frac{1}{2} \left[ 1 - \frac{1}{t} \right] \Delta - \frac{1}{2} \left[ 1 + \frac{t}{t} \right] \left( 2k - \Delta \right)^2 > 2kt = e_1' \]

\[ \Leftrightarrow [1 - 2t + 2t^2] \Delta^2 - 4kt[1 - t + t^2] \Delta + 4k^2 t^2 < 0. \]

\( \Leftrightarrow [\Delta - 2kt][1 - 2t + 2t^2] \Delta - 2kt < 0 \Leftrightarrow [1 - 2t + 2t^2] \Delta < 2kt \).

If \( t \leq \frac{1}{2} \), then \( 1 - t \geq 1 - 2t + 2t^2 \). Therefore, \( \frac{1}{4k^2} \leq \frac{1}{4k^2} \). Furthermore, \( \Delta < \frac{2k}{t} \leq \frac{2k}{t} \). If \( t > \frac{1}{2} \), then \( 1 - 2t + 2t^2 < t \). Also, \( t < \frac{1}{2t} \leq 1 \). Therefore \( [1 - 2t + 2t^2] \Delta < 2kt \). That is, \( e_1 > e_1' \) when \( \Delta \leq 2k \) and \( t \in \left( \frac{1}{e_1}, \frac{1}{\Delta} \right) \).

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From (A10), when $\Delta > 2k$, $e_t \geq e_t^*$ implies $2k - \frac{1}{2} \frac{\Delta}{\varepsilon} \Delta \geq 2kt \Rightarrow 4kt \geq \Delta$.

**Proof of Proposition 3.** If the agent supplies effort $2k$, output is at least $\beta + 2k \geq \frac{1}{2}[\beta + \beta + 2k]$ because $\beta + 2k \geq \frac{1}{2} \frac{\beta}{\varepsilon} + \beta + 2k] \Leftrightarrow \Delta \leq 2k$. Therefore, when the agent supplies effort $2k$, the payment to the agent is always nonnegative, the principal’s certain return is $\frac{1}{2}[\beta + \beta + 2k]$, and the agent’s expected utility is $\frac{1}{2}[\beta + \beta] + 2k - \frac{1}{2}[\beta + \beta + 2k] - \frac{1}{2}2k^2 = k - k = 0$. Therefore, the contract specified in Lemma 3 will secure the first-best outcome if it induces the agent to supply the first-best effort. It will do so when $\Delta \leq 2k$ because if the agent delivers sufficient second-period effort to ensure output of at least $\frac{1}{2}[\beta + \beta + 2k]$, he retains the entire incremental surplus generated by his effort (because $P(x) = 1$). Therefore, he will supply the efficient second-period effort $2k[1 - t]$. From (A2), this effort entails personal cost $\frac{1}{\varepsilon} [2k(1 - t)]^2 = k[1 - t]$.

When $\beta + e_t = \frac{1}{2}[\beta + \beta + k] + kt$, second-period effort $2k[1 - t]$ generates reward $\frac{1}{2}[\beta + \beta] + 2k[1 - t] + \frac{1}{2}[\beta + \beta + 2k] = k[1 - t]$. Consequently, the agent will secure an expected utility of $0$ by delivering second-period effort $2k[1 - t]$ when $\beta + e_t = \frac{1}{2}[\beta + \beta] + kt$.

When $\beta + e_t > \frac{1}{2}[\beta + \beta] + kt$, the agent can secure a strictly positive payoff by supplying first-best effort $2k[1 - t]$ in the second period. Under this contract,

$$U_e(e_t | \beta) = \begin{cases} 0 & \text{ for } \beta + e_t < \frac{1}{2}[\beta + \beta] + kt \\ \beta + e_t - \frac{1}{2}[\beta + \beta] - kt & \text{ for } \beta + e_t \geq \frac{1}{2}[\beta + \beta] + kt. \end{cases}$$

The agent chooses first-period effort $e_t$ to maximize $\int_0^1 U_e(e_t | \beta) \beta d\beta - C_0(e_t)$, where, recall, $C_0(e_t) = \frac{\varepsilon^2}{4k}$. It is readily verified that this objective function is maximized at either $e_t = 0$ or $e_t = 2kt$. Both effort levels provide an expected payoff of zero. Therefore, the agent will be willing to supply first-best effort $2kt$ and the principal can secure the first-best outcome with the contract specified in Lemma 3 when $t \geq \frac{1}{\varepsilon}$. From (A10), when $\Delta > 2k$ and $t = \frac{\Delta}{2k} \geq \frac{\Delta}{2k}$, $e_t = 2k - \frac{1}{2} \frac{\Delta}{\varepsilon} \Delta$. From (A13), the principal’s expected gain in this case is $\frac{1}{\varepsilon} [12k^2 - \Delta^2 + (\frac{1}{2})^{2}]$. As $t$ approaches 1 from below, $e_t$ approaches 2k, as does $e_t^* = 2kt$. Furthermore, the principal’s expected gain approaches $k$, which is the expected gain from efficient production.

To show that the optimal contract converges to the contract identified in Lemma 3 when $\Delta > 2k$, recall that when $\Delta > 2k$ and $t > \frac{\Delta}{2k}$, the agent’s second-period effort is as specified in (A8). Also, (A9) and (10) imply that when $\Delta > 2k$ and $t > \frac{\Delta}{2k}$, the equilibrium payment $(P(\beta) = u_e(e_t(\beta)) + C_x(e_t(\beta)))$ to the agent when innate output is $\beta$ is

$$u_e(e_t | \beta) = \begin{cases} 0 & \text{ for } \beta \leq \beta - 2k \\ \frac{1}{4k[1 - t]}(2k + \beta - \beta)^2 & \text{ for } \beta \in (\beta - 2k, \beta - 2k + \varepsilon) \Delta \\ \frac{1}{4k[1 - t]} (2k + \beta - \beta - (\frac{1}{2} - \frac{1}{t}) \Delta \frac{1}{\varepsilon}) & \text{ for } \beta \in (\beta - 2k + \frac{1}{\varepsilon} \Delta, \beta - 2k). \end{cases}$$

Therefore, when $\Delta > 2k$ and $t > \frac{\Delta}{2k}$, the equilibrium payment $(P(\beta) = u_e(\beta) + C_x(\beta))$ to the agent when innate output is $\beta$ is

$$P(\beta) = \begin{cases} 0 & \text{ for } \beta \leq \beta - 2k \\ \frac{1}{2k[1 - t]}(2k + \beta - \beta)^2 & \text{ for } \beta \in (\beta - 2k, \beta - 2k + \varepsilon) \Delta \\ \frac{1}{2k[1 - t]} (2k + \beta - \beta - (\frac{1}{2} - \frac{1}{t}) \Delta \frac{1}{\varepsilon}) & \text{ for } \beta \in (\beta - 2k + \frac{1}{\varepsilon} \Delta, \beta - 2k). \end{cases}$$

From (A8) and (A10), the corresponding output $(x(\beta) = \beta + e_t + e_x(\beta))$ is

$$x(\beta) = \begin{cases} \beta + 2k - \frac{1}{2} \frac{1}{t} \Delta & \text{ for } \beta \leq \beta - 2k \\ 2\beta + 4k - \beta + \frac{1}{2} \frac{1}{t} \Delta & \text{ for } \beta \in (\beta - 2k, \beta - 2k + \varepsilon) \Delta \\ \beta + 2k + \frac{1}{2} \frac{1}{t} \Delta & \text{ for } \beta \in (\beta - 2k + \frac{1}{\varepsilon} \Delta, \beta - 2k). \end{cases}$$

(A17) implies that as $t \to 1$,

$$P(\beta) \to \begin{cases} 0 & \text{ for } \beta \leq \beta - 2k \\ \frac{\Delta}{2k} (\beta - 2k + 2k) & \text{ for } \beta \in [\beta - 2k, \beta]. \end{cases}$$

(A18) implies that as $t \to 1$,

$$x(\beta) \to \beta + 2k.$$
The contract in (A19) and (A20) is the contract identified in Lemma 3, where \( P(x) = 0 \) for \( x \leq \Delta \) (\( = \bar{\beta} \) when \( \Delta > 2k \)) and \( P(x) = \frac{\Delta}{4} \) for \( x > \Delta \).

**Proof of Corollary 1.** From Propositions 1 and 2, \( e_1 > e_1^* \) when \( t \in (\frac{\Delta}{\Delta e_1}, 1) \).

When \( \Delta > 2k, \frac{\Delta}{\Delta e_1} \leq \frac{\Delta}{\Delta e_1^*} \). From (A10), when \( \Delta > 2k \) and \( t \in (\frac{\Delta}{\Delta e_1^*}, 1) \), \( e_1 = 2k - \frac{1}{2} \frac{t}{\Delta e_1} \Delta \) and \( e_1^* = \frac{\Delta}{2} \left( \frac{1}{\Delta e_1^*} \right)^2 \). Because \( e_1 \) and \( e_1^* \) are declining with \( \Delta \), \( e_1 \) and \( e_1^* \) attain their highest values in this region at \( \Delta = 2k \).

When \( \Delta \leq 2k, \frac{\Delta}{\Delta e_1} \geq \frac{\Delta}{\Delta e_1^*} \). From Proposition 1, \( e_1 = \frac{\Delta}{\Delta e_1} \left( \frac{1}{\Delta e_1^*} \right) \Delta \) and \( e_1^* = \frac{\Delta}{\Delta e_1^*} \Delta \) when \( t \leq \frac{\Delta}{\Delta e_1} \). Because \( e_1 \) and \( e_1^* \) are increasing in \( t \), \( e_1 \) and \( e_1^* \) attain their highest values in this region at \( t = \frac{\Delta}{\Delta e_1^*} \).

Therefore, \( e_1^* \) achieves its highest value when \( \Delta \leq 2k \) and \( t \in (\frac{\Delta}{\Delta e_1^*}, \frac{\Delta}{\Delta e_1}) \). From Proposition 2, when \( \Delta \leq 2k \) and \( t \in (\frac{\Delta}{\Delta e_1^*}, \frac{\Delta}{\Delta e_1}) \), \( e_1 = 2k - \frac{1}{2} \frac{t}{\Delta e_1} \Delta - \frac{1}{2} \frac{t}{\Delta e_1^*} \left( \frac{\Delta}{\Delta e_1^*} \right)^2 (2k - \Delta)^2 \).

The value of \( \Delta \) for which \( e_1^* \) is maximized is readily shown to be \( \Delta = 2k \sqrt{\frac{2}{\beta}} - \frac{e_1^*}{\beta} \). At this value of \( \Delta, \frac{e_1}{\Delta e_1^*} \) is \( \frac{1}{\sqrt{\beta}} \), which is a decreasing function of \( \Delta \).

**References**


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